SOCIAL MOBILITY AT THE TOP: WHY ARE ELITES SELF-REPRODUCING?

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Abstract

This paper proposes an explanation for the decrease in social mobility that has occurred in the last two decades in a number of advanced economies, as well as for the divergence in mobility dynamics across countries. Within an intergenerational framework, we show that a two-tier higher education system with standard and elite universities generates social stratification, high social immobility and self-reproduction of the elite. Moreover, we show that the higher the relative funding for elite universities, the higher the elite self-reproduction, and the lower social mobility.

We also analyse the impacts of changes in the weight of the elite and of the middle class upon social mobility. Our findings provide theoretical bases for the inverted-U profile of social mobility experienced in several countries since World War II and to the 'Great Gatsby Curve' relating social mobility to inequality.

Keywords: Elite, Higher Education, Selection, Social mobility, Social stratification.

JEL Classification: I21, J62, O15, Z13.

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1. Introduction

This paper proposes an explanation for the decrease in social mobility that has occurred in the last two decades in a number of advanced economies, as well as for the divergence in mobility dynamics across countries.

The most recent works on social mobility bring to the conclusion that countries display divergent social mobility dynamics. For countries like the UK, the US and France, intergenerational mobility has followed an inverted-U profile since World War II, with a clear decrease in the last generations. In contrast, Nordic countries do not exhibit such a profile and they still present an increase in intergenerational social mobility.

A noticeable fact is that the countries displaying a decrease in social mobility are typically those characterised by a two-tier higher education system. In these countries, tertiary education comprises standard as well as elite universities that essentially differ in both their selection procedures and their budgets.

This paper shows that a *two-tier higher education* system provides an explanation for the observed decline in social mobility. On the one hand, a dual higher education system generates social stratification as well as a self-reproduction of the elites. On the other hand, changes in expenditures per student and in the tightness of selection lead to changes in social mobility. Thus, the decrease in social mobility observed in the last decades can be to a large extent explained by a relative change in expenditures per student in both types of universities.

In other words, this paper focuses on the effects of a two-tier higher education system with significant differences in expenditure between standard and elite universities upon social mobility and elites' self-reproduction. It should be noted that the emergence of a dual higher education system characterised by the concomitance of standard and elite universities appeared at the time of the democratisation and 'massification' within several advanced countries, which took place around the 1970s.

The two main differences between elite and standard universities are the tightness of their selection and the level of their expenditures per student. First, elite universities select a limited number of the best students and this number has not changed much in these last decades. In contrast, the number of students in standard universities has substantially increased. Moreover, despite a meritocratic recruitment, there is an increase in social stratification across students, with the students in elite universities essentially originating

from the top social classes.[1] Second, expenditures per student are significantly higher in elite universities than in standard ones, and this gap has increased in the last decades in several advanced countries.

In this paper, we show that the observed changes in the higher education system provide a key explanation for the observed changes in social mobility in the last decades. To analyse the impact of a dual higher education system on human capital accumulation, social stratification and social mobility, we develop a simple intergenerational model with two major assumptions. We firstly assume the existence of standard and elite universities that differ in their selection procedures and expenditures per student, with these characteristics defining a two-tier higher-education system.

Secondly, the individuals' human capital is assumed to be determined by three elements that differ across individuals: (i) their family backgrounds, i.e., their parents' human capital; (ii) their personal innate abilities that are randomly distributed, and (iii) their curriculum, i.e., the course of study they pursue.

This model permits to explain three main facts related to social mobility. A first result of the model is that the higher-education system and the human capital intergenerational dynamics determine a three-group social stratification. The lower class gathers the individuals with basic education only, the middle class the individuals with a standard university degree, and the upper class, i.e. the 'elite', is composed of those graduated from the elite university.

A second key result of our model is that a two-tier higher education system tends towards a '*steady* social stratification' in which each social group remains inside a steady human capital segment and the size of each group remains constant. Nevertheless, the existence of a steady social stratification can come with social mobility, and we determine the conditions for the existence of social mobility at the top, i.e., between the middle class and the elite.

The third and major finding is that an increase in expenditures per student in the elite university relative to the standard one *reduces social mobility*: fewer students from the middle class can enter the elite, and the elite self-reproduction is reinforced. The reason for this is as follows. An increase in the funding to the elite university enlarges the human capital gap between the elite and the middle class, which augments the weight of the cultural backgrounds transferred inside the family and reduces thereby the relative impact of

[1] See Arrow et al. (2000). Indeed, both selection procedures are meritocratic in the sense that they select students based on excellence and ability and not on social origin.

personal ability. Thus, entering the elite university becomes more difficult for the most able students from the middle class.

The simulations performed in the last section by adopting plausible values of the parameters show that the differences in expenditure per student between the two types of university have indeed a powerful impact on intergenerational social mobility.[2]

Finally, we also analyse changes in the weights of social groups. We find that when the share of the middle class in the population increases, social mobility rises in the generation when it occurs, but it lowers the upward mobility of the middle class later on. This is because the middle class enlargement firstly necessitates the passage of young from the lower to the middle class. But it afterwards increases the weight of the students from the middle class, reducing thereby their probability to join the elite.

These findings provide an interpretation of the main changes in education attainments and social mobility observed in advanced economies since World War II, as well as an explanation of the 'Great Gatsby curve'.

The paper is divided in five sections. In Section 2, the theoretical and empirical literature on the subject is reviewed. The model is developed in Section 3, and we analyse social mobility at the top in Section 4. The intergenerational dynamics is simulated and analysed in section 5. We discuss the main findings and conclude in section 6.

2. Empirical Regularities and Related Literature

We present the empirical regularities and the related literature on the two subjects associated with our model: (i) intergenerational social mobility, and (ii) the dual-system of higher education and social stratification.

2.1. Intergenerational Mobility and Social Stratification

In both the economic and sociological fields, studies on intergenerational mobility are many and diverse.[3] From an intergenerational point of view, the economic literature on social mobility started with the pioneering work of Becker and Tomes (1979) and Loury (1981). In

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[2] E.g., when moving from equal expenditure per student in both universities to twice higher expenditure in the elite university, this increases the elite self-reproduction from 17% to 67%, i.e., what Brezis and Crouzet (2006) find in the case of France.

[3] For reviews on the economic literature: Bjorklund and Jantti (2000, 2009), Black and Devereux (2011), and Chusseau and Hellier (2013).

these approaches, education and human capital dynamics lead to a convergence of dynasties towards the same level of education.[4] Consequently, the system leads to social equalization.

The theoretical economic literature has subsequently explored intergenerational models in which educational dynamics do not converge and generate social stratification. In these models, a number of dynasties remain lastingly or perpetually under-educated. The literature has emphasized several factors that create such under-education traps: a fixed cost of education with a credit constraint (Galor and Zeira, 1993; Barham et al., 1995); an S-shaped education function (Galor and Tsiddon, 1997b); local externalities and neighbourhood effects (Benabou, 1993, 1994, 1996a, 1996b; Durlauf, 1994, 1996) etc.

As regards empirical studies, both economic and sociological literatures lead to similar conclusions (Morgan et al., 2006). Following the works of Solon (1992), the former is essentially based on the calculation of intergenerational elasticities. Intergenerational earnings elasticity β (henceforth *IGE*) are calculated by estimating equation $y_{it} = \alpha + \beta y_{it-1} + \gamma x_{it} + u_{it}$, where y_{it} is the individual *i*'s income or educational attainment (in log), y_{it-1} the same variable for her parents, x_{it} a vector of control variables (age, gender, etc.) and u_{it} the error term. Intergenerational social mobility is then measured by the value $1-\beta$.

So as to account for changes in the deviation in incomes (or education attainments) across generations, a number of works also calculate the intergenerational correlation (*IGC*), $\rho = \beta\sigma_{t-1} / \sigma_t$, where σ_t is the studied variable standard deviation at generation t.

The numerous works on this subject reveal the following empirical regularities:

(i) intergenerational elasticities (IGE) significantly differ across countries, the lowest mobility being found in Italy, the UK, the US and France, with β between 0.4 and 0.5, and the highest in Nordic countries, with β below 0.3 (Corak, 2012).

(ii) There is a decrease in intergenerational mobility (increase in β) in the last two decades in the UK (Blanden et al., 2004, 2007; Nicoletti and Ermisch, 2007), France (Lefranc, 2011; Ben-Halima et al., 2013), and the US (Aaronson and Mazumder, 2008), three countries with a clear dual higher education system. One can note that before the development of the dual system, there had been a slight increase in mobility in these countries (Lee and Solon, 2009; Breen, 2009; Lefranc, 2011). In these countries, social mobility can thus be depicted by an inverted-U curve with a pick for the generations born around 1960-1970.

[4] This convergence can however be slowed down when the market for credit is imperfect.

