Optimal Reimbursement Schemes in Contests *

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Abstract

Many contests, such as innovation races or litigation, often involve reimbursement of expenses. This study examines optimal reimbursement schemes in two-player Tullock contests, analyzing four reimbursement structures: external and internal mechanisms targeting the contest winner or loser. We assess the implications on participant effort, winning probabilities, and designer payoff, under two key conditions: fairness (preserving initial win chances) and viability (positive efforts from players). We find that while external reimbursement for the loser ensures both fairness and viability, full reimbursement to the winner fails to meet these criteria. Additionally, the findings indicate that optimal reimbursement structures and proportions vary depending on the contest designer's objectives, such as maximizing effort or personal payoff.

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1. Introduction

Competition is the lifeblood of economic activities, business, sports, war, R&D races, promotional tournaments, and litigation. In these situations, participants invest resources to secure a prize, which is the essence of a ‘contest’. In many such contests, a player may be reimbursed for their expended resources. For instance, the Defense Advanced Research Projects Agency (DARPA) has employed reimbursement strategies in its Grand Challenges, such as the Self-Driving Car Challenge, to encourage bold technological leaps (Goodrich & Olsen, 2003). Similarly, the XPRIZE Foundation has utilized milestone prizes to offset development costs and catalyze innovation in fields like space exploration and environmental sustainability (Diamandis & Kotler, 2012). The U.S. Department of Energy, the NASA and the European Innovation Council have all implemented reimbursement programs to promote innovation in energy efficiency, sustainable space technologies, and blockchain. Beyond R&D, reimbursement schemes are employed in various domains, including legal, military, and organizational contexts. In legal proceedings, the "English Rule" mandates that the losing party reimburses the winning party's legal costs, thereby discouraging frivolous lawsuits and promoting fairness (Baye et al., 2005). In military history, war indemnities, where the defeated nation compensates the victor, have served as both a deterrent and a means of reparation (Ferguson, 2001). Reimbursement is often utilized in internal competitions, such as promotional tournaments, to incentivize employee development and enhance organizational performance.

Reimbursement in contests can be made in several ways that vary in terms of the extent of reimbursement (full or partial), the source of reimbursement (external reimbursement by a third party, or internal reimbursement by one of the players), and the outcome-contingent target (the winner or the loser). The reimbursement scheme is usually determined by a contest designer who wishes to attain certain goals by affecting the players’ incentives. An innovation authority, for example, may select the reimbursement scheme that best promotes the viability of the R&D race, i.e., maximizes the likelihood of a technological breakthrough. The preferred scheme may also be justified by social norms or by guiding and binding principles of fair conduct. For example, in the context of litigation – a natural application of a two-player augmented contest – the justification of complete reimbursement of costs is based on the idea that the tort system promotes ‘corrective justice’ and on the rule that assigns the burden of proof to the plaintiff: *semper necessitas probandi* *incumbit ei qui agit (*Babylonian Talmud, Tractate Baba Kama 35b). The former idea justifies full coverage of the plaintiff's expenses when he wins; the second rule rationalizes complete reimbursement of the defendant's expenses when the plaintiff loses (and the defendant wins).

Although reimbursement schemes are inherently important in these contests, there is a scarcity in the literature to systematically analyze and compare various reimbursement schemes. In this study we theoretically focus on four schemes of partial or full reimbursement. In the first two schemes, a third party (the designer) reimburses the winner or the loser, whereas in the latter two schemes one player reimburses the cost to the other. Moreover, we consider a set of desirable properties such as participation constraint, and fairness for each of the schemes and compare their performance in terms of the total effort generated (resources spent), and the designer's payoff.

Reimbursement schemes in contests have been investigated in the literature earlier, although not that systematically. Chowdhury and Sheremeta (2011a) constructed a Tullock (1980) contest with effort spillovers under complete information where contingent upon winning or losing, the payoff of a player is a linear function of prizes, own effort, and the effort of the rival. This generic structure nests several existing contests in the literature and is used as a special case to analyze models of internal reimbursement. Similar structure of spillover and reimbursement was studied by Baye et al. (2005, 2012), in an All-pay auction set-up, focusing on the designer's payoff and the players' efforts under symmetric prize valuations. Loser reimbursing the winner has been studied in the literature (Baumann & Friehe, 2012; Carbonara et al., 2015; Luppi & Parisi, 2012; Matros & Armanios, 2009; Yates, 2011) focusing on Tullock contests. Specific models of external reimbursement are also analyzed by Cohen and Sela (2005), Matros (2012), and Thomas and Wang (2017).[1] Matros and Armanios (2009) generalized the external reimbursement by allowing reimbursement of the winner and the loser, jointly or separately, focusing on the designer's payoff under symmetric prize valuations. Xiao (2018) proved that an all-pay contest with additively separable spillovers has a unique Nash equilibrium.

1.1. Four schemes of reimbursement

In this study, we introduce a two-player Tullock contest that incorporates the option for effort reimbursement in four distinct reimbursement schemes. These schemes are categorized based on

[1] Thomas and Wang (2017) studied a Tullock contest with external subsidization regardless of who wins. An important characteristic of this setting is the assumption that there is a limited amount of resource that can be used for subsidies.

the source (external or internal) and target (winner or loser) of the reimbursement. While each scheme is suitable for particular contexts, contest designers may also face choices between them. Thus, it becomes essential to identify which scheme best fits a given context and whether improvements can be made. Our key questions include which reimbursement scheme is the most desirable, and whether any of them meet multiple desirable criteria simultaneously. A central focus in contest design is determining how designers can achieve their objectives (e.g., designer payoff or total effort) through interventions that influence the contest. In this study we answer these for the first time. Table 1 summarizes the four reimbursement schemes for clarity.

Table 1. Reimbursement schemes

<table>
<tr><td colspan="2" rowspan="2">Schemes</td><td colspan="2">Reimbursement source</td></tr>
<tr><td>External (from the designer)</td><td>Internal (from the opponent)</td></tr>
<tr><td rowspan="2">Reimbursement target</td><td>Winner</td><td>Scheme A</td><td>Scheme B</td></tr>
<tr><td>Loser</td><td>Scheme C</td><td>Scheme D</td></tr>
</table>

Scheme A: reimbursement of the winner's expenses by a third party (external reimbursement). This reimbursement scheme is common in competitions between employees for a position or job or for promotion to a certain rank. The manager often wishes to motivate the participants by covering the winner's expenses (Thomas & Tung, 1992). Such a reimbursement scheme is also common in R&D races because the contest designer wants to increase the profitability of winning. This scheme has also been studied in contests by Cohen and Sela (2005) and was expanded by Matros and Armanios (2009) and Matros (2012).

