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SOCIAL MOBILITY AT THE TOP

AND

THE HIGHER EDUCATION SYSTEM

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Abstract

This paper relates social mobility and social stratification to the structure of higher education. We develop an intergenerational model which shows that a two-tier higher education characterised by a division between elite and standard universities can be a key factor in generating permanent social stratification, social immobility and self-reproduction of the 'elite'. In our approach, low mobility at the top is essentially explained by the differences in quality and in selection between elite and standard universities.

A key result is that the wider the quality gap and the difference in per-student expenditures between elite and standard universities, the less social mobility. This is because a larger quality gap reinforces the weight of family backgrounds at the expense of personal ability. Our simulations show that this impact can be large. These findings provide theoretical bases for the differences in social mobility at the top observed between advanced countries.

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1. Introduction

This paper relates social mobility and social stratification to the structure of higher education. We show that a two-tier higher education characterised by a division between elite and standard universities leads to permanent social stratification, social immobility and self-reproduction of the elite. In our approach, low mobility at the top is essentially explained by differences in quality and in selection between elite and standard universities.

Democratization in higher education is a world-wide phenomenon. In most advanced economies, the share in a generation of the individuals pursuing tertiary education has shifted from about 10% in the decade following World War II up to more than 60% in the present period (Restuccia and Vandenbroucke, 2013, for the US). Moreover, most universities have a meritocratic recruitment, which is not based on family ties. Still, despite democratization and meritocracy, countries as the UK, the US and France display low intergenerational mobility, and several works suggest that mobility has decreased compared to the post-World War II period. In addition, they exhibit even lower mobility at the top. In contrast, Nordic countries exhibit comparatively high mobility with no decrease over time.[1]

A key divergence between higher education systems is their elitist orientation which results in duality, i.e., large differences between elite and standard universities, which are essentially twofold: differences in the tightness of selection and in quality of education.

First, elite universities admit a limited number of the best students whereas standard universities are significantly less restrictive in their selection. For the US, Hoxby (2009, p. 98) finds that, from 1962 to 2007, the SAT scores of those accepted by the colleges have increased for the most selective and decreased for the least selective.[2]

Elite and standard universities also differ in their quality, which affects the human capital of their students. A good indicator of quality is per-student expenditure which is significantly higher in elite universities than in standard ones, and this gap has widened in the recent decades in several advanced countries.

Finally, there are significant differences across countries. The UK, the US and France display large differences in their selection tightness and their per-student expenditures between the two types of university and are thereby highly dual, which is not the case in Nordic countries.

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[1] Corak (2013) and Lee and Solon (2009). In addition, several works show that social mobility is lower at the upper tail of income distribution, with a particularly low likelihood to move downward for the sons of the highest earners in the US and the UK (Jäntti et al., 2006). Empirical regularities are detailed in the next section.

[2] Moreover, differences in selection have widened over time. Elite universities have not increased much their intakes in recent decades whereas the intakes have substantially risen in standard universities (Su et al., 2012).

The correspondence between differences in intergenerational mobility and differences in duality of higher education across countries suggests that there could be a link between these two phenomena.

Our paper develops an intergenerational model which shows that low mobility at the top can be generated by a two-tier higher education system, wherein higher education is divided into two major categories – elite and standard universities – that significantly differ in their selection procedures and their quality. Consequently, a highly dual tertiary education should lead to low intergenerational mobility at the top.

The model is based on two main assumptions.

First, the system of higher education is dual: there are standard and elite universities that differ both in their quality (identified by their per-student expenditures) and in the rigor of their selection process. Moreover, in all higher education institutions, admission is meritocratic and based solely on the human capital level.

The second assumption is that human capital formation, which determines lifetime income, depends on three factors that differ across individuals: (i) the family backgrounds, i.e., the parents' human capital; (ii) the personal innate ability which is randomly distributed, and (iii) the type of study the individual pursues which depends on her choice within the education system.

Intuitively, the three factors act as follows. Innate ability fosters social mobility because its distribution is independent from social origins. In contrast, family background acts against mobility. Finally, the education system stimulates mobility if it reinforces the impact of innate ability, whereas it stimulates immobility if it strengthens the influence of family background.

Our results are developed in two steps. First, we show that the dual higher education system and the human capital intergenerational dynamics define a three-group social stratification. The lower class draws together the individuals with basic education only, the middle class those with a standard university degree, and the upper class, i.e. the 'elite', is comprised of elite university graduates. We show that this division tends toward a '*steady* social stratification' wherein each social group remains inside a steady human capital segment, with the size of each group remaining constant.

However, steady stratification does not prevent social mobility. We show that individuals can climb from the middle class to the elite, and the opposite, i.e., descend from the elite to the middle class, and the condition for social mobility at the top, i.e., between the middle class and the elite, is determined in the model.

In a second step, we analyse the effects on social mobility of changes in the education system, i.e., in universities' quality and admission rules. To this end, we define several indicators of social mobility at the top.

Regarding quality, we show that social mobility at the top decreases when the quality gap between universities increases. The greater this gap, the less social mobility between the middle class and the elite. The reason for this effect is that a relative increase in the quality of the elite universities widens the human capital gap between the middle class and the elite. This augments the weight of skills transferred inside the family and reduces thereby the relative impact of personal ability in human capital formation. Thus, getting admitted to an elite university becomes more difficult for the most able students born into the middle class.

Regarding admission rules, we find that a tighter admission to the elite universities drives down the middle class upward mobility. We also show that when the rigor of admissions to standard universities is relaxed, social mobility rises in the generation wherein it occurs, yet the upward mobility of the middle class decreases in the following generations.

We finally implement a series of simulations that corroborates our theoretical findings and show that the impact of higher education duality on mobility at the top is large.

The paper is divided into five sections. In Section 2, the literature on the subject is reviewed. The model is developed in Section 3, and we analyse social mobility at the top in Section 4. Simulations are performed in Section 5, and Section 6 concludes.

2. Literature

We briefly present the literature on the two subjects related to our approach: (i) intergenerational mobility and social stratification, and (ii) the higher education structure.

2.1. Intergenerational mobility, mobility and social stratification

In economics and sociology, studies on intergenerational mobility are numerous and diverse, and they lead to similar conclusions. The economic literature has been reviewed by Bjorklund and Jantti (2000, 2009), Fields (2008), Causa and Johansson (2009), Bjorklund and Salvanes (2011), Black and Devereux (2011). Chusseau and Hellier (2013) present a synthesis of the theoretical literature. Hertz et al. (2007) estimate the relation between education and intergenerational mobility for many countries in a rather long period. These reviews and studies show that the impact of the structure of higher education on intergenerational mobility is rarely mentioned.

2.1.1. Theoretical approaches

The early approaches diagnosed a human capital convergence of dynasties, with however a slowing-down of the convergence when the credit market is imperfect (Becker and Tomes, 1979; Loury, 1981).

The subsequent literature has analysed the mechanisms that lead to transitory or permanent under-education traps, and thereby social stratification. These mechanisms are numerous: a fixed cost of education with imperfections on the credit market (Galor and Zeira, 1993; Barham et al., 1995), an S-shaped education function (Galor and Tsiddon, 1997), neighbourhood effects (Benabou, 1993, 1994, 1996; Durlauf, 1994, 1996), differences in altruism (Das, 2007). Most of these approaches focus on market imperfections that result in non-convexities (the function that relates the individual's human capital to that of her parents is not concave).

Finally a more recent literature relates under-education traps and social segmentation to the structure of education systems. These approaches are reviewed in section 2.2.

2.1.2. Empirical studies

A comparison of empirical methods and studies in economics and sociology can be found in Blanden (2013) and Breen et al (2016).

Following the works of Solon (1992), the economic literature is essentially based on the calculation of intergenerational elasticities (*IGE*), i.e., the elasticity β of the individuals' earnings (or education) in relation to their parents. Intergenerational mobility is then measured as $1-\beta$. Other indicators are often utilised, as the intergenerational correlation which permits to erase the impact of intergenerational changes in intra-generational inequality, and the rank-rank slope which measures the relation that binds the rank of children to that of their parents (e.g., Chetty et al., 2014).

Sociologists often utilise mobility tables (e.g., Duncan, 1966 and Breen, 2009), which provide appropriate measures when the population is divided in different groups (earnings deciles, education levels, social classes, etc.). Absolute mobility measures the variation in attainment (income, education) from one generation to the next whereas relative mobility (social fluidity) indicates the probability to switch groups from one generation to the next. The mobility matrix $\{a_{ij}\}$ depicts the proportion of individuals in group i with parents in group j.

Whatever the selected indicator, the numerous works on this subject reveal the following empirical regularities:

1. Intergenerational earnings mobility significantly differs across countries (Corak, 2013; Mazzonna, 2014). The US, the UK, France, and Italy display the lowest mobility (IGEs between 0.4 and 0.5), Canada and Nordic countries the highest (IGEs below 0.25).

2. Intergenerational mobility is lower at the upper tail of the income distribution (Jäntti et al., 2006; Björklund et al., 2012). In particular, Jäntti et al. (2006) found that in the US and the UK, sons of highest earners have a low likelihood to move downward the income ladder compared to the Nordic countries (Denmark, Finland, Norway, Sweden). Hence, the US and the UK display low social mobility at the top.

3. In several countries, there seems to have been a decrease in intergenerational mobility in the last generations compared to those born after the war. Several works suggest such a decrease in the UK (Blanden et al., 2004, 2007; Nicoletti and Ermisch, 2007), in France (Lefranc, 2011; Ben-Halima et al., 2014), and in the US (Aaronson and Mazumder, 2008)[3], whereas these countries exhibited a slight increase in mobility in the generations before (Lee and Solon, 2009; Breen, 2009; Lefranc, 2011). In these countries, social mobility can thus be depicted by an inverted-U curve with a peak for the generations born around 1960.

