

# COOPERATION AND COMPETITION IN A DUOPOLY R&D MARKET\*

DAMIANO BRUNO SILIPO

*Dipartimento di Economia Politica*

*Università della Calabria,*

*87036 Arcavacata di Rende*

*Cosenza*

*Italy*

and

AVI WEISS

*Department of Economics*

*Bar-Ilan University*

*52900 Ramat-Gan*

*Israel*

---

\* We thank Huw Dixon, John Hey and especially Gianni De Fraja for comments on a previous version of this paper. The usual disclaimer applies. Financial support from the Italian MURST, from the Samuel Neaman Institute for Advanced Studies in Science and Technology, and from the Bar-Ilan University Research Center is gratefully acknowledged.

## **Abstract**

In a general setting with uncertainty and spillovers in R&D activity, we consider the incentive to cooperate among firms at any or all of the following three stages. Firms can jointly agree on the level of R&D expenditures, they can set up joint research facilities, and/or they can engage in an information sharing agreements, by which they agree to share any findings with the other firm. We compare expenditures on R&D, profit levels, and welfare levels across the different possible cooperative and competitive setups and offer antitrust implications. Our model differs from previous analyses in three important ways. First, most studies consider only research aimed at lowering production costs, and therefore consider only situations where total profits fall as spillovers increase. We allow for the possibility of product innovation, and define the concepts of offsetting spillovers (falling total profits) and incremental spillovers (when total profits increase as spillovers increase). Second, we consider a wider variety of cooperation possibilities than do most prior studies. Finally, we use far more general functional forms than is usual in the literature.

## I. Introduction

It is well recognized that there is a market failure in the provision of innovations that is related to the nature of R&D activity. The failure is attributed to several factors. First, there is uncertainty about the outcome of R&D activity. Firms that devote resources to research into a new product or technique do not know whether they will succeed, or how long the research will take. Second, due to spillover effects, successful firms may not be able to appropriate all of the rents from the outcome of R&D activity. There are various ways in which imitation of a novel idea can take place: property rights may be only broadly attributed; researchers may move between firms transferring knowledge from successful to unsuccessful firms, and so on. Unsuccessful firms can therefore benefit from successful R&D without paying the full cost. One way to mitigate the detrimental effects of this market failure is to have firms cooperate in R&D activities.

There is a considerable amount of work on the issue of how uncertainty and spillovers affect R&D activity. In a seminal paper, d'Aspremont and Jacquemin (1988) compared the cooperative and non-cooperative levels of R&D expenditure when there are spillovers from R&D activity. They found that for a high value of the spillover parameter R&D expenditure is higher when firms cooperate in R&D activity. The result was achieved using a simple model, in which two firms undertake cost reduction R&D activity with no uncertainty. Subsequent papers supported the robustness of their results over a much wider class of models, including uncertainty in R&D activity (Choi, 1989), oligopolistic industries (Suzumura, 1992), product innovation (Motta, 1992 and Rosenkranz, 1995) and several forms of cooperation in R&D (Kamien et al., 1992). All these results are based on the assumption that the incentive to cooperate is not affected by uncertainty and spillovers. Marjit (1991) and Combs (1992) examined the role of uncertainty on the incentive to cooperate, and showed that cooperation will take place only when the probability of success is relatively high. Choi (1993) studied the incentive to cooperate in a duopoly R&D market where there is uncertainty and spillovers. He showed that if the spillover parameter is high, cooperation in R&D is more likely to take place, and that the cooperative level of R&D expenditure is higher than the competitive level.

The model presented in this paper is most closely related to Choi's (1993) in that we consider an industry with two potential producers, each attempting to "discover" a new product or process, and we use completely general functional forms in analyzing the issues.

Our analysis, however, differs from Choi's in two important aspects. First, we look at a broader range of definitions of "cooperation" than did Choi. Choi assumes that cooperation between firms means that the parties agree on how much to spend on research, but that each firm retains sole-proprietorship of the results from the research (aside from the amount that spills-over to the other firm). We investigate additional possibilities by analyzing the effects of cooperation between firms at three different points in the R&D process. We allow for: a) firms agreeing on the level of expenditures (as in Choi); b) firms setting up joint research facilities (denoted below joint ventures), which lend themselves to the exploitation of synergies by, for example, eliminating any duplication of research efforts; and c) firms entering into information sharing agreements, whereby any firm that is successful in its research efforts is obligated to share the results with the other firm (an ex-ante cross-licensing agreement). We do not allow for collusion in the product market. Since each of the three types of cooperation can either exist or not, there are eight possible permutations. We consider only six of them, since we believe that when research is conducted in the same facilities, it would be impossible not to share the results of the research. In all cases we assume that any agreement is costlessly enforced.<sup>1</sup>

The second major difference between our paper and Choi's is that his conclusions are based on the assumption that total industry profits decline as the spillover rate increases, due to intensified competition in the product market. However, spillovers may also enlarge the scope for use of a discovery, and if this effect dominates the former, profits can increase as spillovers increase. To this end, we introduce the concepts of offsetting and incremental spillovers, and show how and under what circumstances this dimension is consequential. In this more general framework, we examine the effects of uncertainty and spillovers on the level of R&D expenditure under the six regimes outlined above. Across these cases we compare equilibrium levels of investment, profitability, in order to discover which setups will be most preferred by the firms, and welfare. Although our generalized functional specification precludes us from giving a complete ranking of all variables in all situations, we are able to derive several insights. The main findings are as follows.

First, it is the type of spillovers, and not merely their extent, that determine relative desirability. Specifically, when spillovers are offsetting competition in R&D tends to be more profitable than cooperation in R&D (through an information sharing agreement or a joint research venture), but with incremental spillovers cooperation is often more desirable. This is because cooperation leads to maximal spillovers (since any findings by one firm are shared

with the other), so if spillovers are incremental cooperation is to the benefit of the firms, and they have an incentive to cooperate, while if they are offsetting competition is more beneficial. The relative level of investment in R&D (cooperation vs. non-cooperation), however, depends mostly on the extent of spillovers and not on the type of spillovers. In addition, cooperation is often, but not always, welfare enhancing. Second, cost sharing usually leads to increased investment, profits and welfare. Finally, joint ventures categorically lead to more investment, higher profits and greater welfare than information sharing agreements.

This paper is organized as follows. Section 2 sets up the basic model. Section 3 considers the level of R&D expenditure and profits under three regimes without cost sharing – competition, information sharing agreements, and joint ventures. We compare between these setups from the producers' perspective, and discuss welfare implications. Section 4 reexamines these three regimes when research costs are decided upon cooperatively, i.e., with cost sharing. We show how this additional layer of cooperation affects behavior in each of the three setups, and compare across the three setups with cost-sharing. The final Section summarizes the results, and raises issues regarding antitrust considerations connected with cooperation, and possible conflicts and concerns that may arise between the cooperating parties. Some policy implications are also discussed.

## **II. The Model – Basic Setup**

Consider two firms undertaking R&D activity, that must decide whether to cooperate and how much to spend on R&D activity. The purpose of the R&D efforts is to discover a new product or process, and if a firm's R&D efforts are successful, i.e., if it has discovered the product or process, it can not reap any additional benefit by discovering the results of the other firm's R&D activities since these will be redundant.<sup>2</sup> Cooperation can be undertaken in the R&D market only, since local antitrust laws prohibit cooperation in the product market. In the absence of cooperation between the firms *and in the absence of spillovers*, success in R&D activities on the part of a single firm will result in a monopoly in the product market, while if both firms succeed, there will be competition between the firms. If, however, the patent system does not guarantee perfect appropriability, research results may spill over to the rival firm who will then be able to appropriate part of the benefits from the innovation even though its own research efforts were unproductive (it will be able to produce an imperfect

substitute). Denoting the degree of spillovers in the project by  $\alpha$ ,  $\alpha \in [0,1]$ , define  $R_1(\alpha)$  as the expected revenue to the successful firm,  $R_2(\alpha)$  the expected revenue to the unsuccessful firm, and  $R_3$  the expected revenue to the duopolist in the case where both firms succeed, either independently or cooperatively. We proceed as in Choi (1993), and make the following natural assumptions

*Assumption 1:* (i)  $\frac{\partial R_1(\alpha)}{\partial \alpha} < 0$ ;  $\frac{\partial R_2(\alpha)}{\partial \alpha} > 0$ ; (ii)  $R_2(0) = 0$ ;

and (iii)  $R_1(1) = R_2(1) = R_3$ .

Part (i) of Assumption 1 states that spillovers are detrimental to the successful firm but help the unsuccessful firm. Part (ii) states that the expected revenue of the unsuccessful firm is zero if there are no spillovers since it does not participate in the product market.<sup>3</sup> Part (iii) says that the revenue with cooperation and discovery is equal to the revenue of each duopolist in the case of noncooperative discovery (by at least one duopolist) and complete spillovers.<sup>4</sup>

In contrast to Choi (1993), we introduce the possibility that spillovers can increase or reduce total industry profits. We thus define:

*Definition 1:* Spillovers are incremental if  $\frac{\partial [R_1(\alpha) + R_2(\alpha)]}{\partial \alpha} > 0$ , and they are

offsetting if  $\frac{\partial [R_1(\alpha) + R_2(\alpha)]}{\partial \alpha} < 0$ .

The above definition states that an increase in the degree of spillovers can increase total industry revenue by enlarging the scope of use of a discovery (on this point see Marjit 1990, and Levin and Reiss, 1988), or it may reduce total industry revenue, due to intensified competition in the product market (Choi, 1993). When the former effect dominates the latter, we call the spillovers "incremental", and we call them "offsetting" if the reverse is true. Spillovers are most likely to be incremental in the case of product innovations, rather than process innovations, since in this case it is more plausible that market demand increases with the number of the firms that market the innovation. Moreover, incremental spillovers are more likely in the case of complementary goods than in the case of close substitutes. Finally, spillovers are more likely to be incremental when they take place across industries rather than within an industry (for intra-industry spillovers see Bernstein and Nadiri, 1988).

