Balancing Democracy: Majoritarianism vs. Expression of Preference Intensity

by

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Abstract

Direct confrontation between majority rule (MR) and the most in-depth studied scoring rules—the Borda rule (BR) and the plurality rule (PR)—on the basis of their fundamental weaknesses (violating one of the two principles: majoritarianism and suitable recognition of preference intensity) has not been attempted. Defining the cost of a rule as the expected erosion of the principle that it violates, this study fills the gap by comparing MR with PR and BR in terms of their mutual costs. The main findings are the evident superiority of PR over MR and the superiority of MR over BR.

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Two eminent principles (ideals) with which social-aggregation rules should comply are majoritarianism and respect of voters' preference intensity. These principles ensure respect of the preferences of a simple majority and the protection of a minority with sufficiently strong preferences, respectively. The tension between them undergirds the debate between supporters of MR and advocates of some scoring rule, notably the Borda rule (BR), see footnote 12, or the most common plurality rule (PR), see opening sentence in section 3.3.

So far, typical arguments in favor of MR, BR, and PR are couched in axiomatic terms: showing that only these aggregation rules satisfy desirable requirements regarding the relationship of individual and social preferences. Two alternative arguments for MR are the epistemic one, based on Condorcet's jury theorem (List and Goodin 2001) and the consequentialist-utilitarian one (Rae 1969, Brighouse and Fleurbaey 2010, Nakada et al. 2021), which is applicable in the constitutional stage, where the veil of ignorance prevails.

However, no direct confrontation has been attempted between MR and either BR or PR on the basis of their fundamental weaknesses, viz., the violation, respectively, of majoritarianism and of some recognition of preference intensity (the two principles presented above). The objective of the current study is to undertake the challenge of filling this gap by comparing, first, MR and BR and then MR and PR in terms of their mutual costs (the cost of a rule is defined as *the erosion of the principle that it violates*). The two main findings reported below are the evident superiority of PR over MR and the superiority of MR over BR. These results shed new light on the old controversy between Condorcet and Borda and their followers, who fiercely attacked PR before defending, respectively, MR and BR.[1]

[1] Notice that Condorcet and Borda presented MR and BR as different solutions to problems that both identified with PR. One of these problems is that under PR a society may end up choosing

1. The novelty of our approach and independence properties of MR and BR

Let society *N* consist of *n* voters and suppose that the set of social alternatives, *X*, has *m* elements. R_i denotes the preference relation of individual *i*, which is assumed to be strict ordering, and $\boldsymbol{R} = (R_1, \ldots, R_n)$ is a preference profile. An aggregation rule is a mapping from the set of possible profiles to the set of possible reflexive and complete social-preference relations. (Here we do allow for indifference and the typical social-preference relation is *R*.)

According to Arrow's (1963) Independence of Irrelevant Alternatives (IIA) axiom, social preferences between any two alternatives must depend only on the individual preferences between them. IIA implies that social preferences disregard information about individuals' preference intensity in regard to these alternatives as well as evaluations or comparisons of these alternatives relative to others. The easily grasped aggregation rule based on majority comparisons is the clearest and most celebrated example of a rule that satisfies IIA. Obviously, in pairwise voting based on MR, the majority is decisive, namely, the social preference is robust to the intensity of the minority preferences.[2] The typical resolution of such decisiveness is by increasing the required majority or augmenting simple-majority rule with constitutional constraints that protect the minority. In the former case, the required special majority becomes greater than one-half, implying that the special-majority rule becomes biased in favor of one of the alternatives, usually the status

a Condorcet loser. The proposed criterion for comparing MR with PR takes this difficulty into account via the significance assigned to the erosion of majoritarianism.

[2] As Baharad and Nitzan (2002) clarify "Although tyranny of the majority and majority decisiveness are different notions and one does not imply the other, they share a common problematic aspect: their existence entails a large set of losers (the members of the permanent minority group or the members of any minority group) likely to be deeply alienated from the political system. Majority decisiveness is nevertheless a weaker, less menacing and, perhaps, even an acceptable property of voting rules relative to the tyranny of the majority for two reasons. First, the incidence of effective concentration of decision-making power implied by majority decisiveness is not restricted to a specific permanent majority group, so it does not imply factionalism. Second, the incidence of majority decisiveness is not restricted to 'unjust' alternatives, so it is not necessarily bad or undesirable."

quo.[3] In the latter case, the augmented rule becomes biased in favor of the minority.