(iii) In contrast, in Nordic countries with a non-dual higher education system, intergenerational mobility of earnings has increased during the last decade, and there is no inverted-U curve of social mobility.[5]

As regards the IGE, the results of the sociological empirical literature are quite similar (Erikson and Goldthorpe, 1992; Breen, 2009). Moreover, in addition to the IGE estimations, the sociological empirical works are based on the calculations of social fluidity and of social mobility tables. In these tables, the class positions are typically depicted by professional occupations and immobility is measured by the sum of the diagonal values (i.e., the proportion of the population remaining in the same social group from one generation to the next).[6]

The same analysis was implemented for education groups. With parental education being represented by using the CASMIN classification, several works show a low probability for children from the lower classes to enter the upper classes (Muller et al., 1989; Braun and Muller, 1997)

The sociological literature has also emphasized the changes in social structures after World War II. In fact, within most advanced countries, the proportion of blue collars in the working population has decreased, and the weight of the middle class has critically risen in the employed population. The sixties and seventies have seen both offspring from the lower classes to upgrade their skill and join the middle class, and offspring from the middle class to join the upper class.

Moreover, the sociological literature on the elite displays regularities somehow similar to the economic literature. They stress that, while the proportion of blue collars entering the elite class in the 1960s was double in the US as compared to Britain, France and Germany (Blau and Ducan, 1967), this mobility has decreased even in the US after the 1980s (see the review by Corcoran, 1995). They also emphasize changes in the mobility at the top: low before World War II, increasing after the war, and decreasing or constant from the 1980s.

Finally, the effects of the structure of educational systems upon social polarization, and particularly the influence of public expenditure for primary, secondary and higher education, were modelled and analysed within intergenerational approaches by Driskill and Horowitz (2002), Su (2004), Bertocchi and Spagat (2004) and Chusseau and Hellier (2011).

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5 Bratberg et al. (2005) for Norway, and Pekkala and Lucas, (2007) for Finland.

6 E.g., Ducan (1966), Parsons et al. (1992), and Breen (2009). Social fluidity, measured by the probability to move between classes, has also been studied (e.g., Breen, 2009).

2.2. Higher Education: Massification and Stratification

2.2.1 Empirical Regularities

Since World War II, education levels and education systems have known dramatic changes in advanced economies. These changes have substantially modified both social and educational structures within these societies. In what follows, we point to five stylised facts on which our approach is based.

1. A key change is the 'massification' of tertiary education. In most advanced economies, the share in a generation of the individuals pursuing tertiary education has shifted from about 10% in the decade following World War II to 60% and more in the present period.

2. In many advanced countries, this massification has come with the emergence of a dual system characterised by the concomitance of both standard and elite universities. Moreover, the huge increase in the numbers of students has essentially concerned standard universities, while selection has remained very narrow in elite establishments.

For the US, Su et al. (2012) report that the public post-secondary colleges, considered as non-elitist, have increased their enrolment by 525% between 1959 and 2008, whereas the enrolment in the elite colleges has increased by 250%.[7]

In France, elite universities are represented by the *Grandes écoles* (henceforth GE) that recruit less than 4% of a generation. On top of the traditional distinction between universities and *Grandes écoles*, Albouy and Wanecq (2003) defines the '*Très Grandes écoles*' (Top GE), which are the most prestigious leading to the highest top executive and public positions. They show that, for men, the share of a generation entering a top GE has decreased from 0.8 to 0.6% for the generations born in 1959-1968 compared to those born in 1929-1938, while in the same time the share of a generation completing tertiary education has been multiplied by more than 3.5 in France.[8]

This difference in the recruitment of students derives from the fact that standard universities have been opened to a large public of students and their selection process is quite lenient. It is sufficient to have successfully completed secondary education to enter a standard university.

In contrast, elite universities recruit a narrow number of students with a very severe selection procedure. Oxby (2009, p. 98) finds that, from 1962 to 2007, the SAT scores of colleges have increased for the most selective and decreased for the least selective.

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[7] based on the National Center for Education Statistics (Digest of Education Statistics 2009, Tables 3 and 189).

[8] The share of a generation entering a *grande ecole* (but not a top GE) has increased from 2.3% to 3.2%.

3. Standard and elite universities also differ in their budgets. The expenditure per student has always been substantially higher in elite universities than in standard ones, and this gap has increased in the last decades in a number of advanced countries. In the US, expenditure per student in elite universities (Ivy League) is more than three times higher than in other universities. In addition, this expenditure increased by 20% in the former from 1999 to 2009, whereas this increase was lower than 8% in the latter (Desrochers and Wellman, 2011).

In France in 2002, the spending per student is on average 3.5 times higher in the top GE than in the standard university.[9]

4. There is a clear social stratification in the access to elite universities. In the US, despite the wish to democratize selection, SAT scores are correlated with family education and wealth.[10] Carnevale and Strohl (2010) show that the top SES quartile represents 70% of the students in the most selective colleges, against 14% for bottom half of the population (sum of the two lowest quartiles). They also show that, from 1982 to 2006, the share of the top quartile in the least selective colleges has substantially decreased, and the share of the bottom quartiles increased.[11]

For France, Albouy and Wanecq (2003) show that, since the end of World War II, the difference in the probability to enter a GE between offspring from the upper class and the 'popular classes' has followed a U-curve. In the 1929-1938 generations, the upper class offspring were 27 times more likely to enter a GE (Top GE included) than those from the popular classes, this ratio falling to 18 for the 1939-1948 generations, 17 for the 1949-1958 generations, and going back up to 20 for the 1959-1968 generations. For the sole '*Très Grandes écoles*' (top GE), the corresponding ratios are 30, 24, 25 and 40. This clearly shows that social segregation in the entry to elite establishments has increased in the last decades.

5. In addition, educational stratification and social stratification are related. Entering an elite university is the natural path to the highest private and public positions. For the US, Temin (1999) showed that, in 1990, almost one-fifth of the 460 CEOs of the Fortune 500 companies were graduated from the Ivy League. Moreover, amongst the 800 chief executives running the largest US public companies in 2003, 87 had MBAs from the three top business

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[9] Spending per student is about 50,000 € in *Ecole des Mines Paris* (a most prestigious GE in engineering) as well as in ENA (the most prestigious GE for public administration), about 20,000 € and more for a number of GE in engineering (Centrale Paris, ENPC, ENSTA), of 25,000 € for HEC and 15,000 for the ESCP (two most prestigious business schools), whereas is of about 7000 € in standard universities (data: *Observatoire Boivigny*).

[10] Works on the variables affecting SAT results are numerous (e.g., Bouchard and McGue, 1981, and Hernnstein and Murray, 1994). They showed that being the child of an alumnus adds the equivalent of 160 SAT point, and increases one's chance of admission by almost 20 percentage points.

[11] In 2000, 25% of students in top universities came from 1% upper tail of income (see Temin, 1999a, p.32, and Kingston and Lewis, 1990, p.111). See also Brezis and Temin (2008).

schools - Harvard, Stanford and Wharton.[12] Similar regularities can be found in France. In 1997, 55% of top executives of the firms inside the French CAC 40 (stock exchange Index) came from the top GE (Baverez, 1998).[13]

Finally, the sociological empirical literature on education shows that the two-tier higher education system has indeed permitted to reduce inequality in the access to higher education, but not the inequality in the access to the highest level, which has even increased in several countries.[14]

2.2.2 Theoretical Approaches

In the last decade, the analysis of massification and stratification of educational systems has known a growing attention in the economic literature. The impacts of stratified education systems upon human capital, income distribution, growth and welfare have been particularly studied. In these approaches, education is modelled as a succession of stages. In most models, there is a basic education whose completion is a prerequisite to enter higher education. A key issue tackled in this literature is the analysis of the distribution of public education expenditure between the different education cycles, particularly basic and higher education (Driskill and Horowitz, 2002; Su 2004, 2006; Blankenau et al., 2007; Viaene and Zilcha, 2013). Moreover Su et al. (2012) distinguish between standard and elite colleges to analyse the U-shape relation between wages and skills observed in the US in the last two decades.

All the above-mentioned facts suggest that social stratification and social mobility are closely related to the structure of higher education, and not only to income and wealth distribution. The following section proposes a modelling of this relation.

3. The model

The model analyses the effects of human capital accumulation upon social mobility to the top within a dual higher education system.

We consider a model of successive generations, in which each individual has one child. A dynasty comprises the successive generations linked by a parent-child relationship, and we

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12 Temin, 1999a, p.32, and Forbes Global, October 13, 2003, p.28. See also Temin (1999b) and Miller (1950).

13 As regards the political elite, 35% of ministers have attended ENA since 1980 (Brezis and Crouzet, 2004, 2006).

14 See a thorough analysis of this literature in Shavit et al (2007). See also the so-called MMI hypothesis, (Raftery and Hout, 1993), which analyses the relationship between two-tier education and stratification.

assume a constant population normalised to 1. When born, the individual is endowed with a minimal human capital l. We denote individual (i,t) the individual of the t-th generation of dynasty i.

We assume that individuals maximise their lifetime labour income, and that income increases with the individual's human capital stock. Human capital is higher (i) when having a university degree than when having basic education only, and (ii) when having an elite university degree than a standard university degree. Consequently, individuals always prefer to enter the university than to remain at the basic education level, and they all prefer to enter an elite university than a standard one.