We assume that the results of R&D activity are uncertain. Denote by  $P(x)$  the probability of success if the firm invests  $x$  in the project. Following Choi (1993), we make the following natural assumption:

*Assumption 2:*  $P'(x) \geq 0$ ;  $P''(x) \leq 0$ ;  $P'(0) = \infty$ ; and  $P'(\infty) = 0$ .<sup>5</sup>

Firms must decide whether or not to cooperate. Cooperation can occur at any or all of three stages. Firms can agree on the level of spending on research (as per Choi), they can choose to conduct the R&D using joint research facilities, and they can choose to share the outcomes of their research efforts.<sup>6</sup> This yields eight possible cooperation configurations. However, two of these can be eliminated, since the use of joint research facilities automatically results in the sharing of research outcomes.

We proceed as follows. We first develop and compare the three cases without spending synchronization – full competition, information sharing only, and research joint ventures (in which information is also shared automatically). In our comparison we address the issues of the level of R&D expenditures, the relative profitability from the firm's perspective, and welfare. We then allow for joint spending decisions, and show how this affects the previous analysis. The six cases are shown in Table 1 in the order in which they will be analyzed. Figure 1 shows all the comparisons made in the paper, and indicates the sections in which the comparisons appear.

### **III. Competition, Information Sharing Agreements, and Research Joint Ventures**

#### **A. Competition (C)**

We consider first the case where there is no cooperation. If firms compete on all fronts, the expected net profit of firm  $i$  is<sup>7</sup>

$$(1) \quad E\Pi_i^c = P(x_i)(1 - P(x_j))R_1(\alpha) + (1 - P(x_i))P(x_j)R_2(\alpha) + P(x_i)P(x_j)R_3 - x_i.$$

The first term is the probability that the first firm alone is successful in its research efforts times the income the firm gets if it alone discovers, and thus earns monopoly profits (given the level of spillovers). The second term is the probability that only the other firm discovers times the income realized in this instance, and the third term shows the probability that both firms discover times the income in that case. If neither firm discovers, revenues are zero. The cost of the research is incurred in all cases.

Firm  $i$  chooses the optimal level of expenditure on the project from maximization of (1) with respect to  $x_i$  for given  $x_j$ . The first order condition for a maximum is

$$(2) \quad G_i = \frac{\partial E\Pi_i^c}{\partial x_i} = P'(\hat{x}_i^c) [R_1(\alpha) - P(x_j)(R_1(\alpha) + R_2(\alpha) - R_3)] - 1 = 0,$$

where  $\hat{x}_i^c \equiv \hat{x}_i^c(x_j, R_1(\alpha), R_2(\alpha), R_3)$  is the solution to (2).<sup>8</sup> An analogous condition holds for firm  $j$ .

Totally differentiating (2) with respect to  $x_i$  and  $x_j$ , we have

$$\frac{d\hat{x}_i^c}{dx_j} = \frac{P'(\hat{x}_i^c)P'(x_j)(R_1(\alpha) + R_2(\alpha) - R_3)}{P''(\hat{x}_i^c)(R_1(\alpha) - P(x_j)(R_1(\alpha) + R_2(\alpha) - R_3))} < 0$$

by the second order conditions. Moreover,

$$\frac{\partial(d\hat{x}_i^c/dx_j)}{\partial x_j} < 0 \quad \text{if} \quad \left[ P'(x_j)^2 - P''(x_j)P(x_j) \right] (R_1(\alpha) + R_2(\alpha) - R_3) > -P''(x_j)R_1(\alpha),$$

which will occur except in extreme instances.<sup>9</sup> If this condition holds, there is guaranteed to be a unique and symmetric Nash equilibrium in the R&D competitive game. If this condition does not hold there may also be non-symmetric equilibria. We focus on the symmetric equilibrium.

Given the Nash equilibrium, we now confirm the following proposition.

**Lemma 1.** *An increase in the degree of spillovers reduces the level of R&D expenditure in a competitive R&D market irrespective of the nature of the spillovers.*

The proof of this lemma is given in the Appendix.

This well-known result continues to hold even with incremental spillovers because, in deciding the level of R&D expenditure, each firm takes into account only its own gain. Since  $\partial R_1(\alpha)/\partial \alpha < 0$  independent of the type of spillovers that exists, each firm reduces the level of R&D expenditure as the degree of spillovers increases.<sup>10</sup>

## **B. Information Sharing Agreements (IS)**

In this case firms decide independently on the amount of resources to invest in R&D and on the type of research to carry out, but they write an enforceable contract to share research



results with the other firm *ex post*.<sup>11,12</sup> Hence, both firms succeed if at least one firm succeeds. This stylized depiction of R&D cooperation allows us to analyze the relationship between the incentive to cooperate and the imperfect appropriability of R&D results separately from other incentives such as the elimination of effort duplication (analyzed in Section III C) or the sharing of R&D costs (analyzed in Section IV and in Choi (1993)).

In the case of an IS, firm  $i$ 's expected net profit is

$$(3) \quad E\Pi_i^s = [P(x_i)(1 - P(x_j)) + (1 - P(x_i))P(x_j) + P(x_i)P(x_j)]R_3 - x_i,$$

where the superscript  $s$  denotes information sharing. The first order conditions for a maximum are

$$(4) \quad L_i = \frac{\partial E\Pi_i^s}{\partial x_i} = P'(\hat{x}_i^s)(1 - P(x_j))R_3 - 1 = 0,$$

where  $\hat{x}_i^s = \hat{x}_i^s(x_j, R_3)$  is firm  $i$ 's optimal amount of R&D expenditure when firms share the results of R&D activity. Since with an IS the spillover parameter is always at its maximum, spillovers do not affect the amount of R&D expenditure in an IS. Rather, such expenditures are affected by the size of total industry profits created by the innovations (i.e.,  $2R_3$ ).

Comparing the level of R&D expenditure in the two cases above we have

**Lemma 2.** *The level of R&D expenditure is higher with competition in R&D activity than with information sharing agreements.*

*Proof:* In a symmetric Nash equilibrium, conditions (2) and (4) become, respectively,

$$(5) \quad G' = P'(\hat{x}^c) [(1 - P(\hat{x}^c))R_1(\alpha) + P(\hat{x}^c)(R_3 - R_2(\alpha))] - 1 = 0$$

and,

$$(6) \quad L' = P'(\hat{x}^s)(1 - P(\hat{x}^s))R_3 - 1 = 0$$

Assume  $\alpha = 1$ . From Assumption 1,  $R_3 = R_2(1) = R_1(1)$ , so substituting into (5) and comparing (5) and (6), it is straightforward to conclude that  $\hat{x}^c = \hat{x}^s$ . On the other hand, if  $0 < \alpha < 1$ ,  $L'$  is unaffected, but since  $\frac{\partial R_1(\alpha)}{\partial \alpha} < 0$  and  $\frac{\partial R_2(\alpha)}{\partial \alpha} > 0$ , then  $G'(\alpha) > 0$ . Since  $\frac{\partial G'}{\partial x} < 0$ , condition (5) is satisfied if and only if  $\hat{x}^c > \hat{x}^s$ .

*Q.E.D.*

This result is well known, and is intuitive since with competition the firm appropriates most (and in some instances all) of the rents that accrue from its research efforts, while with an information sharing agreement the rents are shared equally, so the incentive to carry out research is diminished.<sup>13</sup> Note, though, that this is one of the results that contrasts sharply with the results in d'Aspremont and Jacquemin (1988) and Choi (1993) who showed that the level of R&D expenditure is higher with cooperation when spillovers are high. The reason for the discrepancy in predictions lies in the definition of cooperation, since in those models the level of R&D expenditures is what is determined cooperatively. Their results will be shown to hold in this case in Section IV below.

We turn now to the more interesting question of the circumstances under which duopolists prefer to cooperate in R&D. Cooperation is preferred if and only if the firm's expected net profit under cooperation is higher than its expected net profit when there is no cooperation in R&D activity. The condition for firm  $i$  to prefer cooperation via an information sharing agreement to no cooperation is

$$(7) \quad \begin{aligned} E\Pi_i^s &= [P(\hat{x}_i^s)(1 - P(\hat{x}_j^s)) + (1 - P(\hat{x}_i^s))P(\hat{x}_j^s) + P(\hat{x}_i^s)P(\hat{x}_j^s)]R_3 - \hat{x}_i^s \geq \\ E\Pi_i^c &= P(\hat{x}_i^c)(1 - P(\hat{x}_j^c))R_1(\alpha) + (1 - P(\hat{x}_i^c))P(\hat{x}_j^c)R_2(\alpha) + P(\hat{x}_i^c)P(\hat{x}_j^c)R_3 - \hat{x}_i^c. \end{aligned}$$

As clear from (7), the equilibrium profit of a competitive firm depends on the degree of spillovers, while that of the cooperative firm does not. In addition, when spillovers are complete both research efforts and profits are equivalent under the two regimes (Lemma 2). To compare expected profits with less than complete spillovers, then, we evaluate the derivative of the expected profits in the non-cooperative case with respect to changes in the degree of spillovers.

Totally differentiating the right hand side of (7), we have

$$(8) \quad \frac{dE\Pi_i^c}{d\alpha} = \frac{\partial E\Pi_i^c}{\partial \hat{x}_i^c} \frac{d\hat{x}_i^c}{d\alpha} + \frac{\partial E\Pi_i^c}{\partial \hat{x}_j^c} \frac{d\hat{x}_j^c}{d\alpha} + \frac{\partial E\Pi_i^c}{\partial \alpha}.$$

By the envelope theorem, the first term on the right hand side of (8) is zero.

With respect to the second term, from Lemma 1 we know that  $d\hat{x}_j^c/d\alpha < 0$ .

Differentiating from (7),

$$(9) \quad \frac{\partial E\Pi_i^c}{\partial \hat{x}_j^c} = -P(\hat{x}_i^c)P'(\hat{x}_j^c)(R_1(\alpha) + R_2(\alpha) - R_3) + P'(\hat{x}_j^c)R_2(\alpha).$$

It is clear from (9) that when  $\alpha = 0$ ,  $\frac{\partial E\Pi_i^c}{\partial \hat{x}_j^c} < 0$  since  $R_2(0) = 0$ , and that when  $\alpha = 1$ ,

$\frac{\partial E\Pi_i^c}{\partial \hat{x}_j^c} > 0$ . Thus, there exists a value of  $0 < \bar{\alpha} < 1$  such that  $\frac{\partial E\Pi_i^c}{\partial \hat{x}_j^c} < 0$  if  $\alpha < \bar{\alpha}$ ,

$\frac{\partial E\Pi_i^c}{\partial \hat{x}_j^c} = 0$  if  $\alpha = \bar{\alpha}$ , and  $\frac{\partial E\Pi_i^c}{\partial \hat{x}_j^c} > 0$  if  $\alpha > \bar{\alpha}$ .<sup>14</sup> Accordingly, the second term in (8) is

positive if  $\alpha < \bar{\alpha}$  and negative if  $\alpha > \bar{\alpha}$ .