4. In contrast, in Nordic countries without high duality in their higher education system, intergenerational mobility of earnings has increased and does not display an inverted-U curve during the last decade (Fochesato and Bowles, 2014, Fig.6, p.20)

The same analysis was implemented for education groups. With parental education being represented by using the CASMIN classification[4], several works show a low probability for children from the lower classes to enter the upper classes (Muller et al., 1989, and Braun and Muller, 1997, for several European countries).

2.2. Education structure, social stratification and higher education

We review the literature on the interplay between education and social stratification. We subsequently describe the differences in higher education between countries and the changes in the system since World War II.

[3] In contrast, Breen and Golthorpe (1999, 2001) found no change in mobility in the UK by comparing cohorts born in 1958 and in 1970. Chetty et al. (2014) found no decrease in intergenerational earnings mobility in the US, but their estimates are based on cohorts born between 1971 and 1993.

[4] The *Comparative Analysis of Social Mobility in Industrial Nations* (CASMIN) educational classification provides a categorical schema that can apply to different countries.

2.2.1. Education structure and Social stratification

The relationship between educational systems and social mobility and stratification has been analysed for a long time by sociologists. Turner (1960) makes a distinction between comprehensive education systems that tend to bring the largest proportion of children to the skill level necessary to come into the elite, from highly selective systems that recruit a limited number of the best students to enter the elite group. A similar classification was proposed by Hopper (1968) who distinguishes different levels of educational stratification based on the selection and differentiation processes.

Kerckhoff (1995) suggests that the effect of family backgrounds could be magnified when the education system is highly stratified and selective. This argument has been confirmed by several empirical works (Hanushek and Woessmann, 2006; Pfeffer, 2008; Dronkers et al., 2011). Most of these works are based on PISA surveys, and are thereby centred on the education system up to secondary school.

In the recent economic literature on education and human capital, there has been a growing interest into the analysis of stratification of educational systems. In these approaches, education is usually modelled as a succession of stages, and basic education is a prerequisite to enter colleges and universities. A key issue in this literature is the analysis of the distribution of public expenditures between the different education cycles, particularly between basic and higher education (Driskill and Horowitz, 2002; Su 2004, 2006; Blankenau et al., 2007; Di Gioacchino and Sabatini, 2009; Viaene and Zilcha, 2013).

The analysis of the impact of the education system structure on social stratification and inequality can be found in Bertocchi and Spagat (2004) who develop a partition between basic and secondary education and show that social stratification is determined by the education system at the different stages of economic development. Chusseau and Hellier (2011) build an intergenerational model with three education cycles (compulsory basic education, vocational studies, and university, with a selection to enter the latter) that can generate very different social stratifications depending on the public funding allocated to each cycle and on the severity of the admission procedure. Finally, a number of works have analysed the economic and social impacts of higher education systems. Several issues have been tackled: training versus signalling objectives (Arrow, 1973; Spence, 1973; Stiglitz, 1975), higher education costs and their effects on modest families (Caucutt and Kumar, 2003, Akyol and Athreya, 2005, Gilboa and Justman, 2009, Anderberg, 2013), admission tightness (Gilboa and Justman, 2005, Gary-Bobo and Trannoy, 2008). Wälde (2000) shows that egalitarian and elitist education systems, by determining the gap between skilled and less skilled workers,

have different impacts on technological change and thereby on inequality. Su et al. (2012) distinguish between standard and elite colleges to analyse the U-shape relationship between wages and skills observed in the US in the last two decades.

2.2.2. Differences and Changes in higher education

Since World War II, the development of education systems has followed rather diverse orientations in advanced economies. In what follows, we highlight some key facts on which our approach is based.

The first is the democratization of tertiary education, with admission procedures based on meritocracy. However, in many advanced countries, this democratization has come with the development of a two-tier system characterised by the concomitance of standard and elite universities. This differentiation between two types of universities has widened over time since the huge increase in the number of students has primarily concerned standard universities, the selection remaining narrow in elite establishments.

In the US, Su et al. (2012) note that, between 1959 and 2008, the non-elitist public post-secondary colleges have increased their enrolment by 525% against 250% in elite colleges. In France, elite universities are represented by the *Grandes écoles* that recruit less than 4% of a generation. Albouy and Wanecq (2003) have shown that there was almost no change in the recruitment of the top *Grandes écoles*, while at the same time the share of a generation completing tertiary education was multiplied by more than 3.5.[5] In contrast, Nordic countries do not exhibit such differences in the selection processes across universities.[6]

The second key fact is that standard and elite universities differ in their budgets, which to a large extent determine their quality. The expenditures per student are substantially higher in elite universities than in standard ones, and this gap has increased in the last decades in a number of advanced countries. In the US, expenditures per student in elite universities (Ivy League) are more than three times higher than in other universities. In addition, from 1999 to 2009, the total operating expenditures increased by 20% in elite universities, and by less than 8% in standard ones (Desrochers and Wellman, 2011). In France in 2002, the spending per student is on average 3.5 times higher in the top GE than in standard universities.[7]

The third key fact is that the access to elite universities is mostly open to the elite's offspring. In the US, SAT scores are highly correlated with family education and wealth

[5] They define the '*Très Grandes écoles*' as the most prestigious schools leading to the highest top executive and public positions. They show that, for men, the share of a generation entering these top GE decreased from 0.8 for the generations born between 1929 and 1938, to 0.6% for those born between 1959 and 1968.

[6] The variance between elite and other universities is lower in Denmark, Finland and Norway (Brezis, 2012).

[7] Data from the *Observatoire Boivigny*.

(Brezis and Temin, 2008). Carnevale and Strohl (2010) show that the top socioeconomic quartile represents 70% of the students in the most selective colleges, against 14% for the bottom half of the population, this difference having significantly risen from 1982 to 2006.

For France, Albouy and Wanecq (2003) show that, since the end of World War II, the difference in the probability to enter a *Grande école* between students from the upper class and the 'popular classes' has followed a U-curve.[8]

These facts clearly indicate that that there is a social segregation in the entry to elite establishments which has increased in the last decades.

Finally, empirical regularities show that entering an elite university is the natural path to the highest private and public positions (Temin, 1999, for the US; Baverez, 1998, for France).

The following section proposes a modelling based on these empirical regularities.

3. The model

The model aims at analysing the impact of the structure of higher education on social mobility. We therefore assume this structure to be exogenous and to differ across countries, and we analyse the education decisions of individuals and their impact on intergenerational mobility and social stratification.

We consider an overlapping generation model in which each individual has one child. A dynasty consists of the successive generations linked by a parent-child relationship, and we assume a constant number of dynasties normalised to 1. We denote individual (i,t) the individual of the t-th generation of dynasty i.

The model is developed as follows. First, we expose the structure of the education system and human capital accumulation. We subsequently define the steady stratification and we show that a two-tier higher education system tends towards a steady stratification. The effects of changes in the structure of higher education on social mobility at the top will be analysed in the following section 4.

3.1. Education System and Human Capital Accumulation

Individuals accumulate human capital through education, and education is comprised of two phases, i.e., basic and higher education.

Without any education, the individual has the minimal human capital $l > 0$.

[8] The upper class offspring were 27 times more likely to enter a GE than those from the popular classes in the generations born in1929-1938, 17 times for the 1949-1958 generations, and 20 times for the 1959-1968 generations.

Being young (child), all individuals receive the same basic education (but their human capital at the end of basic education differs because of differences in ability and family background) and their basic needs are provided by their parents. At the end of basic education, an individual (*i,t*) has accumulated a human capital level denoted h_{it}^{B} and she becomes an adult. Once adult the individual lives one period of time and chooses whether to study at the university for a time $\varphi < 1$, or to join directly the labour market.

When completing her overall education (basic education, or higher education if she enters a university), individual (*i,t*) possesses the final human capital level denoted h_{it}^{F}. Then, she spends the whole of her remaining time working.

3.1.1. Basic education

The State provides all individuals with basic education. The individual's human capital at the end of basic education, h_{it}^{B}, depends on two elements:

1) Her family-related ability that represents the impact of intra-family externalities and transfers. These externalities and transfers can act through several channels: the intra-family direct transmission of human capital; the intra-family transmission of capacity to learn, i.e., capacity of analysis, capacity to organise studying activity; information about the education system; education strategies; affiliation with influential networks etc. All these intra-family externalities and transfers are directly linked to the parent's overall human capital h_{it-1}^{F}.

2) Her personal innate ability, a_{it} for individual (*i,t*). As in Maoz and Moav (1999), personal ability is independent from family backgrounds, randomly distributed across individuals within each generation and it belongs to the segment $\left[\underline{a}, \overline{a}\right] \subset \mathrm{R}_{+}^{*}$.[9]

In consequence, we assume that the human capital at the end of basic education h_{it}^{B} is given by the simple functional form:

$$h_{it}^{B} = a_{it}\left(h_{it-1}^{F}\right)^{\eta}, \qquad \text{with} \quad 0 < \eta < 1. \tag{1}$$

We finally assume $\underline{a} > l^{1-\eta}$, which ensures that the skill attainment at the end of basic education is higher than her human capital when she was born, *l*.

3.1.2. Higher Education

We assume a two-tier higher education, namely, students can enter either standard universities (*S*) or elite universities (*E*). In addition, higher education is meritocratic: students are only selected on their human capital when completing basic education.

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[9] In contrast with Maoz and Moav (1999), ability is not measured by differences in education costs.

We do not formally model the decisions of each type of establishment in terms of admission procedure and per-student expenditure.[10] There are three reasons for this. First, our objective is to analyse the impact of the division of tertiary education between standard and elite establishments as observed in the US, the UK and France. If the three countries share the characteristic of having a two-tier tertiary education, they substantially differ in terms of tuition fees, public versus private higher education, weight of the state in the funding of establishments, way to select the managing staff etc. Any utility function of the elite establishment would then be highly controversial because, if it fits with one country, it would not fit with another. Nevertheless, despite differences in objectives and in competition across universities, the three countries share the specificity of having a limited number of highly selective establishments benefiting from high per-student expenditures. Second, in the three countries, the elite tertiary education includes several competing establishments and their selection rules and budgets derive to a large extent from this between-establishment competition. Our objective is not to analyse this competition process but to focus on the impact of the differences between elite and standard universities. Third, we have observed that all advanced countries have implemented policies leading to democratization in tertiary education. Several reasons explain these policies (technological changes, globalization, pro-growth policies, social goals etc.) and, based on these explanations, we assume the existence of standard universities admitting a large intake of students at the end of basic education.

