Credibility, Pre-Production and Inviting Competition in a Network Market

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Abstract. In this paper we considered a new solution to the credibility problem present in network industries. This problem arises because the value of a network good to its owner depends positively on the number of consumers who buy the good. Because of this property, it is in the interest of the producer to try to convince consumers that the market will be large, even if he knows it is untrue. Consumers, in turn, will disregard producer claims, and will, instead, try to reason out what size the market will attain. As a result, a lower than optimal quantity, both for consumers and producers, will be produced, i.e., the resulting equilibrium is Pareto inefficient. Katz and Shapiro (1985) and Economides (1996) suggest a solution to this problem in which the firm invites competitors to share their technology and enter the market, thus voluntarily giving up their monopoly position. We suggest an alternative remedy of pre-producing the good. This has the effect of changing the firm's cost structure in a manner that causes consumers to believe that the amount they will optimally sell (and hence the market size) will increase, again leading to higher profitability. The two strategies are compared, and we show the conditions under which each is preferable. We then consider combinations of these two strategies in two different manners. In the first the leader produces and then invites competitors who then also pre-produce, and in the second the leader pre-produces but the fringe firms do not; rather, they produce in the same period in which they sell. Surprisingly we found that the latter dominated the former, which led us to a better understanding about how and why each strategy works. In short, inviting competitors creates a positive externality that benefits all firms, while pre-producing helps only the firm doing the pre-producing and harms all other firms. Thus, the leader invites competitors so he can benefit from the positive externality, but he is better off if the competitors do not pre-produce.

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I. Introduction

One of the main characteristics of network goods is that the value of the product to each consumer increases with the number of consumers who own the product. Because of this property, the willingness of any particular consumer to buy the product will depend not only on the price of the good, but also on the size of the market. When a new product is *introduced* to a market, however, consumers do not know what size the market will attain, and so must base their decisions on beliefs. One source from which consumers can obtain information is from the declarations made by the producer about his production intentions. However, the producer has a clear incentive to overstate the expected market size in order to be able to increase the price, so these declarations may not be credible. Thus, the consumer may discount this information, and try instead to reason what the firm will do optimally given consumer behavior. The result of this is that a Pareto inferior equilibrium will be attained, while if the producer could somehow convince the consumers that his intentions are sincere, and, in fact, he carried through on those intentions, all parties would be made strictly better off.[1]

Katz and Shapiro (1985) and Economides (1996) address this issue. They suggest that one way to overcome, at least partially, this confidence problem, is for the firm to voluntarily relinquish its monopoly position by inviting competitors into the market. The result of the increased competition is to increase consumer expectations about the size of the market because of the effect competition usually has on production. Consequently, the quantity the firm can sell increases, and under certain conditions the firm's profits increase.

In this paper we suggest an alternative strategy. The main tenet is that the firm may be able to take steps that will change its cost structure in a way that convinces consumers that a larger quantity will be supplied. In particular, if the firm can lower marginal costs the amount consumers believe will be produced increases, and, in turn,

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[1] Another source of uncertainty is with respect to the behavior of other consumers. Specifically, consumers must not only be convinced about producer behavior, they must be convinced that other consumers will buy the product. When each consumer's decision to buy depends on the purchase of the others, this requires common knowledge of beliefs for purchase to actually occur. Etziony and Weiss (2001) address this issue in an experimental setting.

so does the quantity actually bought. While there may be a number of ways to lower marginal costs, the one we investigate is pre-production by the firm. This has the effect of lowering marginal costs to zero, and, in fact, turning the variable costs into fixed costs. This strategy was suggested by Spence (1977), Dixit (1980), Ware (1983) and Allen (1993), among others, as a way of convincing potential competitors about actions. We show how this strategy can also be used to convince consumers about actions, which will consequently benefit the firm. Interestingly, using this tool in the manner we suggest leads to an increase in both producer and consumer welfare, which also distinguishes our results from those in previous studies. After developing the model, we compare between this strategy and that of inviting competition. We also combine the strategies and show the optimal combination under different cost and demand conditions.

We begin by demonstrating the credibility problem faced by producers in a network market in Section II. Section III contains the solution of pre-production, and in Section IV we compare this solution with that of inviting competitors as developed in Economides (1996). In Section V the two strategies are combined under two possible scenarios. In the first, all firms produce before the selling stage, although the inviting firm produces before the other firms, and is thus a Stackelberg leader in the market. In the second, only the inviting firm pre-produces. All strategies are then compared in Section VI, and insight into the way each strategy works is derived from the comparison. A short summary and discussion are presented in Section VII.

II. The Credibility Problem in Network Markets

A credibility problem in a network market is said to exist in a situation in which a producer of a good has an incentive to misstate his intentions in order to affect the behavior of consumers or of other producers. An example of the latter is what has been termed “vaporware” – an announcement by a firm about the expected release date of a new product or about the capabilities of a new product, even when these statements are false. The purpose of these announcements is to depress competition (see Levy, 1996, Cass and Hylton, 1999, Dranove and Gandal, 2000, and Bayus, 2001 for analyses).

With respect to misleading consumers, the essence of network goods makes exaggerated statements about the size of the market particularly valuable to the producer if he can get consumers to believe him. This is because of the effect purchase by others has on the value each consumer attributes to the good. Consumers, recognizing this incentive, will discount any statements made by the producer. The result of this is that consumers will not believe some statements of intent, which, if carried out, would be beneficial to all parties. This will be the case when it is not in the producer's interest to carry out these actions ex-post, even if it is in his interest ex-ante.

Most previous studies of this problem considered a multi-period setting with a basic durable good and complementary goods. Consider, for instance, a consumer considering buying a home video-game system. The reason a consumer purchases such a product is because of the games he will be able to play, and not because of the hardware. If there is only a small stock of existing games, and he believes there will be few games made for the machine in the future, he may be reluctant to make the purchase. In addition, if he believes the cost of games will be increased significantly, he may not purchase the system. The producer could attempt to convince consumers that games will be made available and that prices will be kept low, but the consumer may believe that the producer will be more concerned with creating the next generation of machines than with the creation of new games for machines that were already purchased, and that there will be nothing compelling the producer not to increase prices once the machines have been bought. The inability of producers to credibly commit to actions in future periods is particularly acute in high-tech industries because of the high rate of technological progress. In the case just discussed, the creation of a large stock of games in the first period, or the presence of competitors willing to make new games can help convince consumers that this will not be a problem.

Farrel and Gallini (1988) discuss a situation in which before purchasing a good, consumers must bear a setup cost in order to benefit from the product. This cost could be a training cost or the purchase of hardware necessary to use the software in which he is really interested (as in the example in the last paragraph). This leads to an ex-post opportunism problem, because the producer can use the fact that the cost has been sunk, and can then increase the price of the good. Klemperer (1987), Farrel and Shapiro (1988, 1989) and Beggs and Klemperer (1992) analyze this possibility in a case in which the firm determines the setup cost, and consumers, after paying this setup cost, must bear a switching cost to move to a competing product (see Klemperer, 1995, for a survey). In Williamson's (1975) terms this industry suffers from a “hold-up” problem – a combination of ex-post small numbers (once the sunk cost is made only the producer can supply the product) and opportunism, a combination that has also been denoted a lock-in problem (e.g., Varian, 1999). This type of problem can, of course, cause the consumer to not buy the product.

All these studies considered indirect network externalities, where the consumption of others is important because of the effect the size of the market has on the availability of complementary goods. Katz and Shapiro (1985) were the first to discuss the credibility problem in a market with direct network externalities, where the base good itself becomes more valuable as the market size increases (as in the case of telephones and fax machines). In their model, consumers' beliefs about the market size are determined prior to the producer deciding how much to produce and sell. Once determined, these beliefs can no longer be changed. The producer takes these beliefs into account in deciding how much to produce. This problem is reexamined and expanded in Economides (1993, 1996) in a setting in which consumer belief-formation and production occur concurrently.

A short exposition of the problem, based on the Economides (1996) model will help bring the salient features of the credibility problem in a one period model into focus. We start with a differentiable, separable inverse demand function for the network good:

$$P(Q,S) \equiv P(Q) + f(S),$$

where Q is the actual quantity of the good sold and S the amount consumers expect to be sold. The first part of the inverse demand curve shows the direct effect of the quantity demanded on price, with $\frac{\partial P(Q,S)}{\partial Q} = \frac{\partial P(Q)}{\partial Q} < 0 \quad \forall Q$. The second part is the network effect, with $\frac{\partial P(Q,S)}{\partial S} = \frac{\partial f(S)}{\partial S} \geq 0 \quad \forall Q,S$. To make the presentation simple, we assume linear functions, so that (1) becomes:

$$P(Q,S) = a - Q + eS, \; e<1.$$

The producer's technology is CRS, with constant marginal costs equal to c. If the producer were accounted full credibility, in that any quantity he announced would be believed *and produced*, then he would face the demand curve:

$$P(Q,Q) = a - (1-e)Q$$

and the optimal quantity would be given by:

(1) $$Q_0 = S_0 = \frac{a-c}{2(1-e)}.$$

Assume, then, that the producer declares that he will produce Q_0 units and that consumers believe him, so that $S_0 = Q_0$. In this case, when deciding how much to actually produce the firm would maximize:

$$P(Q, S_0) = a - Q + eS_0,$$

and the quantity produced would be

(2) $$Q^*(S_0) = \frac{a-c+eS_0}{2}.$$

By substituting (1) into (2) and comparing, it is easy to see that $Q^*(S_0) < Q_0$. Thus, the firm overstates the amount it will produce, and the credibility problem is born. The problem is demonstrated in Figure 1. The fulfilled expectations demand curve is more elastic than the demand curve actually facing the producer. This gives him an incentive to produce less than consumers expect.