The third term in (8) denotes the direct effect of  $\alpha$  on firm  $i$ 's profit. By Definition 1,  $\frac{\partial E\Pi_i^c}{\partial \alpha} > 0$  if spillovers are incremental, and  $\frac{\partial E\Pi_i^c}{\partial \alpha} < 0$  if they are offsetting (since we are changing only  $\alpha$  and not the amount of research or the probability of success).

Combining these effects, for  $\alpha \geq \bar{\alpha}$  and offsetting spillovers, an increase in the degree of spillovers reduces the expected profit of the competitive firm. Conversely, if  $\alpha \leq \bar{\alpha}$  and spillovers are incremental, the expected profit of the competitive firm increases as the degree of spillovers increases. If, however,  $\alpha < \bar{\alpha}$  and spillovers are offsetting, or if  $\alpha > \bar{\alpha}$  and spillovers are incremental, the sign of (8) depends on the relative strength of these two counteracting effects.

Given this, we now stipulate

**Proposition 1.** *If spillovers are sufficiently high and are offsetting, then firms prefer competition to an information sharing agreement. Conversely, if spillovers are sufficiently low and are incremental, then firms tend to prefer an information sharing agreement to competitive research programs.*

(Proof in Appendix.)

These results are demonstrated in Figures 2 and 3, each of which show three different paths for expected competitive profits. Figure 2 shows the situation with offsetting spillovers. The first two panels show the case where competition is always preferred to cooperation because of the spillovers. This result is natural, since, with offsetting spillovers, total profits fall with an increase in the number of firms producing the good, so the incentive to achieve a monopoly position is great. The third panel presents a case where for low amounts of spillovers cooperation is preferable, but once spillovers become too high,

competition ensues. Note that the level of spillovers at which the change occurs must be less than  $\bar{\alpha}$ , as defined above, since the slope of the expected profit must be positive at that point. For this to be the case, it must be that when spillovers are low, competitive firms invest heavily in R&D in order to attempt to capture monopoly profits, so it is preferable to cooperate in order to cut down on excessive R&D expenditures. When spillovers are great research expenditures are closer to the cooperative level, so this consideration is inconsequential.

In Figure 3 incremental spillovers are presented. The first panel shows a case in which cooperation is always preferred due to the incremental nature of the spillovers. In the second panel cooperation is preferable only for low values of the spillover parameter, since for high values, most of the incremental benefits from multiple producers have already been realized because of the spillovers, and the benefits from increased research expenditures in competition outweigh the benefit from cooperation. Panel 3 shows the case where the second part of Proposition 1 does not hold - competition is always preferred, due to the great benefit to be gained by increased R&D expenditures.

### **C. Research Joint Ventures (JV)**

As an alternative to information sharing agreements, firms can conduct cooperative R&D by undertaking research joint ventures, that is, by conducting R&D in only one research lab or coordinate their research strategies as well as sharing the results of R&D activity.<sup>15</sup> In this instance, the probability of one firm succeeding is not independent of the actions of the other. This form of cooperation allows the two firms to save costs due to the presence of complementary assets and the elimination of any duplication of effort. Despite this we continue to maintain the assumption that firms decide upon their levels of investment independently, and discuss how joint determination of investment expenditures affect the results in the next Section.

We do not model the cost savings from the JV explicitly. Rather, we make the following assumption, which captures the "spirit" of these cost savings:

*Assumption 3:*  $P(2x) > 1 - (1 - P(x))^2 \forall x$ .

This assumption states that for the same level of expenditures on research in each regime, the probability of success under a JV is greater than the probability of success by *at*

least one firm when research is carried out independently. This means that by avoiding duplication of efforts, the JV increases the probability of success for any given size investment. We assume that this relationship holds for changes also, so that:

*Assumption 3a:*  $P'(2x) > P'(x)(1 - P(x)) \forall x$

This assumption means that at each level of investment, an additional dollar spent in a joint venture is more productive than an additional dollar in separate research facilities. This assumption continues to capture the essence of avoiding the duplication of efforts. Note that Assumptions 3 and 3a will be used only when comparing a JV to other forms of organization.

In a JV, firm i's expected profit is

$$(10) \quad E\Pi_i^v = P(x_i + x_j)R_3 - x_i,$$

and the first order condition for a maximum is

$$(11) \quad M_i = \frac{\partial E\Pi_i^v}{\partial x_i} = P'(\hat{x}_i^v + x_j)R_3 - 1 = 0.$$

From (11) we obtain firm i's optimal level of R&D expenditure,  $\hat{x}_i^v \equiv \hat{x}_i^v(x_j, R_3)$ , when firms form a research joint venture. Note that each firm still determines its own level of expenditures. In what follows, we assume a symmetric Nash equilibrium.

Comparing investment levels in a JV with those in competition and an IS agreement, we arrive at the following proposition.

**Proposition 2.** *There exists a value of  $\alpha$ , denoted  $\hat{\alpha}$ ,  $0 < \hat{\alpha} < 1$ , such that if  $\alpha > \hat{\alpha}$  then  $\hat{x}^v > \hat{x}^c \geq \hat{x}^s$ , with equality when  $\alpha = 1$ , if  $\alpha = \hat{\alpha}$  then  $\hat{x}^v = \hat{x}^c > \hat{x}^s$ , and if  $\alpha < \hat{\alpha}$  then  $\hat{x}^c > \hat{x}^v > \hat{x}^s$ .*

(Proof in Appendix.)

One implication of this proposition for antitrust policy is that the general belief that cooperation in research between firms leads to a lessening of research efforts, is not necessarily true. Rather, it depends on the type of cooperation and on the degree of spillovers.

Consider now expected profits. In evaluating a JV, we consider the payoff functions when firms behave symmetrically. We begin by comparing a JV to an IS. Comparing (3) and (10), the condition for a JV to be preferred to an IS in equilibrium is:

$$(12) \quad E\Pi^v = P(2\hat{x}^v)R_3 - \hat{x}^v > E\Pi^s = \left[ 2P(\hat{x}^s)(1 - P(\hat{x}^s)) + P(\hat{x}^s)^2 \right] R_3 - \hat{x}^s .$$

From this condition it is easy to show that:

**Proposition 3.** *A JV is always preferred to an IS agreement.*

(Proof in Appendix.)

This occurs because in both cases information is always shared, while in a JV synergies in research are also realized. Thus a JV can be viewed as an IS with the additional benefit of avoiding the duplication of costs.

Compare now a JV with competition. The condition for preferring a JV to competition is:

$$(13) \quad E\Pi^v = P(2\hat{x}^v)R_3 - \hat{x}^v \geq E\Pi^c = P(\hat{x}^c)(1 - P(\hat{x}^c))[R_1(\alpha) + R_2(\alpha)] + P(\hat{x}^c)^2 R_3 - \hat{x}^c .$$

Unfortunately, this does not allow for a full ranking without further specification of the functions. However, some insight can be attained, and is presented in the following proposition.

**Proposition 4.** *A JV will be preferred to competition when spillovers are incremental and low ( $\alpha \leq \hat{\alpha}$ ). Competition will tend to be preferred over a JV when spillovers are offsetting and high ( $\alpha \geq \hat{\alpha}$ ), and the probability of success in research efforts is low.*

(Proof in Appendix.)

Proposition 4 shows that the choice between a JV and competition in R&D depends on the nature and magnitude of spillovers. Offsetting spillovers provide an incentive to compete in R&D because they increase the relative value of being the sole producer. By contrast, if spillovers are incremental, there are benefits to be had by the presence of multiple producers, so firms do not have strongly conflicting interests. In this instance, cooperation is of particular importance when spillovers are low because the gain from cooperation is great, while if spillovers are high cooperation adds little. Cases other than those stated in the proposition require more precise functional specification in order to evaluate whether a JV or competition is more profitable.

## **D. Welfare**

There are three welfare-related issues that differ across the different regimes. The first is the amount of R&D conducted. The results of this research determines whether the product will be available, so this welfare issue is unrelated to the welfare issues once the product exists (the amount of competition). There are three parties that stand to gain from the existence of the product – the two producers and consumers. This means that firms do not reap the full benefits from their research efforts, and so under-invest in R&D.<sup>16</sup> Thus, the more R&D carried out, the closer the economy will be to the optimal level of R&D, and the greater will be welfare. The second issue is related to competition in the product market – the more competition the better off the economy from a welfare perspective. The third issue is the possibility of cost-savings from joint ventures. Denoting welfare in competition, IS agreements and JVs by  $W^c$ ,  $W^s$  and  $W^v$ , respectively, we can conclude:

**Proposition 5.** *a)  $W^v > W^s$ ; b) if  $\alpha \geq \hat{\alpha}$  then  $W^v > W^c$ ; c) if  $\alpha = 1$  then  $W^c = W^s$ .*

(Proof in Appendix.)

In all other cases clear conclusions cannot be reached without more precise functional specifications. In particular, when comparing welfare in competition and an IS when  $\alpha < 1$ , we note that the former has more research, but there are less spillovers. Thus, neither is clearly superior.

## **IV. Cost Sharing Agreements**

We now add the possibility of cost sharing agreements to our previous analysis. We assume that expenditures can be costlessly monitored. As is intuitively clear, cost sharing makes the firms more profitable, and so will be desired by firms. Such setups, however, may be untenable because of monitoring difficulties and/or because of legal (antitrust) prohibitions as discussed in the next Section. In our development below we compare each case with its parallel case without cost sharing, and compare between the three regimes with cost sharing. In all that follows, we denote cost sharing by a second superscript c.