In summary, we assume that (i) the standard universities' objective is defined by public authorities who wish to bring a rather large share of a generation to pursue tertiary education, and (ii) the elite universities' goal is to select a limited number of the best students to train the elite at each generation. Consequently, we suppose a division of tertiary education between standard an elite establishments which differ in their admission and in their budgets to show that this division leads to permanent stratification and low mobility.

The two types of universities firstly differ in their admission procedures. Entering the standard university requires a minimum level of human capital, $\hat{h}$, at the end of basic education. On top of the minimum required level $\hat{h}$, the elite university selects the best students at the end of basic education and decides on the quota, which must remain narrow so as to ensure the specificity of elite universities. This assumption is in line with the observation highlighted in the previous section that the increase in the number of students is significantly lower in elite establishments. In consequence:

[10] Haupt (2012) proposes a model of public spending on higher education, but he does not distinguish standard from elite universities.

1) To enter the standard university (*S*), individual (*i,t*)'s human capital at the end of basic education, h_{it}^{B}, must be at least equal to $\hat{h}$.

2) To enter the elite university (*E*), in addition to the above condition, the individual must belong to the top α in terms of human capital at the end of basic education, h_{it}^{B}. Since the population is constant and normalized to 1, α depicts both the number of students accepted in the elite university and the share of the population having an elite university degree.

Consider now an individual who enters the university. Her human capital at the end of higher education, h_{it}^{F}, depends on two factors: (i) her human capital at the end of basic education, h_{it}^{B}, and (ii) the quality of the university where she studies, denoted V_j, $j = S, E$. Quality is a synthetic indicator of the diverse elements provided by elite establishments to promote the career and social position of their students. A good measure of quality is the per-student expenditures of the university. Higher funding means better and more numerous staff, better infrastructures, and finally better education. It also permits to generate influential networks that signal the belonging to the elite. Consequently, and following the stylised facts exposed in Section 2, we assume that the expenditure per student, and hence the 'quality', is higher in the elite university than in the standard one: $V_E > V_S$.

We assume a simple multiplicative higher education function. More complex functions could be considered, but they would not modify the main results.[11] Therefore, we have:

$$h_{it}^{F} = \begin{cases} h_{it}^{S} = V_S h_{it}^{B} = V_S a_{it} \left(h_{it-1}^{F} \right)^{\eta} & \textit{if the individual enters a standard university} \quad (2) \\ h_{it}^{E} = V_E h_{it}^{B} = V_E a_{it} \left(h_{it-1}^{F} \right)^{\eta} & \textit{if the individual enters an elite university} \quad (3) \end{cases}$$

where h_{it}^{S} and h_{it}^{E} are individual (*i,t*)'s overall human capital at the end of higher education when completing the standard and elite university respectively.

3.1.3. Educational choice

Once they have achieved their basic education, individuals possess one unit of time they allocate either to working, or to studying in tertiary education.

Individual (*i,t*) maximises her lifetime income I_{it} by making the following discrete choice:

$$\hat{I}_{it} = \max \left\{ I_{it}^{j},\ j = B, S, E \right\}$$

[11] Two features are necessary to produce our outcomes: the human capital h_{it}^{j}, $j = E, S$, must be (i) a continuous monotonically increasing function of the quality V_j, and (ii) a continuous monotonically increasing and concave function of the human capital attainment at the end of basic education. The simple form adopted here permits to generate simple analytical results and conditions, which shorten and simplify the presentation.

The incomes I_{it}^{j} depend on human capital, on earnings per unit of human capital, and on the working time during adult life. We denote w_t the after-tax earnings[12] per unit of human capital at the beginning of generation *t*'s adult life. To simplify, we assume an exogenous and constant rate of growth υ of unit earnings and we denote *r* the discount factor.

Consider individual (*i,t*) with human capital h_{it}^{B} at the end of basic education. Then:

$I_{it} = I_{it}^{B} = \int_0^1 w_t e^{(\upsilon - r)\theta} h_{it}^{B} d\theta$ if she joins directly the labour market.

$I_{it} = I_{it}^{S} = \int_\varphi^1 w_t e^{(\upsilon - r)\theta} V_S h_{it}^{B} d\theta$ if she enters a standard university

$I_{it} = I_{it}^{E} = \int_\varphi^1 w_t e^{(\upsilon - r)\theta} V_E h_{it}^{B} d\theta$ if she enters an elite university

As $V_E > V_S$, it is clear that students always prefer an elite university to a standard one.

Lemma 1. *All individuals prefer entering the U-university to joining directly the labour market if* $V_U > \underline{V} = \left(e^{\upsilon - r} - 1\right)\left(e^{\upsilon - r} - e^{(\upsilon - r)\varphi}\right)^{-1}$, $U = S, E$, *and they all prefer to join directly the labour market in the opposite case*.

Proof. Individual (*i,t*) prefers entering the *U*-university to joining the labour market if $I_{it}^{U} > I_{it}^{B} \Leftrightarrow V_U > \left(e^{\upsilon - r} - 1\right)\left(e^{\upsilon - r} - e^{(\upsilon - r)\varphi}\right)^{-1}$, $U = S, E$.

Condition $V_U > \underline{V}$ signifies that the efficiency of the *U*-university, i.e. its per-student expenditure V_U, must be sufficient to cover the opportunity cost linked to studying time.

As $V_S < V_E$, three situations are possible. When $V_E < \underline{V}$, no-one chooses higher education. When $V_S < \underline{V} < V_E$, no-one enters the standard university and the best α students at the end of basic education enter the elite university, provided that they reach the minimum threshold $\hat{h}$. Finally, when $V_S > \underline{V}$, all the children with a human capital higher than $\hat{h}$ enter the university, and the best α among them the elite university. This last situation is the only one which is relevant for our purpose and which corresponds to observed facts. Consequently, we shall henceforth suppose that the following condition holds:

$$V_S > \underline{V} \qquad (4)$$

[12] There is an income tax to fund education which is freely provided by the State. The tax is not explicitly modelled because it does not affect the individuals' education decisions.

This leads to the following Lemma on the educational choice:

Lemma 2. *Individual (i,t)'s human capital at the end of her education,* h_{it}^F , *is:*

$$h_{it}^F = \begin{cases} h_{it}^B & if \quad h_{it}^B < \hat{h} \\ h_{it}^S & if \quad \hat{h} \le h_{it}^B < h_{\alpha,t} \\ h_{it}^E & if \quad \hat{h} < h_{\alpha,t} \le h_{it}^B \end{cases} \tag{5}$$

The case of additional education costs on top of the above-considered opportunity cost is discussed in Section 6.

3.2. Social stratification and intergenerational mobility

Lemma 2 points to the fact that individuals are differentiated according to the type of education they receive. This defines the following three social groups:

Definition 1. Within each generation, we define as:

1) <u>*Lower class*</u> the set of individuals who have only achieved basic education.
2) <u>*Middle class*</u> the set of individuals who have a standard university degree.
3) <u>*Elite*</u> the individuals with an elite university degree.

The three social groups are defined by the educational attainment of individuals. This typically reproduces the facts underlined in Section 2 showing the correspondence between social stratification and stratification in higher education.

By definition, the size of the elite α is given. We denote γ the size of the middle class and $\varepsilon = 1 - \alpha - \gamma$ the size of the lower class. Except α, these sizes can change over time.

3.2.1. Steady segments

We now show that each type of study (*B*,*S*,*E*) is characterized by a specific steady segment. In this purpose, let us define function $h_t^F = H_a^j(\cdot)$ for a given *a*, such that:

$$h_t^F = H_a^j\left(h_{t-1}^F\right) = V_j a\left(h_{t-1}^F\right)^\eta, \quad \forall t, \text{ for } j = B, S, E \tag{6}$$

with $V_B = 1$.

$H_a^j(\cdot)$ is the function that relates the human capital at the end of the type of study *j* at any generation *t* to the preceding generation's overall human capital h_{t-1}^F *for a given personal ability* a . The shapes of the curves $H_a^j(\cdot)$ are depicted in Figure 1.

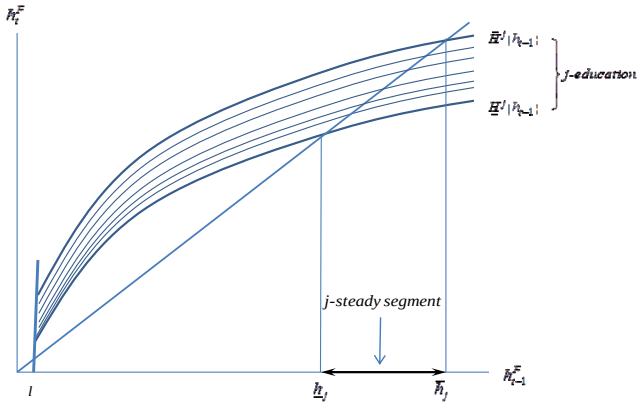

Figure 1. The *j*- steady segment, $j = B,S,E$

Function $H_a^j(\cdot)$ is denoted $\underline{H}^j(\cdot)$ for $a = \underline{a}$ and $\overline{H}^j(\cdot)$ for $a = \overline{a}$. Functions $\overline{H}^j(\cdot)$ and $\overline{H}^j(\cdot)$ respectively determine the lowest and highest human capital a child can reach at the end of education *j* when all her ancestors pursued education *j*. All the curves $H_a^j(\cdot)$ are located between curves $\underline{H}^j(\cdot)$ and $\overline{H}^j(\cdot)$ and any two curves never intersect.