However, the amelioration of majority decisiveness, which requires unanimity among members of the majority regarding the best alternative, can be attained by scoring rules that allow some restricted expression of preference intensities (Baharad and Nitzan 2002).[4] The restriction has two aspects. First, the implied cardinal and interpersonally comparable utilities take a particular arbitrary form that depends just on the ranking of the alternatives. Second, the individuals are assumed to share the same ranking-dependent utilities.

Our combined approach of ordinal and ranking-based utilitarianism is very different from the typical unrestricted utilitarian approach that works mostly with the standard principles (Benthamite, Rawlsian, etc.), which is very difficult to apply in comparing alternative voting rules.[5] More explicitly, the typical utilitarian approach compares the expected social welfare (Benthamite, Rawlsian, etc,) obtained under different voting rules. In contrast, in our approach, the social planner compares MR to a scoring rule on the basis of expected deviation from the two fundamental democratic principles that are assumed to be of equal significance (not on the basis of some standard utilitarian principle). Since MR implies ordinal utilities, the proposed measure of expected deviation of a scoring rule from the

[3] Bahard and Nitzan (2016) identify a unique flexible scoring rule that may be consistent with any special majority, creating a plausible minimal requirement for decision rules inspired by majority-based utilitarianism.

[4] An alternative rule that has the flavor of a standard scoring rule has been proposed by Hortala-Vallve (2012) in order to protect the minority by allowing the intensity of voters' preferences to find expression. This voting rule is applied in a restricted non-standard social-choice setting where voters face a number of independent issues, each of which has to be approved or dismissed. Another rule, storable voting, was proposed by Casella (2005) for an even more limited setting of binary decisions. It allows voters to express their preference intensity by shifting votes not among issues but inter-temporally. The concern to select a voting rule that incorporates in some way preference intensity also underlines the majority judgement approach of Balinski and Laraki (2010) and the more recent implicit utilitarian- voting approach, Caragiannis et al. (2017).

[5] In particular, it is very different than the growing recent approach that tries to understand how simple voting mechanisms may help to communicate the intensities of individual preferences about social states, Ambuehl and Bernheim (2021).

majority principle is naturally defined in terms of the number of individuals who prefer one alternative to another one, and not in terms of cardinal interpersonally non-comparable individual utilities. Since scoring rules imply some restricted form of cardinal and personally comparable utilities, the proposed measure of expected deviation of MR from the principle of allowing expression of preference intensity is naturally defined in terms of the ranking-based utilities of the two scoring rules on which we focus.[6] Both measures of deviation from the fundamental principles obviously depend on the assumed preference distributions in the population and they take into account all possible preference profiles and all possible compared alternatives.

Hereinafter, we focus on the most widely studied monotonic scoring rule, BR, and the most common weakly monotonic scoring rule, PR. Given the plausibility of a complete veil of ignorance in the constitutional stage, BR is a reasonable representative scoring rule.[7] Thus, we first compare MR with BR and then compare MR with PR.

The effectiveness of a given scoring rule in resolving the majority-decisiveness problem is valid when voters sincerely reveal their preferences as well as when they resort to variable degrees of coordinated strategic voting. In both cases, BR is immune to simple-majority decisiveness if the number of alternatives is larger than two.[8] This rule does not satisfy IIA but it does satisfy the weaker Modified IIA (MIIA) recently proposed by Maskin (2022), which allows a particular form of

[6] Interestingly, Bossert and Suzumura (2017) have shown that, with their alternative articulation of the Benthamite greatest-happiness of-the-greatest-number principle and with ordinally measurable and interpersonally non-comparable utilities, the social decision rule chooses those alternatives that maximize the number of individuals who end up with their greatest element. This rule is tantamount to PR.

[7] For further enthusiastic support of BR, see Saari (2006). The two most recent axiomatizations of BR appear in Heckelman and Ragan (2020) and Maskin (2022).

[8] The simple-majority decisiveness studied by Baharad and Nitzan (2002) requires that the majority voters endorse the same best alternative.

preference-intensity expression. This condition requires that if two profiles and two alternatives x and y are given, and if each individual ranks the two alternatives the same way in both profiles and ranks the same number of other alternatives between x and y in both profiles, then the social preference between these two alternatives is the same for both profiles.