The model is developed as follows. First, we expose the structure of human capital accumulation. We subsequently define the steady stratification and we show that a two-tier higher education system tends towards a steady stratification with possible social immobility.

3.1. Human Capital Accumulation

Individuals accumulate human capital through education. Education comprises two phases, i.e., basic and higher education. We assume that all individuals acquire basic education, and both basic and higher education are freely provided by the government, so that there are no costs for the individuals who decide to pursue tertiary education.[15]

At the end of basic education, individual (i,t) has accumulated the human capital level h_{it}^B. At the conclusion of her education, she possesses the final skill level h_{it}^F that depends on her course of study (basic education only, standard university, or elite university).

3.1.1. Basic education

As in Galor and Tsiddon (1997a), the individual's accumulation of human capital at the end of basic education, h_{it}^B, depends on two elements:

1) Her family-related ability that represents the impact of intra-family externalities and transfers. These externalities and transfers can act through several channels: the intra-family direct transmission of human capital; the intra-family transmission of capacity to learn, i.e., capacity of analysis, capacity to organise the studying activity etc.; information about the

[15] This simplifying assumption is tailored to draw attention to the sole structure of the education system as determining skill attainment and social stratification. In particular, the impact of credit market imperfections when education is costly is not tackled, this issue having given rise to an abundant economic literature.

education system, the education strategies, affiliation to influential networks etc. All these intra-family externalities and transfers are directly linked to the parent's human capital h_{it-1}^F.

2) The individuals' personal innate ability (a_{it} for individual (i,t)) which is randomly distributed across individuals within each generation and independent from family backgrounds. We assume that personal abilities belong to the segment $[\underline{a},\bar{a}] \subset \mathrm{R}_+^*$.

In consequence, we assume that the human capital at the end of basic education h_{it}^B is given by the simple functional form:[16]

$$h_{it}^B = a_{it}\left(h_{it-1}^F\right)^{\eta}, \qquad \text{where} \quad 0<\eta<1. \tag{1}$$

3.1.2. Higher Education

The system of higher education is meritocratic in the sense that students are selected depending on their skills at the end of basic education only. We assume a two-tier higher education, namely, standard universities (S) and elite universities (E), as suggested by the facts highlighted in the Section 2.

The two types of universities differ in their selection procedures. Entering the standard university requires a minimum level of human capital, $\hat{h}$, at the end of basic education.

In contrast, on top of the minimum required level $\hat{h}$, the elite university selects the best students at the end of basic education and decides on the quota, which must remain narrow so as to ensure the specificity of elite universities. This results in the following rules:

1) To enter the standard university (S), individual (i,t) must possess h_{it}^B higher than $\hat{h}$ at the end of basic education;

2) To enter the elite university (E), she must, in addition to the above condition, belong to the top α in terms of human capital at the end of basic education, h_{it}^B. Since the population is constant and normalized to 1, α depicts both the number of students accepted in the elite university and the share of the population having an elite university degree.

Let us rank students by ascending order of human capital at the end of basic education. Then, for each generation t, there is a unique level $h_{\alpha,t}$ such as there are α students with a level higher than or equal to $h_{\alpha,t}$, and thus $1-\alpha$ students with a level lower than $h_{\alpha,t}$, at the end of basic education. The value $h_{\alpha,t}$ is thus the human capital threshold that separates the students who can enter the elite university from those who cannot, at the end of basic education.

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[16] In addition, we assume $\underline{a} > l^{1-\eta}$, which ensures that the skill attainment at the end of basic education is higher than when the individual was born.

3.1.3. Human Capital at the end of Higher Education

The individual's human capital at the end of higher education h_{it}^F depends on two factors: (ii) her human capital at the end of basic education, h_{it}^B, and (ii) the expenditure per student in the university where she studies, denoted V_j, $j = S, E$. Higher expenditure per student means better and more numerous staff, better infrastructures and finally better education.

We assume a simple multiplicative higher education function. More complex functions could be considered[17], but they would typically not modify the model main results (only the thresholds and the form of the conditions would be changed).

Following the stylised facts exposed in Section 2, we assume that the expenditure per student in elite university, V_E, is higher than in the standard one, V_S. Moreover, we assume V_S >1, which ensures that individuals prefer entering the university than the labour market at the end of basic education. Therefore, we have:

$$h_{it}^F = \begin{cases} h_{it}^S = V_S h_{it}^B = V_S a_{it} \left(h_{it-1}^F\right)^{\eta} & \text{if the individual enters the standard university} \quad (2) \\ h_{it}^E = V_E h_{it}^B = V_E a_{it} \left(h_{it-1}^F\right)^{\eta} & \text{if the individual enters the elite university} \quad (3) \end{cases}$$

where h_{it}^S and h_{it}^E are individual (*i,t*)'s human capital endowment at the end of higher education when pursuing the standard and elite university respectively.

The individual maximises her lifetime income. As we have assumed no education costs, the individual behaviour comes down to selecting the study that provides for the highest human capital h_{it}^F. As we assume $1 < V_S < V_E$, then individuals who can enter the university always prefer the elite university to the standard one.

This leads to the following Lemma, summarizing the educational behaviour:

Lemma 1. *Individual (i,t)'s human capital at the end of her education,* h_{it}^F, *is:*

$$h_{it}^F = \begin{cases} h_{it}^B & \text{if} \quad h_{it}^B < \hat{h} \\ h_{it}^S & \text{if} \quad \hat{h} \le h_{it}^B < h_{\alpha,t} \\ h_{it}^E & \text{if} \quad \hat{h} < h_{\alpha,t} \le h_{it}^B \end{cases} \quad (4)$$

[17] In particular, the function could be log-linear in h_{it}^B and V_j and the weights of each factor could differ between standard and elite universities.

3.2. Social stratification and social mobility

Lemma 1 points to the fact that, within the model, individuals are differentiated according to the type of education they receive. This allows defining the following three social groups:

Definition 1. Within each generation, we define as:

1) *Lower class* the set of individuals who have only followed basic education.
2) *Middle class* the set of individuals who have a standard university degree.
3) *Elite* the individuals with an elite university degree.

The three social groups are defined by the educational attainment of individuals. This typically reproduces the observed facts underlined in Section 2, which has shown the correspondence between social stratification and stratification in higher education.

By definition, the size of the elite α is given. We denote γ the size of the middle class. Hence, the size of the lower class is $\delta = 1-\alpha-\gamma$.

3.2.1. Steady segments

We show now that each type of study (*B,S,E*) is characterized by a specific steady segment, defined hereafter.

Let us define function $H_a^j(\cdot)$ such that:

$$h_t^F = H_a^j\left(h_{t-1}^F\right) = V_j a\left(h_{t-1}^F\right)^\eta, \quad \forall t, \quad \text{where } V_j = 1,\ V_S,\ V_E \text{ for } j = B,\ S,\ E. \tag{5}$$

$H_a^j(\cdot)$ is the function that binds the human capital at the end of the education cycle *j* at any time *t* to the preceding generation's human capital h_{t-1}^F *for a given personal ability* a. The shapes of the curves $H_a^j(\cdot)$ are depicted in Figure 1. Note that all these curves move upwards when the expenditure per student V_j increases.

For $a = \underline{a}$, function $H_a^j(\cdot)$ is denoted $\underline{H}^j(\cdot)$, and $\bar{H}^j(\cdot)$ for $a = \bar{a}$. All the curves of functions $H_a^j(\cdot)$ are thus located between curves $\underline{H}^j(\cdot)$ and $\bar{H}^j(\cdot)$.

For the given ability *a* and the given type of study *j*, the steady state of the dynamics (5) is the fixed point h_j^a of function $H_a^j(\cdot)$ defined by equality:

$$h_a^j = H_a^j\left(h_a^j\right) \Rightarrow h_a^j = \left(V_j a\right)^{1/(1-\eta)} \tag{6}$$

Graphically, the steady state is at the intersection of the $H_a^j(\cdot)$ curve and the 45° curve (recall that all $H_a^j(\cdot)$ are concave). As $a \in [\underline{a}, \bar{a}]$, then all the steady states are located in the segment $[\underline{h}_j, \bar{h}_j]$, as depicted in Figure 1.