Scheme B: reimbursement of the winner's expenses by the loser (internal reimbursement). As noted in the context of legal tort disputes, this scheme of reimbursement is recognized by the lawmaker based on legal principles of justice or fairness. The principle of "restoring the status quo ante" is a case in point. Contrary to the American rule, which holds each party liable for its legal expenses, the English (continental-global) rule stipulates that the loser pays the winner's legal expenses. The English rule has been found to have significant advantages over the American rule (Baumann & Friehe, 2012; Baye et al., 2012; Carbonara et al., 2015; Hause, 1989; Hughes & Snyder, 1995; Katz, 1987; Luppi & Parisi, 2012). This scheme is also applied in war when victors impose war indemnities on the vanquished side to recoup the costs of warfare and punish the defeated population (Sullo & Wyatt, 2014). In other words, the rationale of reparations is based on the right of the winning party to claim compensation for war damage and expenses. It is no wonder that peace treaties between warring nations often resort to legal justifications such as coverage of war expenses. In this scheme, the maximal possible effective reimbursement that results in the participation of the players cannot be complete.[2]

Note that Carbonara et al. (2015) and Luppi and Parisi (2012), who studied this kind of reimbursement, assumed that the prize for the winner is taken from the loser's wealth. We assume, as Baye et al. (2005), who studied a dispute between two parties on an indivisible asset, that in Scheme B (similarly to the other three Schemes and as in the standard models of Tullock), the prize is not taken from the loser's wealth, since sometimes players compete for a resource that does not belong to them, e.g. the prize may be provided by the designer.

Scheme C: reimbursement of the loser's expenses by a third party (external reimbursement). The most common application of this reimbursement is insurance. An insurance company reimburses the loser for their expenses. This reimbursement scheme is plausible in various contexts, such as education, where underperforming students may receive support after failing a test; intra-firm competitions, where designers incentivize participation by offsetting losses; and R&D races aimed at stimulating innovation. Minchuk and Sela (2020) studied this scheme in the context of insurance, where the designer can offer players an insurance option that requires them to pay a premium to cover potential losses. They found that this scheme may be profitable for the designer. This scheme of reimbursement was also studied in Matros and Armanios (2009).

Scheme D: reimbursement of the loser's expenses by the winner (internal reimbursement). This scheme of reimbursement is observed in the aftermath of war when the victor is required to compensate the loser after being convicted of war crimes or when there is an agreement (imposed by the designer or between the parties) on the importance of supporting the weak or needy. The most common example of such contests are post-war situations in which the victorious front help rebuild the losing party.

[2] The rate of reimbursement is less than 100%; it is equal to the maximal rate that yields an interior contest equilibrium.

In Tullock's (1980) celebrated contest, the players invest efforts to win a particular prize, and the winning probabilities increase with their efforts. Applying this setting (see Bevia & Corchon, 2024; Konrad, 2009), our objective is to clarify the preferred scheme of reimbursement for specific objectives of the contest designer, while focusing on two desirable properties, namely fairness and viability. Below we define and discuss those properties further.

Definition: Fairness - a reimbursement scheme is fair if it does not reverse the pre-scheme winning probabilities.

Fairness (or plausibility) is defined by a single condition: the ratio of players' winning probabilities in the initial, unaltered contest must not be reversed by any intervention. There are two main reasons for this requirement. First, from a political standpoint, a policy that drastically alters winning probabilities is likely to be infeasible, as the initially stronger player would have the incentive and means to resist a shift that reduces their status from prospective winner to likely loser. Second, even if such a policy were enacted, the stronger player could choose to withdraw from the contest altogether, effectively nullifying the designer's intentions. To avoid this outcome, the designer refrains from implementing policies that would lead to this reversal. This criterion aligns with Groh et al. (2012), who also characterize it as a fairness-condition.

Definition: Viability – a reimbursement scheme is viable if all players' post-scheme equilibrium efforts are positive.

In any contest, particularly in a two-player setting, achieving a viable equilibrium that satisfies players' participation constraints is crucial. Without viability, the contest designer's objectives – which frequently rely on players' efforts – cannot be realized. For instance, in litigation contests, the designer aims to uphold the quality of the justice system by encouraging litigants to exert sufficient effort to reveal the truth based on evidence, thus ensuring justice. More broadly, the contest-design literature commonly assumes that designers seek to maximize players' efforts. This assumption is especially relevant in sports competitions, where designers recognize that high-effort contests are more appealing to spectators, driving demand and enhancing ticket sales or revenue. Thus, maximizing participant effort aligns naturally with the designer's commercial objectives in these settings (Szymanski, 2003).[3]

[3] In a war contest, positive efforts can be undesirable. But in such a case, the existence of a corner equilibrium, of zero efforts, is from the outset desirable for the designer so there is no room for their intervention.

1.2. The optimal reimbursement scheme and rate

Our aim is to provide a theoretical tool with potential applications for finding the optimal scheme of reimbursement, including the optimal reimbursement rates, for various objectives: designer payoff, and total effort. The analysis considers the possibility that the designer is interested, or not interested, in full reimbursement, fairness, and viability. To simplify the illustration of the comparison of the reimbursement schemes, we assume a two-player Tullock contest and a linear cost. These assumptions are common in the literature (e.g. Baye et al., 2005, 2012; Carbonara et al., 2015; Luppi & Parisi, 2012). Reimbursement schemes in contests were earlier analyzed by Cohen et al. (2023). However, they focus on internal funding by the loser, uniform mixed funding, and non-uniform external funding - and demonstrate equivalence in terms of designer profit and player outcomes. But they do not consider viability or fairness constraints. Additionally, unlike the current analysis, the prize values were symmetric in Cohen et al., (2023).