### **A. Competition with Cost Sharing**

With cost sharing, but competition in all other stages, expected profits are given by:

$$(14) \quad E\Pi^{cc} = P(x)(1 - P(x))R_1(\alpha) + (1 - P(x))P(x)R_2(\alpha) + P(x)P(x)R_3 - x.$$

The optimal level of R&D expenditures is then found through the first-order condition:

$$(15) \quad \frac{\partial E\Pi^{cc}}{\partial x} = P'(\hat{x}^{cc}) \left[ R_1(\alpha) + R_2(\alpha) - 2P(\hat{x}^{cc}) \left( R_1(\alpha) + R_2(\alpha) - R_3 \right) \right] - 1 = 0.$$

Comparing investment levels with those in competition without cost sharing, we arrive at the following:

**Lemma 3.** *There exists a value of  $\alpha$ , denoted  $\tilde{\alpha}$ ,  $0 < \tilde{\alpha} < 1$ , such that if  $\alpha < \tilde{\alpha}$  then*

*$\hat{x}^c > \hat{x}^{cc}$ , if  $\alpha = \tilde{\alpha}$  then  $\hat{x}^c = \hat{x}^{cc}$ , and if  $\alpha > \tilde{\alpha}$  then  $\hat{x}^c < \hat{x}^{cc}$ .*

(Proof in Appendix.)

Comparing profit levels, we find that:

**Lemma 4.**  $E\Pi^{cc} \geq E\Pi^c$ , with equality when  $\alpha = \tilde{\alpha}$ .

*Proof:* When  $\hat{x}^c = \hat{x}^{cc}$  (14) and (1) are identical, so profits under the two regimes are equal.

If  $\hat{x}^c \neq \hat{x}^{cc}$  it is clear that  $E\Pi^{cc} > E\Pi^c$ , since the investment level is chosen so that, at  $\hat{x}^{cc}$ ,  $\partial E\Pi^{cc} / \partial x = 0$ . *Q.E.D.*

The logic behind this result is immediate. Since the firms with cost sharing can always choose the level of investment chosen without cost sharing, their profits cannot fall below those without cost sharing. Lemmas 3 and 4 are the same as the results in Choi (1993).

### **B. Information Sharing Agreements with Cost Sharing**

In this case the firm's expected profits are:

$$(16) \quad E\Pi^{sc} = \left[ 2P(x) - P(x)^2 \right] R_3 - x,$$

and in equilibrium:

$$(17) \quad 2P'(\hat{x}^{sc}) \left( 1 - P(\hat{x}^{sc}) \right) R_3 = 1.$$

Comparing an IS with cost sharing with and IS without cost sharing, it is immediate to show that:



**Lemma 5.**  $\hat{x}^{sc} > \hat{x}^s$ , and  $E\Pi^{sc} > E\Pi^s$ .

*Proof:* To find the investment levels we must compare:

$$(18) \quad 2P'(\hat{x}^{sc})(1 - P(\hat{x}^{sc}))R_3 = P'(\hat{x}^s)(1 - P(\hat{x}^s))R_3.$$

Since  $P' > 0$  and  $P'' < 0$ , it is clear that  $\hat{x}^{sc} > \hat{x}^s$ . Profits are also greater with cost sharing since the level chosen without cost sharing is available with cost sharing, and is rejected in favor of a higher, and more profitable, investment level. *Q.E.D.*

The reason there is more investment with cost sharing than without is because there is less free riding, and the reason profits are greater is because, as above, the level of investment chosen without cost sharing could always be chosen with cost sharing.

Comparing competition with cost sharing, with IS with cost sharing, we find that:

**Proposition 6.** *When  $\alpha = 1$ ,  $\hat{x}^{sc} = \hat{x}^{cc}$ . When  $\alpha < 1$ , if spillovers are incremental and the probability of success is lower than  $1/2$ , or spillovers are offsetting and the probability of success is greater than  $1/2$ , then  $\hat{x}^{sc} > \hat{x}^{cc}$ . In all other cases the sign is not clear.*

(Proof in Appendix.)

Note that this result is different from the case when there was no cost sharing (Lemma 2), where there was always more research with competition than with cooperation. The reason for the difference is that under the current regime the competitor is matching each dollar of research expenditures with a dollar of his own, so spillovers become less problematic. Nevertheless, it is clear from the proof that it is still likely that there will always be more investment in competition than with an IS. Such a determination, however, cannot be made in such a generalized setting.

Turning to a comparison of profits, we have:

**Proposition 7.** *With cost sharing, competition and an IS are identical when  $\alpha = 1$ . When  $\alpha < 1$ , an IS is preferred if spillovers are incremental, and competition is preferred if spillovers are offsetting.*

(Proof in Appendix.)

This result is logical. Recall that costs are shared in either case, and that each firm has an *ex-ante* equivalent probability of discovering. Therefore, when spillovers are offsetting the firms, *ex-ante*, prefer as little spillovers as possible, and so prefer competition. Conversely, when spillovers are incremental, they are better off, *ex-ante*, with full spillovers, so cooperation is the preferred venue.

### **C. Research Joint Ventures with Cost Sharing**

Expected profits with a JV and cost sharing are:

$$(19) \quad E\Pi_i^{vc} = P(2x)R_3 - x.$$

The first order condition for a maximum is given by:

$$(20) \quad 2P'(2\hat{x}^{vc})R_3 = 1.$$

Comparing (20) and (11) it is immediate to conclude that:

**Lemma 6.**  $\hat{x}^{vc} > \hat{x}^v$ , and  $E\Pi^{vc} > E\Pi^v$ .

*Proof:* From (20) and (11) we see that  $2P'(2\hat{x}^{vc})R_3 = P'(2\hat{x}^v)R_3$ . Thus, it is clear that the first part of the Proposition holds. The second part holds since, as in Lemmas 4 and 5, the cost sharing firms could have chosen to have the same level of expenditures as they did without cost sharing. *Q.E.D.*

Comparing a JV with cost sharing with an IS with cost sharing, we see that:

**Lemma 7.**  $\hat{x}^{vc} > \hat{x}^{sc}$ , and  $E\Pi^{vc} > E\Pi^{sc}$ .

*Proof:* Comparing (20) and (17), we need:

$$2P'(2\hat{x}^{vc})R_3 = 2P'(\hat{x}^{sc})(1 - P(\hat{x}^{sc}))R_3,$$

and from Assumption 3a it is clear that this can only occur if  $\hat{x}^{vc} > \hat{x}^{sc}$ .

Since there is cost sharing, it is clear that  $E\Pi^{vc}(x = \hat{x}^{vc}) > E\Pi^{vc}(x = \hat{x}^{sc})$ . Consequently, it is sufficient to show that  $E\Pi^{vc}(x = \hat{x}^{sc}) > E\Pi^{sc}(x = \hat{x}^{sc})$ . This amounts to showing that:

$$P(2\hat{x}^{sc})R_3 - \hat{x}^{sc} > [2P(\hat{x}^{sc}) - P(\hat{x}^{sc})^2]R_3 - \hat{x}^{sc},$$

which holds by Assumption 3. *Q.E.D.*

The logic behind this result is that a joint venture has an advantage over an IS since it has all the benefits of an IS, plus the added benefit of cost-savings. Note that this is exactly analogous to Proposition 3 and parts of Proposition 2.

Our final comparison is between a JV with cost sharing and competition with cost sharing.

**Lemma 8.** *If a) spillovers are incremental and the probability of success is lower than  $1/2$ ; b) spillovers are offsetting and the probability of success is greater than  $1/2$ ; or c)  $\alpha = 1$ , then  $\hat{x}^{vc} > \hat{x}^{cc}$ .*

(Proof in Appendix.)

Note that there is no instance in which we can conclusively state that there is more research in competition than in a JV, and, in fact, this may never occur. However, without specifying the functions a more precise statement is not available.

Finally comparing profits under a JV with cost sharing and competition with cost sharing we get:

**Proposition 8.** *If spillovers are incremental, firms prefer a JV with cost sharing to competition with cost sharing. A JV is also preferred with offsetting spillovers if  $\alpha$  is large.*

(Proof in Appendix.)

Note that the situation with offsetting spillovers is not, in general, clear. Since costs are shared, there are two issues affecting the choice between the regimes. With a JV there are cost savings, and, in addition, there are maximum spillovers. When spillovers are incremental both of these effects favor a JV over competition, so the result is clear. With offsetting spillovers, however, there is a tradeoff between the two effects, and the relative desirability cannot, in general, be discerned. If, however, spillovers are large, the benefit from competition is mitigated, and cooperation becomes more profitable.

## **D. Welfare**

Because the probability of success function is concave, it would seem that there would be a distinct advantage to cost sharing, since it will maximize the probability of success for any given size investment. Nevertheless, in equilibrium there was always an equal amount of R&D done by the two firms, so cost sharing does not increase efficiency from this perspective. Thus, no new welfare issues arise when cost sharing is added. The issues to be considered, then, are the level of expenditures, the amount of competition in the product market, and cost savings. We divide the welfare comparisons into two propositions, the first showing how cost sharing affects welfare, and the second comparing the different alternatives with cost sharing.

**Proposition 9.** *Cost sharing increases welfare, except in the case of competition with low spillovers when the converse occurs.*

*Proof:* Spillovers and cost savings are unaffected by the presence of cost sharing, so the only determining factor is the level of investment. The proposition is thus shown to hold by Lemmas 3, 5 and 6. *Q.E.D.*

**Proposition 10.** *a)  $W^{vc} > W^{sc}$ ; b)  $W^{vc} > W^{cc}$  if spillovers are incremental and there is at most a 50% probability of success, if they are offsetting and there is at least a 50% probability of success, or if  $\alpha = 1$ ; and c)  $W^{sc} > W^{cc}$  if spillovers are incremental and if at most a 50% probability of success, or spillovers are offsetting and there is at least a 50% probability of success ( $W^{sc} = W^{cc}$  when  $\alpha = 1$ ).*

*Proof:* a) With both a JV and an IS there are complete spillovers, but with a JV there are cost savings, and from Lemma 7 there is more research in a JV than in an IS.

b) A JV has higher spillovers, cost savings, and according to Lemma 8, more R&D expenditures under the conditions stated in the proposition.

c) When  $\alpha < 1$  there is more competition in the product market with an IS, and from Proposition 6 there is also more research under the conditions stated in the proposition. When  $\alpha = 1$  an IS and competition are identical. *Q.E.D.*

Comparing these results with those in Proposition 5, we are more able to determine welfare rankings with cost sharing than without, but we still cannot give a complete ranking.