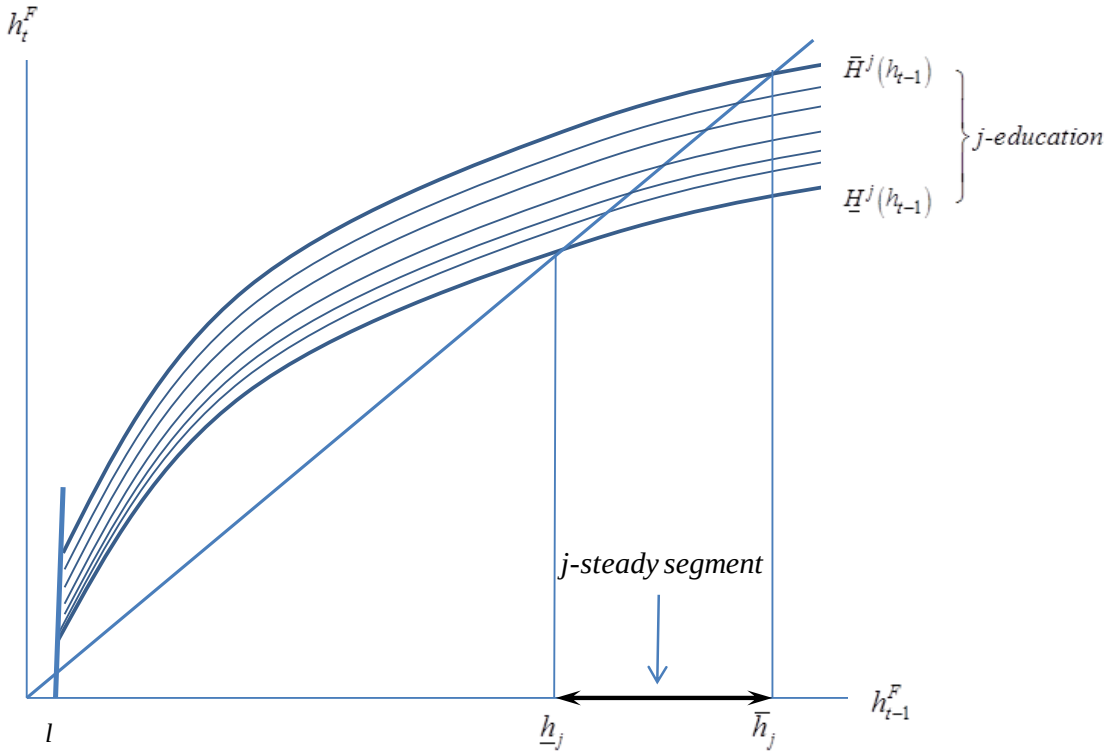

Figure 1. The *j*- steady segment, $j = B, S, E$

Definition 2. The segment $[\underline{h}_j, \bar{h}_j]$ is called *j-steady segment*, for $j = B,S,E$.

The human capital dynamics engender three steady segments: $[\underline{h}_B, \bar{h}_B]$, $[\underline{h}_S, \bar{h}_S]$ and $[\underline{h}_E, \bar{h}_E]$.[18] These steady segments (*B*, *S* and *E*) are depicted in Figure 2. Note that the *S*-steady segment and the *E*-steady segment move to the right, when V_S and V_E increase respectively.

Remark 1: Condition $\hat{h} \leq \left(\underline{a} V_S^{\eta}\right)^{1/1-\eta}$ is a sufficient condition for all the successive generations of a dynasty to enter the university once one generation of this dynasty entered the

[18] with: $[\underline{h}_B, \bar{h}_B] = [\underline{a}^{1/(1-\eta)}, \bar{a}^{1/(1-\eta)}]$, $[\underline{h}_S, \bar{h}_S] = [(V_S \underline{a})^{1/(1-\eta)}, (V_S \bar{a})^{1/(1-\eta)}]$ and $[\underline{h}_E, \bar{h}_E] = [(V_E \underline{a})^{1/(1-\eta)}, (V_E \bar{a})^{1/(1-\eta)}]$.

university.[19] In what follows, we assume that this condition holds, which allows focusing on the mobility between the middle and the upper class.

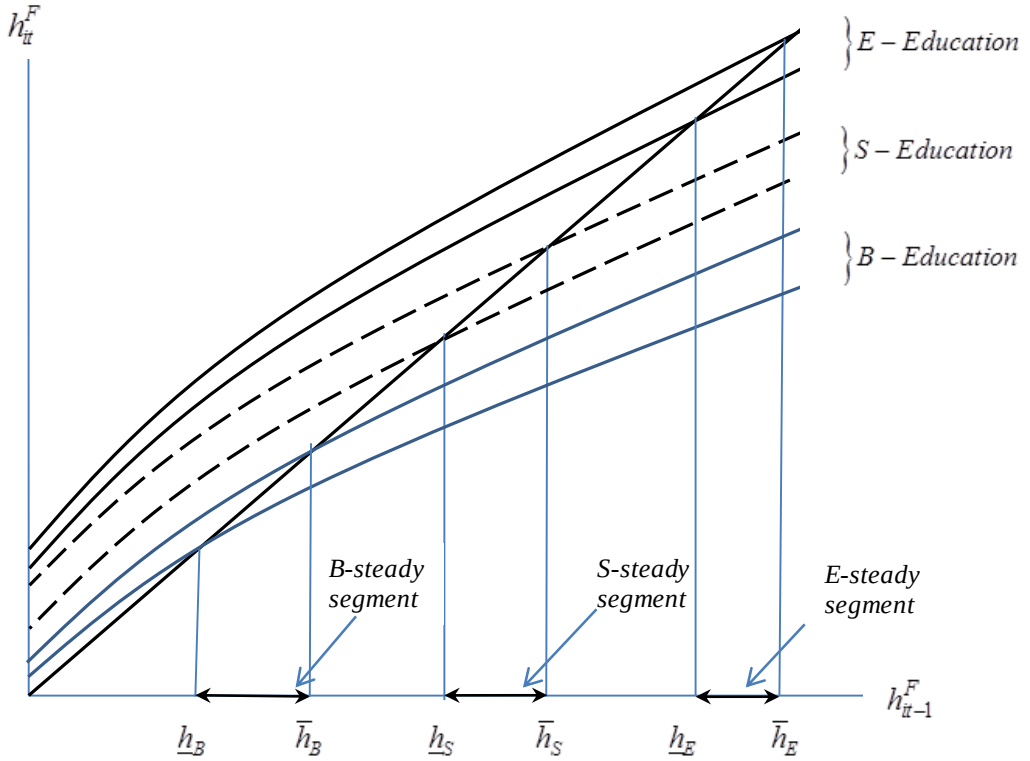

Figure 2. Steady segments

3.2.2. Steady stratification

In the preceding sub-section, we have defined the steady segment corresponding to each type of study. In this section, we define a 'steady stratification' and we subsequently show that a 'higher education system' always tends towards a steady stratification.

Definition 3. A *higher education system* $\left(\hat{h}, \alpha, V_S, V_E\right)$ is defined by four parameters, i.e., (i) the two selection procedures $\hat{h}$ and α, and (ii) the couple of public expenditures per student in the dual system V_S and V_E.

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[19] Proof: The least skilled individual who can enter *S* is such that $h_{it}^B = \hat{h}$ and $h_{it}^F = V_S \hat{h}$ at the end of study *S*. To enter the university, her child must have a skill higher than $\hat{h}$ at the end of basic education, even when having the lowest ability $\underline{a}$.

Hence: $\underline{a}\left(V_S \hat{h}\right)^{\eta} \geq \hat{h} \Rightarrow \underline{a} V_S{}^{\eta} \geq \hat{h}^{1-\eta} \Rightarrow \hat{h} \leq \left(\underline{a} V_S{}^{\eta}\right)^{1/1-\eta}$.

Definition 4. A *Steady stratification* is a situation in which:

1) The size of each social group remains unchanged from one generation to the next, i.e., γ is constant (α is constant by definition).

2) At each generation, all individuals are inside a steady segment and (i) all the dynasties inside the lower class have a human capital belonging to the *B*-steady segment $\left[\underline{h}_B, \overline{h}_B\right]$, (ii) all the dynasties inside the middle class a human capital inside the *S*-steady segment $\left[\underline{h}_S, \overline{h}_S\right]$, and (ii) all the dynasties within the elite have a human capital inside the *E*-steady segment $\left[\underline{h}_E, \overline{h}_E\right]$.

It is essential to note that being in a steady stratification does not mean that the dynasties indefinitely remain in the same segment. When there is social mobility, dynasties inside the middle class can move to the elite and *vice versa*. However, the number of dynasties in each segment, and thereby in each social group, remains unchanged.

Given Definitions 3 and 4, we now establish the following:

Lemma 2. *Consider a higher education system* $\left(\hat{h}, \alpha, V_S, V_E\right)$ *characterised by the functions (1)-(3). Then, whatever the initial distribution of human capital across dynasties, this system tends towards a steady stratification*.

Proof. Appendix A.

We can distinguish two cases depending on the position of threshold $\hat{h}$ in relation to $\overline{h}_B$ (see Appendix A). When $\hat{h} > \overline{h}_B$, the steady stratification typically comprises three social groups, i.e., there is a lower class.[20] If $\hat{h} < \overline{h}_B$, the steady stratification is characterised by two social groups only because the lower class vanishes during the transitional dynamics to the steady stratification. Then, the steady stratification is characterised by $\gamma = 1 - \alpha$.

Having shown that a higher education system always tends towards a steady stratification, we now place ourselves in a situation of steady stratification to analyse social mobility at the top.

[20] Then, the steady state depends on both the characteristics of higher education system and the initial (generation 0) distribution of human capital across dynasties.

4. Social Mobility at the Top

We firstly determine the condition for mobility to occur. We subsequently define the mobility rates utilised to study mobility at the top, i.e., between the middle class and the elite. We finally analyse the impacts of the characteristics of higher education, i.e. the expenditures per student and the selection process in each university, upon social stratification and social mobility.