Consumers, anticipating this result, do not believe that the producer will produce the amount defined in (1). There exists, however, an equilibrium at which the amount consumers believe will be produced equals the amount the firm optimally produces. To find this equilibrium graphically, we construct two reaction functions, that of the informed consumer who knows how much will be produced, i.e., $R_C : S = Q$, and that of the producer as per (2): $R_F : Q = \frac{a-c}{2} + \frac{e}{2}S$. Equilibrium is attained when these are equal:

$$Q_E = \frac{a-c}{2-e}.$$

This equilibrium is demonstrated in Figure 2. This equilibrium is attained mathematically by maximizing profits for a fixed level of *S*, and then setting *S*=*Q*. Note that in this instance the firm suffers from a total lack of credibility, and is unable to affect consumer beliefs at all. Clearly both consumers and producers would be better off at Q_0 than at Q_E, but because of the credibility problem an inferior equilibrium is attained.

III. Pre-Production

To allow for comparison with the strategy of inviting competition developed in Katz and Shapiro (1985) and Economides (1996), we use the same functional forms used in the Economides model. Consider first a monopolist that is unable to either pre-produce or invite competition. The monopolist produces a new network good, and faces the inverse demand curve $P(Q,S) = a - Q + f(S)$, where Q is total industry purchases and S is the amount consumers believe will be purchased, $f' > 0$. The production technology is assumed to be CRS, so marginal costs are constant and equal to c. The firm's optimization problem is:

$$\operatorname*{Max}_{Q} \Pi = \operatorname*{Max}_{Q} (a - Q + f(S))\, Q - cQ \,.$$

Note that because of the credibility problem, S is a parameter in this objective function. The quantity and profits of the producer in equilibrium, denoted by Q^{ns} and Π^{ns} (the superscripts stand for "no strategy"), respectively, are:

$$(3) \qquad Q^{ns} = \frac{a + f(Q^{ns}) - c}{2}; \qquad \Pi^{ns} = \left[\frac{a + f(Q^{ns}) - c}{2}\right]^2,$$

with the condition that $f'(Q^{ns}) < 1$ for this to be a maximum, since if $f'(Q^{ns}) > 1$ it is always worthwhile to increase production.

Consider now pre-production (pp). The game now consists of two periods. In the first period (denoted period 0) the firm has the option to produce any quantity of the product at marginal cost c. In period 1 the firm can sell some or all of what was produced, and increase production with the same marginal cost. For simplicity we assume that there is no storage cost.[2] Since the firm is, in essence, limiting its options by producing a large quantity of the good before marketing, it would seem to be harmful to the firm because it reduces the firm's strategy space. However, as in Spence (1977), Dixit (1980), Ware (1983) and Allen (1993), we show that this actually leads to increased profits. As mentioned above, the difference in our setting from the setting in those earlier studies is that here the source of the benefit is from the effect on consumers, while in the earlier studies it is because of the effect on

[2] We make this assumption because of the complexity of the solution. Simplification of the model might allow for explicitly including a storage cost, and one of the central questions would be how this cost affects the incentive to, and profitability of, pre-producing.

potential competitors. In addition, one of the results of our model is that both producer and consumer welfare increases when pre-production occurs, which also distinguishes our results from those in previous studies.

As a result of pre-production, the firm's marginal cost structure in the selling period is

$$(4)\qquad \begin{cases} mc = 0 \quad for \;\; Q^{pp} \le Q_0^{pp} \\ mc = c \quad for \;\; Q^{pp} > Q_0^{pp} \end{cases}$$

where Q_0^{pp} is the amount pre-produced and Q^{pp} is the amount actually sold. As will be shown shortly, the quantity produced optimally in the pre-production stage will be greater than that when the firm cannot pre-produce. Thus, there is never any reason for the firm to produce also in the selling period, and the bottom part of (4) becomes irrelevant. To simplify the exposition, we therefore assume that the firm cannot produce in the selling period.

We will solve the firm's objective function in stages. Assume, first, that the firm produced Q_0^{pp} units in period 0, and let us assume for now that this quantity is sufficient to supply the firm with all the units it will desire to sell in period 1. The firm has thus borne a cost equal to cQ_0^{pp}. Denote by $\overline{Q}^{pp}$ the amount the firm chooses to sell in the selling period under these circumstances. The firm's objective function given consumer expectations is given by:

$$\underset{\overline{Q}^{pp}}{Max}\, \Pi = (a - \overline{Q}^{pp} + f(S))\, \overline{Q}^{pp} - cQ_0^{pp} .$$

Solving this yields an optimal quantity sold of

$$\overline{Q}^{pp} = \frac{a + f(S)}{2} .$$

Consumers, on their part, know that if the firm has produced $\overline{Q}^{pp}$ units it will sell them all, but the firm will never sell more than $\overline{Q}^{pp}$ units even if it has already produced those units. Thus, $S \le \overline{Q}^{pp}$. In the case of equality we can better define $\overline{Q}^{pp}$ by:

$$\overline{Q}^{pp} = \frac{a + f\left(\overline{Q}^{pp}\right)}{2}$$

Consider now what happens if the firm produces less than $\overline{Q}^{pp}$ units, i.e., $Q_0^{pp} < \overline{Q}^{pp}$. In this case, consumers will certainly believe that the firm will sell its entire pre-produced quantity, since it would want to sell $\overline{Q}^{pp}$ units in period 1, were that quantity available. Thus, consumer expectations can be summed up as follows:

$$S = \begin{cases} Q_0^{pp} & \text{if} \quad Q_0^{pp} \leq \overline{Q}^{pp} \\ \overline{Q}^{pp} & \text{otherwise} \end{cases}$$

These expectations are demonstrated in Figure 3.

Given these expectations, we now return to the firm's strategy in period 0. Since consumers will never believe that a quantity greater than $\overline{Q}^{pp}$ will be sold, even if it has been produced, the firm will never produce more than $\overline{Q}^{pp}$ units. Below that quantity, consumers believe that the quantity produced will all be sold. Thus, dropping the subscript, the firm's objective function is:

$$\text{(5)} \qquad \begin{aligned} &\text{Max } \left[a - Q^{pp} + f(Q^{pp}) - c\right] Q^{pp} \\ &\quad \text{s.t.} \quad 0 \leq Q^{pp} \leq \overline{Q}^{pp} \end{aligned}$$

Which yields the following solution:

$$\text{(6)} \qquad Q^{pp} = \begin{cases} 0 & \text{if } \ a + f(Q^{pp}) < c \\[2ex] \dfrac{a + f(Q^{pp}) - c}{2 - f'(Q^{pp})} & \text{if } \ \dfrac{f'(Q^{pp})\left[a + f(Q^{pp})\right]}{2} < c \leq a + f(Q^{pp}) \\[2ex] \dfrac{a + f(\overline{Q}^{pp})}{2} & \text{otherwise} \end{cases}$$

It is easy to verify that $Q^{pp} > Q^{ns}$ for all positive levels of production. Thus, as stated above, the firm never has an incentive to produce in the selling period. Profits are given by:

$$\text{(7)} \quad \Pi^{pp} = \begin{cases} 0 & \text{if } a + f(Q^{pp}) < c \\[2ex] \left(\dfrac{a + f(Q^{pp}) - c}{2 - f'(Q^{pp})}\right)^2 \left[1 - f'(q_L^0)\right] & \text{if } \ \dfrac{f'(Q^{pp})\left[a + f(Q^{pp})\right]}{2} < c \leq a + f(Q^{pp}) \\[2ex] \dfrac{a + f(Q^{pp})}{2}\left(\dfrac{a + f(Q^{pp})}{2} - c\right) & \text{otherwise} \end{cases}$$

IV. Inviting Competition

In this section we briefly present a model of inviting competition (*ic*) based on Economides (1996), and then compare the outcome with pre-producing. Assume the firm has an option to invite competitors to join the market, and that the entering firms will have the same technology and costs as the existing firm (the firm shares its technology with the other firms).[3] The total number of firms in the market is denoted n, with the number of invited firms, therefore, equaling n-1. The inviting firm will be treated as a price leader, with the remaining n-1 firms being fringe firms. Total industry production will be given by $Q = Q^{ic} + \sum_{i=1}^{n-1} q_f^i$, where Q^{ic} is the amount produced by the leader, and q_f^i is the production level of fringe firm i.