BR violates the majority principle in those instances (preference profiles and pairs of compared alternatives) where it protects the minority effectively by taking into account its higher preference intensity rather than the majority's lower preference intensity. While the emphasis in Baharad and Nitzan (2002) is on the different degrees of majority-decisiveness amelioration that different scoring rules provide, in the current study the focus is on the comparison between MR and the two most common scoring rules on the basis of the severity of the problems they cause: disregarding expression of preferences (which implies prevention of effective expression of preference intensity by the minority) and violating the majority principle. Let us now define the severity of the two problems in a way that enables comparison of the "costs" associated with applying the two alternative democratic voting rules and, in turn, the preference of MR or BR.[9]

2. The severity of violating the two fundamental principles

One possibility is to measure the severity of a problem by the probability of its occurrence. Baharad and Nitzan (2007, 2011) take such an approach, focusing on comparing alternative scoring rules. Here, in contrast, we focus on the Condorcet–Borda (binary–positional) controversy and our approach is different in two substantive respects. First, in our study, the problem of disregarding preference intensity (and, in particular, the intensity of the minority's preferences) arises not only under majority decisiveness, in which the majority members share a common view regarding the best alternative, but under all circumstances in which the

[9] Analogous definitions apply for the comparison of MR and PR.

majority preference between any two alternatives is respected. Second, the severity of a problem is defined *not* in terms of the probability of its occurrence, taking into account the emergence of all possible preference profiles and any pair of alternatives, but in terms of the expected erosion of the two principles. Erosion of the majority principle takes into account all possible preference profiles and any pair of alternatives where the majority's preference is overlooked. Erosion of the second principle (respect of voters' preference intensity) takes into account all possible preference profiles and any pair of alternatives where the disregard of preference intensity implies that the minority's preference intensity is disregarded, despite the fact that it is larger than that of the majority.

Our analysis may yield a flexible, "case-dependent" choice between the two aggregation rules. One rule may be superior for certain preference profiles and pairs of alternatives whereas the other rule may prevail for others.[10] The implementation of such flexibility, requiring the practical partitioning of the set of pairs of alternatives, may involve considerable difficulties. Our objective, therefore, is to identify the preferred aggregation rule (just one of the two rules and not a flexible, case-dependent rule) on the basis of its lower expected violation of a fundamental principle. That is, the expected severity of the problem that it raises should be lower than that of the problem associated with using the alternative aggregation rule. The expected severity of the compared rules is referred to as their expected cost. The main contribution of our study is the clarification of the expected costs of the rules and the use of these expected costs to determine the superiority of one of them. In other words, given a plausible definition of the expected cost of using the two rules,

[10] The justification of deviating from MR in order to protect the minority is typically deemed plausible when the two alternatives result in substantially different long-run irreversible outcomes and is usually implemented by applying a qualified majority rule. Recently, Barberá et al. (2021) studied the hybrid rule proposed by Daunou, which deviates from MR when a Condorcet winner does not exist by applying PR after eliminating the Condorcet losers. In this case, the reconciliation of conflicting desiderata is based on accommodating them lexicographically.

we propose the selection of one of them, MR or BR, for any preference profile in $\mathfrak{R}^n$ and every x and y in X .

Given a specific situation, namely preference profile **R** and pair of alternatives x and y, we first measure the corresponding cost of applying BR in terms of the erosion of the first principle (taking into account preference intensity and, in particular, the minority's preference intensity), C(BR, **R**, x**,** y); then we measure the cost of applying MR in terms of the erosion of the second principle, C(MR,**R**, x, y). The application of MR is warranted in a specific situation, if C(MR, **R**, x**,** y) < C(BR, **R**, x**,** y); the application of BR is warranted if C(BR, **R**, x**,** y) < C(MR, **R**, x**,** y). And in case C(MR, **R**, x**,** y) = C(BR, **R**, x**,** y), the use of either MR or BR is justified.[11,12] This may ensure an ideal flexible situation-dependent balanced democracy that applies in every particular situation the aggregation rule associated with the lower cost. As already noted, however, such a flexible situation-dependent aggregation rule is difficult to implement because it requires information about the voters' actual preference profile. Therefore, we impose the restriction that the same aggregation rule must be applied to any pair of alternatives and any preference profile. Given this restriction, let us turn to the comparison of MR with BR.