## V. Summary of Results and Discussion

In this paper we considered cooperation among firms involved in R&D activities with spillovers and uncertainty. We used more generalized functional forms than usual in the literature, and considered the possibility of cooperation at any or all of the following three stages. Firms can jointly agree on the level of R&D expenditures, they can set up joint research facilities, and/or they can engage in an information sharing agreement, by which they agree to share any findings with the other firm. One of the novelties of our research is that we introduce the concepts of offsetting and incremental spillovers. An offsetting spillover is the usual situation (and the situation considered by most prior researchers), in which spillovers makes the profits of the benefiting firm increase less than the profits of the discovering firm fall. This tends to occur when the firms are in direct competition in the product market (for example, compare the profits of a monopolist to those of Cournot competitors). An incremental spillover occurs when we allow for the possibility that spillovers may increase total industry profit. This is likely to arise when the product of R&D activity can be exploited in different directions. In this case, there is a new incentive to cooperate in R&D due to the creation of additional markets by the increase in the number of producers.<sup>17</sup> On the other hand, if spillovers reduce profit, there is a strategic incentive for non-cooperative firms to behave aggressively in order to get the monopoly profit arising from the innovation.

Table 2 summarizes the results. A clear pattern emerges. First, when spillovers are offsetting competition tends to be preferred to cooperation, but with incremental spillovers cooperation tends to be more desirable. This is intuitive, for the reasons discussed in the last paragraph. This same tendency, however, does not exist when considering the level of investment in R&D, where the type of spillovers has little effect, but the extent of spillovers is often crucial.

Other patterns also emerge. Cost sharing usually leads to increased investment, profits and welfare. Cooperation in a joint venture without cost sharing is generally welfare enhancing, but can be more or less profitable and more or less investment oriented than competition. With cost sharing, however, cooperation tends to be more profitable and leads to more investment. A similar pattern holds for a comparison of an IS agreement and competition, with the caveat that welfare comparisons are more difficult to make when there

is no cost sharing. Finally, joint ventures lead to more investment, higher profits and greater welfare than information sharing agreements.

In principle, all of these implications, and any additional insight that can be derived from Table 2, is empirically testable. Thus, for instance, we are more likely to see cooperation in research efforts when spillovers are incremental (such as when there are multiple uses for the discovery) than when they are offsetting (such as when the research is geared at lowering the costs of an existing technology).

Care must be taken, however, in taking these conclusions to the extreme. For instance, the model suggests that we should never see an IS agreement since a JV is superior, and there should almost always be cost sharing. And yet there are many instances in which firms cooperate without using joint research facilities, and without sharing costs. Thus, it would seem that we could conclude that the model is wrong. To understand why such a conclusion would be erroneous, we must look beyond the model presented, and consider other issues.

Governments tend to not trust cooperation between firms. The fear is that while cooperation in R&D may increase the probability of discovery, cooperation at the research stage of the process could well turn into collusion at the production stage. Thus, any type of cooperation is viewed skeptically, and is often not permitted. But even assuming cooperation is permitted, some types may seem more conducive to collusion than others. Thus, for instance, when engaged in a joint venture, members of the different firms spend much time and effort working in tandem, and this type of contact may create particularly fertile ground for discussions and decision-making in other realms (such as price setting). Information sharing, on the other hand, requires a less intimate setting, and thus may be preferred by authorities. Thus, we may see IS agreements flourish despite their inferiority.

Firms also tend not to trust one another. Thus, firms may feel that in a joint venture they will be giving away too many of their secrets, and prefer to stay at arms length. Similarly, cost sharing without a joint venture may require that each firm monitor the accounting books of the other firm – a clear intrusion into their private matters. Even if firms are willing to bear this intrusion, the monitoring process itself may be problematic. Issues of questionable cost allocations, fabrication of expenditures, and overstatement of efforts could plague the relationship. Each firm has a clear incentive to agree on a high level of investment, and then spend less. And, from an antitrust perspective, such monitoring may be an instrument that can pave the way to collusion. Finally, information sharing agreements may be difficult or costly

to enforce. Although *ex-ante* firms desire such cooperation, *ex-post* the discovering firm has a clear incentive to attempt to renege on the agreement, and enforcement through the court system is likely to be costly and lengthy.

If any of these concerns are present, the scope of possibilities available to the firms may be limited, and firms will have to choose from among the remaining options. For this reason we expect to see different types of agreements in use, and for this reason it is important to obtain as complete a ranking as possible.

The bottom line of this research, we believe, lies in its implications for antitrust legislation and litigation. It is important to recognize the circumstances that lead to one type of setup being superior to another (especially from a welfare perspective). Antitrust officials should be aware that there are often real benefits to be had from allowing certain types of cooperation, with the exact type of cooperation depending mainly on the degree and nature of spillovers. The challenge they face is to find ways to allow for such cooperation when it is beneficial, while at the same time buckling down on antitrust infringements at the product level.

## References

- Bernstein, J. and Nadiri, M.I., 1988, "Interindustry R&D Spillovers, Rate of Return, and Production in High Technology Industries", *American Economic Review*, 78, 429-34.
- Choi, J.P., 1989, "An Analysis of Cooperative R&D", Working Paper, Department of Economics, Harvard University.
- Choi, J.P., 1993, "Cooperative R&D with product market competition", *International Journal of Industrial Organization*, 11, 553-71.
- Combs, K.L., 1992, "Cost sharing vs. multiple research projects in cooperative R&D", *Economics Letters*, 39, pp. 353-7.
- d'Aspremont, C. and A. Jacquemin, 1988, "Cooperative and Noncooperative R&D in Duopoly with Spillovers", *American Economic Review*, 5, 1133-7.
- De Bondt R. and R. Veugelers, 1991, "Strategic Investment with Spillovers", *European Journal of Political Economy*, 7, 345-66.
- Kamien, I. M., Muller, E. and Zang, I., 1992, "Research Joint Ventures and R&D Cartels", *American Economic Review*, 82, 1293-1306.
- Katz, M.L., 1986, "An analysis of cooperative research and development", *Rand Journal of Economics*, 4, 527-43.
- Levin, R. C. and Reiss, P. C., 1988, "Cost-reduction and demand-creating R&D with spillovers", *Rand Journal of Economics*, 19, 538-56.
- Marjit, S., 1990, "On A Non-cooperative Theory of Technological Transfer", *Economics Letters*, 33, 293-8.
- Marjit, S., 1991, "Incentives for cooperative and non-cooperative R and D in duopoly", *Economics Letters*, 37, 187-91.
- Motta, M., 1992, "Cooperative R&D and vertical product differentiation", *International Journal of Industrial Organization*, 10, 643-61.
- Rosenkranz, S., 1995, "Innovation and Cooperation Under Vertical Product Differentiation", *International Journal of Industrial Organization*, 13, 1-22.
- Suzumura, K., 1992, "Cooperative and Noncooperative R&D in a Oligopoly with Spillovers", *American Economic Review*, 82, 1307-20.



## Appendix

### PROOFS OF PROPOSITIONS

#### Proof of Lemma 1

Totally differentiating (2) we have,

$$\frac{d\hat{x}_i^c(\alpha)}{d\alpha} = - \frac{\frac{\partial G_j}{\partial x_j} \frac{\partial G_i}{\partial \alpha} - \frac{\partial G_i}{\partial x_j} \frac{\partial G_j}{\partial \alpha}}{\frac{\partial G_i}{\partial x_i} \frac{\partial G_j}{\partial x_j} - \frac{\partial G_j}{\partial x_i} \frac{\partial G_i}{\partial x_j}}.$$

At a symmetric equilibrium  $\frac{\partial G_i}{\partial \alpha} = \frac{\partial G_j}{\partial \alpha}$ ,  $\frac{\partial G_i}{\partial x_i} = \frac{\partial G_j}{\partial x_j}$ , and  $\frac{\partial G_i}{\partial x_j} = \frac{\partial G_j}{\partial x_i}$ ,  $i, j=1,2$ , so

$$\frac{d\hat{x}_i^c(\alpha)}{d\alpha} = - \frac{\frac{\partial G_i}{\partial \alpha}}{\frac{\partial G_i}{\partial x_i} + \frac{\partial G_i}{\partial x_j}} < 0$$

since  $\frac{\partial G_i}{\partial x_i} < 0$  by the second order conditions,

$$\frac{\partial G_i}{\partial \alpha} = P'(\hat{x}_i^c) \left[ (1 - P(x_j)) \frac{\partial R_1(\alpha)}{\partial \alpha} - P(x_j) \frac{\partial R_2(\alpha)}{\partial \alpha} \right] < 0 \text{ from Assumption 1, and}$$

$$\frac{\partial G_i}{\partial x_j} = -P'(\hat{x}_i^c) P'(x_j) [R_1(\alpha) + R_2(\alpha) - R_3] < 0. \quad Q.E.D.$$

#### Proof of Proposition 1

As shown in the preceding discussion, when spillovers are offsetting  $\frac{dE\Pi^c}{d\alpha} < 0$ , and when  $\alpha = 1$  profits are identical in the two regimes. Thus, in the vicinity of  $\alpha = 1$ , the first part of the proposition follows.

For the second part of the proposition to hold, it is necessary to show that with incremental spillovers  $E\Pi^c(\alpha = 0) < E\Pi^s$ . Evaluating (7) at  $\alpha = 0$  and rearranging, we find that this occurs if and only if

$$(A1) \quad P(\hat{x}^c)R_1 - 2P(\hat{x}^s)R_3 - P(\hat{x}^c)^2(R_1 - R_3) + P(\hat{x}^s)^2R_3 < \hat{x}^c - \hat{x}^s.$$

From Lemma 2 we know that  $\hat{x}^c > \hat{x}^s$ , so the RHS is positive. We denote  $P(\hat{x}^c) = P(\hat{x}^s) + \lambda$ ,  $\lambda > 0$ . Rewriting (A1):

$$(A2) \quad P(\hat{x}^s) \left(1 - P(\hat{x}^s)\right) (R_1 - 2R_3) + \lambda \left[ R_1 - \left(2P(\hat{x}^s) + \lambda\right) (R_1 - R_3) \right] < \hat{x}^c - \hat{x}^s.$$

From Assumption 1,  $R_1(1) = R_2(1) = R_3$ , so if spillovers are incremental  $R_1(\alpha) + R_2(\alpha) < 2R_3 \forall \alpha < 1$ . Thus, the first term on the LHS is negative. The second term can be positive or negative, depending on how productive are the additional expenditures under competition ( $\lambda$ ), the probability of success, and the difference between monopoly revenues and competitive revenues. In particular, if monopoly profits are far greater than competitive profits ( $R_1 \gg R_3$ ),  $P(x)$  is high or  $\lambda$  is low, then the second term will be certainly be negative, and  $E\Pi^s > E\Pi^c$ . In addition, even if this term is positive, but not big enough to overcome the other terms, (A2) continues to hold. *Q.E.D.*

### Proof of Proposition 2

Recall that in Lemma 2 we already showed that  $\hat{x}^c \geq \hat{x}^s \forall \alpha \leq 1$  (with equality when  $\alpha = 1$ ).