This research contributes to the management and contest literature by identifying the optimal reimbursement scheme and rate tailored to the nature of the contest and the designer's objectives. By systematically comparing Schemes A, B, C and D under asymmetric prize valuations, we address various designer objectives, including profit maximization, total effort maximization, and the pursuit of fairness and viability. This comprehensive approach offers actionable insights for contest designers seeking to determine reimbursement strategies with their specific goals.

In patent races, the designer's primary goal is often to maximize companies' efforts, enhancing the likelihood of high-quality outputs, such as patents for COVID-19 vaccines. Scheme A, where the designer provides reimbursement to the winner, generates the highest total effort by removing financial barriers to participation, making it suitable when outcome quality is prioritized over fairness. However, Scheme A introduces an imbalance, as weaker competitors may win despite stronger rivals, which may undermine intrinsic motivation in the long term.

Scheme C, where the designer reimburses the loser, preserves fairness and achieves a high, though slightly lower, total effort than Scheme A. It is preferable when maintaining balanced competition is a priority. For cases where fairness is crucial, Scheme C offers a viable compromise that promotes steady effort and intrinsic motivation without disrupting the equilibrium.

In litigation contests, fairness and truth-seeking are vital. Scheme B, commonly used in these settings, aligns with the English Rule that requires the losing party to reimburse the winner's expenses. This scheme encourages evidence collection and enhances truth-seeking by balancing the incentives for effort. However, full reimbursement is not achieved in Scheme B to ensure viability; otherwise, competition might weaken, or political feasibility may suffer. If full reimbursement for the winner is essential, the designer might opt for a less balanced Scheme A or a hybrid approach that combines Schemes B and A by reimbursing the losing party partially. In cases without constraints of fairness or viability, the scheme ranking provided in the next section offers the designer guidance in selecting the most effective reimbursement approach, balancing the competing demands of effort, payoff, and equitable participation in contest environments.

In the next section we first set up the model. We then cover equilibria in winner reimbursement and in loser reimbursement. The designer's payoff and the total effort are analyzed next, before comparing the four schemes for each criterion. Section 3 concludes by discussing the current results and possibilities of future work.

2. Contests with alternative reimbursement schemes

2.1. Model set-up

To set up the baseline, consider a Tullock (1980) contest with two players ($i = 1,2$) who compete for a prize that Player i values at $V_i > 0$ by exerting costly effort $x_i \geq 0$. With no loss of generality, let $V_1 \geq V_2$ and that players face no fixed cost and unit marginal cost of effort. The probability that Player i wins is: $p_i = \frac{x_i}{x_1+x_2}$ for $(x_1 + x_2) > 0$, and ½ otherwise. The payoff functions in this contest is: $\pi_i^T = V_i \frac{x_i}{x_1+x_2} - x_i$. The existence and uniqueness of equilibrium follows from Szidarovszky and Okuguchi (1997) and Chowdhury and Sheremeta (2011b). Following standard procedure, in equilibrium, $x_1^{*T} = \frac{(V_1{}^2 V_2)}{(V_1+V_2)^2}$, $x_2^{*T} = \frac{(V_1 V_2{}^2)}{(V_1+V_2)^2}$, total effort is $X^{*T} = x_1^{*T} + x_2^{*T} = \frac{(V_1 V_2)}{(V_1+V_2)}$ and the players' winning probabilities are $p_1^{*T} = \frac{V_1}{(V_1+V_2)}$ and $p_2^{*T} = \frac{V_2}{(V_1+V_2)}$, so $\frac{p_1^{*T}}{p_2^{*T}} = \frac{V_1}{V_2} \geq 1$.

Note that we denote the basic Tullock set-up with superscript T. Now, given this set-up, for a reimbursement scheme S $(= A, B, C, D)$ if we denote the equilibrium variables with superscript S, then viability means: $x_1^{*S} > 0, x_2^{*S} > 0$ and fairness means $p_1^{*S}/p_2^{*S} \geq 1$. Let α denote the fraction of effort reimbursed, sourced either from the designer or the opponent. When the reimbursement comes from the designer, then the payoff of the designer is the difference between the total effort after reimbursement, and some function of the prize-values. Since the prize values are fixed, for all practical purposes, we can ignore that part. The subsequent analysis introduces the reimbursement schemes, evaluating their performance based on viability, fairness, and the designer's payoff.

2.2. Winner reimbursement: viability and fairness

Lemma 1. Scheme A, i.e., external reimbursement to the winner, is viable but may not be fair.

Proof: When the funder is external and the winner is the recipient (Scheme A), then the payoff functions are:

$$\pi_1^A = (V_1 + \alpha x_1)\left[\frac{x_1}{x_1+x_2}\right] - x_1 \qquad (1)$$

$$\pi_2^A = (V_2 + \alpha x_2)\left[\frac{x_2}{x_1+x_2}\right] - x_2 \qquad (2)$$

where the designer reimburses the winner for α proportion of her expenses.

Viability can be drawn directly from Chowdhury and Sheremeta (2011a) who show interior solutions. However, as shown by Cohen and Sela (2015), there is an interior equilibrium when $\alpha = 1$, that is, full reimbursement and viability are satisfied. In such a case, the players' efforts are $x_1^{*A} = V_2$, $x_2^{*A} = V_1$, $p_1^{*A} = \frac{V_2}{V_1+V_2}$, $p_2^{*A} = \frac{V_1}{V_1+V_2}$, so $\frac{p_1^{*A}}{p_2^{*A}} = \frac{V_2}{V_1} \leq 1$. Hence, when the players have asymmetric prize valuations, fairness is violated. **Q.E.D.**

Note that this result was extended in Matros (2012) by showing that, under $\alpha = 1$, there are corner equilibria in which a player invests more than the prize value of the other player and drives him out of the contest.

Lemma 2. Scheme B, i.e., internal reimbursement to the winner, cannot be both complete and viable. To attain viability, the rate of reimbursement, α, must satisfy $\alpha \leq \frac{V_2^2}{(V_1^2+{V_2}^2)} \leq \frac{1}{2}$.