2.1. Proposed measures of erosion of the two principles

Consider, first, the cost of applying MR. It involves the possible erosion of the principle of allowing expression of preference intensity, which implies that the minority can win when its preference intensity exceeds that of the majority (Principle 1). This principle is eroded when, given a specific preference profile, MR

[11] An analogous criterion is applied in Section **3,** in the comparison between MR and PR where the cost of BR, C(BR,**R**,x,y), is replaced by the cost of PR, C(PR,**R**,x,y).

[12] The endogenous partition of the set of profiles takes into account the costs of the rules assigning equal weights to these costs. But one can easily enrich the approach by assigning different weights to the costs C(MR, **R)** and C(BR,**R)** that are associated with the application of MR and BR.

and BR yield different social preferences between two alternatives *x* and *y*. Let us present a natural and intuitive measurement of the erosion.

Suppose that, given a specific preference profile, alternative *x* is preferred over *y* under MR and that alternative *y* is preferred over *x* under BR because the score of *y*, B(*y*), is higher than the score of *x*, B(*x*). The positive difference between these scores, (B(y) - B(x)), is referred to as the unrealized advantage of the preference intensity of the minority. The first measure is the share of the minority's preference intensity that actually erodes due to the use of MR that yields the social preference of alternative *x*, despite its inferiority to *y* according to BR. This inferior alternative should not be preferred according to Principle 1. The proposed measure of erosion of Principle 1, for a particular preference profile and two given alternatives *x* and *y*, given that MR and BR result in different social preferences is the unrealized advantage of the preference intensity of the minority relative to its total intensity.

Measure 1: (B(y) - B(x)) / B(*y*)

Given a specific preference profile and pair of alternatives, Measure 1 represents the erosion of the minority's ability to effectively express its preference intensity and ensure the superiority that its preferred alternative would have enjoyed had BR been used as the aggregation rule.

When the erosion of Principle 1 materializes under specific profile **R** and two alternatives *x* and *y*, the majority prefers *x* over *y*, the minority prefers *y* over *x*, and the Borda count of *y* exceeds that of *x*, B(*y*)>B(*x*).[13] Note that the advantage in terms of preference intensity of *y* over *x*, B(*y*)-B(*x*), is equal to the difference between the

[13] According to the Borda rule (m-1), points are assigned to the best alternative ranked at the top, (m-2) points are assigned to the second-best alternative, and so on. (No points are assigned to the worst alternative.)

minority's preference intensity of *y* over *x* and the majority's preference intensity of *x* over *y*. So when Principle 1 is not respected, this advantage is not reflected in the social preference because *x* is socially preferred over *y* under MR.

To sum up, the proposed measure of the cost of MR in a particular situation is the rate of reduction in the more intense minority's preference of the socially inferior alternative in terms of preference intensity relative to the actually socially superior alternative under MR.

Consider now the second majority principle: it requires that *x* is socially preferred over *y* when a majority prefers *x* over *y*. The cost of applying BR involves the possible erosion of this principle, Principle 2. When such a possibility is realized under specific profile **R** and two alternatives *x* and *y*, the Borda count of *x* exceeds that of *y*, that is, $(B(x)-B(y))>0$, and a majority prefers *y* to *x*, that is, the number of individuals who prefer *y* to *x*, $N(y,x)$, is larger than the number of individuals who prefer *x* to *y*, yet *x* is socially preferred to *y* by BR because the minority's preference intensity of *x* over *y* exceeds the majority's preference intensity of *y* over *x*. Analogously to the measurement of erosion of Principle 1, for a specific preference profile and pair of alternatives *x* and *y*, the measurement of erosion of Principle 2 takes the form:

Measure 2 = $(N(y,x)-N(x,y))/N(y,x)$

The proposed measure of the cost of BR is the disregarded advantage of the majority obtained by alternative *y* (which is superior to alternative *x* under MR), divided by the actual majority of *y*,.[14] In other words, Measure 2 represent the erosion of the majority's ability to effectively express its preference and ensure the superiority that its preferred alternative would have enjoyed had MR been used as the aggregation rule.

[14] Analogous measures are applied for PR.