In a symmetric equilibrium, optimization in a JV requires:

$$(A3) \quad M' = P'(2\hat{x}^v)R_3 - 1 = 0.$$

Comparing (6) with (A3), and using Assumption 3a, we see that

$$P'(2\hat{x}^s) > P'(\hat{x}^s) \left(1 - P(\hat{x}^s)\right) = P'(2\hat{x}^v)$$

which can only occur if  $\hat{x}^v > \hat{x}^s$ .

Compare now (5) and (A3). If  $\alpha = 1$  then  $\hat{x}^c = \hat{x}^s$ , so, from the last result,  $\hat{x}^v \geq \hat{x}^c$ . If  $\alpha = 0$ , (5) and (A3) require that:

$$P'(\hat{x}^c) \left(1 - P(\hat{x}^c)\right) R_1(0) + P'(\hat{x}^c) P(\hat{x}^c) R_3 = P'(2\hat{x}^v) R_3.$$

Defining  $R_1 \equiv (1 + \lambda)R_3$  and simplifying, this can be rewritten as:

$$(A4) \quad P'(\hat{x}^c) + P'(\hat{x}^c) \left(1 - P(\hat{x}^c)\right) \lambda = P'(2\hat{x}^v).$$

If  $\hat{x}^v \geq \hat{x}^c$  then  $P'(\hat{x}^c) > P'(2\hat{x}^v)$  so (A4) fails. Hence, we conclude that at  $\alpha = 0$ ,  $\hat{x}^c > \hat{x}^v$ .

Thus, there exists a value  $\hat{\alpha}$ ,  $0 < \hat{\alpha} < 1$ , at which  $\hat{x}^c = \hat{x}^v$ , so that  $\hat{x}^c > \hat{x}^v$  if  $\alpha < \hat{\alpha}$ , and  $\hat{x}^c < \hat{x}^v$  if  $\alpha > \hat{\alpha}$ . Q.E.D.

Proof of Proposition 3

From (12) the optimal level of investment in a JV is given by:

$$2P'(2x^v)R_3 = 1.$$

Comparing this to the level chosen by the firm (See Equation 11) it is clear that  $\hat{x}^v$  is sub-optimal. Hence, since, from Proposition 2,  $\hat{x}^v > \hat{x}^s$ , it is clear that  $E\Pi^v(x = \hat{x}^v) > E\Pi^v(x = \hat{x}^s)$ . Consequently, it is sufficient to show that  $E\Pi^v(x = \hat{x}^s) > E\Pi^s(x = \hat{x}^s)$ . This amounts to showing that:

$$P(2\hat{x}^s)R_3 - \hat{x}^s > \left[ 2P(\hat{x}^s)(1 - P(\hat{x}^s)) + P(\hat{x}^s)^2 \right] R_3 - \hat{x}^s,$$

which, simplified, yields:

$$P(2\hat{x}^s) > 2P(\hat{x}^s) - P(\hat{x}^s)^2,$$

which holds by Assumption 3. Q.E.D.

Proof of Proposition 4

Rearranging (13), the condition for undertaking a JV becomes

$$(A5) \quad \left[ R_1(\alpha) + R_2(\alpha) - \frac{P(2\hat{x}^v)}{P(\hat{x}^c)} R_3 \right] - P(\hat{x}^c) [R_1(\alpha) + R_2(\alpha) - R_3] < \frac{\hat{x}^c - \hat{x}^v}{P(\hat{x}^c)},$$

and competition is preferred if the inequality is reversed.

Consider first the case where  $\hat{x}^c \geq \hat{x}^v$  (where  $\alpha \leq \hat{\alpha}$ , see Proposition 2). In this case the right hand side of (A5) is non-negative. If the LHS is non-positive, a JV is preferred to competition.

From Assumption 3 we can conclude that:

$$\frac{P(2\hat{x}^c)}{P(\hat{x}^c)} > 2 - P(\hat{x}^c).$$

Replacing this in the LHS of (A5), a sufficient (but not necessary) condition for a JV to be preferred to competition is:

$$R_1(\alpha) + R_2(\alpha) - (2 - P(\hat{x}^c))R_3 < P(\hat{x}^c)[R_1(\alpha) + R_2(\alpha) - R_3].$$

Rearranging, this becomes:

$$(A6) \quad (1 - P(\hat{x}^c))[R_1(\alpha) + R_2(\alpha) - 2R_3] < 0.$$

By definition when  $\alpha < 1$ , when spillovers are incremental  $R_1(\alpha) + R_2(\alpha) < 2R_3$ , and the opposite when spillovers are offsetting. Thus, if spillovers are incremental (A6) holds and a JV is preferred to competition.

Reversing the inequality in (A5), note that if  $\alpha \geq \hat{\alpha}$  the RHS is non-positive, so it sufficient that the LHS be positive for competition to be preferred to a JV. This becomes:

$$(A7) \quad R_1(\alpha) + R_2(\alpha) - \frac{P(2\hat{x}^v)}{P(\hat{x}^c)}R_3 > P(\hat{x}^c)[R_1(\alpha) + R_2(\alpha) - R_3].$$

Because of the direction of the inequality, we cannot substitute from Assumption 3, however we note that if  $\hat{x}^v$  is not too much greater than  $\hat{x}^c$ ,  $\frac{P(2\hat{x}^v)}{P(\hat{x}^c)} < 2$ , and the LHS of (A7) is positive when spillovers are offsetting. If, concurrently,  $P(\hat{x}^c)$  is small, (A7) will hold.

*Q.E.D.*

### Proof of Proposition 5

a) This follows since, from Proposition 3,  $\hat{x}^v > \hat{x}^s$ , so a JV has more research, the same amount of competition in the product market if discovery occurs, and cost savings from the use of joint resources.

b) When  $\alpha \geq \hat{\alpha}$ , from Proposition 2,  $\hat{x}^v \geq \hat{x}^c$ . Thus, in this case, there is more research with a JV, more competition in the product market (since with a JV there are full spillovers), and cost savings.

c) When  $\alpha = 1$ , from Lemma 2,  $\hat{x}^c = \hat{x}^s$ , there are full spillovers in both regimes, and there are no cost-savings from joint facilities.

*Q.E.D.*

Proof of Lemma 3

Comparing (5) and (15), we get:

$$(A8) \quad \begin{aligned} & P'(\hat{x}^c) [R_1(\alpha) - P(\hat{x}^c) (R_1(\alpha) + R_2(\alpha) - R_3)] = \\ & P'(\hat{x}^{cc}) [R_1(\alpha) + R_2(\alpha) - 2P(\hat{x}^{cc}) (R_1(\alpha) + R_2(\alpha) - R_3)]. \end{aligned}$$

If  $\alpha = 1$  this reduces to:

$$P'(\hat{x}^c) (1 - P(\hat{x}^c)) = 2P'(\hat{x}^{cc}) (1 - P(\hat{x}^{cc})),$$

which can only hold if  $\hat{x}^c < \hat{x}^{cc}$ . If, alternatively,  $\alpha = 0$ , (A8) becomes:

$$P'(\hat{x}^c) [R_1(0) - P(\hat{x}^c) (R_1(0) - R_3)] = P'(\hat{x}^{cc}) [R_1(0) - 2P(\hat{x}^{cc}) (R_1(0) - R_3)].$$

If  $\hat{x}^c = \hat{x}^{cc}$  then this amounts to requiring that  $P(R_1(0) - R_3) = 0$ , which can only occur when  $P=0$ . If  $\hat{x}^c < \hat{x}^{cc}$  the RHS is clearly less than the LHS. Thus, equality can be attained only if  $\hat{x}^c > \hat{x}^{cc}$ .

Since the functions are all continuous in  $\alpha$  it follows that there exists a value of  $\alpha$  for which  $\hat{x}^c = \hat{x}^{cc}$ . *Q.E.D.*

Proof to Proposition 6

Comparing competition and information sharing under cost sharing, we get the following:

$$(A9) \quad 2P'(\hat{x}^{sc}) (1 - P(\hat{x}^{sc})) R_3 = P'(\hat{x}^{cc}) [R_1(\alpha) + R_2(\alpha) - 2P(\hat{x}^{cc}) (R_1(\alpha) + R_2(\alpha) - R_3)].$$

When  $\alpha = 1$  (A9) reduces to:

$$2P'(\hat{x}^{sc}) (1 - P(\hat{x}^{sc})) R_3 = 2P'(\hat{x}^{cc}) (1 - P(\hat{x}^{cc})) R_3,$$

so clearly  $\hat{x}^{sc} = \hat{x}^{cc}$ .

To evaluate what occurs when  $\alpha < 1$ , note that the LHS of (A9) does not change when  $\alpha$  changes, but the RHS does. Thus, we investigate the sign of the derivative of the RHS with respect to  $\alpha$ . The derivative is given by:

$$(A10) \quad \begin{aligned} & P'(\hat{x}^{cc}) \left( \frac{\partial(R_1 + R_2)}{\partial \alpha} \right) (1 - 2P(\hat{x}^{cc})) + P'' [R_1 + R_2 - 2P(\hat{x}^{cc}) (R_1 + R_2 - R_3)] \frac{\partial \hat{x}^{cc}}{\partial \alpha} \\ & - 2P'(\hat{x}^{cc})^2 (R_1 + R_2 - R_3) \frac{\partial \hat{x}^{cc}}{\partial \alpha}. \end{aligned}$$

The second and third terms are clearly positive since, by Lemma 1, investment in R&D falls when spillovers increase. The first term can be positive or negative. It will clearly be positive (or zero) if either of the condition in the proposition hold. Thus, in those instances, lowering  $\alpha$  below 1 will cause the RHS of (A9) to fall, so that investments will have to fall to bring equality in (A9). Thus,  $\hat{x}^{sc} > \hat{x}^{cc}$ . If the conditions stated in the proposition do not hold the first term in (A10) will be positive, and the derivative cannot be signed.