Using the same demand schedule and cost function as above, the quantities produced by each of the producers will be:

$$Q^{ic}(S) = \frac{a + f(S) - c}{2}; \quad q_f(S) = \frac{a + f(S) - c}{2n},$$

and total production and the price will be given by:

$$Q(S) = \frac{[a + f(S) - c](2n - 1)}{2n}; \quad P(S) = \frac{a + f(S) - c}{2n}.$$

The fulfilled expectations equilibrium will therefore be:

$$(8) \qquad S^* = Q(S^*) = \frac{[a + f(S^*) - c](2n - 1)}{2n}.$$

Replacing (8) in the firm's objective function yields profits:

$$(9) \qquad \Pi^{ic}(n, S^*) = \frac{nS^{*2}}{(2n - 1)^2}.$$

Totally differentiating (9) with respect to the number of firms yields:

$$\frac{d\Pi^{ic}(n, S^*)}{dn} = \frac{[(2n + 1)f'(S^*) - 2n]S^{*2}}{(2n - 1)^2[2n - (2n - 1)f'(S^*)]}.$$

Since the minimum number of firms is 1 (the leader), the optimal number of firms is given by:

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[3] Note that in Economides (1996) costs were set equal to zero. A positive marginal cost is necessary in our model for pre-production to be beneficial.

$$n^* = \begin{cases} \dfrac{f'(S^*)}{2\left(1\text{-}f'(S^*)\right)} & \text{if } \dfrac{2}{3} < f'(S^*) < 1 \\ 1 & \text{if } f'(S^*) \le \dfrac{2}{3} \end{cases}$$

and profits are equal to:

$$\text{(10)} \quad \Pi^{ic} = \begin{cases} \dfrac{\left[a + f(S^*) - c\right]^2 \left[1 - f'(S^*)\right]}{2f'(S^*)} & \text{if } \dfrac{2}{3} < f'(S^*) < 1 \\ S^{*2} & \text{if } f'(S^*) \le \dfrac{2}{3} \end{cases}$$

Thus, we see that as long as $\frac{2}{3} < f'(S^*) < 1$, it is beneficial for the firm to invite competitors. The explanation of this finding is that the competition credibly increases consumer expectations, and thus leads to greater demand. For this to increase producer profits, however, the network externality must be sufficiently strong.

We turn now to a comparison of the two strategies, and also compare them to the no strategy case. Clearly the no strategy case will be strictly dominated since the choice not to pre-produce and the choice not to invite competitors is always an option in the other cases. In order to make the comparison tractable, we specify a linear form for the network effect, $f(S) = eS$. In this case, profits in the no-strategy case (Equation (3)) become

$$\Pi^{ns} = \frac{(a\text{-}c)^2}{(2\text{-}e)^2},$$

profits with pre-production (Equation (7)) are:

$$\Pi^{pp} = \begin{cases} 0 & \text{if } a < c \\ \dfrac{(a-c)^2}{4(1-e)} & \text{if } \dfrac{ea}{2-e} < c \le a \\ \dfrac{a(a-2c+ec)}{(2-e)^2} & \text{otherwise} \end{cases}$$

and profits with inviting competition (Equation (10)) are:

$$\Pi^{ic} = \begin{cases} \dfrac{(a-c)^2}{8e(1-e)} & \text{if } \dfrac{2}{3} < e < 1 \\ \dfrac{(a\text{-}c)^2}{(2\text{-}e)^2} & \text{if } e \le \dfrac{2}{3} \end{cases}$$

Table 1 shows the condition under which each is preferred (more profitable).

The table demonstrates the following. When marginal costs equal zero there is no benefit in pre-producing. Thus, if the network effect is small (<2/3) all strategies are identical, while if it is large inviting competition becomes dominant. If, however, marginal costs are positive, pre-producing is always preferred to no strategy. If the network effect is small, it is preferred to inviting competition also, while if it is large, the sizes of the parameters will determine which is more profitable.

For demonstration purposes, in Figure 4 we present profits as a function of the marginal cost, with the other parameters chosen as a=10 and e=0.8.

As seen, both inviting competition and pre-production are always at least as good as no strategy. At low marginal costs, the benefit to be had from pre-production is minimal since the effect of lowering marginal costs to zero on consumer beliefs is not significant. However, when the marginal cost is high the benefit becomes more substantial, and pre-producing becomes even more effective than inviting competition.

V. Combining the Strategies

In this section we consider the possibility of combining the strategies of pre-producing and inviting competitors. There are three manners in which this can be done. The firm could pre-produce and then invite competitors who also pre-produce; the firm could invite competitors and then have them all pre-produce simultaneously; or the firm could pre-produce and then invite competitors who could not pre-produce. We consider only the first and the third, since the second is clearly inferior to the first from the firm's perspective. This is because with the first case the firm can produce the same amount he would produce if the second were chosen. Thus, he can always do at least as well by pre-producing before the other firms do (thus becoming a Stackelberg leader instead of a Cournot competitor). We begin with all firms pre-producing, and then look at pre-production by the leader alone.

A. Pre-production by all firms

We proceed in the same manner as in Section III above. Denote the leader by superscripts "pp_n" (pre-production by all n firms) and fringe firms by the same superscript and subscript "f". First, note that since the objective functions for all followers are identical, they will each produce the same amount. The leader and each of the followers' objective functions in the selling stage (after pre-producing) are, respectively,

$$\underset{Q^{pp_n}}{Max}\ Q^{pp_n} P(Q^{pp_n}, q_f^{pp_n}, n, S) - cQ_0^{pp_n}; \quad and$$
$$s.t.\quad Q^{pp_n} \le Q_0^{pp_n}$$
$$\underset{q_f^{pp_n}}{Max}\ q_f^{pp_n} P(Q^{pp_n}, q_f^{pp_n}, n, S) - cq_{f,0}^{pp_n}$$
$$s.t.\quad q_f^{pp_n} \le q_{f,0}^{pp_n}$$

where $Q_0^{pp_n}$ and $q_{f,0}^{pp_n}$ are the amounts pre-produced by the leader and followers, respectively. We assume that these constraints are non-binding, i.e., that the firm produced at least the amount it desires to sell. Using the same functions as above, the first order conditions show that each of the followers' reaction functions (to the amount produced by the leader, and assuming that all fringe firms produce the same amount) is given by:

$$q_f^{pp_n} = \frac{a + f(S) - Q^{pp_n}}{n}.$$

Replacing this in the leader's objective function yields the maximal amount the firm will want to sell when it lacks credibility:

$$\overline{Q}^{pp_n} = \frac{a + f(S)}{2}, \tag{11}$$

and the maximal amount each of the fringe firms will produce is:

$$\overline{q}_f^{pp_n} = \frac{a + f(S)}{2n}. \tag{12}$$

Maximal total sales will equal:

$$\overline{Q}_T^{pp_n} = \frac{[a + f(S)](2n-1)}{2n},$$

and if this amount has been produced, consumers will believe that this quantity will be sold, so $S = \overline{Q}_T^{pp_n}$. Any units produced greater than this amount will not help convince consumers that the network size will be larger than $\overline{Q}_T^{pp_n}$. If less is produced, consumers will believe that everything that has been produced will be sold.

We turn now to the production period. To find the optimal quantity produced by the leader and the optimal number of firms to invite, we first establish the following Proposition.

Proposition 1: *If fringe firms are invited to enter the market, they will never choose to produce less than* $\overline{q}_f^{pp_n}$.

(Proof in Appendix.) What this proposition states is that if demand and cost conditions are such that fringe firms prefer to produce less than this quantity, these same conditions guarantee that it is not in the leader's interest to invite competition. In other words, any time it is in the interest of the leader to invite competition the entering firms will produce the amount given by (12). Thus, in calculating the optimal production level of the leader and the number of firms to invite, the leader can take the reaction function in (12) as given.

The leader's objective function in its production period, after substituting in (12), is, therefore:

(13)

$$\underset{Q^{pp_n},n}{Max}\prod_L = \underset{Q^{pp_n},n}{Max}\left[a-\left(Q^{pp_n}+(n-1)\frac{a+f(Q_T^{pp_n})}{2n}\right)+f\left(Q^{pp_n}+(n-1)\frac{a+f(Q_T^{pp_n})}{2n}\right)-c\right]Q^{pp_n}$$

$$\text{s.t.}\quad \begin{aligned} Q^{pp_n} &\leq \overline{Q}^{pp_n} \\ n &\geq 1 \end{aligned}$$

where $Q_T^{pp_n}$ is the total amount actually produced, The constraint says this quantity will credibly be sold if and only if $Q_T^{pp_n} \leq \overline{Q}_T^{pp_n}$.