Given a preference profile and a pair of alternatives, comparison of the costs of applying MR and BR—the costs being measured by the relative erosion of the principle that is not respected by the rule—is the basis for selecting one aggregation rule over the other. The preferred rule is the less costly one.[15]

2.2. Illustration

Example 1

Suppose $N = \{1,2,3\}$, $X = \{ w,x,y,z\}$ and the preference profile is $\mathbf{R}=(R_1, R_2, R_3)$:

$$R_1: yR_1zR_1wR_1x$$
$$R_2: xR_2yR_2zR_2w$$
$$R_3: zR_3yR_3wR_{31}x$$

Let us determine simple majority relation R^{maj} and Borda social preference relation R^B corresponding to profile $\boldsymbol{R}$:

It can be verified that

$$y\ R^{maj}\ z\ R^{maj}\ w\ R^{maj}\ x$$

Let B(s) denote the Borda score of alternative s. In the above example, B(w)=2, B(x)=3, B(z)=6 and B(y)=7. Hence,

$$y\ R^B\ z\ R^B\ x\ R^B\ w$$

Case 1: Consider the comparison of y and z. In such a case,

$$C(MR,\mathbf{R})=C(BR,\mathbf{R})=0.$$

Thus, either rule may be used.

[15] Our criterion of comparison between aggregation rules disregards operational simplicity and degree of manipulability. By using pairs of alternatives as our standard of comparison, we avoid the need to take lack of transitivity into account.

Case 2: Consider the comparison of *w* and *x*. In such a case,

C(MR,**R**)=(B(*x*)-B(*w*))/B(*x*)=(3-2)/3=1/3.

C(BR,**R**)=(N(*w*,*x*)) - N(*x*,*w*) / N(*x*,*w*) =(2-1)/2=1/2. Since C(MR,**R**) = 1/3 < C(BR,**R**) = 1/2, the justified aggregation rule for comparison of *x* and *w* is MR.

Example 2: Suppose that $N = \{1,2,3, \ldots,11\}$, $X = \{ x,y,z,v,t\}$ and the preference profile is $\boldsymbol{R} = (R_1, R_2, \ldots, R_{11})$, such that the ranking of the alternatives by voters 1,…,6 is:

x

t

z

v

y

and the ranking of the alternatives by voters 7, …, 11 is:

t

y

z

v

x

In this case, comparing *x* with *t*, we get:

$x \; R^{maj} \; t$ yet $t \; R^{B} \; x$, because B(*t*) = 38 and B(*x*) = 24.

C(MR,**R**)=(B(*t*)-B(*x*))/B(*t*)=(38-24)/38=14/38=7/19.

C(BR,**R**)=(N(*x*,*t*) – N(*t*,x)) /N(*x*,*t*)=(6-5)/6=1/6.

Since C(BR,**R**) < C(MR,**R**), the justified aggregation rule for comparison between *x* and *t* is BR.

3. Main findings

Given a particular pair of alternatives, the application of MR seems justified when MR is the preferred rule in the majority of preference profiles. Exclusive reliance on MR seems plausible if this justification is valid for most pairs of alternatives. (The application of BR is justified for the complementary set of pairs of alternatives.) Certainly, MR is especially appealing if its use is justified for all possible pairs of alternatives. Similar arguments may, of course, justify the use of BR. If preference profiles and pairs of alternatives are generated by a plausible probabilistic model of voters' preferences and alternatives, then when selecting an aggregation rule in the constitutional stage, we may justify the use of either MR or BR for determining the social preference of any pair of alternatives. The preferred rule is the one associated with the smaller expected cost—the erosion it is expected to cause to the principle that the alternative non-adopted rule respects. An alternative possibility is to take into account not the expected costs of the two rules but their a priori likelihood of being adopted in case of divergence between the outcomes of MR and BR. The preferred rule now becomes the one associated with the greater likelihood of being the preferred (less costly) one. The conclusion according to the two possible criteria may hinge on the number of voters n and the number of alternatives m. The question is how n and m affect the desirability of MR and BR under the veil of ignorance in the constitutional stage regarding the actual preference profile and the compared alternatives. Before turning to the identification of the preferred rule according to the two possible criteria, we describe the alternative statistical models used to generate the preference profiles of the voters and the alternatives.