Proof: When the loser reimburses the winner (Scheme B), then the payoff functions are:

$$\pi_1^B = (V_1 + \alpha x_1)\left[\frac{x_1}{x_1+x_2}\right] - \alpha x_2\left[1 - \frac{x_1}{x_1+x_2}\right] - x_1 \qquad (3)$$

and

$$\pi_2^B = (V_2 + \alpha x_2)\left[\frac{x_2}{x_1+x_2}\right] - \alpha x_1\left[1 - \frac{x_2}{x_1+x_2}\right] - x_2 \qquad (4)$$

where the losing player reimburses the winner for α percent of her expenses. First, let us prove that to obtain a unique interior equilibrium, it is necessary that $\alpha < 1$. In other words, full reimbursement must be violated.

Notice that (3) and (4) can be presented as:

$$\pi_1^B = V_1 \frac{x_1}{x_1+x_2} - x_1 + \alpha\,(x_1 - x_2) \qquad (5)$$

$$\pi_2^B = V_2 \frac{x_2}{x_1+x_2} - x_2 + \alpha\,(x_2 - x_1) \qquad (6)$$

The first-order equilibrium conditions are:

$$\frac{\partial \pi_1^B}{\partial x_1} = \alpha + \frac{V_1\, x_2}{(x_1+x_2)^2} - 1 = 0 \qquad (7)$$

$$\frac{\partial \pi_2^B}{\partial x_2} = \alpha + \frac{V_2\, x_1}{(x_1+x_2)^2} - 1 = 0 \qquad (8)$$

Given that the second-order conditions are satisfied, the players' equilibrium efforts are:

$$x_1^{*B} = \frac{{V_1}^2\, V_2}{(1-\alpha)(V_1+V_2)^2} \qquad (9)$$

$$x_2^{*B} = \frac{V_1\, {V_2}^2}{(1-\alpha)(V_1+V_2)^2} \qquad (10)$$

To ensure positive efforts, $\alpha < 1$. Substituting (9) and (10) into (5) and (6), we get:

$$\pi_1^{*\,B} = \frac{V_1\left({V_1}^2(1-\alpha) - \alpha {V_2}^2\right)}{(1-\alpha)(V_1+V_2)^2} \qquad (11)$$

$$\pi_2^{*\,B} = \frac{V_2\left({V_2}^2(1-\alpha) - \alpha {V_1}^2\right)}{(1-\alpha)(V_1+V_2)^2} \qquad (12)$$

It can be readily verified to ensure that these payoffs are not negative,

$$\alpha \leq \frac{V_2^2}{(V_1^2 + {V_2}^2)} \leq \frac{1}{2} \qquad (13)$$

Hence, viability is satisfied only when (13) is satisfied, that is, full reimbursement is given up.

Note that partial reimbursement satisfies the fairness imperative because $\frac{p_1^{*B}}{p_2^{*B}} = \frac{x_1^{*B}}{x_2^{*B}} = \frac{V_1}{V_2}$. **Q.E.D.**

Note that reimbursement scheme B was studied also by Baye at el. (2005, 2012) in an All-Pay-Auction set-up. Baye at el. (2005) find that there exists an interior equilibrium under the British system of full reimbursement ($\alpha = 1$), and not just under the Continental system (partial reimbursement, $\alpha < 1$) as in our setting with a stochastic CSF.

Lemmas 1 and 2 imply the following impossibility rule:

Theorem 1. No scheme of reimbursement for the winner (Schemes A and B) can satisfy full reimbursement, fairness, and viability.

2.3. Loser reimbursement: viability, and fairness

In contrast to the theorem above we now show that consistency among the three properties (full reimbursement, fairness, and viability) is possible under reimbursement of the loser's costs.

Theorem 2. Both the internal reimbursement or external reimbursement scheme of reimbursement for the loser (Schemes C and D) can satisfy the full reimbursement, fairness, and viability criteria.
Proof: We will prove this into two parts. First, for external reimbursement and then for internal reimbursement.

Part 1. Suppose that the funder is external, and the loser is the recipient (Scheme C). In this case, the payoff functions are:

$$\pi_1^C = V_1 \left[\frac{x_1}{x_1+x_2}\right] + \alpha x_1 \left[1 - \frac{x_1}{x_1+x_2}\right] - x_1 \tag{14}$$

$$\pi_2^C = V_2 \left[\frac{x_2}{x_1+x_2}\right] + \alpha x_2 \left[1 - \frac{x_2}{x_1+x_2}\right] - x_2 \tag{15}$$

Or, alternatively,

$$\pi_1^C = (V_1 - \alpha x_1) \left[\frac{x_1}{x_1+x_2}\right] - x_1(1-\alpha) \tag{14'}$$

$$\pi_2^C = (V_2 - \alpha x_2) \left[\frac{x_2}{x_1+x_2}\right] - x_2(1-\alpha) \tag{15'}$$

where the designer reimburses the loser for α percent of her expenses.

(14) and (15) imply that $x_i^C = 0 \Rightarrow \pi_i^C = 0$, so $0 < x_i^C < V_i^C$ results in positive payoffs.

The first-order equilibrium conditions are:

$$\frac{\partial \pi_1^C}{\partial x_1} = \frac{x_2\,(V_1+\alpha x_2)}{(x_1+x_2)^2} - 1 = 0 \tag{16}$$

$$\frac{\partial \pi_2^C}{\partial x_2} = \frac{x_1\,(V_2+\alpha x_1)}{(x_1+x_2)^2} - 1 = 0 \tag{17}$$

Given that the second-order conditions are satisfied, by (16) and (17), the relationship between the players' positive equilibrium efforts is:

$$x_2^{*C} = \frac{-V_1+\sqrt{V_1^2+4\alpha x_1^*(\alpha x_1^*+V_2)}}{2\alpha} \tag{18}$$

In the symmetric case, where $V_1 = V_2 = V$, we would have obtained:

$$x_2^{*C} = \frac{-V+\sqrt{V^2+4\alpha x_1^*(\alpha x_1^*+V)}}{2\alpha} = \frac{-V+\sqrt{(2\alpha x_1^*+V)^2}}{2\alpha} = x_1^{*C} \tag{19}$$

Matros and Armanios (2009) studied the symmetric version ($V_1 = V_2$) of this reimbursement scheme but allowed reimbursement of both the winner and the loser. They showed that there is an interior equilibrium for any α. They also obtained that under external reimbursement to the loser, the designer's payoff declines in α. Instead, we study situations that allow asymmetry between the prize valuations and show (in Section 2.2) that the optimal α for the designer's payoff decreases as the gap between the asymmetries increases.