The Kuhn-Tucker conditions yield three solutions.[4] The first solution is identical to that found in Section III above, where the firm pre-produces and does not invite competitors. In this case n=1 and $Q^{pp_n} \leq \frac{a+f(S)}{2}$, which occurs when $c \geq \frac{f'(Q^{pp_n})\left[a+f(Q^{pp_n})\right]}{2}$. Another way to designate this possibility is that if the maximization problem in (5) *without the constraint* yields $Q^{pp} \leq \overline{Q}^{pp}$, then no competitors will be invited. In this case, because of the high marginal costs, the amount the leader desires to sell is credible, and so it is not in the firm's interest to invite competitors. The amount produced by the leader, and the profits he attains will

[4] The fourth solution, in which n>1 and $Q^{pp_n} < \frac{a+f(S)}{2}$ cannot be optimal because the leader's profits are monotonically decreasing in the number of firms.

be as in (6) and (7), respectively (taking into account the range of marginal costs for which this solution is applicable).

The other two solutions occur when the maximization problem in (5) without the constraint yields $Q^{pp} > \overline{Q}^{pp}$. In these solutions $Q^{pp_n} = \frac{a + f(S)}{2}$ and either n=1 or n>1. In the first case the firm desires producing exactly the maximum that is credible, while in the second the firm would like to produce more, but consumers do not believe more will be sold. Thus, the producer invites competition, thereby increasing consumer expectations. Solving (13), the optimal number of firms invited in this instance will be given by:

$$n^* = \begin{cases} \dfrac{f'(Q_T^{pp_n})(f(Q_T^{pp_n}) + a)}{2\{[a + f(Q_T^{pp_n})][1 - f'(Q_T^{pp_n})] + cf'(Q_T^{pp_n})]\}} & \text{if } \; c \leq \dfrac{(a + f(Q_T^{pp_n}))\left[3f'(Q_T^{pp_n}) - 2\right]}{2f'(Q_T^{pp_n})} \\ 1 & \text{otherwise} \end{cases}$$

Combining these cases with the first solution, the total size of the network market will be given by:

$$Q_T^{pp_n} = S^* = \begin{cases} \dfrac{[2f'(Q_T^{pp_n}) - 1](a + f(Q_T^{pp_n})) - cf'(Q_T^{pp_n})}{f'(Q_T^{pp_n})} & \text{if } \; c \leq \dfrac{(a + f(Q_T^{pp_n}))\left[3f'(Q_T^{pp_n}) - 2\right]}{2f'(Q_T^{pp_n})} \\ \dfrac{a + f(Q^{pp_n})}{2} & \text{if } \; \dfrac{(a + f(Q^{pp_n}))\left[3f'(Q^{pp_n}) - 2\right]}{2f'(Q^{pp_n})} < c \leq \dfrac{f'(Q^{pp_n})\left[a + f(Q^{pp_n})\right]}{2} \\ \dfrac{a + f(Q^{pp_n}) - c}{2 - f'(Q^{pp_n})} & \text{if } \; \dfrac{f'(Q^{pp_n})\left[a + f(Q^{pp_n})\right]}{2} < c < a + f(Q^{pp_n}) \\ 0 & \text{if } a + f(Q^{pp_n}) \leq c \end{cases}$$

and the leader's profit will be:

$$\Pi^{pp_n} = \begin{cases} \dfrac{[a + f(Q_T^{pp_n})]^2[1 - f'(Q_T^{pp_n})]}{2f'(Q_T^{pp_n})} & \text{if } c \leq \dfrac{(a + f(Q_T^{pp_n}))\left[3f'(Q_T^{pp_n}) - 2\right]}{2f'(Q_T^{pp_n})} \\ \dfrac{a + f(Q^{pp_n})}{2}\left(\dfrac{a + f(Q^{pp_n})}{2} - c\right) & \text{if } \; \dfrac{(a + f(Q^{pp_n}))\left[3f'(Q^{pp_n}) - 2\right]}{2f'(Q^{pp_n})} < c \leq \dfrac{f'(Q^{pp_n})\left[a + f(Q^{pp_n})\right]}{2} \\ \left(\dfrac{a + f(Q^{pp_n}) - c}{2 - f'(Q^{pp_n})}\right)^2\left[1 - f'(Q^{pp_n})\right] & \text{if } \; \dfrac{f'(Q^{pp_n})\left[a + f(Q^{pp_n})\right]}{2} < c < a + f(Q^{pp_n}) \\ 0 & \text{if } a + f(Q^{pp_n}) \leq c \end{cases}$$

The solution is demonstrated in Figure 5 for the linear case presented above, in which $f(S) = eS$, a=10, e=0.8 and $c \in (0,10)$. $Q_f^{pp_n}$ denotes total production by the fringe firms. In this figure we clearly see the different regions of solutions. When costs are low (c<4.1667) the firm invites competitors and produces more than the fringe firms, but this amount falls as costs increase. Concurrently the number of fringe firms decreases and the total amount produced by the fringe firms also decreases.[5] Interestingly, the amount produced by each firm increases. This is a result of a combination of fewer firms in the industry and lower production by the leader. For the intermediate range of costs (4.1667<c<6.6667) no competitors are invited into the industry, and the leader is limited to the maximal credible quantity. Note that in this range, price is not sensitive to changes in marginal costs. If marginal costs are in the upper range (c>6.6667) the amount the firm desires to produce optimally is credible. In this region higher costs are accompanied by lower production and higher prices.

B. Pre-production by the leader only

We consider now the outcome if the leader pre-produces but the invited firms do not, i.e., the leader invites competitors, but not at a point in time at which they can pre-produce. First, we note again that the leader will not produce units he does not intend to sell or cannot sell. In addition, as clear from above, any quantity he does sell is pre-produced – i.e., there is no production by the leader in the selling period. We proceed as in the last section, first calculating the maximal amount the leader will be able to sell in the selling period (given it has pre-produced) taking into account the number of firms and the reaction function of each firm, and then consider the problem again in the production period when the firm must decide how much to produce and how many firms to invite.

In the selling period, the objective functions of the leader and the fringe firms, respectively, are:

[5] Note that the number of fringe firms in this example is never greater than 1. This occurs because of the numbers chosen. If, for instance, we were to change the example to e=0.9, the largest number of invited firms would be 3.5.

$$\underset{Q^{pp_1}}{Max}\prod^{pp_1} = \underset{Q^{pp_1}}{Max}\ \left(a + f(S) - Q^{pp_1} - (n-1)q_f^{pp_1}\right)Q^{pp_1} - cQ_o^{pp_1}$$
$$s.t.\quad Q^{pp_1} \le Q_o^{pp_1}\ ;\quad \text{and}$$
$$\underset{q_{f,i}^{pp_1}}{Max}\prod_f^{pp_1} = \underset{q_{f,i}^{pp_1}}{Max}(a + f(S) - Q^{pp_1} - (n-2)q_{f,-i}^{pp_1} - q_{f,i}^{pp_1} - c)q_{f,i}^{pp_1} \quad i = 1,..,n\text{-}1$$

where Q^{pp_1} is the amount sold by the leader (with the superscript denoting pre-production by only one firm), $Q_0^{pp_1}$ is the amount pre-produced by the leader, $q_{f,i}^{pp_1}$ is the amount produced by fringe firm i, and $q_{f,-i}^{pp_1}$ is the amount produced by each of the other fringe firms.[6] The amount optimally produced by each fringe firm, given symmetry, is given by:

(14) $$q_f^{pp_1} = \frac{a + f(S) - Q^{pp_1} - c}{n}.$$

Replacing this in the leader's objective function, the maximal amount consumers believe the leader will sell in the selling period is:

(15) $$\overline{Q}^{pp_1} = \frac{a + f(S) + (n-1)c}{2}.$$

Replacing (15) in (14), optimal production by each fringe firm will be:

(16) $$q_f^{pp_1} = \frac{a + f(S) - (n+1)c}{2n}.$$

Returning to the production period, the leader's objective function is:

(17) $$\begin{aligned} &\underset{Q^{pp_1},n}{Max}\prod_L = \underset{Q^{pp_1},n}{Max}\left[a - \left(Q^{pp_1} + (n-1)q_f^{pp_1}\right) + f\left(Q^{pp_1} + (n-1)q_f^{pp_1}\right) - c\right]Q^{pp_1} \\ &\text{s.t.}\quad Q^{pp_1} \le \overline{Q}^{pp_1} \\ &\qquad\quad n \ge 1 \end{aligned}$$

The Kuhn-Tucker conditions yield multiple solutions as in the last Section. Unfortunately, the algebraic complexity does not allow us to give clear conditions on c for each range like we did above, but we are still able to give some insight into the solution. First, all solutions in which n=1 are identical to the solution in the last case and to that when there is only pre-production. The only case in which the solution differs is when n>1 and $Q^{pp_1} = \frac{a + f(S) + (n-1)c}{2}$. Since, in equilibrium, expectations

[6] This presentation has used the fact each fringe firm produces the same amount. Were this not so, a summation sign would be used instead of multiplying the quantity produced by each firm by the number of firms.