3.1. The simulation

Preference profiles can be generated by several probabilistic models. We base our results on three: the impartial anonymous and neutral culture model, the Cubic Uniform population, and the spatial Euclidean Box model. These approaches have the common feature of generating preferences so as to reflect or approximate data samples in real elections.[16] The results presented below are based on the Euclidean Box model (the results for the Cubic Uniform and the impartial anonymous and neutral culture model are essentially the same). For a particular number of voters, n, and m alternatives (the candidates' positions), we independently and uniformly sample the alternatives and the positions of the voters from the Box on the assumption that a voter's utility for a candidate is a decreasing function of the distance between the candidate's position and the voter's position.[17] We have generated 100,000 such preference profiles and then computed the expected cost of the compared aggregation rules (the expected erosion of the fundamental principles applying measures 1 and 2) over all pairs of alternatives.[18]

3.2. The superiority of MR over BR

The simulation results establish the superiority of MR in terms of its lower mean cost taking into account all cases (each case in the simulation comprises of a

[16] See https://francois-durand.github.io/svvamp/.

[17] According to Merrill (1984) and Tideman and Plassmann (2013), when generating alternatives (candidates) and voters by means of simulations based on a spatial model, outcomes come very close to describing the distribution of actual outcomes, and ranking data simulated with the spatial model are very similar to observed ranking data. The spatial-model results thus tend to be more realistic.

[18] The random selection of preference orderings may not capture well the presence of proactive, highly motivated minority members who may strategically coordinate their reports of preferences. At the constitutional stage, it is difficult to capture this possibility without adding extra parameters such as an exogenous polarization factor. Therefore, our analysis disregards such possible strategic considerations.

preference profile and a pair of alternatives). This is true for the odd numbers of voters, n=3,5,7,9,11,13,15,21,31,41,51, and the numbers of alternatives, m=3,4,…,10. For any combination of number of alternatives m and number of voters n, MR is superior to BR in terms of its expected cost (viz., the expected erosion of Principle 1 by MR is smaller than the expected erosion of Principle 2 by BR), taking into account all possible comparisons of alternatives and preference profiles.

TABLE 1: RATIO OF THE EXPECTED COSTS OF BR AND MR

VOTERS / CANDIDATES	3	5	7	9	11	13	15	21	31	41	51
3	-	1.76	1.58	1.41	1.33	1.26	1.24	1.15	1.11	1.13	1.1
4	2.28	1.98	1.77	1.64	1.53	1.49	1.44	1.37	1.34	1.31	1.31
5	2.53	2.15	1.91	1.78	1.69	1.63	1.59	1.51	1.48	1.46	1.42
6	2.71	2.27	2.03	1.89	1.8	1.75	1.7	1.64	1.58	1.55	1.53
7	2.84	2.36	2.11	1.97	1.88	1.82	1.79	1.71	1.66	1.64	1.62
8	2.94	2.42	2.18	2.04	1.95	1.89	1.85	1.78	1.72	1.7	1.69
9	3.01	2.48	2.22	2.09	2	1.94	1.9	1.82	1.77	1.74	1.73
10	3.06	2.52	2.27	2.13	2.04	1.98	1.94	1.87	1.81	1.78	1.76

Table 1 presents the positive ratio of the expected costs of BR and MR. This ratio, referred to as the relative advantage of MR, ranges from 1.103 when m=3 and n=51 to 3.064 when m=10 and n=3.

Two unequivocal findings come to light in regard to the effect of m and n on the advantage of MR. For a given number of voters n, the advantage and relative advantage of MR rises with the number of alternatives m. For a given number of alternatives m, the advantage of MR falls (with one negligible exception) with the number of voters n.

Table 2 presents the results that illustrate the superiority of MR in terms of an alternative criterion: the a priori likelihood of being adopted when the two aggregation rules yield different outcomes. This likelihood is the percentage of pairs of alternatives under which MR is the superior aggregation rule, under all preference profiles that result in the erosion of Principles 1 and 2. (MR and BR yield different preferences between the compared alternatives.) These results confirm all the findings adduced on the basis of the first criterion. Note, however, that for a small number of alternatives and a large number of voters, the superiority of MR tends to be insignificant. For example, for three alternatives and fifty-one voters, the likelihood of MR being the preferred rule is 0.529. This suggests that when the electorate is even remotely large, the aggregation rules on which we focus are almost equally good in terms of the two applied criteria.