*Q.E.D.*

*Proof of Proposition 7*

An information sharing agreement with cost sharing will be preferred to competition with cost sharing if:

$$(A11) \quad P(\hat{x}^{sc}) \left( 2 - P(\hat{x}^{sc}) \right) R_3 - \hat{x}^{sc} > P(\hat{x}^{cc}) \left[ R_1(\alpha) + R_2(\alpha) - P(\hat{x}^{cc}) \left( R_1(\alpha) + R_2(\alpha) - R_3 \right) \right] - \hat{x}^{cc}.$$

Competition will be preferred if the inequality is reversed.

When  $\alpha = 1$  it is clear from Proposition 6 that there is equality. Differentiating the RHS of (A11) with respect to  $\alpha$  and using the Envelope Theorem:

$$\frac{\partial E\Pi^{cc}}{\partial \alpha} = \frac{\partial(R_1 + R_2)}{\partial \alpha} P(\hat{x}^{cc}) \left( 1 - P(\hat{x}^{cc}) \right).$$

Since this is, by definition, positive with incremental spillovers and negative with offsetting spillovers, the result follows. *Q.E.D.*

*Proof of Lemma 8*

From (20) and (15) we require that:

$$(A12) \quad 2P'(2\hat{x}^{vc})R_3 = P'(\hat{x}^{cc}) \left[ R_1(\alpha) + R_2(\alpha) - 2P(\hat{x}^{cc}) \left( R_1(\alpha) + R_2(\alpha) - R_3 \right) \right]$$

If  $\alpha = 1$ , then (A12) becomes:

$$2P'(2\hat{x}^{vc})R_3 = 2P'(\hat{x}^{cc}) \left[ 1 - P(\hat{x}^{cc}) \right] R_3,$$

so  $\hat{x}^{vc} > \hat{x}^{cc}$  by Lemma 3.

Differentiating the RHS of (A12) by  $\alpha$ , we find that

$$\frac{\partial RHS}{\partial \alpha} = P'(\hat{x}^{cc}) \frac{\partial(R_1 + R_2)}{\partial \alpha} \left( 1 - 2P(\hat{x}^{cc}) \right).$$

Since a lowering in  $\alpha$  that leads to a lowering of the RHS of (A12) will cause  $\hat{x}^{cc}$  to fall further below  $\hat{x}^{vc}$ , we can conclude that if  $P(\hat{x}^{cc}) < 1/2$  and spillovers are incremental, or  $P(\hat{x}^{cc}) > 1/2$  and spillovers are offsetting, then  $\hat{x}^{vc} > \hat{x}^{cc}$ . *Q.E.D.*

*Proof of Proposition 8*

For a JV with cost sharing to be preferred to competition with cost sharing we require, from (19) and (14)

$$(A13) \quad P(2\hat{x}^{vc})R_3 - \hat{x}^{vc} > P(\hat{x}^{cc})[R_1(\alpha) + R_2(\alpha) - P(\hat{x}^{cc})(R_1(\alpha) + R_2(\alpha) - R_3)] - \hat{x}^{cc}.$$

When  $\alpha = 1$  this reduces to:

$$P(2\hat{x}^{vc})R_3 - \hat{x}^{vc} > [2P(\hat{x}^{cc}) - P(\hat{x}^{cc})^2]R_3 - \hat{x}^{cc}.$$

We extend this equation to:

$$P(2\hat{x}^{vc})R_3 - \hat{x}^{vc} > P(2\hat{x}^{cc})R_3 - \hat{x}^{cc} > [2P(\hat{x}^{cc}) - P(\hat{x}^{cc})^2]R_3 - \hat{x}^{cc}.$$

The first inequality holds since  $\hat{x}^{vc}$  maximizes  $E\Pi^{vc}$ , and the second holds by Assumption 3. Thus, when  $\alpha = 1$  a JV with cost sharing is preferred to competition with cost sharing.

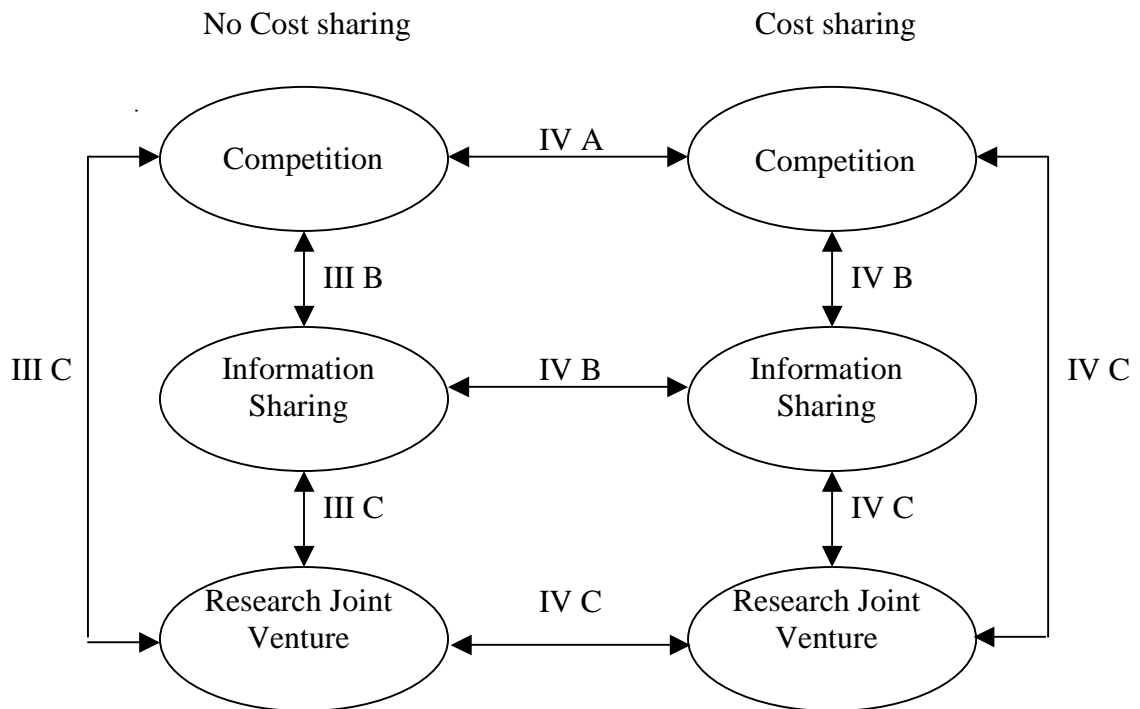
Differentiating the RHS of (A13) with respect to  $\alpha$ , and using the envelope theorem, we find that  $\partial E\Pi^{cc} / \partial \alpha = P(\hat{x}^{cc})(1 - P(\hat{x}^{cc})) \frac{\partial (R_1 + R_2)}{\partial \alpha}$ . The sign of this derivative depends on the nature of the spillovers. If the spillovers are incremental this is positive, so by lowering  $\alpha$  the profits under competition fall, so a JV continues to be more profitable. If, however, spillovers are offsetting, it is possible that at some level of  $\alpha$  the two are equal, so that when  $\alpha$  is small competition is preferred, while when  $\alpha$  is large a JV is preferred.

*Q.E.D.*

**Table 1**

		Investment agreement	Joint research facilities	Information sharing
1	Competition			
2	ISA			X
3	JV		X	(X)
4	Cost	X		
5	Cost-ISA	X		X
6	Cost-JV	X	X	(X)

**Figure 1**





**Table 2**

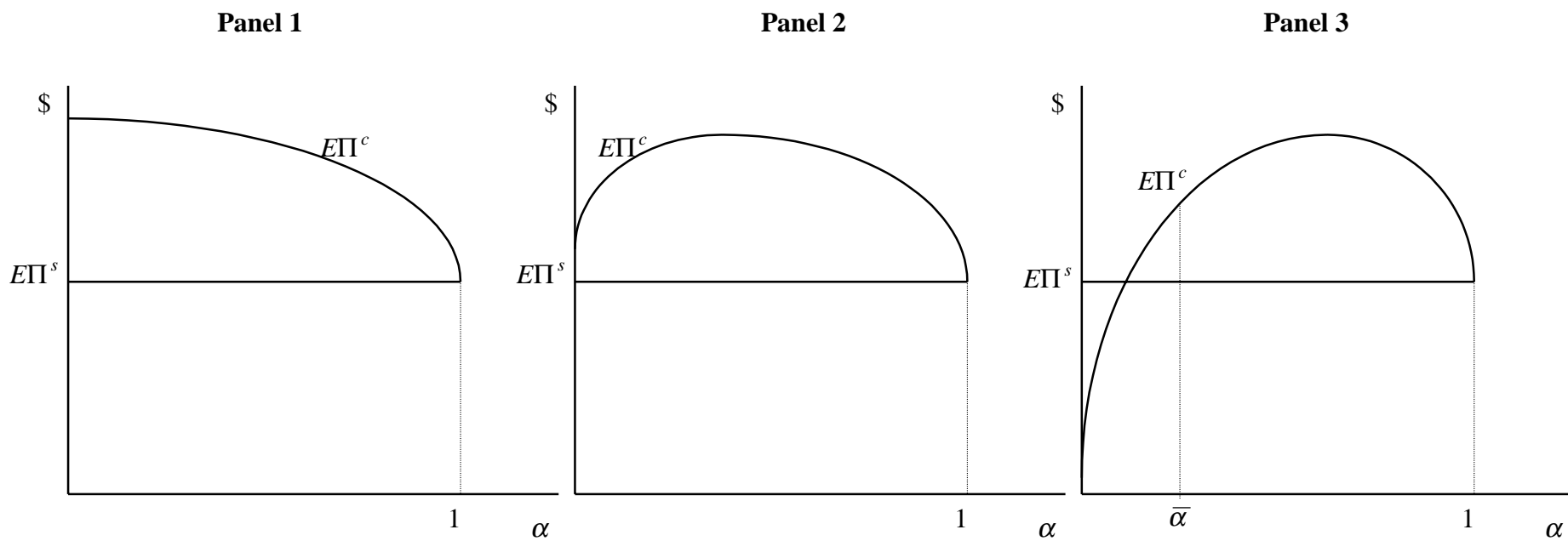
**R&D Expenditures, Profits and Welfare**