4.1. The conditions for mobility

Social mobility is defined as a situation in which a number of students from the middle class accede to the elite university, while the equivalent number of students from the elite enter the standard university, falling thereby in the middle class (since the size of the elite is constant by definition). These moves depend on human capital at the end of basic education, h_{it}^{B}, which determines the individuals' position and hence the type of university they can join.

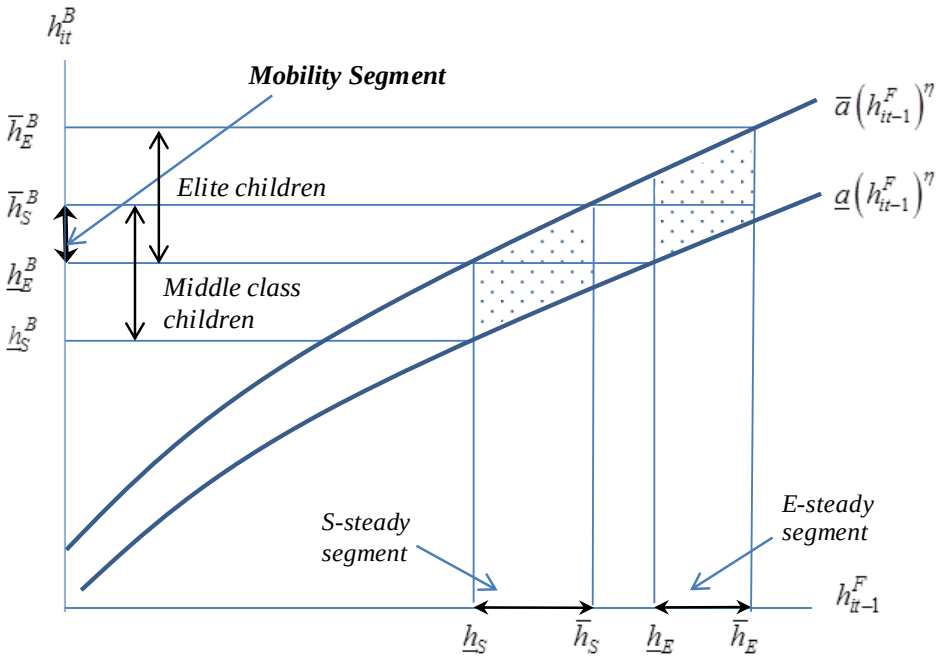

Figure 3. The mobility segment

Figure 3 depicts the rationale of mobility between the middle class and the elite at the end of basic education. On the *x*-axis, the middle class parents are in the *S*-steady segment and the elite parents on the *E*-steady segment. The *y*-axis displays the position of the offspring at

the end of basic education: The middle class offspring are in segment $\left[\underline{h}_S^B, \overline{h}_S^B\right]$ and the elite offspring in segment $\left[\underline{h}_E^B, \overline{h}_E^B\right]$.

The condition for students from the middle class to accede the elite university is that, at the end of basic education, the most skilled from the middle class have a higher level than the least skilled from the elite, i.e., $\overline{h}_S^B > \underline{h}_E^B$ (otherwise, all the students from the elite enter the elite university and leave thereby no opportunity for middle class students to enter the elite).

We define Condition 1 as:

Condition 1:

$$V_E / V_S < (\overline{a} / \underline{a})^{1/\eta}$$

This establishes the following proposition highlighting the effect of relative budgets on social mobility:

Proposition 1. *Assume that we are at a steady stratification. Then, social mobility exists if Condition* 1 *is fulfilled.*

Proof. There is social mobility if students from the middle class can enter the elite, i.e., $\underline{h}_E^B < \overline{h}_S^B \Leftrightarrow \underline{a}\left(\underline{h}_E\right)^\eta < \overline{a}\left(\overline{h}_S\right)^\eta \Leftrightarrow \underline{a}^{1/(1-\eta)} V_E^{\eta/(1-\eta)} < \overline{a}^{1/(1-\eta)} V_S^{\eta/(1-\eta)} \Leftrightarrow V_E / V_S < (\overline{a} / \underline{a})^{1/\eta}$.

When Condition 1 is not fulfilled, students from the middle class remain in the middle class and students from the elite remain in the elite. This is the case when segments $\left[\underline{h}_S^B, \overline{h}_S^B\right]$ and $\left[\underline{h}_E^B, \overline{h}_E^B\right]$ do not overlap. Each group is then fully insulated from the other. In contrast, when Condition 1 is met, there is room for social mobility at the top.

It should be emphasized that Condition 1 depends on the difference in expenditure per student between universities (V_E / V_S) and on the difference in ability between the most and the least able student ($\overline{a} / \underline{a}$). In fact, for social mobility to occur, the difference in per-student expenditures must not be too large compared to the difference in abilities. The reason for this is that the difference in per-student expenditures determines the difference in attainment at the end of basic education due to social origins (elite parents received an elite education V_E whereas middle class parents had a standard education V_S) which lessens mobility. In contrast, a larger interval of personal abilities fosters mobility provided that abilities are independent from social origin. Consequently, the condition for social mobility critically depends on the relative budget per student between the two types of universities.

Definition 5. Assuming that Condition 1 is fulfilled, we define the segment $\left[\underline{h}_E^B, \overline{h}_S^B\right]$ as the *mobility segment*.

The mobility segment is the set in which the attainments of the middle class offspring and of the elite offspring overlap at the end of basic education (Figure 3). [21] It is thus the set in which social mobility can occur. From Figure 3, it can be noted that, the mobility segment can exist even when the *S*-steady segment and the *E*-steady segment do not overlap.[22]

Lemma 3. *Assume that the mobility segment does exist. Then the threshold* $h_{\alpha,t}$ *which separates the students entering the elite university from those entering the standard university is inside this segment.*

Proof. See Appendix B, in which the diagrammatic determination of $h_{\alpha,t}$ is also exposed.

When the mobility segment exists, all the students from the middle class with human capital h_{it}^B higher than $h_{\alpha,t}$ enter the elite university and become thereby members of the elite. Symmetrically, all the students from the elite with human capital lower than $h_{\alpha,t}$ at the end of basic education enter the standard university and 'fall' thereby in the middle class. This generates social mobility.

4.2. Mobility rates

We analyse mobility from the double point of view of the middle class and of the elite. A straightforward measurement of the upward mobility of students from the middle class consists in computing the proportion of those who join the elite group. Identically, the downward mobility of students from the elite can be measured by the proportion of those who join the middle class and the elite self-reproduction by the proportion of students from the elite who remain in this group. Consequently:

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21 After calculation, we obtain : $\underline{h}_E^B = \left(\underline{a} V_E{}^{\eta}\right)^{1/(1-\eta)}$ and $\overline{h}_S^B = \left(\overline{a} V_S{}^{\eta}\right)^{1/(1-\eta)}$.

22 when the *S*-steady segment and the *E*-steady segment overlap, the mobility segment always exists.

Definition 6. We define:

1) *Elite replacement rate* the proportion ρ of students from the elite who fall in the middle class.
2) *Elite self-reproduction rate* the proportion $\beta = 1 - \rho$ of students from the elite who remain in the elite.
3) *Middle class upward mobility rate* the proportion μ of students from the middle class who enter the elite.

It can be noted that ratio β / μ measures the difference in opportunity to enter the elite between the offspring from the elite and those from the middle class. This ratio is the inverse of the fluidity index analysed by sociologists.

Let us set ourselves at the steady stratification and assume that *Condition* 1 is fulfilled, i.e., the mobility segment does exist.

Let $f_S(h)$ and $f_E(h)$ be the density functions of human capital at the end of basic education for the middle class and elite offspring respectively. Thus:

1) $f_S(h)$ is defined on segment $\left[\underline{h}_S^B, \overline{h}_S^B\right]$, and $\int_{\underline{h}_S^B}^{\overline{h}_S^B} f_S(h)dh = 1$;

2) $f_E(h)$ is defined on segment $\left[\underline{h}_E^B, \overline{h}_E^B\right]$, and $\int_{\underline{h}_E^B}^{\overline{h}_E^B} f_E(h)dh = 1$.

Note that functions f_S and f_E typically change from one generation to the next.[23] The above-defined mobility rates are:[24]

$$\mu = \int_{h_\alpha}^{\overline{h}_S^B} f_S(h)dh \; ; \qquad \rho = \int_{\underline{h}_E^B}^{h_\alpha} f_E(h)dh ; \quad \beta = \int_{h_\alpha}^{\overline{h}_E^B} f_E(h)dh . \tag{7}$$

Finally note that h_α is such that the number of students from the middle class who enter the elite is equal to the number of students from the elite who fall back to the middle class:

$$\mu\gamma = \rho\alpha \tag{8}$$

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[23] Mobility rates vary at each generation and depend on the distribution of the children's skill attainments at the end of basic education (see simulations). Here, we make no particular assumptions on the density functions.