TABLE 2: PERCENTAGE OF SIMULATIONS IN WHICH MR IS SUPERIOR TO BR

VOTERS / CANDIDATES	3	5	7	9	11	13	15	21	31	41	51
3	-	1	0.91	0.83	0.77	0.71	0.67	0.59	0.54	0.55	0.53
4	1	0.94	0.89	0.84	0.79	0.75	0.72	0.65	0.62	0.59	0.58
5	0.97	0.94	0.9	0.85	0.81	0.77	0.74	0.68	0.64	0.62	0.61
6	0.98	0.95	0.9	0.86	0.82	0.79	0.76	0.71	0.66	0.64	0.63
7	0.98	0.95	0.91	0.87	0.83	0.8	0.77	0.72	0.68	0.66	0.65
8	0.98	0.95	0.91	0.87	0.84	0.8	0.78	0.73	0.69	0.67	0.66
9	0.98	0.95	0.91	0.87	0.84	0.81	0.78	0.73	0.69	0.67	0.66
10	0.98	0.96	0.92	0.88	0.84	0.81	0.79	0.74	0.7	0.68	0.67

Clearly, the easier rule to implement, MR, with its less demanding information requirements, is a plausible choice.

Finally, notice that, given the relatively low prevalence of variance in the results obtained by MR and BR when pairs of alternatives are compared across all possible preference profiles (percentage of pairs with erosion), which, ranges from 2 percent to 10 percent, the superiority of MR in terms of the two criteria is understandably much more relevant and significant across the erosion cases.

3.3. The superiority of PR over MR

The most common and best known scoring rule is the plurality rule, PR. Under this rule, one point is assigned to the most preferred alternative and no points accrue to the remaining alternatives (from the second-best to the worst). This makes PR an extreme weakly monotonic scoring rule (in which only the score assigned to the best alternative exceeds that of the second-best and all other alternatives) that assigns significance only to every voter's most preferred alternative. Applying the same methodology of comparing MR and PR on the basis of their costs (the extent of erosion of the principle they violate), PR emerges unambiguously as the superior aggregation rule.

Our findings stablish the superiority of PR in terms of its lower mean cost, all simulation outcomes taken into account. That is, for any combination of number of alternatives m and number of voters n, PR outperforms MR in terms of its expected cost (the expected erosion of Principle 2 is smaller than the expected erosion of Principle 1), all possible comparisons of alternatives and preference profiles taken into account. The positive and smaller-than-1 expected ratio of the costs of PR and MR is referred to as the relative advantage of PR. This relative advantage, presented in Table 3, ranges from 0.417 (the largest relative advantage), when m=10 and n=9, to 0.754 (the smallest relative advantage), when m=3 and n=51.

Table 3 yields two unequivocal findings regarding the effect of *m* and *n* on the advantage of PR. For a given number of voters *n*, the relative advantage of PR increases with the number of alternatives *m*. For a given number of alternatives *m*, when the number of voters is sufficiently large, $n \geq 9$, the relative advantage of PR decreases with *n*.

TABLE 3: RATIO OF THE EXPECTED COSTS OF PR AND MR

VOTERS / CANDIDATES	3	5	7	9	11	13	15	21	31	41	51
3	0.5	0.52	0.55	0.58	0.6	0.62	0.63	0.68	0.71	0.74	0.75
4	0.5	0.49	0.5	0.53	0.55	0.57	0.59	0.63	0.68	0.7	0.73
5	0.5	0.47	0.47	0.49	0.51	0.53	0.54	0.58	0.64	0.67	0.69
6	0.5	0.46	0.45	0.46	0.48	0.49	0.51	0.55	0.6	0.64	0.66
7	0.5	0.45	0.44	0.45	0.46	0.47	0.48	0.52	0.57	0.61	0.64
8	0.5	0.45	0.43	0.43	0.44	0.45	0.46	0.49	0.54	0.58	0.61
9	0.5	0.44	0.43	0.42	0.43	0.44	0.44	0.47	0.52	0.55	0.58
10	0.5	0.44	0.42	0.42	0.42	0.42	0.43	0.45	0.5	0.53	0.56

Table 4 illustrates the superiority of PR in terms of the alternative criterion: the a priori likelihood of being adopted, when the two aggregation rules