For a given ability a, and a given type of study *j*, the steady state of the dynamics (6) is the fixed point h_j^a of function $H_a^j(\cdot)$ defined by the equality:

$$h_a^j = H_a^j\left(h_a^j\right) \Rightarrow h_a^j = \left(V_j a\right)^{1/(1-\eta)} \tag{7}$$

Since $a \in \left[\underline{a}, \overline{a}\right]$, all the steady states are inside the segment $\left[\underline{h}_j, \overline{h}_j\right]$ as depicted in Figure 1.

Definition 2. The segment $\left[\underline{h}_j, \overline{h}_j\right]$ is called *j-steady segment*, for *j* = *B*,*S*,*E*.

The human capital dynamics engender three steady segments: $\left[\underline{h}_B, \overline{h}_B\right] = \left[\underline{a}^{1/(1-\eta)}, \overline{a}^{1/(1-\eta)}\right]$, $\left[\underline{h}_S, \overline{h}_S\right] = \left[\left(V_S \underline{a}\right)^{1/(1-\eta)}, \left(V_S \overline{a}\right)^{1/(1-\eta)}\right]$ and $\left[\underline{h}_E, \overline{h}_E\right] = \left[\left(V_E \underline{a}\right)^{1/(1-\eta)}, \left(V_E \overline{a}\right)^{1/(1-\eta)}\right]$. These steady segments (*B*, *S* and *E*) are depicted in Figure 2. Note that the *S*-steady segment and the *E*-steady segment move to the right when V_S and V_E increase respectively.

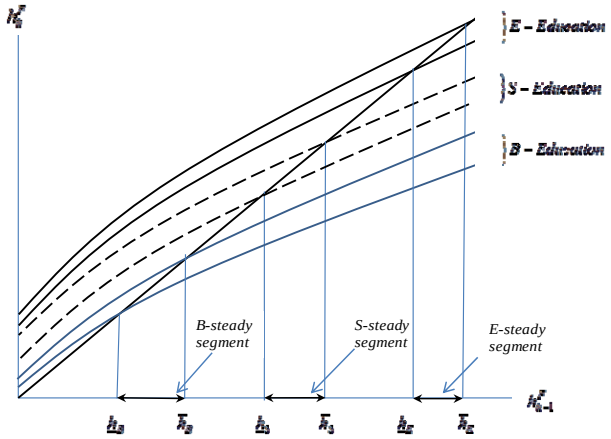

Figure 2. Steady segments

So as to restrict the analysis to the mobility between the middle and the upper class, we assume that once one generation of a dynasty enters the university, then all the successive generations of this dynasty also enter the university. The least skilled individual who enters the university is such that $h_{it}^{B} = \hat{h}$ and $h_{it}^{F} = V_S \hat{h}$ at the end of study *S*. To enter the university, her child must possess at least the human capital $\hat{h}$ at the end of basic education, even when having the lowest ability $\underline{a}$. Hence: $\underline{a}\left(V_S \hat{h}\right)^{\eta} \geq \hat{h} \Rightarrow \hat{h} \leq \left(\underline{a} V_S{}^{\eta}\right)^{1/1-\eta}$. In what follows, we thus assume that the following condition holds:

$$\hat{h} \leq \left(\underline{a} V_S{}^{\eta}\right)^{1/1-\eta}$$

3.2.2. Steady stratification

In the preceding sub-section, we have defined the steady segment corresponding to each type of study. In this section, we define a 'steady stratification', and we subsequently show that a 'higher education system' always tends towards a steady stratification.

Definition 3. A *higher education system* $\left(\hat{h}, \alpha, V_S, V_E\right)$ is defined by four elements, i.e., (i) the two selection procedures $\hat{h}$ and α, and (ii) the respective qualities, V_S and V_E.

Definition 4. A *Steady stratification* is a situation in which:

1) The size of each social group remains unchanged from one generation to the next.

2) At each generation, all individuals are inside a steady segment and (i) all the individuals inside the lower class have a human capital belonging to the *B*-steady segment $\left[\underline{h}_B, \overline{h}_B\right]$, (ii) all the individuals inside the middle class have a human capital inside the *S*-steady segment $\left[\underline{h}_S, \overline{h}_S\right]$, and (iii) all the individuals within the elite have a human capital inside the *E*-steady segment $\left[\underline{h}_E, \overline{h}_E\right]$.

It is essential to note that being in a steady stratification does not mean that dynasties indefinitely remain in the same segment. When there is social mobility, dynasties inside the middle class can move to the elite and *vice versa*. However, the number of dynasties in each segment, and thereby in each social group, remains unchanged. Given Definitions 3 and 4, we can establish the following:

Lemma 3. *Consider a higher education system* $\left(\hat{h}, \alpha, V_S, V_E\right)$ *characterised by the functions (1)-(3). Then, regardless of the initial distribution of human capital across dynasties, this system tends towards a steady stratification*.

Proof. Appendix A.

Lemma 3 shows that a two-tier higher education leads to a steady social stratification. This can be a two-group or a three-group stratification depending on the position of threshold $\hat{h}$ in relation to $\overline{h}_B$ (see Appendix A). When $\hat{h} \geq \overline{h}_B$, the steady stratification typically comprises the three social groups and the size of each group depends on both the characteristics of higher education and the initial (generation 0) human capital distribution across dynasties. If $\hat{h} < \overline{h}_B$, the steady stratification is characterised by two social groups only because the lower class vanishes during the transitional dynamics to the steady stratification which is then characterised by $\gamma = 1 - \alpha$. Hence, when the admission threshold $\hat{h}$ is sufficiently low, we end up with two social groups.

4. Social Mobility at the Top

The emergence of steady stratifications does not mean that there is no mobility across social groups. We now place ourselves in a situation of steady stratification to determine the condition for mobility to occur. By studying mobility at the steady stratification, in which the weight of each social group in the population remains constant, we escape from the usual problem concerning relative mobility when the weights of social groups change across generations (Blanden, 2013, p.42). In addition mobility in the transitional dynamics to the steady stratification crucially depends on the initial distribution of human capital and on the successive distributions of abilities.

4.1. The conditions for mobility

Social mobility at the top is defined as a situation in which a number of students from the middle class accede to the elite university, while the equivalent number of students from the elite enter the standard university, falling thereby in the middle class (since the size of the elite is constant by definition). These movements depend on human capital at the end of basic education, h_{it}^B, which determines the individuals' position and hence the type of university they can join.

Figure 3 depicts the rationale of mobility between the middle class and the elite at the end of basic education. On the *x*-axis, the middle class parents are in the *S*-steady segment and the elite parents in the *E*-steady segment. The *y*-axis displays the position of the offspring at the end of basic education: The middle class offspring are in segment $\left[\underline{h}_S^B, \overline{h}_S^B\right]$ and the elite offspring in segment $\left[\underline{h}_E^B, \overline{h}_E^B\right]$.

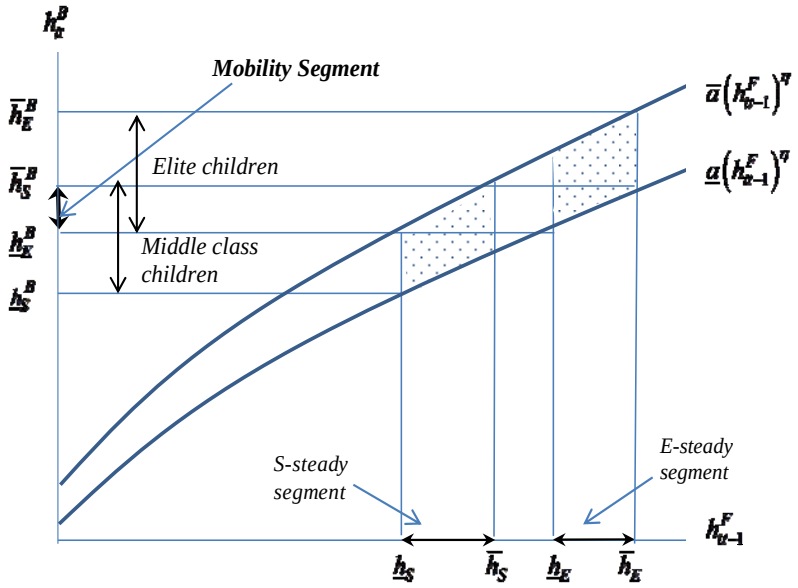

Figure 3. The mobility segment

The condition for students from the middle class to accede to an elite university is that, at the end of basic education, the most skilled from the middle class have a higher level than the least skilled from the elite, i.e., $\overline{h}_S^B > \underline{h}_E^B$ (otherwise, all the students from the elite enter the elite university and leave no opportunity for middle class students to enter the elite).

We define Condition 1 as:

Condition 1: $V_E / V_S < (\overline{a} / \underline{a})^{1/\eta}$

This establishes the following proposition on social mobility:

Proposition 1. *Assume that we are at a steady stratification. Then, social mobility exists if Condition* 1 *is fulfilled.*

Proof. There is social mobility if some students from the middle class can enter the elite, i.e., $\underline{h}_E^B < \overline{h}_S^B \Leftrightarrow \underline{a}\left(\underline{h}_E\right)^\eta < \overline{a}\left(\overline{h}_S\right)^\eta \Leftrightarrow \ \underline{a}^{1/(1-\eta)} V_E^{\eta/(1-\eta)} < \overline{a}^{1/(1-\eta)} V_S^{\eta/(1-\eta)} \Leftrightarrow V_E / V_S < (\overline{a} / \underline{a})^{1/\eta}$.

When Condition 1 is not fulfilled, students from the middle class remain in the middle class and students from the elite remain in the elite. This is the case when segments $\left[\underline{h}_S^B, \overline{h}_S^B\right]$ and $\left[\underline{h}_E^B, \overline{h}_E^B\right]$ on the *y*-axis do not overlap. Each group is then fully insulated from the other. In contrast, when Condition 1 is met, there is room for social mobility at the top.