[24] For simplification, subscript t is omitted. Remember that $h_{\alpha,t}$ is the human capital which separates those who enter the elite university from those who enter the standard university at the end of basic education.

In the next sub-section, we study the impact of changes in the expenditure policy on mobility rates.

4.3. Budget per student in higher education

Proposition 2. *At the steady stratification, an increase in expenditures per student in the elite university* V_E*, and/or a decrease in the expenditure per student in the standard university* V_S*:*

1) reduces the middle class mobility rate μ*,*

2) reduces the elite replacement rate ρ*, increasing thereby the elite self-reproduction* β*.*

Proof. Appendix C.

A change in higher education policy that fosters the funding of elite universities increases the human capital of the elite, and therefore their future income. On top of that, Proposition 2 shows that it lowers social mobility. Fewer students from the middle class can now join the elite, i.e., the self-reproduction of the elite is reinforced. This derives from the fact that, with the rise in the funding allocated to the elite university, the middle class will turn out to possess less human capital relative to the elite. As a consequence, fewer highly able students from the middle class will surpass the human capital level of the low able students from the upper class because of the growing difference in intra-family transfers between the two social groups. Thus, more students from the elite remain in the elite and social mobility at the top is lessened.

Obviously, the impact on social mobility of a decrease in the expenditure per student in the standard university V_S is similar to the impact of an increase in V_E. Decreasing V_S lowers the skill level of the middle class, providing their offspring with lower intra-family human capital transfers and thus a worsened position in the competition for the entry in the elite university.

A major implication of Proposition 2 is that an increase in the funding of higher education that primarily benefits elite universities *does not foster social mobility*. Quite the opposite, it reduces the middle class upward mobility and reinforces the elite self-reproduction.

Finally, if the increase in V_E and/or the decrease in V_S are sufficiently high so that Condition 1 no longer holds, i.e., $V_E / V_S > (\bar{a} / \underline{a})^{1/\eta}$, then, based on Proposition 1, we get no social mobility at all, with a total self-reproduction of the elite.

4.4. Changes in the selection patterns

We now analyze exogenous changes (i) in the selection to enter the elite university, α, which is also the share of the elite class in total population, and (ii) in the selection to enter higher education $\hat{h}$, which increases the size of the middle class, γ.

Proposition 3. *An increase in the share α of students recruited by elite university, with γ constant, increases both the middle class upward mobility rate μ and the elite self-reproduction rate β.*

Proof. Appendix D.

This result is of interest since lessening the tightness of recruitment and permitting the elite to become a larger part of the society leads to both an increase in upward mobility and an increase in self-reproduction of elites, which could be seen as contradictory effects. The explanation for this is as follows. The increasing effect of α upon μ is straightforward because more offspring from the middle class join the elite group whereas the size of the middle class remains constant. But why should β increase with α? On the one hand, the increase in α tends to decrease the elite self-reproduction by increasing the number of elite members, i.e., the denominator of coefficient β. On the other hand, there is an increase in the number of elite offspring entering elite universities which is high enough to over-compensate the increase in the number of elite offspring, raising thereby the elite self-reproduction (see the proof in Appendix D). We can finally note that, from a general point of view, the effect upon 'social mobility' is ambiguous.

Let us now analyse the impact of a release in the selection threshold $\hat{h}$ which modifies the size of the middle class γ. At the beginning of Appendix E, we discuss the conditions for such a release to increase the size of the middle class.

Proposition 4. *Changes in $\hat{h}$ that increases the size γ of the middle class (without change in the size α of the elite) leads to:*

1) a decrease in the elite self-reproduction rate β,

2) a decrease in the middle class upward mobility rate μ.

Proof. Appendix E.

Proposition 4 shows that exogenous changes in $\hat{h}$ leading to the enlargement of the middle class tends to jeopardise its upward mobility, μ. This is because, as the middle class gets larger, the number of its offspring candidate to the elite university augments whereas the number of students accepted in the elite university remains constant. In addition this enlargement also diminishes the elite self-reproduction, β, because more offspring from the middle class accede to the elite. So, the increase in the weight of the middle class (without increase in the weight of the elite) jeopardises both the positions of the middle class and that of the elite. Even if there are more students from the middle class who rise to the elite, their proportion however declines because of the increase in the middle class population. In addition, this reduces the number of students from the elite who remain in the elite.

It must however be underlined that these results concern the steady stratification only. At the time when the increase in γ occurs, the middle class is better off compared to its parents. This is because: (i) the enlargement of the middle class operates by the bottom, i.e., offspring from the lower class accede to the middle class, being thus in a better social position than their parents; (ii) the top side of the middle class and the bottom side of the upper class are not impacted yet during this first generation, and the mobility between the two groups is thus unchanged compared to what would have occurred without enlargement. Compared to the previous generation, the increase in γ is experienced as an improvement by the middle class (because of the improvement of its bottom side) and as no change by the elite. It is only in the following generations that the middle class suffers a decrease in its upward mobility and the elite a decrease in its self-reproduction.

5. Dynamics and simulations

The model developed in the preceding sections has essentially analysed social mobility at the steady stratification. This is because the paths to the steady stratification are numerous and they crucially depend on the initial distribution of human capital across individuals. In this section, simulations are implemented so as to illustrate the dynamics leading to the various steady stratifications with plausible values of the model parameters. More precisely, we start from a purely egalitarian situation in which all individuals are initially endowed with the same human capital and we analyze the dynamics of social stratification and self-reproduction of the elite group. We show that, even when starting from perfect equality, the higher education system generates social stratification with low social mobility.

This exercise is carried out within three scenarios picturing different types of education systems that differ in their elitist orientation. Finally, for each generation in each scenario, we

calculate, the coefficients μ (middle class upward mobility rate) and β (elite self-reproduction rate) and the ratio β / μ that measures the difference in opportunity to enter the elite between the offspring from the elite and those from the middle class (and which is the inverse of the fluidity index).

Note that additional simulations were implemented with a non-egalitarian initial distribution of human capital. Available upon request, these simulations illustrate the results found in Section 4 when introducing changes over time in the higher education system.

5.1. Three scenarios

We start from a situation (generation 0) in which human capital is lower than $\hat{h}$ and all individuals possess the same human capital, i.e., perfect equality: $h_{i0}^F = h_0^F$, $\forall i$. From this initial situation, we study the dynamics that derive from three higher education systems that differ in their expenditure per student in the two universities, V_S and V_E.

Table 1 depicts the parameters and values common to the three scenarios and Table 2 those corresponding to each scenario.

Table 1. Parameters and values common to the three scenarios

h_0^F	$\hat{h}$	α	η	$\underline{a}$	$\overline{a}$	$\underline{h}_B$	$\overline{h}_B$	$(\overline{a} / \underline{a})^{1/\eta}$ *
0.5	0.6	5%	0.3	0.6	1	0.54	1	5.49

*Remember that $V_E / V_S < (\overline{a} / \underline{a})^{1/\eta}$ is the condition for mobility.

From generation 1, we assume that the elite represent 5% of the population. The value $\eta = 0.3$ corresponds to the average level of the intergenerational earnings elasticity (IGE) in advanced countries. The ability interval corresponds to a ratio $\overline{a} / \underline{a} = 1.666$ which can be seen as a rather large gap between the highest and the lowest innate ability, given that these are randomly distributed across individuals.

Table 2. The Three scenarios

	V_S	V_E	$\underline{h}_S$	$\overline{h}_S$	$\underline{h}_E$	$\overline{h}_E$	$(\underline{a} V_S^{\eta})^{1/1-\eta}$ *
Egalitarian (*Equality*)	2.1	2.1	1.39	2.88	1.39	2.89	0.66
Elitist (*Elite*)	1.82	7.4	1.13	2.35	8.41	17.45	0.62
In-between (I-B)	2	4	1.30	2.69	3.49	7.25	0.65

* $\hat{h} \leq \left(\underline{a} V_S^{\eta}\right)^{1/1-\eta}$ is the condition for offspring from the middle class never to fall in the lower class.

In the three scenarios, the parameters were chosen such that: (i) $\underline{h}_B < \hat{h} < \overline{h}_B$, which signifies that there is no lower class at the steady stratification,[25] (ii) $\hat{h} \leq \left(\underline{a} V_S^{\eta}\right)^{1/1-\eta}$, which means that a dynasty never goes back to the lower class once it left it (see Remark 1) , and (iii) Condition 1 holds, i.e., $V_E / V_S < (\overline{a} / \underline{a})^{1/\eta}$, which ensures that the mobility segment does exist.[26]

The three scenarios are:

1. The egalitarian scenario ('*Equality*') which stipulates equal funding in both universities. This scenario is crucial because it constitutes a benchmark for the two cases with a dual higher education. In fact, we show that even within the unified system with equal funding, offspring whose parents are in the top 5% are more likely to be in the top 5% because of intra-family externalities.

2. The elitist scenario ('*Elite*') which assumes that ratio V_E / V_S is equal to 4. This corresponds to the upper limit of what is observed in the most elitist systems, as in the US, the UK and France, in which this ratio is between 3 and 4.