are fulfilled, it must be the case that $S = Q_T^{pp_1}$ (where the latter is total production). Thus, in equilibrium,

$$(18) \qquad S = Q_T^{pp_1} = \frac{\left[a + f(Q_T^{pp_1})\right](2n-1) - (n-1)c}{2n} \Rightarrow a + f(Q_T^{pp_1}) = \frac{2nQ_T^{pp_1} + (n-1)c}{2n-1}.$$

Replacing (15), (16) and the right hand side of (18) in (17), we can rewrite profits as:

$$(19) \qquad \prod^{pp_1} = \frac{n\left[S + c(n-1)\right](S - cn)}{(2n-1)^2}.$$

To find the optimal number of firms, we totally differentiate (19), and optimization requires that $\frac{\mathrm{d}\prod^{pp_1}}{\mathrm{d}n} = \frac{\partial \prod^{pp_1}}{\partial n} + \frac{\partial \prod^{pp_1}}{\partial S}\frac{\mathrm{d}S}{\mathrm{d}n} = 0$. Thus, the optimal number of firms will be the value of n that solves:

$$\frac{2n[(c^2n^2 + S^2 - cn)(f'(S) - 1)] + c^2n[-3nf'(S) + 2n + 2f'(S) - 1] + Sf'(S)(S - c)}{(2n-1)^2[2n - f'(S)(2n-1)]} = 0.$$

We again demonstrate the solution with the linear example we used above, when $f(S) = eS$, a=10, e=0.8 and $c \in (0,10)$.[7] The results are depicted in Figure 6. As seen, there is still a range over which the leader does not invite competition and keeps its quantity (and price) constant, but this range has shrunk considerably to 6.1<c<6.667. Thus, the firm is able to make use of the benefits from inviting competitors in more instances when fringe firms do not pre-produce.

VI. Comparing the Strategies

In this section we compare all five strategies (no strategy, pre-production, inviting competition, pre-production by all firms and inviting competition, pre-production by the leader and inviting competition) using the same example analyzed above. Clearly the two combination strategies will weakly dominate the first three strategies as they increase the range of possibilities facing the producer. The interesting comparison is between the two combined strategies.

A comparison of profits is presented in Figure 7, with the functions behind the graph presented in Table 2. As seen in this Figure, the last strategy we analyzed –

[7] A complete solution of the general linear case is available from the authors upon request.

where only the leader pre-produces, weakly dominates all other possibilities.[8] At first glance this seems surprising. The purpose of inviting firms is so that consumers will believe more will be produced. Combine this with the fact that fringe firms produce more when they pre-produce than when they do not, and it would seem that pre-producing by the fringe firms should be beneficial to the leader. To understand why this is not the case, we must take a deeper look at how pre-producing and how inviting competition improve the leader's profitability.

To start, it is important to note that the fulfilled expectations demand curve is strictly downward sloping. Thus, increased quantities are always accompanied by lower prices. Hence, the benefit to the firm from either inviting competition or pre-producing lies in the increased volume *despite* the lower prices. In fact (as will be seen in Figure 11) prices are highest when no strategy is played.

Consider inviting competition. This has the effect of increasing consumer expectations. For this strategy to help the leader, it must be the case that consumer expectations increase by *more* than the competitor produces, for were this not so the leader would surely lose since he would produce no more, and prices would fall. The competitor thus creates a positive externality on the leader, which is what allows the leader to increase production. Note, however, that this externality affects *all* firms in the industry and not only the leader. Thus, only part of the externality benefits the producer, while any effect on other producers leads to lower prices, and is thus detrimental to the leader.

Pre-producing, on the other hand works differently. It leads to an increase in the amount consumers believe this specific firm will produce, but does not affect how much they believe others will produce unless the price changes. Since the price clearly falls with pre-production, it is expected that other firms will *lower* production when a firm pre-produces – a negative externality. Thus, while the firm invites competitors in order to create the positive externality, it does not desire that they pre-produce since this will harm its profitability.

This understanding is strengthened by looking at the number of competitors, the level of production of the leader, the size of the network market, and the price in each

[8] While we were unable to prove this conjecture in a general setting, we were able to find no examples in which this did not occur.

strategy. The number of competitors is depicted in Figure 8. As per the explanation above, competitors are far more useful to the leader if they do not pre-produce. Thus, we find that more firms are invited when the fringe firms do not pre-produce than when they do. In fact, there is a range of costs over which no competitors are invited when the fringe firms pre-produce, but they are invited when they do not.

Similarly, we see in Figure 9 that the dominant strategy is such because the quantity produced by the leader is the greatest in this case. Interestingly, at low levels of costs the amount produced by the leader in the dominant strategy actually increases with costs (although this is difficult to see in the Figure). There are two effects of an increase in costs. On the one hand, when costs increase pre-production becomes relatively more valuable so there is a switch from inviting competitors to pre-production, a switch that leads to increased production. On the other hand, increased costs directly lead to lower production. When costs are low, then, the former effect overcomes the latter.

Figure 10 shows the size of the network market (total production by all firms), and Figure 11 shows the price. Note that, as per the discussion above, the quantity is highest and the price is lowest in the dominant strategy. Thus, this strategy is not only optimal for the producer; it is also optimal for the consumers as they get a more valuable good (since the market size is greater) for a lower price.

VII. Summary and Discussion

In this paper we considered a new solution to the credibility problem present in network industries. This problem arises because the value of a network good to its owner depends positively on the number of consumers who buy the good. Because of this property, it is in the interest of the producer to try to convince consumers that the market will be large, even if he knows it is untrue. Consumers, in turn, will disregard producer claims, and will, instead, try to reason out what size the market will attain. As a result, a lower than optimal quantity, both for consumers and producers, will be produced, i.e., the resulting equilibrium is Pareto inefficient.

The only remedy to this problem presented in previous literature is that of Katz and Shapiro (1985) and Economides (1996) in which the firm invites competitors to share their technology and enter the market, thus voluntarily giving up their monopoly position. The result of this invitation is to convince the consumers that the market will be large, which, in turn, leads to increased volume and profitability on the part of the inviting firm. The remedy we suggest is one of pre-producing the good, i.e., creating a large stock of goods that will be supplied to the market when the good is first introduced. This has the effect of changing the firm's cost structure in a manner that causes consumers to believe that the amount they will optimally sell (and hence the market size) will increase, again leading to higher profitability. The two strategies are compared, and we show the conditions under which each is preferable.

We then consider combinations of these two strategies in two different manners. In the first the leader produces and then invites competitors who then also pre-produce, and in the second the leader pre-produces but the fringe firms do not; rather, they produce in the same period in which they sell. Surprisingly we found that the latter dominated the former, which led us to a better understanding of how and why each strategy works. In short, inviting competitors creates a positive externality that benefits all firms, while pre-producing helps only the firm doing the pre-producing and harms all other firms. Thus, the leader invites the competitors so he can benefit from the positive externality, but he is better off if the competitors do not pre-produce.

In our model we chose pre-production in order to lower marginal costs to zero. This, of course, is not the only way to achieve the desired results. For instance, we could think of a firm making an R&D investment aimed at lowering production costs. If the R&D process is not discrete, the optimal investment size and, as a result, the optimal degree by which marginal costs should be lowered, could be endogenized. In addition, including a storage cost for pre-produced goods (we assumed there was no storage cost) could allow for a tradeoff between pre-production and other measures the firm could take to affect consumer beliefs.

Finally, we believe that the model we have developed is applicable for many industries, but perhaps it is best to start with those industries for which applicability is limited. In many advanced-technology industries, most of the costs of production are fixed costs. For instance, it costs almost nothing to create another computer chip, or CD, or, even, computer game. Since the value of pre-producing is realized through the lowering of marginal costs, little will be gained if marginal costs are negligible. This was seen in the model, where the benefit of pre-producing depended on the presence of significant marginal costs.

On the other hand, there are many industries, which are not necessarily network markets but which have similar traits, in which marginal costs are significant. Consider, for example, creating a city or a neighborhood, or even just a residential project. This industry will exhibit indirect network externalities, since the larger the size of the city (up to a limit), the more amenities (schools, movie theaters, restaurants, etc.) will be available to the residents. This is known as the “city effect” (Cicerchia, 1999). An entrepreneur interested in raising such a project will have to convince potential residents that these amenities will exist, and this, in turn, will depend upon the population being sufficiently large to support such amenities. The entrepreneur could use either or both of the strategies analyzed in this paper to advance the project; by pre-producing either housing units or complementary products, such as schools, shopping centers, etc. it becomes clear to the purchasers that the entrepreneur will do all in his power to sell the units (since the marginal costs have been lowered dramatically), and introducing competition will make potential residents believe that the entrepreneur will not be able to dramatically raise prices after they have bought, thus stunting growth. While we do not develop this example here, it is part of our continuing research on network industries.

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Appendix

Proof of Proposition 1

To prove this proposition we show the condition under which a fringe firm would want to produce less than the amount suggested when all other firms are producing the amount in the suggested equilibrium. We then show that this condition cannot hold.