TABLE 4: PERCENTAGE OF SIMULATIONS IN WHICH MR IS SUPERIOR TO PR

VOTERS / CANDIDATES	3	5	7	9	11	13	15	21	31	41	51
3	0	0	0.12	0.13	0.18	0.23	0.22	0.3	0.36	0.38	0.4
4	0	0	0.11	0.1	0.18	0.23	0.21	0.28	0.34	0.36	0.38
5	0	0	0.1	0.08	0.15	0.21	0.17	0.24	0.32	0.34	0.36
6	0	0	0.08	0.06	0.13	0.19	0.14	0.22	0.29	0.32	0.34
7	0	0	0.06	0.05	0.1	0.16	0.12	0.19	0.27	0.3	0.32
8	0	0	0.05	0.04	0.09	0.14	0.1	0.17	0.25	0.28	0.3
9	0	0	0.04	0.03	0.07	0.12	0.08	0.15	0.23	0.26	0.28
10	0	0	0.04	0.02	0.06	0.11	0.07	0.13	0.21	0.24	0.26

yield different outcomes. This likelihood is the percentage of pairs of alternatives under which PR is the superior aggregation rule, among all preference profiles that cause Principles 1 and 2 to erode (MR and PR yield different preferences between the compared alternatives). These results confirm the superiority of PR.

Our results provide a novel justification for the widespread use of PR beyond its practical advantages. There are certainly scoring rules that are superior to MR. The identification of this set of scoring rules is a task worth pursuing in future research.

An uncommon but well known scoring rule is the inverse plurality rule (IPR) (Baharad and Nitzan 2005, 2007), also known as the negative plurality rule (De Sinopoli et al. 2015). Under the IPR, one point is assigned to all alternatives but the worst one, which is assigned no points. This makes the IPR yet another extreme

weakly monotonic scoring rule (only the score assigned to the worst alternative is smaller than the score assigned to all other alternatives) that gives significance only to every voter's least preferred alternative. A comparison of MR and IPR based on their costs (the extent of erosion of the principle they violate) has revealed that MR unambiguously superior to IPR despite its structural similarity to PR, the two aggregation rules assigning different scores to just one alternative (the most preferred or the least preferred). This provides justification for the rare use of IPR despite its simplicity and practical advantages, which resemble those of PR.

5. Conclusion

The essential and underlying etiology of the heated debate surrounding the rationale of using the simple-majority rule or a scoring rule is the tension between the two fundamental principles on which we have focused: allowing voters to express their preference intensity and thus providing some protection to the minority (Principle 1) and protecting the majority by respecting majoritarianism (Principle 2). The two scoring rules that attracted the most attention are the Borda method of counts (a monotonic scoring rule) and the plurality rule (the common weakly monotonic rule). Since the two principles cannot be respected simultaneously, satisfying Principle 1 (Principle 2) implies the violation of Principle 2 (Principle 1). In other words, if MR is used, Principle 1 is violated and if BR or PR is used, Principle 2 is breached. These violations may be viewed as the costs associated with the use of these most thoroughly studied aggregation rules. Applying a plausible measure of these costs (the expected cost of the two rules in case of divergence of outcomes) and using an intuitive natural concept of relative erosion of Principles 1 and 2, our study makes its main contribution by establishing that MR is superior to BR yet inferior to PR in terms of the proposed measures of relative erosion of the principles. Our findings are attested on the assumption that both principles are

assigned equal weight.[19] They are based on simulations that apply the impartial anonymous and neutral culture model as well as two alternative more plausible realistic probabilistic models to stimulate the alternatives and voters' preferences. The results enrich the vast literature on the merits and disadvantages of the three most thoroughly examined aggregation rules by providing a novel rationalization for preferring PR over MR and MR over BR in the constitutional stage, where the veil of ignorance prevails. They shed new light on the appeal of PR and BR, assuming that the alternative to each of these scoring rules is MR. The first finding rationalizes the actual revealed superiority of PR over MR. With the second finding, it becomes possible to decide between the approaches of Condorcet and of Borda and to explain the low prevalence of BR relative to MR

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[19] The results extend to the case of asymmetric weight assignment, of course. Here, MR remains the superior aggregation rule as long as the ratio of costs associated with the use of BR and MR exceeds the ratio of the weights assigned to Principle 2 and Principle 1.

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