3. A scenario 'in-between' (*IB*) where $V_E / V_S = 2$.

Finally, expenditures in both universities were chosen so that total expenditure in higher education is identical in the three scenarios. We assume 500 dynasties and we calculate human capital and social stratification corresponding to the three above-described education systems for 13 successive generations following the initial generation 0.

5.2. Results and discussion

As expected, the system generates a steady stratification without lower class, which is attained at generation six in the scenarios *Equality* and *IB*, and seven in scenario *Elite* [27] (from then, the size of each social group remains constant, all middle class members are inside the *S*-steady segment and all the elite members inside the *E*-steady segment). The size of each social class is reported in Tables A1-A3 in Appendix F.

It should be recalled that, without intra-family externality (i.e., with $\eta = 0$), the elite self-reproduction rate β and the middle class upward mobility μ would be identical and equal

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[25] Starting from a low and perfectly equally distributed human capital at generation 0, assuming $\hat{h} > \overline{h}_B$ would induce that all dynasties perpetually remain in the lower class (none ever attains threshold $\hat{h}$), and assuming $\hat{h} < \underline{h}_B$ would result in a very quick disappearing of the lower class and a very quick setting of the two-group steady stratification.

[26] It can be noted that, given the values of the parameters selected, there is no mobility at all when expenditures per student in the elite university are more than 5.5 times higher than those in the standard university.

[27] This limited number of generations to attain the steady stratification, although we start from total equality, can justify the fact that the analysis is implemented in a state of steady stratification.

to 0.05 since the elite gathers the top 5% of the population in terms of human capital. This corresponds to a perfect equality of opportunity.

The results of the simulations at each generation are depicted in Tables A1-A3 in Appendix F. Table 3 depicts the elite self-reproduction rate, the middle class upward mobility rate, and the ratio β / μ, on average at the steady stratification in the three scenarios.

Table 3. Mobility at the steady stratification

	Equal Opportunity	*Equality* (benchmark)	*In-Between*	*Elite*
(1) Average β at the Steady-Strat (%)	5	17.7	66.9	96.6
(2) Average μ at the Steady-Strat (%)	5	4.3	1.7	0.2
β / μ = (1) / (2)	1	4.1	38.4	483

In the case of non-dual higher education (*Equality*), the elite self-reproduction rate (in percent) is equal to 17.7, i.e., 3.5 times higher than its value corresponding to equal opportunity (which is 5). This typically reproduces the impact of intra-family skill externality. Despite the fact that elite offspring has 4 times more chance to enter the elite than middle class offspring ($\beta / \mu = 4.1$), the middle class does not suffer much the impact of the intra-family skill externality since the middle class upward mobility rate is 4.3, i.e., only slightly lower than in the case of equal opportunity.

In the 'in-between' scenario, i.e., with an expenditure per student twice higher in the elite university than in the standard one, the elite are noticeably self-reproducing since the 2/3 of the elite children remain in the elite against less than 1.7% of middle class children who enter the elite. In this case, students from the elite are about 40 times more likely to be in the elite than those from the middle class, and the middle class upward mobility rate is substantially lessened (1.7% compared to 4.3% in the 'equal' scenario and 5% with equal opportunity).

The 'Elitist' scenario significantly amplifies the previous results. The elite self-reproduction is now almost total since coefficient β is higher than 95%, and the middle class' opportunity to go up the social ladder is almost nil ($\mu = 0.2\%$), which means that entering the elite is 483 times more likely for elite offspring than for middle class offspring.

In summary, the simulations implemented with plausible values of the parameters show that dual higher education is a powerful factor of social immobility. Compared to the situation with a unified tertiary education (scenario '*Equality*'), the two-tier system makes the elite self-reproduction move from 17.7% up to 67% when the expenditure per student is twice higher in the elite university than in the standard one, and to 97% when it is four times

higher. Moreover, the middle class upward mobility rate falls from 4.3% (i.e., rather close to its equal opportunity value, 5%) to 1.7% and 0.2% respectively.

The core of our results lies in the respective intensities of personal ability (a_{it}) and intra-family externality and transfers (measured by the parents' human capital h^F_{it-1}). Within our model, when the human capital gap between the middle class and the elite is not too large because of limited differences in budget allocations between elite and standard universities, then the effects of intra-family externality are not too large, and personal ability prevails, which results in social mobility. In contrast, when allocations to the elite university are high compared to the standard one, the difference in social backgrounds prevails, which boost the self-reproduction of elites.

6. Conclusion

This paper shows that social stratification and social mobility are closely related to the structure of Higher Education. More precisely, an education system characterised by the division of higher education between elite and standard universities leads to social stratification between the middle class and the elite, the latter being to a large extent self-reproducing. Moreover, our paper highlights that a higher education system always tends towards a steady stratification and our simulations show that this state is attained after a limited number of generations.

Our main result is that, the higher the difference in expenditure per student between the elite and standard universities, the lower the upward social mobility of the middle class, and the more self-reproducing the elite group. Our simulations using plausible values of the parameters show that this impact is huge.

As a consequence, in contrast to the popular view, an increase in public spending on higher education can lessen social mobility. In particular, the policies implemented in several countries in the last decades that have increased the higher education budgets so as to promote social mobility, can paradoxically lead to the opposite result. This happens when the increase in budget essentially benefits elite universities. This has been the case in those countries that display a decrease in intergenerational mobility over the last two decades.

We obtain two additional results concerning the modifications of the selection procedures and the related changes in the sizes of social groups. First, an enlargement of the elite (i.e., an increase in the proportion of elite households in the population) increases both the middle class upward mobility and the elite self-reproduction. Second, a change in the selection procedure which increases the share of the middle class in the population (without change in

the share of the elite) lowers social mobility, except for the generation when this increase occurs. These results provide explanations for a number of facts observed since World War II.

The post-World War II period experienced a rapid democratization of tertiary education leading to an increase in the shares of the middle class and, to a lesser extent, of the elite group. These changes have been at the beginning twice beneficial to the middle class. First, the middle class newcomers had parents from the lower class, and their access to university was indubitably a promotion to them. Second, the enlargement of the elite allowed the more able and more educated students from the middle class to join the elite, promoting social ascension for the most able from the middle class. However, these pro-middle class changes were transitional.

Once individuals from the lower class had joined the middle class, their children did not continue going up the social ladder. Quite the opposite, the longer term (and steady stratification) negative impact of the enlargement of the middle class now lessens the middle class upward mobility. In addition, over the last decades, the gap between the elite and standard universities in terms of budgets per student has increased in several countries. This increase has reduced the middle class upward mobility, jeopardised its social position (in line with Easterly, 2001), and boosted the self-reproduction of the elite group.

Finally, the model predicts that growing differences in budgets per student between elite and standard universities result in both (i) higher differences in skill levels between the middle class and the elite, leading to higher differences in income and to higher inequality, and (ii) lower social mobility.

These two results provide a new explanation for the 'great Gatsby curve' put forward by Corak (2012, 2013) and Krueger (2012). In addition to the usual explanation in which inequality explains low mobility, particularly through education costs with imperfections on the credit market, our model shows that both higher inequality and lower mobility can be derived from the specific structure of the education system when the budget gap between elite and standard universities increases. It thus predicts that the countries with more elitist educational systems should experience both higher inequality and lower mobility, which is what is observed. [28]

In conclusion, the paper shows that the differences in expenditure per students and in selection between standard and elite universities are key factors generating social stratification.

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[28] This interpretation is also supported by the fact that the rise in inequality essentially derives from the upper incomes (Atkinson and Piketty, 2007; Autor et al., 2004; Kopczuk et al., 2010).

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Appendix A. The education system always tends towards a steady stratification

A steady stratification is characterised by the two following features (Definition 4):

1) the size of each social group remains unchanged from one generation to the next;

2) at each generation, all the dynasties inside the lower class have a human capital inside the *B*-steady segment, all individuals inside the middle class a human capital inside the *S*-steady segment and all individuals in the elite a human capital inside the *E*-steady segment.

1) *Feature* 1

Let us denote α, γ and δ the respective sizes of the elite, the middle class and the lower class, with $\alpha+\gamma+\delta=1$.

The size α is given by definition. Hence, this size is constant.

Since $\alpha+\gamma+\delta=1$, it is sufficient to show that the size of one additional social group tends towards a constant value to show Feature 1. We show that δ tends towards a constant value. For this, we consider the three cases, $\hat{h}<\underline{h}_B$, $\hat{h}>\overline{h}_B$ and $\underline{h}_B<\hat{h}<\overline{h}_B$:

(i) $\hat{h}<\underline{h}_B$: Then all dynasties can enter the university after a limited number of generations. From this time, $\delta=0$.

(ii) $\hat{h}>\overline{h}_B$: Then (a) all the dynasties with an initial human capital (i.e., at generation 0) below $\hat{h}$ perpetually remain under-educated (they never enter the university) and (b) all the dynasties with an initial human capital above $\hat{h}$ enter the university from the first generation, and all their offspring also pursue higher education. Consequently, δ is constant and equal to the number of parents with an initial human capital below $\hat{h}$.