Given that $V_2 < V_1$, the inequality $0 < x_2^{*C} < x_1^{*C}$ is necessarily satisfied. Tullock CSF then ensures that Player 1's winning probability remains larger than Player 2's: $p_2^* < p_1^*$. In addition, the reimbursement can be complete ($\alpha = 1$) in this equilibrium. In other words, fairness and viability are satisfied, and full reimbursement can be satisfied as well.

Part 2: When the winner reimburses the loser (Scheme D). Then the payoff functions are:

$$\pi_1^D = (V_1 - \alpha x_2)\left[\frac{x_1}{x_1+x_2}\right] + \alpha x_1\left[1 - \frac{x_1}{x_1+x_2}\right] - x_1 \tag{20}$$

$$\pi_2^D = (V_2 - \alpha x_1)\left[\frac{x_2}{x_1+x_2}\right] + \alpha x_2\left[1 - \frac{x_2}{x_1+x_2}\right] - x_2 \tag{21}$$

Since $\frac{x_1}{x_1+x_2} + \frac{x_2}{x_1+x_2} = 1$,

$$\pi_1^D = V_1\left[\frac{x_1}{x_1+x_2}\right] - x_1 = \pi_1^T \tag{20'}$$

and

$$\pi_2^D = V_2\left[\frac{x_1}{x_1+x_2}\right] - x_2 = \pi_2^T \qquad (21')$$

This brings us back to Tullock's benchmark contest, in which full reimbursement, fairness, and viability are all satisfied. **Q.E.D.**

To sum up, Theorems 1 and 2 clarify why full reimbursement of the winner's expenses is unlikely to be realized under certain desirable properties, whereas it is possible when the loser is the recipient. This is because to get a viable contest, there should be some room for competition. Reimbursing the winner may discourage the players, whereas repaying the loser in full provides incentives to "enter" the contest while repaying the winner does not.

2.4. Designer payoff

Consider a contest designer who wishes to maximize their payoff ($\pi_d^{*\ S}$): the difference between the players' total effort and the amount of the reimbursement to one of them in Scheme S, while upholding both viability and fairness. Such an objective is plausible when efforts are transferred to the designer, as often seen in rent-seeking literature or where the effort is a direct source of revenue for the designer as in sports.

Finding 1. When the prize value asymmetry (V_1/V_2) is not sufficiently large, the optimal reimbursement parameter α for designer payoff maximization in case of internal reimbursement for the loser (Scheme C) is $\alpha = 0$. But when it is sufficiently large, the optimal α is $= 1$.

Finding 2. When the reimbursement is internal, reimbursement for the winner (Scheme B) is always more payoff dominant for the designer than reimbursement for the loser (Scheme D).

Finding 3. When the reimbursement is external and the prize value asymmetry is (is not) sufficiently large, then reimbursement for the winner (Scheme A) is payoff dominated (dominant) for the designer than reimbursement for the loser (Scheme C with optimal α).

Proof and simulations: Note that due to the mathematical-algebraic difficulty in extracting the equilibrium of Scheme C. Finding 2 is proven while Findings 1, 3 are illustrated by simulations.

By applying Reimbursement Scheme A, in interior equilibrium with a full reimbursement, the designer can secure the following maximal expected net payoff (Matros & Armanios, 2009):

$$\pi_d^{*\,A} = x_1^{*A} + x_2^{*A} - x_1^{*A}\frac{x_1^{*A}}{x_1^{*A}+x_2^{*A}} - x_2^{*A}\frac{x_2^{*A}}{x_1^{*A}+x_2^{*A}} = \frac{2V_1V_2}{V_1+V_2} = 2X^T \qquad (22)$$

Note that in Scheme A, the total effort and the designer's payoff are not directly tied to the number of players, meaning they can either increase or decrease (Liu & Dong, 2019).

By Theorem 1, Scheme A of external reimbursement of the winner is inconsistent with viability and fairness. In other words, a designer who insists on upholding these properties cannot apply Scheme A. By Theorem 2, these properties are satisfied when the designer applies Scheme C of external reimbursement to the loser. In this case, their expected net payoff is:

$$\pi_d^{*\,C} = x_1^{*C} + x_2^{*C} - \alpha x_1^{*C}\frac{x_2^{*C}}{x_1^{*C}+x_2^{*C}} - \alpha x_2^{*C}\frac{x_1^{*C}}{x_1^{*C}+x_2^{*C}} = x_1^{*C} + x_2^{*C} - \frac{2\alpha x_1^{*C} x_2^{*C}}{x_1^{*C}+x_2^{*C}} \qquad (23)$$

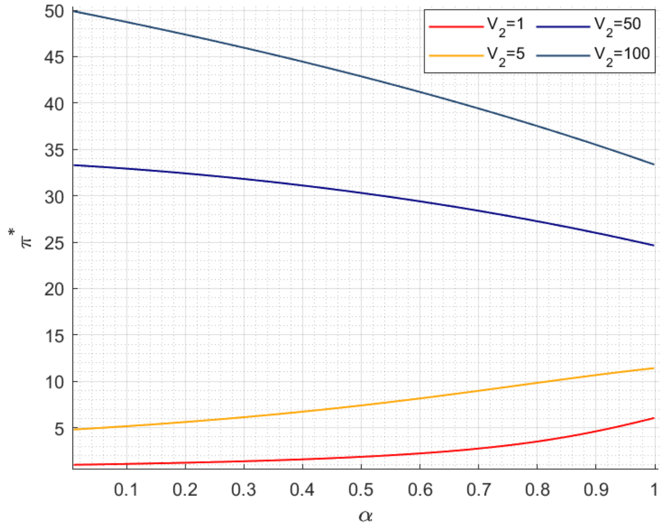
Given the algebraic difficulty of extracting the players' equilibrium efforts as a function of the prize values from Eq. 18, we use simulation to compare the Designer's payoff margins between Scheme C with other schemes. We start by comparing Scheme C with Scheme A.