Assume, then, that the leader has invited *n*-1 firms, and that leader produces the amount in (11) and each of the other fringe firms produces the amount in (12). The objective function facing the fringe firm in the production period (when *n* is a constant) is

(A1) $$\underset{q_f^{pp_n}}{Max}\left[a-\left(\overline{Q}^{pp_n}+(n-2)+\overline{q}_f^{pp_n}+q_f^{pp_n}\right)+f\left(\overline{Q}^{pp_n}+(n-2)+\overline{q}_f^{pp_n}+q_f^{pp_n}\right)-c\right]q_f^{pp_n}.$$

The first order condition, after simplification and using (11) and (12), yields

(A2) $$q_f^{pp_n}=\frac{a+f-nc}{n(2-f')}.$$

For (12) not to be an equilibrium, it must be the case that the quantity in (A2) is *less than* that in (12). (Recall that the quantity in (12) is the *maximum* that can credibly be supplied, so if the quantity in (A2) is greater than that in (12), the equilibrium holds.) Rearranging, this reduces to

(A3) $$c>a+f.$$

This condition clearly cannot hold, since a+f is the intercept of the demand curve, and no firm can profitably produce when (A3) holds, unless $f'>1$, which is ruled out (see the text).

Q.E.D.

We would like to make two notes about this proof. First, if n<2 it is difficult to speak of how a single fringe firm can lower production since there is no single firm to do so. If we consider the case of n=2, the condition in (A3) also reduces to $f'>1$, which is precluded if the equilibrium is to be finite (as discussed in the text). Second, if we consider the linear case in the text, the condition can be stated either as that in (A3) or that $f'\equiv e>1$.