It should be emphasized that Condition 1 depends on the quality gap between universities (V_E / V_S) and on the difference in ability between the most and the least able student ($\overline{a} / \underline{a}$). In fact, for social mobility to occur, the quality gap between the two types of

universities must not be too large compared to the difference in abilities. The reason for this is that the quality gap determines the difference in attainment at the end of basic education due to social origins (elite parents received an elite education V_E whereas middle class parents had a standard education V_S) which lessens mobility. In contrast, a larger interval of personal abilities fosters mobility, provided that abilities are independent from social origin. Consequently, the condition for social mobility critically depends on the quality gap between elite and standard universities.

Definition 6. Assume that Condition 1 is fulfilled. Then, the segment $\left[\underline{h}_E^B, \overline{h}_S^B\right]$ is defined as the *mobility segment*.

The mobility segment $\left[\underline{h}_E^B, \overline{h}_S^B\right] = \left[\left(\underline{a}V_E{}^{\eta}\right)^{1/(1-\eta)}, \left(\overline{a}V_S{}^{\eta}\right)^{1/(1-\eta)}\right]$ is the set in which the attainments of the middle class offspring and of the elite offspring overlap at the end of basic education (Figure 3). It is thus the set in which social mobility can occur. Figure 3 shows that the mobility segment can exist even when the *S*-steady segment and the *E*-steady segment do not overlap on the *x*-axis. When the *S*-steady segment and the *E*-steady segment overlap, the mobility segment does exist.

4.2. Indicators of mobility at the top

We assume that *Condition* 1 is fulfilled, i.e., the mobility segment does exist, and we set ourselves at the steady stratification.

Definition 7. We define the following three indicators of mobility:

1) The *Elite self-reproduction rate* is the proportion ρ of students from the elite who remain in the elite.
2) The *Middle class upward mobility rate* is the proportion μ of students from the middle class who enter the elite.
3) The *Mobility at the top index* is the ratio $\tau = \mu / \rho$.

We analyse mobility from the double point of view of the middle class and of the elite. As our approach generates social classes, we construct mobility indicators which measure between-class relative mobility, i.e., the likelihood of children born in one class (i) to remain in this class and (ii) to enter another class. Since we analyse mobility at the top, we focus on mobility between the middle class and the elite. Therefore we define an elite self-

reproduction indicator as well as a middle class upward mobility indicator. In addition, the 'Mobility at the top' index $\tau = \mu / \rho$ measures the difference in the opportunity to enter the elite group between the offspring from the middle class and those from the elite. Its inverse $1/\tau$ measures how many times a child from the elite is more likely to belong to the elite once adult compared to a child from the middle class. When $\tau = 1$, this opportunity is the same and there is full mobility at the top. When $\tau = 0$ the self-reproduction of the elite is total and mobility at the top is nil.

Note that these ratios are in line with the usual measurements made by sociologists from mobility tables, which is consistent with the study of between-class mobility.[13] It is clear that, as we focus on between-class mobility, the IGE is not an appropriate measure of mobility within our theoretical framework. In fact, a simple OLS estimation would provide the IGE η when controlling for the per-student expenditure on education (ability would then be part of the error term).

We now study the impacts of changes in the structure of higher education on mobility indicators.

4.3. Quality and mobility

Proposition 2. *An increase in the elite university quality* V_E *and/or a decrease in the standard university quality* V_S*:*

1) increases the elite self-reproduction ρ *.*

2) decreases the middle class mobility rate μ *.*

3) decreases the mobility at the top index τ *.*

Proof. Appendix B. Opposite changes in V_E and V_S have the opposite effects.

Proposition 2 shows that 'ceteris paribus' an increase in the funding allocated to elite universities that rises their quality lowers social mobility at the top. Fewer students from the middle class can join the elite, i.e., the elite self-reproduction is reinforced. This derives from the fact that, with the rise in the quality of the elite university, the middle class will turn out to possess less human capital relative to the elite. As a consequence, fewer highly able students from the middle class will surpass the human capital level of the low able students from the upper class because of the growing difference in intra-family transfers between social groups. Thus, more students from the elite remain in the elite and social mobility at the top is lessened.

[13] Blanden (2013) pp. 41-42 for a discussion.

This Proposition shows that an increase in the funding for elite universities not only increases the elite's human capital and future income, but also it reduces social mobility.

Similarly, decreasing V_S lowers the skill level of the middle class, providing their offspring with fewer intra-family human capital transfers and thus a worsened position in the competition for the entry in the elite university.

Proposition 2 has two major implications:

1. Ceteris paribus, countries in which the quality gap between universities is bigger because elite universities benefit from higher per student expenditures should exhibit lower mobility at the top and thereby lower intergenerational mobility.

2. An increase in the funding of higher education that primarily benefits elite universities *does not foster social mobility*. Quite the opposite, it reduces the middle class upward mobility and reinforces the elite self-reproduction.

Finally, if the increase in V_E and/or the decrease in V_S are sufficiently high so that Condition 1 no longer holds, i.e., $V_E / V_S > (\overline{a} / \underline{a})^{1/\eta}$, then, based on Proposition 1, we get no social mobility at all, with a complete self-reproduction of the elite.

4.4. Changes in admission rules

We now introduce exogenous changes in the admission rules so as to determine their impacts on social mobility between the middle class and the elite. We firstly analyse the effect of a change in the selection to enter the elite university (α), and subsequently a change in the admission to the standard university ($\hat{h}$).

Proposition 3. *An increase in the share α of students recruited by the elite university without change in the admission to the standard university $\hat{h}$ entails:*

1) *an increase in the elite self-reproduction rate ρ,*
2) *an increase in the middle class upward mobility rate μ, and*
3) *an ambiguous impact on the 'mobility at the top' index, τ.*

Proof. Appendix C.

Proposition 3 shows that reducing the tightness of recruitment to elite university, making thereby the elite larger, leads to both an increase in the middle class upward mobility and an increase in self-reproduction of elites. The explanation is as follows.

The increasing effect of α upon μ is straightforward because more offspring from the middle class join the elite group and the size of the middle class decreases (since α increases

and $\hat{h}$ remains unchanged. As regards ρ there are two opposite effects. On the one hand, the rise in α tends to decrease the elite self-reproduction by increasing the number of elite members, i.e., the denominator of coefficient ρ. On the other hand, there is an increase in the number of elite offspring entering the elite university, which is high enough to over-compensate the increase in the number of elite offspring, raising thereby the elite self-reproduction.

Finally, as mobility increases for the middle class but lessens for the elite, the effect on mobility at the top, τ, is ambiguous.

The analysis of a change in the admission level to enter the standard university, $\hat{h}$, leads to the following results (proofs in Appendix D):

Proposition 4. *A decrease in the admission level to enter the standard university, $\hat{h}$, which augments the size γ of the middle class, without change in α, induces at the steady stratification:*

1) *A decrease in the elite self-reproduction ρ.*
2) *A decrease in the middle class upward mobility μ.*
3) *An ambiguous impact on the 'mobility at the top' index τ.*

Firstly note that if the decrease in $\hat{h}$ that makes children from the lower class move in the middle class is permanent, then the lower class vanishes. For the middle class to increase without disappearing of the lower class, the decrease is $\hat{h}$ must be temporary. This feature is explained and discussed in Appendix D.

Proposition 4 shows that a lower $\hat{h}$ leading to the enlargement of the middle class tends to jeopardise its upward mobility. This is because, as the middle class gets larger, the number of its offspring who are candidate to the elite university augments whereas the number of students accepted in this university remains constant. This enlargement also lowers the elite self-reproduction because more offspring from the middle class accede to the elite. Note that, if the relative upward mobility of the middle class decreases, its mobility increases in absolute terms since the number of middle class children who accede to the elite augments.

It must finally be underlined that these results concern the steady stratification only. At the time when the increase in γ occurs, the new middle class members are better off compared to their parents. Hence, compared to the previous generation, the increase in γ is experienced as an improvement by the middle class (because of the improvement of its bottom side) and as no change by the elite (because the top side of the human capital distribution is not impacted yet). It is only in the following generations that the middle class suffers a decrease in its upward mobility and the elite a decrease in its self-reproduction.

We did not formally tackle an increase in population in our model, but our results can be easily applied to the analysis of the impact of a growing population in the following way. First, if the number of students recruited by the elite universities, α, increases at the same rate as the population, then all the results are unchanged. In contrast, if this number remains constant, or if it grows at a lower rate than the population, then this is equivalent to a decrease in α in our model. Hence, population growth accompanied with a lower growth of the number of students admitted to elite universities reduces the mobility to the top, because fewer children from the middle class can join the elite.

5. Dynamics and simulations

The model developed in the preceding sections has essentially analysed social mobility at the steady stratification. This is because the paths to the steady stratification are numerous and they crucially depend on the initial distribution of human capital across individuals. In this section, calibrations are implemented to illustrate the dynamics leading to the various steady stratifications with plausible values of the model parameters.

More precisely, we start from a purely egalitarian situation in which all individuals are initially endowed with the same human capital and we analyse the dynamics of social stratification and the self-reproduction of the elite group. We show that, even when starting from perfect equality, the two-tier higher education system generates social stratification with low social mobility.

This exercise is carried out within three scenarios picturing different types of education systems that differ in their elitist orientation. Finally, for each generation in each scenario, we calculate, the coefficients μ (middle class upward mobility rate), ρ (elite self-reproduction rate), and τ (mobility at the top index).

It must be emphasized that the calibrations implemented here cannot portray the situation of one country. They solely show that, with plausible values of the parameters, the impact of the structure of higher education is significant regardless of the funding constraint. Note that additional simulations were made with a non-egalitarian initial distribution of human capital. These simulations illustrate the results found in Section 4 when introducing changes over time in the higher education system.

5.1. Three scenarios

We start from a situation (generation 0) in which human capital is lower than $\hat{h}$ and all individuals possess the same human capital, i.e., perfect equality: $h_{i0}^F = h_0^F < \hat{h},\ \forall i$. From

this initial situation, we study the dynamics that derive from three higher education systems that differ in their expenditures per student in the two universities, V_S and V_E .
Table 1 depicts the parameters and values common to the three scenarios and Table 2 those corresponding to each scenario.