Let $V_1 = 100$. Assuming 12 alternative prize valuations V_2 of Player 2 and using the first-order conditions (16) and (17), we obtain by simulation the equilibrium x_1^*, x_2^* and π_d^* of Scheme C, when $0 \le \alpha \le 1$ (but, for convenience, only $\alpha = 0$, $\alpha = 0.5$ and $\alpha = 1$ are shown). We also show the equilibrium x_1^*, x_2^* , and π_d^* of Scheme A and Scheme D (which is equal to the classic Tullock model) and Scheme C (when $\alpha = 0$). See Table A1 in the Appendix.

Matros and Armanios (2009) showed that symmetric external reimbursement for the winner is payoff dominant over external reimbursement for the loser. We extend their result by simulation and show in Figure 1 that when there is an asymmetry in prize valuations, and it is sufficiently high, then reimbursing the loser is payoff dominant (for the designer) than reimbursing the winner. That is, when the asymmetry (V_1/V_2) is sufficiently high, the result of Matros and Armanios is reversed and the optimal α for the designer's payoff decreases is 0.

In the simulation, when the asymmetry is sufficiently large ($V_1/V_2 > 6.786$), the Designer's payoff is larger in Scheme C than in Scheme A, $\pi_d^{*\,C} >$, when $\alpha = 1$. In such a situation, Scheme C is not only consistent with viability and fairness but gives the designer an advantage (added value) relative to Scheme A. Recall that this advantage exists by Theorem 2, avoiding the reversal of the players' winning probabilities.

Figure 1. Designer's payoff in Scheme C, when $V_1 = 100$ and V_2 takes values 1, 5, 50, and 100



We now compare the Designer's payoff when he applies Schemes B and D. In the former case, the payoff is equal to the total effort, so by (9) and (10),

$$X^{*B} = x_1^{*B} + x_2^{*B} = V_1 V_2/[(1-\alpha)(V_1+V_2)] \qquad (24)$$

As noted in Section 2.1, since positive efforts and payoffs constrain the reimbursement rate α (it cannot be complete), the maximal payoff is obtained at the largest possible α (see Eq. 13). Hence:

$$\alpha_{max}^B = V_2^2/(V_1^2+V_2^2) \qquad (25)$$

which yields the maximal payoff:

$$\pi_d^{*\,B} = \left[V_2\left({V_1}^2+{V_2}^2\right)\right]/[V_1\,(V_1+V_2)] \qquad (26)$$

Since Scheme D, with a full reimbursement, is equivalent to no intervention (the benchmark Tullock contest, see Part 2 of the proof of Theorem 2), the maximal payoff is:

$$\pi_d^{*\,D} = V_1V_2/(V_1+V_2) = X^T$$

Hence, Scheme B with $\alpha = \alpha_{max}^B$ is payoff dominant over Scheme D. That is:

$$\pi_d^{*\,B} = \frac{V_2\,({V_1}^2+{V_2}^2)}{V_1\,(V_1+V_2)} > \frac{V_2\,({V_1}^2)}{V_1\,(V_1+V_2)} = \frac{V_1V_2}{V_1+V_2} = \pi_d^{*\,D}.$$

Q.E.D.

Finding 4. When reimbursement goes to the winner, external reimbursement (Scheme A) is always payoff-dominant over the most effective internal reimbursement scheme (Scheme B).

Finding 5. When reimbursement goes to the loser and prize value asymmetry is not sufficiently large, then the internal reimbursement (Scheme D) is payoff-dominant over the external reimbursement (Scheme C).

Finding 6. External reimbursement for the winner (Scheme A) is always payoff-dominant than internal reimbursement for the loser (Scheme D).

Proof and simulations: Note that due to the mathematical-algebraic difficulty in extracting the equilibrium of Scheme C, Findings 4, 6 are proved while Finding 5 is obtained by simulation.

Let us start by comparing Scheme A with Scheme B. By (22) and (26), ${\pi_d^*}^A$ is always greater than ${\pi_d^*}^B$. Now, applying the simulation as in the previous section and noting that ${\pi_d^*}^D = X^T$ and ${\pi_d^*}^A = 2X^T$, which means ${\pi_d^*}^A > {\pi_d^*}^D$, we get Table A1 (see Appendix A), in which ${\pi_d^*}^C$can be compared with ${\pi_d^*}^D$. It turns out that only when the prize value asymmetry is sufficiently large, i.e., $V_1/V_2 > 23.421$ (solved by simulations, see also Columns 1, 2, 5, and 8 in Table 2), Scheme C is payoff dominant to Scheme D. That is, the designer would rather reimburse the loser than let the winner do it. **Q.E.D.**

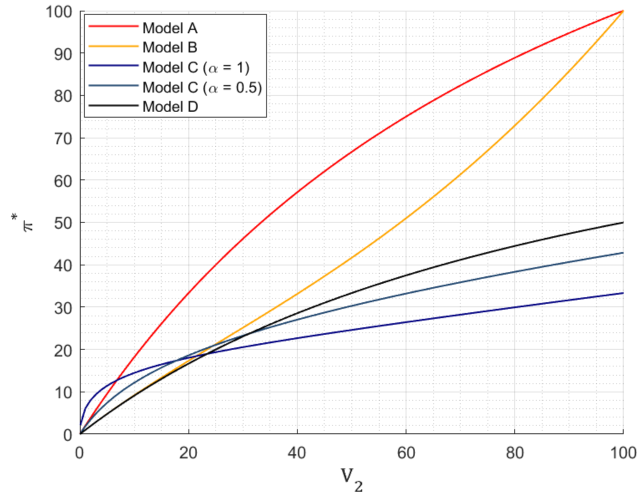
Finding 4 might appear to be counterintuitive – as the designer is willing to reimburse the winner with their own fund rather than allowing for the loser do it for themself. However, note that an internal reimbursement scheme will reduce the incentive to compete as the loser payoff will be too low. This discouragement is alleviated when the designer reimburses the winner. Similarly, when the prize value is more symmetric, then the players expend more resources than when it is not. When the designer will have to reimburse it, it outweighs the gain of the extra revenue. Hence, as we see in Finding 5, an internal scheme provides higher payoff for the designer.

By Findings 2–6, we get:

Result 1. When the prize value asymmetry is not large enough, the reimbursement schemes can be ranked in terms of designer payoff as: $A \succ B \succ D \succ C$.