Figure 1

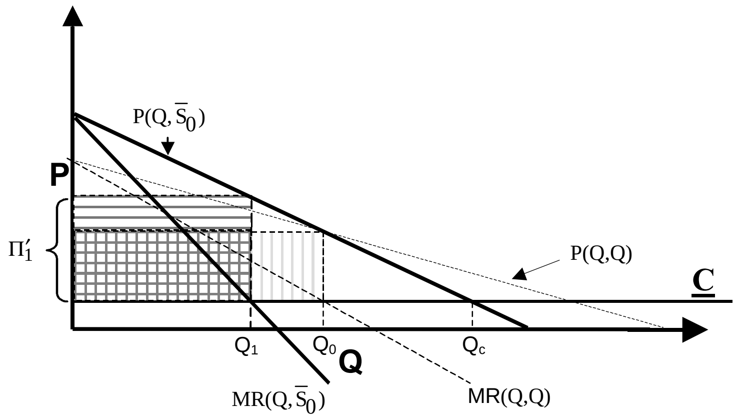

Figure 2

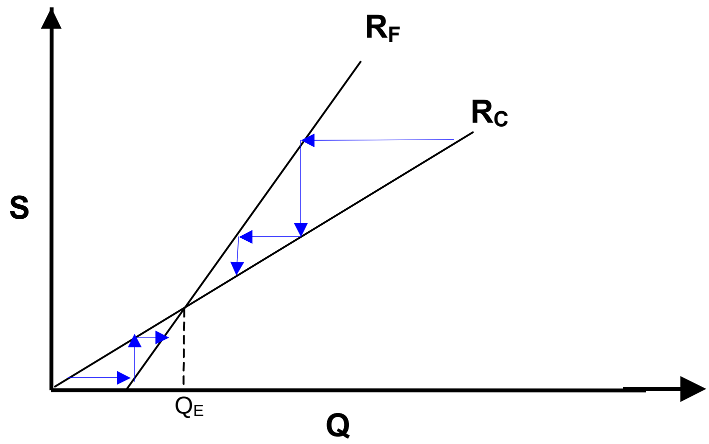

Figure 3

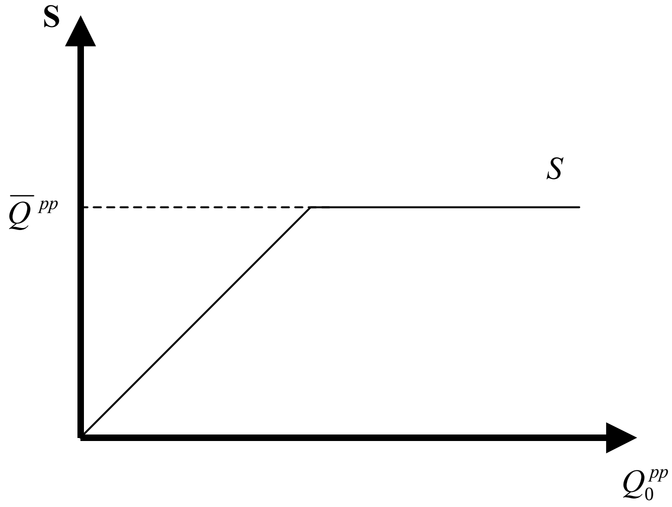

Figure 4

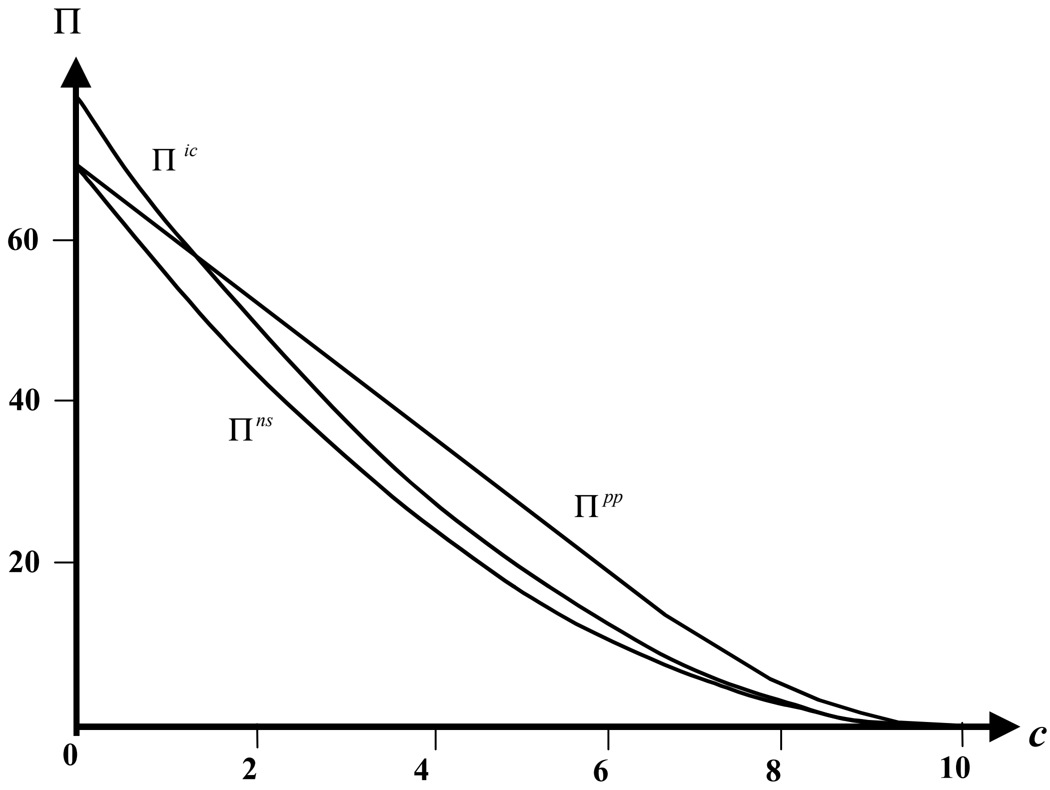

Figure 5

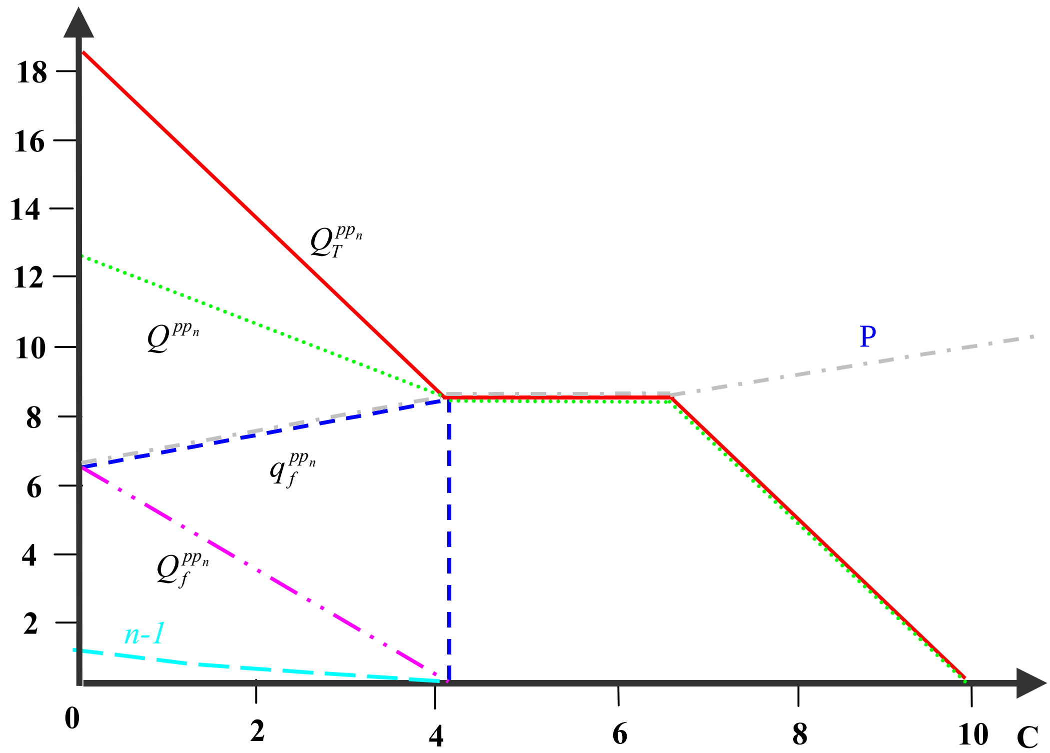

Figure 6

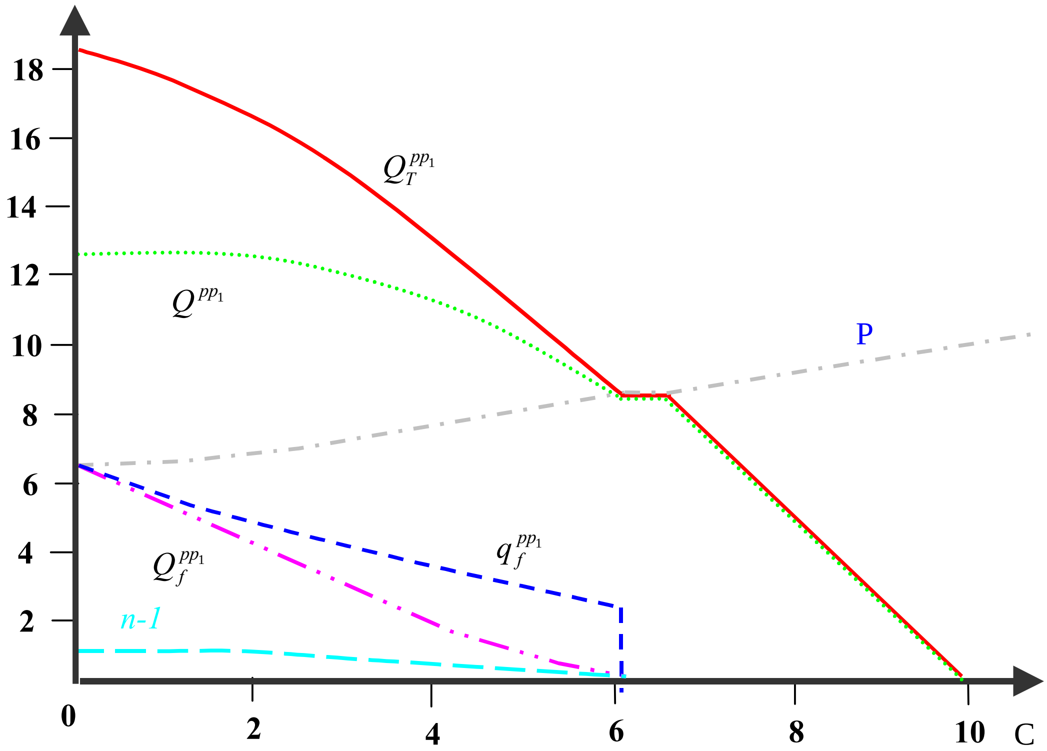

Figure 7

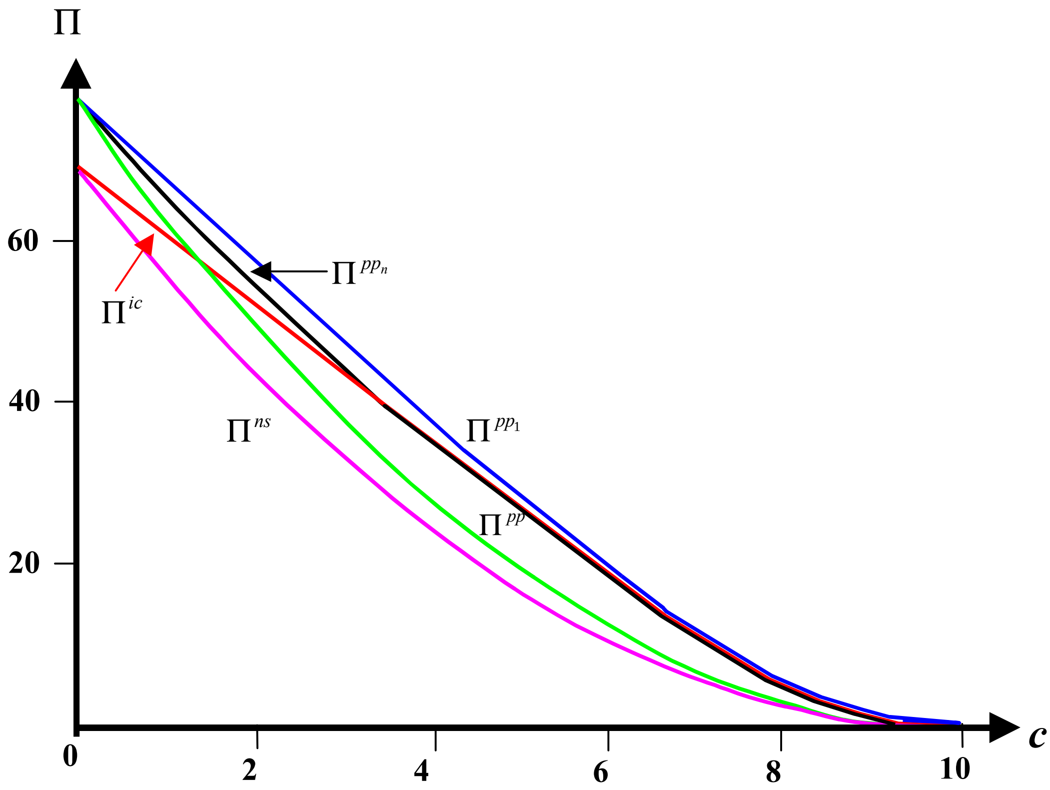

Figure 8
Number of Firms

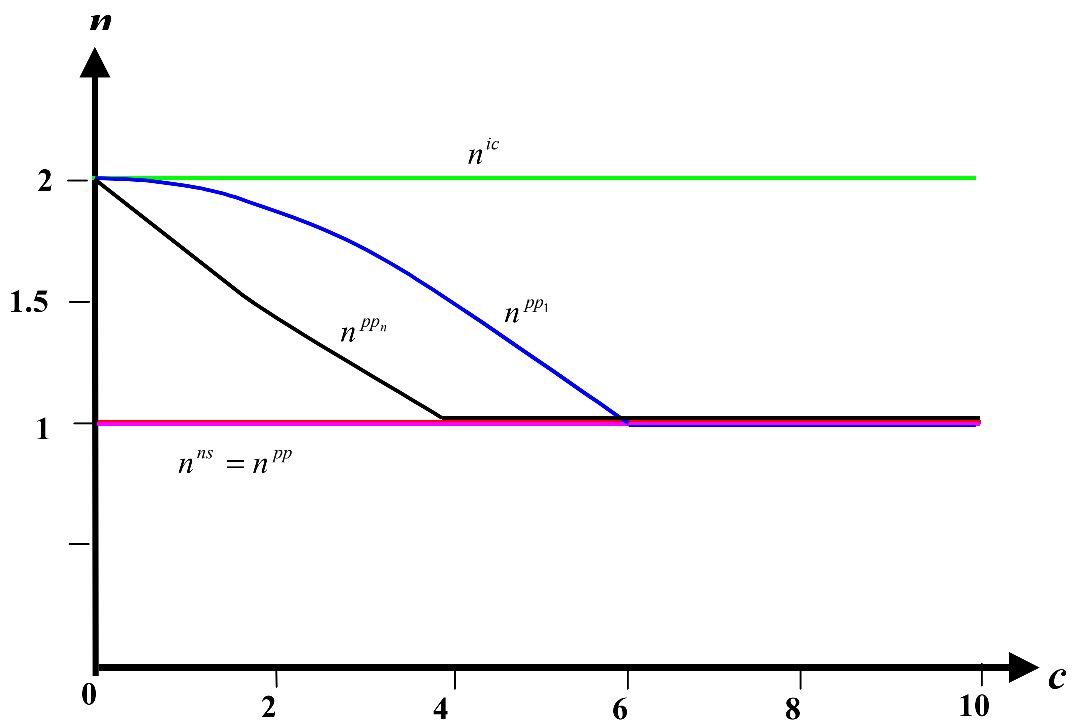

Figure 9
Production by the Leader

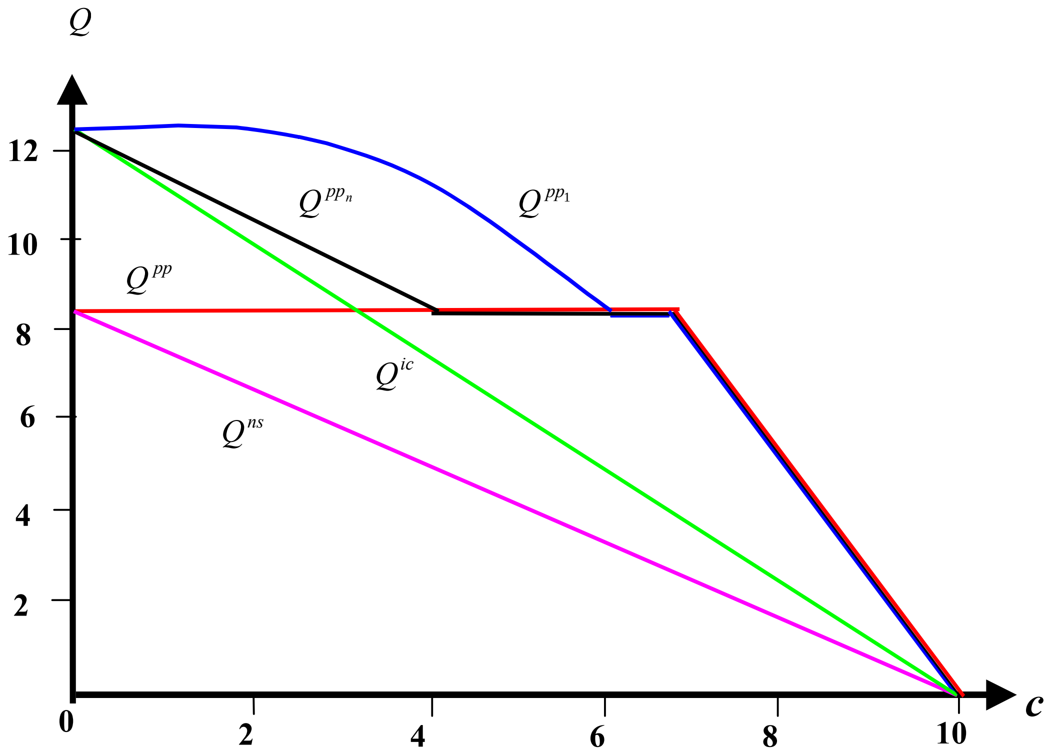

Figure 10
Market Size

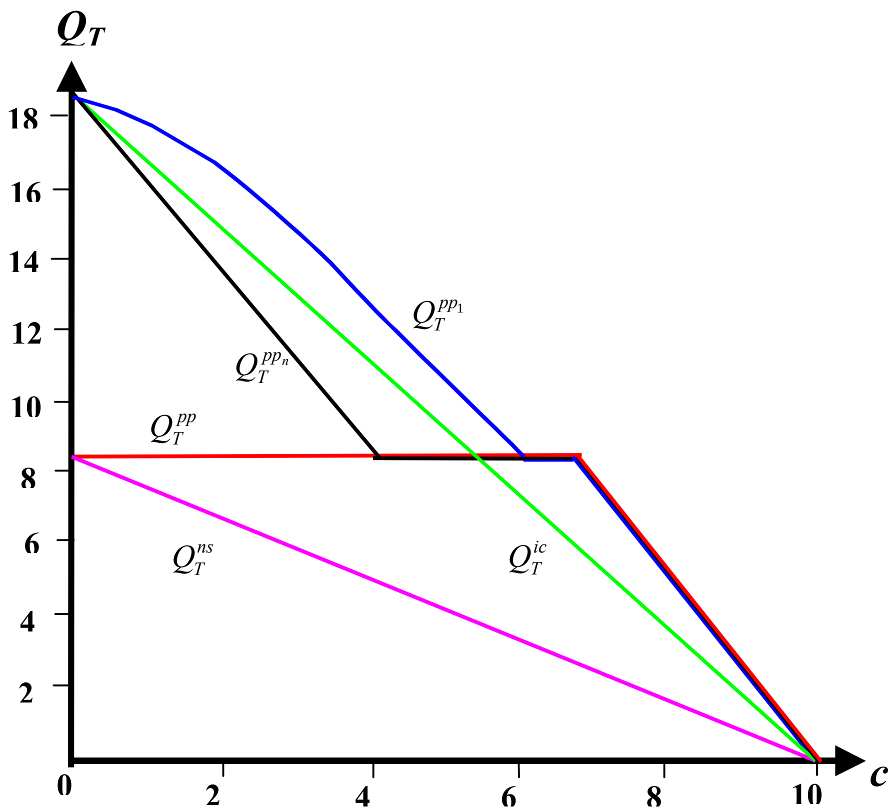

Figure 11
Price

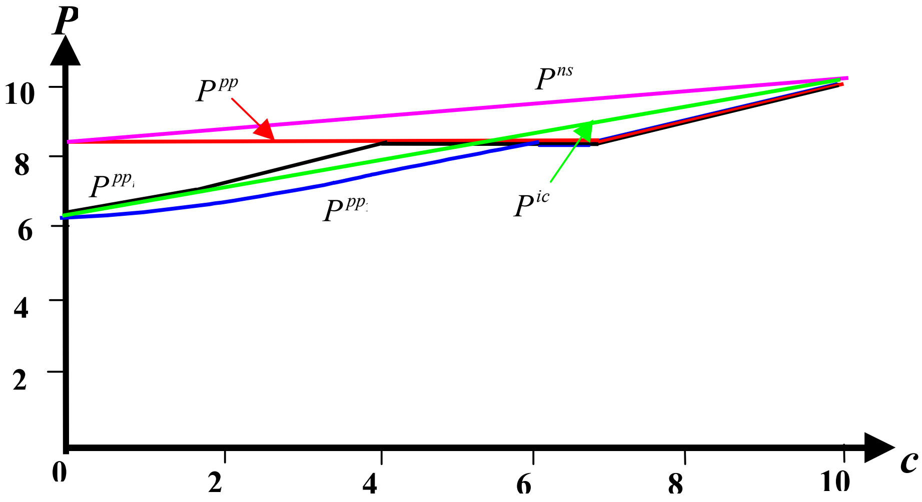

Table 1

Preference order	e	c
$pp = ic = ns$	$e \le \frac{2}{3}$	c=0
$pp \succ ic = ns$	$e \le \frac{2}{3}$	0<c<a
$ic \succ pp = ns$	$\frac{2}{3} < e < 1$	c=0
$ic \succ pp \succ ns$	$\frac{2}{3} < e < 1$	$0 < c < \frac{\left(-4e^2 + 5e - 2 + 2\sqrt{4e^4 - 10e^3 + 8e^2 - e}\right)a}{e-2}$
$pp \succ ic \succ ns$	$\frac{2}{3} < e < 1$	$\frac{\left(-4e^2 + 5e - 2 + 2\sqrt{4e^4 - 10e^3 + 8e^2 - e}\right)a}{e-2} < c < a$