Table 1. Parameters and values common to the three scenarios

h_0^F	$\hat{h}$	α	η	$\underline{a}$	$\overline{a}$	$\underline{h}_B$	$\overline{h}_B$	$(\overline{a}/\underline{a})^{1/\eta}$ *
0.5	0.6	5%	0.3	0.6	1	0.54	1	5.49

* $V_E / V_S < (\overline{a}/\underline{a})^{1/\eta}$ is the condition for mobility.

Table 2. The Three scenarios

	V_S	V_E	$\underline{h}_S$	$\overline{h}_S$	$\underline{h}_E$	$\overline{h}_E$	$(\underline{a}V_S^{\eta})^{1/1-\eta}$ *
Egalitarian (*Equality*)	2.1	2.1	1.39	2.88	1.39	2.89	0.66
Elitist (*Elite*)	1.82	7.4	1.13	2.35	8.41	17.45	0.62
In-between (I-B)	2	4	1.30	2.69	3.49	7.25	0.65

* $\hat{h} \leq \underline{h}_S^B = \left(\underline{a}V_S^{\eta}\right)^{1/1-\eta}$ is the condition for offspring from the middle class never to fall in the lower class.

From generation 1, we assume that the elite represent 5% of the population. The value $\eta = 0.3$ corresponds to the average level of the intergenerational earnings elasticity (IGE) in advanced countries.[14] The ability interval corresponds to a ratio $\overline{a}/\underline{a} = 1.666$ which can be seen as a plausible gap between the highest and the lowest innate ability, given our hypothesis that these are randomly distributed across individuals.[15]

In the three scenarios, the parameters were chosen such that: (i) $\underline{h}_B < \hat{h} < \underline{h}_S^B < \overline{h}_B$, which implies that a dynasty never goes back to the lower class once it has left it,[16] and (ii) Condition 1 holds, i.e. $V_E / V_S < (\overline{a}/\underline{a})^{1/\eta}$, which ensures that the mobility segment does exist.[17]

The three scenarios are:

1. The egalitarian scenario ('*Equality*') which stipulates equal funding in both universities. This scenario is crucial because it constitutes a benchmark for the two other cases. In fact, even with equal funding, offspring whose parents are in the top 5% are more likely to be in the top 5% because of intra-family externalities.

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[14] Hence, an econometric estimation from the human capital values determined by the model after controlling for the school quality (1, V_S and V_E) provides an IGE of 0.3.

[15] 95.6 % of children have an IQ between 70 and 129 according to Gregory (1995, Table 4), but (i) the distribution is Gaussian with about 50% inside the IQ interval 90-109 and (ii) IQ is not fully independent from social origins.

[16] Because of the low and perfectly equally distributed human capital at generation 0, (i) assuming $\hat{h} > \overline{h}_B$ would induce that all dynasties perpetually remain in the lower class, i.e. none ever attains $\hat{h}$, and (ii) assuming $\hat{h} < \underline{h}_B$ would very rapidly lead to the vanishing of the lower class and the setting of the two-group steady stratification.

[17] For the values of the parameters, there is no mobility at all when expenditures per student in the elite university are more than 5.5 times higher than those in the standard university.

2. The elitist scenario ('*Elite*') which assumes a ratio V_E / V_S equal to 4. This corresponds to the upper limit of what is observed in the most elitist systems, as in the US, the UK and France, in which this ratio is between 3 and 4.

3. A scenario 'in-between' (*IB*) where $V_E / V_S = 2$.

Finally, expenditures in both universities were chosen so that total expenditure in higher education is identical in the three scenarios. We assume 500 dynasties and we calculate human capital and social stratification corresponding to the three above-described education systems for 13 successive generations following the initial generation 0.

5.2. Results

As expected, the system generates a steady stratification without lower class, which is attained at generation six in the scenarios *Equality* and *IB*, and seven in scenario *Elite*. From then, the size of each social group remains constant. The size of each social group is reported in Tables E1-E3 in Appendix E.

It should be recalled that, without intra-family externality (i.e., with $\eta = 0$), the elite self-reproduction rate ρ and the middle class upward mobility μ would be identical and equal to 0.05 since $\alpha = 5\%$. This corresponds to a perfect mobility and equality of opportunity.

Table 3 depicts, for each of the three scenarios, the elite self-reproduction rate, the middle class upward mobility rate, and the mobility at the top index, on average at the steady stratification. We also add the inverse of the mobility index, $1/\tau$, which represents how many times a child born in an elite family has more chance to be in the elite group once adult compared to a middle-class child. (The results of the simulations at each generation are depicted in Tables E1-E3 in Appendix E).

Table 3. Mobility at the steady stratification

	Equal Opportunity	*Equality* (benchmark)	*In-Between*	*Elite*
Average ρ (%)	5	17.7	66.9	96.6
Average μ (%)	5	4.3	1.7	0.2
Average $\tau = \mu / \rho$	1	0.244	0.026	0.0021
Average $1/\tau$	1	4.1	38.4	483

In the case of equal per student expenditures (*Equality*), the elite self-reproduction rate is equal to 17.7%, i.e., 3.5 times higher than its value corresponding to equal opportunity (which is 5%). This typically reproduces the impact of intra-family human capital externality and transfers. Despite the fact that elite offspring have 4 times more likelihood to enter the

elite than middle class offspring ($1/\tau = 4.1$), the middle class does not suffer much from the intra-family externality (since the middle class upward mobility rate is 4.3%, i.e., only slightly lower than in the case of equal opportunity).

In the 'in-between' scenario, i.e., with expenditures per student in elite universities twice as large as that in standard ones, the elite are noticeably self-reproducing since 2/3 of elite offspring remain in the elite versus less than 1.7% of middle class offspring who enter the elite. In this case, students from the elite are about 40 times more likely to be in the elite than those from the middle class, and the middle class upward mobility rate is substantially reduced (1.7% compared to 4.3% in the 'equal' scenario, and to 5% with equal opportunity).

The 'Elitist' scenario significantly amplifies the previous results. The elite self-reproduction is now almost total, since ρ is higher than 95%, and the middle class' opportunity to go up the social ladder is almost nil ($\mu = 0.2\%$), which means that entering the elite is 483 times more likely for elite offspring than for middle class offspring.

In summary, the simulations implemented with plausible values of the parameters show that dual higher education is a powerful factor of social immobility. Compared to the situation with a unified tertiary education (scenario '*Equality*'), the two-tier system makes the elite self-reproduction move from 17.7% up to 67% when the expenditure per student is twice higher in the elite university than in the standard one, and up to 96.6% when it is four times higher. Moreover, the middle class upward mobility rate falls from 4.3% (i.e., rather close to its equal opportunity value, 5%) to 1.7% and 0.2% respectively.

The core of our results lies in the respective intensities of personal ability (a_{it}) and intra-family transfers (measured by the parents' human capital h_{it-1}^{F}). These results show that when differences in budget allocations between elite and standard universities are not large, then the effects of intra-family transfers are also not too large and personal ability prevails, which results in social mobility. In contrast, when budget allocations to the elite university are high compared to the standard one, the difference in social backgrounds prevails, which boost the self-reproduction of elites.

6. Discussion and conclusion

This paper shows that social stratification and social mobility are closely related to the structure of higher education. An education system characterised by a division of higher education into elite and standard universities leads to permanent social stratification between the middle class and the elite, the latter being to a large extent self-reproducing. This is even true in case of democratization and meritocracy in tertiary education. Moreover,

we find that a two-tier higher education always tends towards a steady stratification and the simulations presented in Section 5 suggest that this stratification could be attained after a limited number of generations.

A major outcome of the paper is that, the greater the difference in quality and per-student expenditures between the elite and standard universities, the lower the upward social mobility of the middle class, and the more self-reproducing the elite group.[18] The simulations using plausible values of the parameters show that this impact can be large.

The core mechanism of this result lies in the respective intensities of personal ability (a_{it}) and intra-family transfers (measured by the parents' human capital h_{it-1}^{F}). When differences in budget allocations between elite and standard universities are not too large, then the effects of intra-family transfers are limited and personal ability prevails, which results in social mobility. In contrast, when budgets allocated to elite universities are large compared to those of standard universities, the difference in social backgrounds prevails, which boosts the self-reproduction of elites. This reveals that an increase in public spending on higher education could drive down social mobility. This happens when the increase in budget primarily benefits elite universities. Indeed, an increase in per-student expenditures in elite universities compared to standard ones hampers social mobility at the top by reinforcing the weight of intra-family human capital transfers at the expense of personal ability.

We obtain two additional results regarding the changes in admission rules. First, a wider recruitment in the elite universities, i.e., an increase in the proportion of elite households in the population α, increases both middle class upward mobility and elite self-reproduction. Second, an easing of the standard universities admission (decrease in $\hat{h}$) lowers the middle class upward mobility, but it also decreases the self-reproduction of elites.

These results predict that countries with more elitist higher education systems should exhibit less mobility at the top and, ceteris paribus, less intergenerational mobility. This is typically what is observed. The post-World War II period experienced a rapid democratization of tertiary education, due to a substantial expansion of admissions to the standard universities (lower $\hat{h}$). This initially fostered social mobility because the middle class newcomers had parents from the lower class, and their access to university drove mobility upward. However, this increase in social mobility was transitional. Once individuals from the lower class had joined the middle class, their children have not continued going up the social ladder. These dynamics have been observed in most advanced

[18] Fabella (2016) studies the determinants of reforms in quality, but she focuses on primary and secondary education.

countries.[19] These facts are in line with our model, which shows that a slackening in the rigor of admission in standard universities leads first to an increase in social mobility, but in the longer term it leads to a decrease in the middle class upward mobility.

In addition to democratization in higher education experienced by all countries, the gap between the elite and standard universities in terms of per-student budgets has widened in several countries over the last two decades. Proposition 2 suggests that this has both reduced the middle class upward mobility and boosted the self-reproduction of the elite. Consequently, the policies of increasing higher education budgets so as to promote social mobility could have paradoxically leaded to the opposite result, i.e. diminishing social mobility at the top, when they essentially benefited to the most prestigious establishments.