This result is visualized in Figure 2. The graphs in Figure 2 specify the designer’s expected net revenue (payoff) when he applies Schemes A, B, C ($\alpha = 0.5$ and $\alpha = 1$), and D.

Figure 2. Comparison of Designer's payoff in Schemes A, B, C, and D, when $V_1 = 100$.



2.5. Player efforts

As mentioned in the Introduction, the designer may wish to maximize the players' total effort, especially in patent (R&D) races in which the quality of the final product, which depends on the effort expended, is the designer's main goal. This is also relevant for the contests with social externalities (such as R&D and education) where more effort benefits society. As shown by Cohen and Sela (2015), the optimal α of Scheme A for the objective of maximization of efforts is $\alpha = 1$.

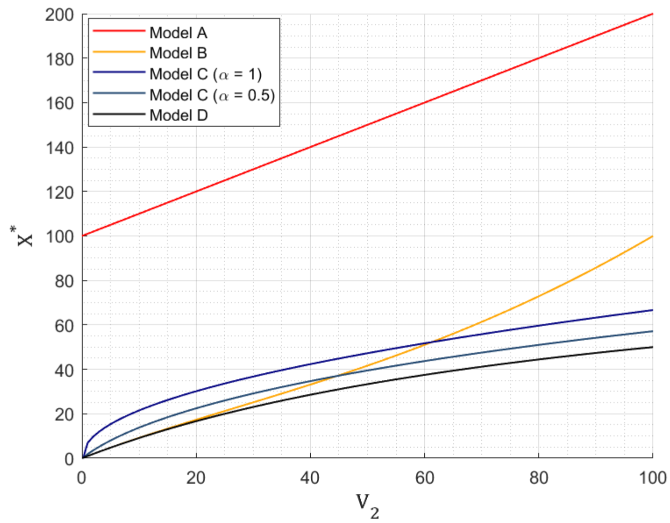
When the reimbursement is internal (Schemes B and D), the total effort is equal to the designer's net payoff (because the designer is not involved in reimbursement), so the optimal α of Scheme B in this case is the maximal alpha, $\alpha = \frac{V_2^2}{(V_1^2+{V_2}^2)}$ (note, though, that the equilibrium of Scheme D is independent of α).[4,5] By Lemma 1, Finding 1, and Theorem 2, total effort under Schemes A, B, and D is $X^{*A} = V_1 + V_2$, $X^{*B} = \frac{V_2\,({V_1}^2+{V_2}^2)}{V_1\,(V_1+V_2)}$, $X^{*D} = \frac{V_1 V_2}{V_1+V_2}$. It is easy to see that $X^{*A} > X^{*B} > X^{*D}$.

By Table 2 and Figure 3, $X^{*A} > X^{*C} > X^{*D}$. The total efforts in Scheme C increases with α, so, here too the optimal α is 1. Our results complement and add to the results of Baye et al. (2012) who studied different types of internal reimbursement in an all-pay auction setting.

[4] Chen and Rodrigues-Neto (2023) applied monetary and emotional preferences in Scheme B and found that if the litigants' relative advantages are sufficiently balanced, an increase in either reimbursement expenses or negative relational emotions increases total efforts in equilibrium.

[5] In contrast to our paper, which focuses on symmetric reimbursement rates, Baik and Shogren (1994) found that Scheme C with asymmetric reimbursement rates reduces effort compared to symmetric rates of return.

Figure 3. Comparison of total effort in Schemes A, B, C, and D, when $V_1 = 100$



Notice that a winner-pay contest corresponds to Scheme C, where the loser receives full external reimbursement ($\alpha = 1$), while the all-pay contest corresponds to Scheme D, which equals a non-intervention Tullock contest. Lagerlöf (2020) studied hybrid winner-pay and all-pay contests and found that the total effort is always lower than in a corresponding all-pay contest with symmetric prizes. This result is consistent with ours. However, in all-pay auctions with variable rewards under incomplete information, where the reward is also a function of the bidder's own bid (a form of reimbursement), there is potential for paradoxical behavior: reducing rewards or increasing costs may lead to a higher expected sum of bids or, alternatively, a higher expected maximum bid (Kaplan et al., 2002). Yates (2011) shows that in winner-pay contests, a pure-strategy Nash equilibrium exists and is unique under weak assumptions on the contest success function.

2.6. Comparison between reimbursement schemes

In this study we investigate four schemes of reimbursement in contests in terms of payoff and effort. We find that reimbursing the winner for their expenses cannot satisfy the three criteria of complete reimbursement, fairness, and viability. Only reimbursing the loser can be consistent with these criteria. Scheme A can be viable but cannot be fair because it reverses the initial winning probabilities. By sufficiently increasing their investment, the weaker player can come away better off. Under complete reimbursement, Scheme B cannot be viable and cannot ensure positive efforts by both rivals. Such a viability requires that the rate of reimbursement does not exceed a certain limit (see Eq. 13). Any rate above this constraint causes the players to refrain from participating because they will have to pay the loser's expenses if they win, making their expected payoff negative. Complete internal reimbursement for the winner eliminates the incentives to enter the contest while repaying the loser does not. If the prize value asymmetry is sufficiently large, then Scheme C provides the highest payoff to the designer. In the usual case in which the gap is not particularly large, Scheme A is payoff dominant. However, the total effort is always the largest in Scheme A and the smallest in Scheme D. Table 2 summarizes the findings presented in Section 2.

Table 2. Comparing the four schemes of reimbursement

Scheme	Full reimbursement	Fair	Viable	Payoff rank		Effort rank
				Low asymmetry	High asymmetry	
A	✓	×	✓	1	2	1
B	×	✓	✓	2	3	2/3
C ($\alpha = 1$)	✓	✓	✓	4	1	2/3
C (α: optimal)	✓	✓	✓	3	1	2/3
D	✓	✓	✓	3	4	4

Note: Scheme D equals to Schemes A, B and C when $\alpha = 0$. For Scheme B full reimbursement, recall that we assume the maximal partial reimbursement rate that ensures viability: $\alpha = V_2^2/(V_1^2 + V_2^2)$.