Comparison	Expenditures	Profits	Welfare
C vs. S	$\hat{x}^c > \hat{x}^s$ (if $\alpha = 1$ then $\hat{x}^c = \hat{x}^s$ )	$E\Pi^c > E\Pi^s$ if offsetting and high $\alpha$ $E\Pi^c < E\Pi^s$ if incremental and low $\alpha$	$W^c = W^s$ if $\alpha = 1$
V vs. S	$\hat{x}^v > \hat{x}^s$	$E\Pi^v > E\Pi^s$	$W^v > W^s$
V vs. C	$\hat{x}^v > \hat{x}^c$ if $\alpha > \hat{\alpha}$ $\hat{x}^v < \hat{x}^c$ if $\alpha < \hat{\alpha}$	$E\Pi^c > E\Pi^v$ if offsetting, high $\alpha$ and low P $E\Pi^c < E\Pi^v$ if incremental and low $\alpha$	$W^v > W^c$ if $\alpha \geq \hat{\alpha}$
CC vs. C	$\hat{x}^c > \hat{x}^{cc}$ if $\alpha < \tilde{\alpha}$ $\hat{x}^c < \hat{x}^{cc}$ if $\alpha > \tilde{\alpha}$	$E\Pi^{cc} > E\Pi^c$ if $\alpha \neq \tilde{\alpha}$	$W^c > W^{cc}$ if $\alpha < \tilde{\alpha}$ $W^c < W^{cc}$ if $\alpha > \tilde{\alpha}$
SC vs. S	$\hat{x}^{sc} > \hat{x}^s$	$E\Pi^{sc} > E\Pi^s$	$W^{sc} > W^s$
VC vs. V	$\hat{x}^{vc} > \hat{x}^v$	$E\Pi^{vc} > E\Pi^v$	$W^{vc} > W^v$
CC vs. SC	$\hat{x}^{sc} > \hat{x}^{cc}$ if incremental and $P < 1/2$ , or offsetting and $P > 1/2$ (if $\alpha = 1$ then $\hat{x}^{sc} = \hat{x}^{cc}$ )	$E\Pi^{sc} > E\Pi^{cc}$ if incremental $E\Pi^{sc} < E\Pi^{cc}$ if offsetting (if $\alpha = 1$ then $E\Pi^{sc} = E\Pi^{cc}$ )	$W^{sc} > W^{cc}$ if incremental and $P < 1/2$ , or offsetting and $P > 1/2$ (if $\alpha = 1$ then $W^{sc} = W^{cc}$ )
VC vs. SC	$\hat{x}^{vc} > \hat{x}^{sc}$	$E\Pi^{vc} > E\Pi^{sc}$	$W^{vc} > W^{sc}$
VC vs. CC	$\hat{x}^{vc} > \hat{x}^{cc}$ if $\alpha = 1$ , or incremental and $P < 1/2$ , or offsetting and $P > 1/2$	$E\Pi^{vc} > E\Pi^{cc}$ if incremental, or offsetting and high $\alpha$	$W^{vc} > W^{cc}$ if $\alpha = 1$ , or incremental and $P < 1/2$ , or offsetting and $P > 1/2$

Legend: C – Competition; S – Information sharing; V – Joint venture; CC – Competition with cost sharing; SC – Information sharing with cost sharing; VC – Joint venture with cost sharing.

P – Probability of success.

**Figure 2**  
**Offsetting Spillovers**



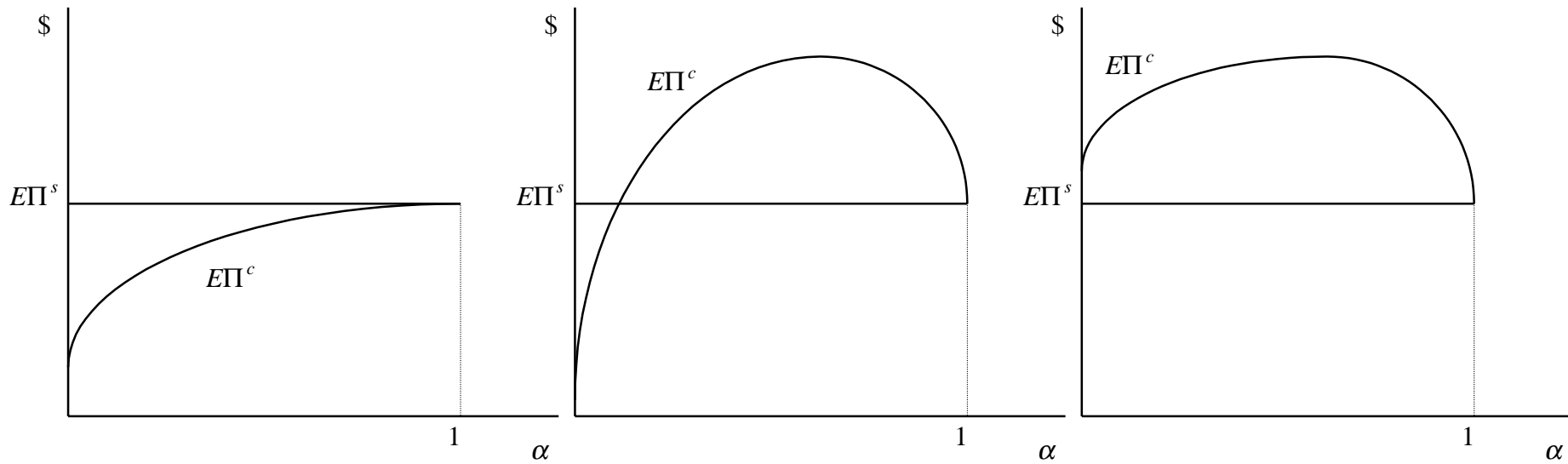
**Figure 3**

**Incremental Spillovers**

**Panel 1**

**Panel 2**

**Panel 3**



## Footnotes

<sup>1</sup> Choi, conversely, considers the effects of monitoring costs.

<sup>2</sup> This is the same setup as in Choi (1993), but it differs from the setup in Kamien et al. (1992), where a firm's costs are reduced by the sum of R&D efforts in the industry when spillovers are increased to their maximal level.

<sup>3</sup> Alternatively, one can view this as "normalizing" profits in this case to be zero.

<sup>4</sup> This assumption differs from that in Kamien et al. (1992) in that in their paper costs are decreased by the sum of research efforts, so that both firms succeeding differs from only one firm succeeding with complete spillovers.

<sup>5</sup> Choi makes the additional assumption that  $P'(x)^2 + P(x)P''(x) \geq 0$ . This is an assumption about the degree of concavity of the cumulative probability function, and does not hold for every function. As will be discussed below, this assumption is a sufficient, but not necessary condition for signing a slope.

<sup>6</sup> Each of these will be described in more detail below.

<sup>7</sup> To simplify, we assume the firms are entrants into the new market; so that there are no profits from current production.

<sup>8</sup> Assumption 2 ensures that a solution to Equation (2) always exists. In addition, the second order conditions for a maximum are satisfied:

$$\partial G_i / \partial x_i^* = P''(x_i^*) \left[ R_1(\alpha) - P(x_j) (R_1(\alpha) + R_2(\alpha) - R_3) \right] < 0,$$

since  $P''(x_i^*) < 0$ , and the expression in square brackets is positive, by the first order condition.

<sup>9</sup> Note that this condition is not related to Choi's (1993) condition (see Footnote 4), because here both terms in the first brackets are positive, while in Choi's condition one is positive and the other is negative.

<sup>10</sup> Notice that Lemma 1 is an extension of Choi's (1993) result to the case of incremental spillovers as well as offsetting spillovers. For the latter case Choi (1993) proved that an increase in the degree of spillovers reduces the level of R&D expenditure.

<sup>11</sup> Our definition of information sharing is similar to the definition in Kamien, et al. (1992), except that in our paper firms do not avoid duplication of R&D activities; rather, each firm decides on its own activities independently.

<sup>12</sup> IS as defined here could be implemented by exchange of researchers between the two firms, in order to avoid the problem of incomplete communication of R&D results.

<sup>13</sup> In a dynamic R&D model, Reinganum (1982) made a similar argument. In a different context Katz (1986) found results similar to ours.

<sup>14</sup> The existence of  $\bar{\alpha}$  was shown in the text; however, there is no guarantee that it is unique. The following is a sufficient condition for uniqueness. Differentiating (9) and imposing symmetry in the Nash equilibrium, we get,

$$(A3) \quad \frac{d \frac{\partial E\Pi_i^c}{\partial \hat{x}_j^c}}{d\alpha} = - \left[ P'(\hat{x}^c)^2 + P(\hat{x}^c)P''(\hat{x}^c) \right] \frac{d\hat{x}^c}{d\alpha} (R_1(\alpha) + R_2(\alpha) - R_3) - P(\hat{x}^c)P'(\hat{x}^c) \frac{dR_1(\alpha)}{d\alpha} + P''(\hat{x}^c) \frac{d\hat{x}^c}{d\alpha} R_2(\alpha) + P'(\hat{x}^c) (1 - P(\hat{x}^c)) \frac{dR_2(\alpha)}{d\alpha}.$$

From Assumptions 1 and 2 and from Lemma 1 all but the first term are positive. Choi (1993) assumes that the first term is also positive, which is a condition on the degree of concavity of the cumulative probability function (See Footnote 1). This condition is sufficient, but certainly not necessary. Hence, unless the first term is sufficiently negative to overcome the other terms, (A3) is positive.

<sup>15</sup> Our definition of a JV is equivalent to the Research Joint Venture Competition case in Kamien et al. (1992).

<sup>16</sup> Note that this may not necessarily hold in a dynamic framework. See, for example, Fudenberg et al. (1983) and Grishagin et al. (2001).

<sup>17</sup> Very recently, Martin (1995) provided theoretical support to the fact that cooperation in the R&D market leads to tacit collusion in the product market. This could help mitigate the loss from losing a monopoly position due to cooperation.