Let us finally discuss the impacts of relaxing certain simplifying assumptions.

First, our results were determined by assuming that the only cost of universities was the opportunity cost linked to the time spent in higher education. Hence, families' incomes and wealth have no impact on educational choices. This simplifying assumption was motivated by the objective to focus on the structure of tertiary education without considering the funding and credit constraints which have been abundantly analysed in the economic literature. It is clear that adding a cost of higher education with imperfections on the credit market would (i) lengthen the transition to steady stratification and (ii) magnify the anti-mobility bias of the two-tier system. This is simply because the constrained families are those with a low income, i.e., with a low human capital of parents.

Second, we have assumed that the admission to elite universities was based on a given number of intakes, α, and not on a human capital threshold. This feature is justified by the observation that, either this is effectively the way they select their students (in the case of the French *grandes écoles*), or the elite establishments have increased their entry requirements to maintain a narrow number of admitted (in the case of the US), which is equivalent to defining a number of intakes. If we assume that the selection to elite universities is based on a given human capital threshold which is stricter than that of standard universities, then the findings are significantly modified. In particular, an increase in the elite university's budget does not lessen the middle class upward mobility, but it significantly enlarges the size of the elite group (demonstration available from the authors upon request).

Third, inserting a human capital externality inside each type of university would also magnify our findings on mobility, because this would reinforce family backgrounds.

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[19] Even in Nordic countries, despite the continuing increase in intergenerational mobility, some works suggest that it is not the case for mobility at the top (Bjorklund et al., 2012)

In conclusion, the paper shows that a two-tier higher education system, with differences in quality and per-student expenditure as well as in admission procedures between standard and elite universities, can be a key factor in generating permanent social stratification and social immobility.

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Appendix A. The education system always tends towards a steady stratification

A steady stratification is characterised by the following two features (Definition 5):

1) The size of each social group remains unchanged from one generation to the next.

2) At each generation, all the dynasties inside the lower class have a human capital inside the *B*-steady segment, all individuals inside the middle class a human capital inside the *S*-steady segment and all individuals in the elite a human capital inside the *E*-steady segment.

1) *Feature* 1

Let us denote α, γ and ε the respective sizes of the elite, the middle class and the lower class, with $\alpha + \gamma + \varepsilon = 1$.

As α is given, it is sufficient to show that the size of one additional social group tends towards a constant value to show Feature 1. We show that ε tends towards a constant value. For this, we consider the three cases, $\hat{h} < \underline{h}_B$, $\hat{h} > \overline{h}_B$ and $\underline{h}_B < \hat{h} < \overline{h}_B$:

(i) $\hat{h} < \underline{h}_B$: Then all dynasties can enter the university after a limited number of generations. From this time, $\varepsilon = 0$.

(ii) $\hat{h} \geq \overline{h}_B$: Then (a) all the dynasties with an initial human capital (i.e., at generation 0) below $\hat{h}$ perpetually remain under-educated (they never enter the university) and (b) all the dynasties with an initial human capital above $\hat{h}$ enter the university from the first generation, and all their offspring also pursue higher education. Consequently, ε is constant and equal to the number of parents with an initial human capital below $\hat{h}$.

(iii) $\underline{h}_B < \hat{h} < \overline{h}_B$: Once a dynasty is above $\hat{h}$, then this dynasty *perpetually* follows higher education. In addition, all dynasties sooner or later move above $\hat{h}$ because the probability for a dynasty never to move above $\hat{h}$ tends towards 0 when time tends toward infinite. Hence, ε tends towards 0.

The above reasoning reveals two situations. First, when $\hat{h} < \overline{h}_B$, the education system determines one unique steady stratification characterised by α individuals inside the elite and $1-\alpha$ inside the middle class. Second, when $\hat{h} \geq \overline{h}_B$, the steady stratification comprises three social groups and it depends on both the characteristics of the higher education system and the initial distribution of human capital across dynasties.

2) *Feature* 2

To show Feature 2, we must prove that:

(i) When an individual is in the *j*-steady segment, *j* = *S*, *E*, then her child is also in this segment, if she follows the same study *j*.

(ii) When an individual is in the middle class (*S*-steady segment) and her offspring enters the elite school, she accede to the elite class and is then inside the *E*-steady segment.

(iii) When an individual is in the elite class (*E*-steady segment), and her offspring enters the standard university, she enters the middle class and is then in the *S*-steady segment.

Condition (i): This is straightforward by definition of the *j*-segment.

Condition (ii): We denote v the child born inside the middle class who possesses the lowest human capital at the end of basic education among those entering the elite university.

Consider a child born inside the elite with a skill at the end of basic education lower than that of child v (this child does exist since v enters the elite university). This child does not

enter the elite university. Would she study in the elite university, she would be inside the *E*-segment (by definition of the *E*-segment), and her skill would be higher than $\underline{h}_E$. Hence child v who pursued studies at the elite university has a final human capital higher than $\underline{h}_E$. In addition, v's parent has a skill level lower than $\overline{h}_E$, and hence v also has a skill level lower than $\overline{h}_E$. Therefore v's final skill belongs to the *E*-steady segment $\left[\underline{h}_E, \overline{h}_E\right]$. This is also the case of all children from the middle class who enter the elite university since their final human capital is higher than h_v^F and lower than $\overline{h}_E$.

Condition (iii): The same proof applies in the case in which a child whose parents belong to the elite enters the standard university. More precisely, this child has a final education lower than that of the most skilled offspring of the middle class, would the latter pursue a standard university study, and higher than the least skilled offspring from the middle class.

Appendix B. Impacts of changes in qualities

We firstly determine the human capital threshold at the end of basic education that separates the children entering the elite university from the others, and we show that this threshold is inside the mobility segment. We subsequently analyse the impact of changes in quality.

1. The human capital threshold from which children enter the elite university

Assume that the mobility segment $\left[\underline{h}_E^B, \overline{h}_S^B\right]$ does exist and let m and e be respectively the number of offspring from the middle class and from the elite in the mobility segment. The children inside the mobility segment, whatever their origin (middle class or elite), are ranked in ascending order of skill at the end of basic education. The number of offspring inside segment $\left[\overline{h}_S^B, \overline{h}_E^B\right]$ is $\alpha - e < \alpha$ and the number of offspring inside segment $\left[\underline{h}_E^B, \overline{h}_E^B\right]$ is $\alpha + m > \alpha$. There is thus a unique $h_\alpha \in \left[\underline{h}_E^B, \overline{h}_S^B\right]$ such that the number of offspring with a skill level higher or equal to h_α at the end of basic education is equal to α.

The determination of h_α (subscript t denoting the generation is omitted to simplify) is as follows.

Consider the variable h inside the mobility segment: $h \in \left[\underline{h}_E^B, \overline{h}_S^B\right]$. We define the following functions:

1) $E(h) =$ Number of offspring from the elite in the segment $\left[\underline{h}_E^B, h\right]$. On the mobility segment, $E(h)$ is continuous and monotonically increasing from 0 to *e*.

2) $M(h) =$ Number of offspring from the middle class in the segment $\left[h, \overline{h}_S^B\right]$. On the mobility segment, $M(h)$ is continuous and monotonically decreasing from *m* to 0.

Hence, h_α is the unique value of h belonging to segment $\left[\underline{h}_E^B, \overline{h}_S^B\right]$ such that $E(h) = M(h)$. The diagrammatic determination of h_α is depicted in Figure B1, where we choose $m > e$ without loss of generality.

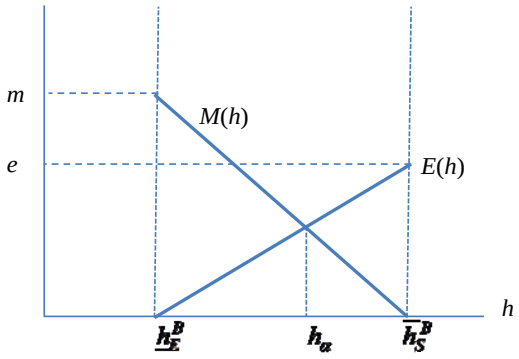

Figure B1. Determination of h_α

2. Changes in qualities

For a given size of the elite (α), the number of middle class offspring who enter the elite university is equal to the number of elite offspring who enter the standard university. As μ is the proportion of middle class children who enter the elite, and $(1-\rho)$ the proportion of children from the elite who leave the elite, we have:

$$\mu\gamma = (1-\rho)\alpha$$

We suppose a continuum of children on the segments $\left[\underline{h}_S^B, \overline{h}_S^B\right]$ and $\left[\underline{h}_E^B, \overline{h}_E^B\right]$.

Let $f_S(h)$ and $f_E(h)$ be the density functions of human capital at the end of basic education for the middle class and elite offspring respectively. Thus, (i) $f_S(h)$ is defined on segment $\left[\underline{h}_S^B, \overline{h}_S^B\right]$ and $\int_{\underline{h}_S^B}^{\overline{h}_S^B} f_S(h)dh = 1$, and (ii) $f_E(h)$ is defined on segment $\left[\underline{h}_E^B, \overline{h}_E^B\right]$ and $\int_{\underline{h}_E^B}^{\overline{h}_E^B} f_E(h)dh = 1$. We make no particular assumption on functions $f_S(h)$ and $f_E(h)$, except that $f_S(h) > 0, \forall h \in \left]\underline{h}_S^B, \overline{h}_S^B\right[$ and $f_E(h) > 0, \forall h \in \left]\underline{h}_E^B, \overline{h}_E^B\right[$ (continuum of children).

1) *Increase in* V_E

An increase in V_E, moves the skill of all individuals in the elite upwards, which increases the post basic education skill of their offspring. In Figure 3 in the text, the *E*-steady segment $\left[\underline{h}_E, \overline{h}_E\right]$ as well as the corresponding post-basic education segment $\left[\underline{h}_E^B, \overline{h}_E^B\right]$ move upwards. Consequently, the threshold h_α moves upwards too. This leads to:

1) A decrease in $\mu = \int_{h_\alpha}^{\overline{h}_S^B} f_S(h)dh$ since h_α increases whereas $\overline{h}_S^B$ remains unchanged.