Table 3. Optimal α in reimbursement schemes

Scheme	Target	Source	Designer objective		
			Payoff (low value asymmetry)	Payoff (high value asymmetry)	Effort
A	Winner	External	$\alpha = 1$		
B	Winner	Internal	$\alpha = V_2^2/(V_1^2 + V_2^2)$		
C	Loser	External	$\alpha = 0$	$\alpha = 1$	
D	Loser	Internal	Unaffected by α		

Table 3 presents the optimal α for each objective function (payoff with various value asymmetry, effort) of the designer in each of the four reimbursement models. If the designer chooses to use Scheme A, the optimal reimbursement rate is $\alpha = 1$ for each of the possible objective functions.[6] While Scheme D is not affected by α at all, the optimal α in Scheme B is the maximal $\alpha =$

[6] For a nonlinear reimbursement function, Lie and Dong (2019) showed that if the effort cost function is concave (convex), the optimal reimbursement scheme is (not) to return the full cost to the winner. In addition, Minchuk (2018) showed that if the effort cost function is concave (convex), then reimbursement increases (decreases) designer payoff.

$V_2^2/(V_1^2 + V_2^2)$. In Scheme C, when the designer is interested in effort or in payoff when prize value asymmetry is not high enough, the optimal alpha is $\alpha = 1$. But when he is concerned with payoff and the value asymmetry is not sufficiently large, the optimal reimbursement rate is $\alpha = 0$.

3. Discussion

In this study, we set out to identify the optimal reimbursement scheme for different objectives: maximizing designer payoff or incentivizing effort. Our analysis addressed the factors of full reimbursement, fairness, and viability, with applications in both patent races and litigation contests as key case studies.

Our contribution builds upon existing literature on reimbursement and spillovers in Tullock contests (e.g., Chowdhury et al., 2011a, 2011b; Matros & Armanios, 2009) by extending the analysis to Schemes B and D and to Scheme C with asymmetric prize valuations. We find that full reimbursement is generally optimal in Scheme C, even in symmetric value cases, except when maximizing payoff is the sole objective. In Schemes A and C under symmetry we derived the optimal reimbursement rates for objectives that include payoff, effort, and considerations for fairness and viability. Notably, our findings confirm that Scheme A, with a full reimbursement rate, is the most effective across almost all objectives, though it falls short in fairness.

In exploring the necessity of the all-pay condition in fair, non-discriminatory contests, we find that the complete elimination of all-pay is achievable by fully reimbursing one player's expenses – a concept partly implemented in civil litigation through partial reimbursement by the losing party to the winner. Our results show that this full reimbursement is only viable when the recipient is the losing-party. This effectively supports the weaker player, though this aligns with the designer's interest rather than any ethical motive.

Given that full reimbursement typically benefits the losing party, we analyzed the designer's preferences between external and internal reimbursement schemes (Schemes C and D). The optimal choice depends on the designer's objectives and the degree of asymmetry in prize valuations. If the designer seeks to reduce disparities in winning probabilities, external Scheme C consistently outperforms internal Scheme D. For objectives focused on maximizing net payoff, Scheme C remains favorable, but only when prize valuation asymmetry is substantial. In contexts where total effort is important, such as R&D competitions and litigation, Scheme C is again preferred, as external reimbursement of the loser's expenses yields greater overall effort than internal reimbursement through Scheme D.

Our investigation can be extended in various ways. First, the analysis can be extended to more than two players for all four schemes of interest. This will reflect the situations in multi-party competition in R&D. Second, different types of spillover effects (as in Baye et al., 2012 or Chowdhury et al., 2011) or beyond what we consider can also be introduced. This will capture situations beyond R&D and litigation. It will also be possible to show strategic equivalence between different types of reimbursement schemes (Chowdhury and Sheremeta, 2015). Third, both the cost function and the utility function can be generalized with nonlinearity with curvature. The latter one, specifically, will allow capturing the effects of the reimbursement schemes on risk-averse preferences (Liu & Liu, 2019). Fourth, although affirmative action is a broad area that goes beyond simple reimbursement (See Chowdhury et al., 2023; Mealem & Nitzan, 2016) in contests, it will be possible to analyze reimbursement scheme as a tool of affirmative action and compare those with other relevant tools. Finally, there is a scarcity in the experimental literature in analyzing reimbursement schemes and relevant contest design issues. Our study can be used as a theoretical benchmark in experiments to understand behavioral foundations in such contests.

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Appendix.

Table A1. Simulated equilibrium expected net payoff under Schemes A, C, and D.

Value		Scheme A			Scheme C ($\alpha = 0.5$)			Scheme C ($\alpha = 1$)			Scheme D=T		
V_1	V_2	X_1^A	X_2^A	π_d^A	X_1^C	X_2^C	π_d^C	X_1^C	X_1^C	π_d^C	X_1^D	X_2^D	$\pi_d^D=X^T$
100	1	1	100	1	0.20	0.003	0.20	6.47	0.48	6.06	0.98	0.01	0.99
100	5	5	100	5	1.85	0.04	1.85	13.03	2.30	13.03	4.54	0.23	4.76
100	10	10	100	10	11.88	1.88	12.14	17.07	4.43	17.07	8.26	0.83	9.09
100	20	20	100	20	17.54	4.92	18.62	21.75	8.38	21.75	13.89	2.78	16.67
100	30	30	100	30	20.97	8.16	23.25	24.69	12.05	24.69	17.75	5.33	23.08
100	40	40	100	40	23.29	11.38	27.02	26.81	15.05	26.81	20.41	8.16	28.57
100	50	50	100	50	24.94	14.53	30.29	28.45	18.79	28.45	22.22	11.11	33.33
100	60	60	100	60	26.15	17.56	33.20	29.75	21.90	29.75	23.44	14.06	37.50
100	70	70	100	70	27.02	20.47	35.84	30.85	24.90	30.85	24.22	16.96	41.18
100	80	80	100	80	27.69	23.28	38.32	31.81	27.82	31.81	24.69	19.75	44.44
100	90	90	100	90	28.20	25.98	40.65	32.62	30.62	32.62	24.93	22.44	47.37
100	100	100	100	100	28.57	28.57	42.86	33.33	33.33	33.33	25.00	25.00	50.00