Table 2
Profits

Π^{ns}	$\dfrac{(10-c)^2}{1.44}$
Π^{pp}	$\dfrac{(10-c)^2}{1.28}$
Π^{ic}	$\begin{cases} \dfrac{10(10-1.2c)}{1.44} & \text{if } c < 6.667 \\ 1.25\,(10\text{-}c)^2 & \text{if } 6.667 \le c \end{cases}$
Π^{pp_n}	$\begin{cases} \dfrac{(10-0.8c)^2}{1.28} & \text{if } c < 4.1667 \\ \dfrac{10(10-1.2c)}{1.44} & \text{if } 4.1667 \le c < 6.667 \\ 1.25\,(10\text{-}c)^2 & \text{if } 6.667 \le c \end{cases}$
Π^{pp_1}	$\begin{cases} \left(10-c-0.2J_3^*\right)\left(10-0.2c+0.2J_3^*\right)\dfrac{J_3^*}{\left(\dfrac{0.333J_1^*}{c}-\dfrac{J_2^*}{cJ_1^*}\right)^2} & \text{if } c < 6.1 \\ \dfrac{10(10-1.2c)}{1.44} & \text{if } 6.1 \le c < 6.667 \\ 1.25\,(10\text{-}c)^2 & \text{if } 6.667 \le c \end{cases}$ where: $J_1^* = \sqrt[3]{(-4.32c^2+144c-1200)(7.2c-2\sqrt{3}\sqrt{4.68c^2-12c+100})}$ $J_2^* = 1.44c^2-48c+400$ $J_3^* = \dfrac{J_1^*}{6c}-\dfrac{J_2^*}{2J_1^*}-0.4$

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