2) Since $\mu\gamma = (1-\rho)\alpha$, and since α and γ are constant, then ρ increases.

3) A decrease in $\tau = \mu / \rho$ since μ decreases and ρ increases.

2) *Decrease in* V_S

A decrease in V_S, moves downwards the S-steady $\left[\underline{h}_S, \overline{h}_S\right]$ as well as the corresponding post-basic education segment $\left[\underline{h}_S^B, \overline{h}_S^B\right]$ in Figure 3. Consequently, h_α also moves downwards. This leads to:

1) A increase in $\rho = \int_{h_\alpha}^{\bar{h}_E^B} f_E(h)dh$ since h_α decreases and $\bar{h}_E^B$ remains unchanged.

2) A decrease in μ because $\mu\gamma = (1-\rho)\alpha$, ρ increases and α and γ are constant.

3) A decrease in $\tau = \mu / \rho$.

Appendix C. Impact of a change in α

Assume an increase in the weight of the elite in the population α which moves from α_1 up to $\alpha_2 > \alpha_1$, without change in $\hat{h}$.

The values $\underline{h}_S^B = \left(\underline{a} V_S^{\eta}\right)^{1/(1-\eta)}$, $\bar{h}_S^B = \left(\bar{a} V_S^{\eta}\right)^{1/(1-\eta)}$, $\underline{h}_E^B = \left(\underline{a} V_E^{\eta}\right)^{1/(1-\eta)}$ and $\bar{h}_E^B = \left(\bar{a} V_E^{\eta}\right)^{1/(1-\eta)}$ are independent from α and $\hat{h}$, and remain thereby constant.

As $\hat{h}$ is constant and since we start from a steady stratification, the number of children who enter the university remains unchanged and $\delta = \alpha + \gamma$ is constant. Hence, the rise in α entails a decrease in γ that moves from γ_1 down to $\gamma_2 < \gamma_1$.

For any density functions $f_S(h)$ and $f_E(h)$ defined in Appendix B, the increase in α moves the thresholds h_α to accede to the elite university downwards. This is because $\alpha = \mu\gamma + \rho\alpha = \mu(\delta - \alpha) + \rho\alpha \Rightarrow \alpha = \dfrac{\mu}{1+\mu-\rho}\delta$. As $\mu = \int_{h_\alpha}^{\bar{h}_S^B} f_S(h)dh$, then $\mu' = \dfrac{\partial \mu}{\partial h_\alpha} < 0$ and $\rho = \int_{h_\alpha}^{\bar{h}_E^B} f_E(h)dh \Rightarrow \rho' = \dfrac{\partial \rho}{\partial h_\alpha} < 0$. Hence: $\dfrac{\partial \alpha}{\partial h_\alpha} = \dfrac{\mu'(1+\mu-\rho) - (\mu' - \rho')\mu}{(1+\mu-\rho)^2} = \dfrac{\mu'(1-\rho) + \rho'\mu}{(1+\mu-\rho)^2} < 0$.

Then:

1) As h_α decreases, then $\mu = \int_{h_\alpha}^{\bar{h}_S^B} f_S(h)dh$ increases, i.e., the middle class upward mobility increases.

2) As h_α decreases, then $\rho = \int_{h_\alpha}^{\bar{h}_E^B} f_E(h)dh$ increases, i.e., the elite self-reproduction rises.

3) The variation of $\tau = \mu / \rho$ is ambiguous, depending on the distribution of the middle class children and the elite children in the mobility segment, and thus on $f_S(h)$ and $f_E(h)$.

Appendix D. Move(s) in $\hat{h}$ increasing γ

We firstly discuss the conditions for a decrease in $\hat{h}$ to entail an increase in the weight of the middle class γ (and thus a decline in the weight of the lower class). This is because the existence of the lower class at the steady stratification supposes that $\hat{h} \geq \bar{h}_B$. Hence, starting from a steady stratification:

1) if the decrease in $\hat{h}$ is insufficient to make $\hat{h}$ move below $\bar{h}_B$, then there is no change in the size of each social group;

2) if this decrease makes $\hat{h}$ move below $\bar{h}_B$, then the new steady stratification is characterised by the vanishing of the lower class and $\gamma = 1 - \alpha$.

Thus, for changes in $\hat{h}$ to induce an increase in γ without vanishing of the lower class, the move must be twofold. Firstly, the decrease in $\hat{h}$ must place it below $\overline{h}_B$ so as to make lower class children enter the university. In a second stage, $\hat{h}$ must move back above $\overline{h}_B$ to avoid the total vanishing of the lower class. If only the first shift occurs, then the size of the middle class finally attains $\gamma = 1-\alpha$ and the lower class disappears.

Assume that, when the change in $\hat{h}$ occurs, the lower class children are distributed in the *B*-steady segment according to density function $f_B(h)$.

To simplify the proof, we assume a downward move of $\hat{h}$ to the value $\hat{h}_t < \overline{h}_B$ at generation *t*, which is followed by an increase in $\hat{h}$ above $\overline{h}_B$ at generation $t+1$. More complex moves can be analysed in a similar way. Since the selection to enter the elite university remains unchanged, α is constant.

Let γ_0 be the size of the middle class before time *t*. The decrease in $\hat{h}$ makes $\lambda = (1-\alpha-\gamma_0)\int_{\hat{h}_t}^{\overline{h}_B} f_B(h)dh$ children from the lower class enter the standard university in *t*. Hence, the size of the middle class at generation *t* moves from γ_0 up to $\gamma' = \gamma_0 + \lambda$.

1) At generation *t* when $\hat{h}_t$ is set, (i) the increase in the middle class only concerns the bottom of this social group (the new members are the λ middle class individuals with the lowest human capital), and (ii) the mobility segment is the same as in the situation in which $\hat{h}$ would not have decreased. Hence, for generation *t*, the situation as regards mobility at the top is unchanged compared to the case without decrease in $\hat{h}$.

At generation *t*, the middle class is thus better off because, (i) its members with the lowest human capital have parents from the lower class and rising to the middle class is an improvement to them, and (ii) the opportunity to reach the elite is unchanged for those whose parents belong to the middle class. Mobility at the top is unchanged whereas mobility at the bottom increases.

2) We know that $\hat{h} \leq \left(\underline{a}V_S^{\eta}\right)^{1/1-\eta} = \underline{h}_S^B$ (section 3.2.1). Hence, the dynasties entering the university at time *t* will subsequently indefinitely enter the university. When $\hat{h}$ moves up to $\hat{h}_{t+1} > \overline{h}_B$ at time *t*+1, a new steady stratification is achieved at generation *t*+1, with $\gamma' > \gamma_0$ individuals in the middle class and α in the elite. For any density functions $f_S(h)$ and $f_E(h)$, the number of middle class children inside the mobility segment moves from $m_0 = \gamma_0 \int_{\underline{h}_E^B}^{\overline{h}_S^B} f_S(h)dh$ in the former steady stratification up to $m' = \gamma' \int_{\underline{h}_E^B}^{\overline{h}_S^B} f_S(h)dh$ in the new steady stratification, with $m' > m_0$ since $\gamma' > \gamma_0$. In Figure B1 (Appendix B) the curve *M*(*h*) revolves to the right around $\overline{h}_S^B$, which increases the value h_α. As h_α increases, both $\mu = \int_{h_\alpha}^{\overline{h}_S^B} f_S(h)dh$ and $\rho = \int_{h_\alpha}^{\overline{h}_E^B} f_E(h)dh$ decrease. Finally, as both μ and ρ decrease, the move in τ is ambiguous, depending on the distribution of the elite and middle class children inside the mobility segment.

Appendix E. Results of the Simulations

Table E1. Scenario '*Equal*'

Generations	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Size of the Lower class	500	152	37	11	4	2	0	0	0	0	0	0	0	0
Size of the Middle class	0	323	438	464	471	473	475	475	475	475	475	475	475	475
Size of the Elite	0	25	25	25	25	25	25	25	25	25	25	25	25	25
moves from LC to MC		323	115	26	7	2	2	0	0	0	0	0	0	0
Moves from MC to E		25*	21	20	22	19	23	21	19	23	23	21	16	21
μ			6.5	4.56	4.74	4.03	4.86	4.42	4	4.84	4.84	4.42	3.37	4.42
ρ			84.0	80.0	88.0	76.0	92.0	84.0	76.0	92.0	92.0	84.0	64.0	84.0

* From the lower class to the elite.

Table E2. Scenario '*In-between*'

Generations	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Size of the Lower class	500	152	61	20	4	2	0	0	0	0	0	0	0	0
Size of the Middle class	0	323	414	455	471	473	475	475	475	475	475	475	475	475
Size of the Elite	0	25	25	25	25	25	25	25	25	25	25	25	25	25
moves from LC to MC		323	91	41	16	2	2	0	0	0	0	0	0	0
Moves from MC to E		25*	9	8	9	9	11	11	9	7	9	7	8	7
μ			2.79	1.93	1.98	1.91	2.3	2.32	1.89	1.47	1.89	1.47	1.68	1.47
ρ			36.0	32.0	36.0	36.0	44.0	44.0	36.0	28.0	36.0	28.0	32.0	28.0

* From the lower class to the elite.

Table E3. Scenario '*Elite*'

Generations	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Size of the Lower class	500	152	44	14	5	3	2	0	0	0	0	0	0	0
Size of the Middle class	0	323	431	461	0.47	472	473	475	475	475	475	475	475	475
Size of the Elite	0	25	25	25	25	25	25	25	25	25	25	25	25	25
moves from LC to MC		323	108	30	9	2	2	2	0	0	0	0	0	0
Moves from MC to E		25*	3	0	1	0	2	1	1	0	1	1	1	1
μ			0.93	0.00	0.22	0.00	0.42	0.21	0.21	0.00	0.21	0.21	0.21	0.21
ρ			12.0	0.0	4.0	0.0	8.0	4.0	4.0	0.0	4.0	4.0	4.0	4.0

* From the lower class to the elite.