(iii) $\underline{h}_B<\hat{h}<\overline{h}_B$: Once a dynasty is above $\hat{h}$, then this dynasty *perpetually* follows higher education. In addition, all dynasties should sooner or later move above $\hat{h}$ when time tends toward infinite because the probability never to go through segment $\left[\hat{h},\overline{h}_B\right]$ tends towards 0 when $t\to\infty$. Therefore, δ tends towards 0.

Finally note that the above reasoning reveals two situations. First, when $\hat{h}\le\overline{h}_B$, the education system determines one unique steady stratification characterised by α individuals inside the elite and $1-\alpha$ inside the middle class. Second, when $\hat{h}>\overline{h}_B$, the steady stratification comprises three social groups and it depends on both the characteristics of the higher education system and the initial distribution of human capital across dynasties.

2) *Feature* 2

To show Feature 2, we must prove that:

(i) When an individual is in the *j*-steady segment, *j* = *S*, *E*, then her child is also in this segment, if she follows the same study *j*.

(ii) When an individual is in the middle class (*S*-steady segment), and her offspring enters the elite school, she then accede to the elite class and is then inside the E-steady segment.

(iii) When an individual is in the elite class (*E*-steady segment), and her offspring enters the standard school, she then enters the middle class and is then in the S-steady segment.

Condition (i): This is straightforward by definition of the *j*-segment. Assume a parent within the *j*-steady segment $\left[\underline{h}_j, \overline{h}_j\right]$. If her offspring pursue study *j*, then his highest possible skill is $\overline{h}_j$ (corresponding to the highest parent's skill $\overline{h}_j$ and the highest offspring's possible ability $\overline{a}$), and the offspring's lowest possible skill is $\underline{h}_j$ (corresponding to the lowest parent's skill and the lowest offspring's ability). Hence, the offspring's skill also belongs to segment $\left[\underline{h}_j, \overline{h}_j\right]$.

Condition (ii): Consider child *i* from the middle class who enters the elite university with skill attainment h_i^B at the end of basic education. We show that her skill at the end of the elite university belongs to the *E*-steady segment $\left[\underline{h}_E, \overline{h}_E\right]$. Let us denote this student as student *v*.

Let us consider a child from the elite with a skill at the end of basic education lower than that of offspring *v* (this offspring does exist since offspring *v* enters the elite university). Would this elite child pursue elite university, she would be inside the *E*-segment (by definition of the *E*-segment), and her skill would be higher than $\underline{h}_E$. Hence offspring *v* who pursued studies at elite university has a final skill level higher than $\underline{h}_E$. In addition, offspring *v*'s parents has a skill level lower than $\overline{h}_E$, and hence offspring *v* also has a skill level lower than $\overline{h}_E$. Therefore offspring *v*'s final skill belongs to the *E*-steady segment.

Condition (iii): The same proof applies in the case in which a child whose parents belong to the elite enters the standard university. More precisely, this elite offspring has a final education lower than that of the most skilled offspring of the middle class, would the latter pursue a standard university study, and higher than the least skilled offspring from the middle class.

Appendix B. Position and determination of h_α

The subscript *t* denoting the generation is omitted for the sake of simplification.

1) h_α is inside the mobility segment

Assume that the mobility segment $\left[\underline{h}_E^B, \overline{h}_S^B\right]$ does exist and let *m* and *e* be respectively the number of offspring from the middle class and from the elite in the mobility segment. The offspring inside the mobility segment, whatever their origin (middle class or elite), are ranked in ascending order of skill

at the end of basic education. The number of offspring inside segment $\left[\bar{h}_S^B, \bar{h}_E^B\right]$ is $\alpha - e < \alpha$ and the number of offspring inside segment $\left[\underline{h}_E^B, \bar{h}_E^B\right]$ is $\alpha + m > \alpha$. There is thus a unique $h_\alpha \in \left[\underline{h}_E^B, \bar{h}_S^B\right]$ such that the number of offspring with a skill level higher or equal to h_α at the end of basic education is equal to α.

2) Diagrammatic determination of h_α

Consider the variable h defined on the mobility segment: $h \in \left[\underline{h}_E^B, \bar{h}_S^B\right]$.

We define the following functions:

1) $E(h) =$ Number of offspring from the elite in the segment $\left[\underline{h}_E^B, h\right]$. On the mobility segment, $E(h)$ is continuous and monotonically increasing from 0 to *e*.

2) $M(h) =$ Number of offspring from the middle class in the segment $\left[h, \bar{h}_S^B\right]$. On the mobility segment, $M(h)$ is continuous and monotonically decreasing from *m* to 0.

The diagrammatic determination of h_α is depicted in Figure B1, where we choose $m > e$ without loss of generality.

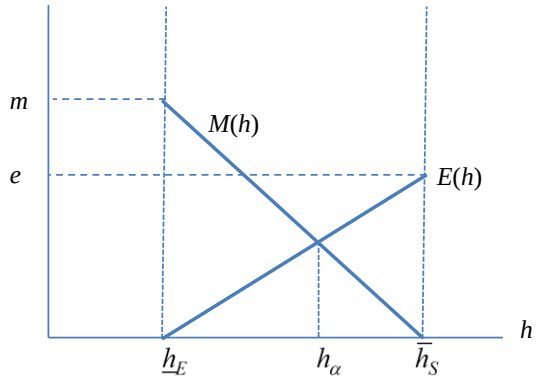

Figure B1. Determination of h_α

Appendix C. Impacts of changes in public expenditures

Remember that, for given sizes of the middle class (γ) and of the elite (α), the number of middle class offspring who enter the elite university is equal to the number of elite offspring who enter the standard university, i.e., $\mu\gamma = \rho\alpha$ (Equation (8) in the text).

1) *Increase in* V_E

An increase in expenditures per student in the elite university, V_E, moves the skill of all individuals in the elite upwards, which in turn increases the post basic education skill of their offspring. In Figure 3 in the text, the E-steady segment $\left[\underline{h}_E, \bar{h}_E\right]$ as well as the corresponding post-basic education segment $\left[\underline{h}_E^B, \bar{h}_E^B\right]$ move upwards. Consequently, the threshold h_α also moves upwards. This leads to:

1) A decrease in $\mu = \int_{h_\alpha}^{\bar{h}_S^B} f_S(h)dh$ since h_α increases whereas $\bar{h}_S^B$ remains unchanged.

2) Since $\mu\gamma = \rho\alpha$, and since α and γ are constant, then ρ decreases, and β increases.

2) *Decrease in* V_S

A decrease in the expenditure per student in the standard university, V_S, moves downwards the S-steady $\left[\underline{h}_S, \bar{h}_S\right]$ as well as the corresponding post-basic education segment $\left[\underline{h}_S^B, \bar{h}_S^B\right]$ in Figure 3. Consequently, the threshold h_α also moves downwards. This leads to:

1) A decrease in $\rho = \int_{\underline{h}_E^B}^{h_\alpha} f_E(h)dh$ since h_α decreases and $\underline{h}_E^B$ remains unchanged, and thus an increase in β.
2) A decrease in μ because $\mu\gamma = \rho\alpha$, ρ decreases and α and γ are constant.

Appendix D. Impact of a change in α

Assume an increase in the weight of the elite in the population α without changes (i) in the weight of the middle class γ and (ii) in the density functions f_S and f_E. The number of middle class offspring and elite offspring entering the elite university are respectively:

$$\mu \times \gamma = \gamma \int_{h_\alpha}^{\bar{h}_S^B} f_S(h)dh \text{ and } \beta \times \alpha = \alpha \int_{h_\alpha}^{\bar{h}_E^B} f_E(h)dh .$$

Let $h_{\alpha 1}$ be the thresholds to accede to the elite university before the increase in α. Without change in $f_E(h)$, the shift in α from α_1 up to $\alpha_2 > \alpha_1$ makes the number of elite offspring in the segment $\left[h_{\alpha 1}, \bar{h}_E\right]$ to increase from:

$$\alpha_1 \int_{h_{\alpha 1}}^{\bar{h}_E^B} f_E(h)dh \text{ up to } \alpha_2 \int_{h_{\alpha 1}}^{\bar{h}_E^B} f_E(h)dh .$$

The corresponding increase is $(\alpha_2 - \alpha_1) \int_{h_{\alpha 1}}^{\bar{h}_E^B} f_E(h)dh$. Concurrently, the increase in the number of offspring recruited by the elite university is $\alpha_2 - \alpha_1$. For the number of middle class offspring who enter the elite university not to rise, we must have $(\alpha_2 - \alpha_1) \int_{h_{\alpha 1}}^{\bar{h}_E^B} f_E(h)dh \geq \alpha_2 - \alpha_1$, i.e., $\int_{h_{\alpha 1}}^{\bar{h}_E^B} f_E(h)dh \geq 1$.

This is impossible when the mobility segment does exist. As the number of middle class offspring is assumed constant, the middle class upward mobility rate increases.

As the middle class upward mobility increases, threshold h_α must decrease and move from $h_{\alpha 1}$ to $h_{\alpha 2} < h_{\alpha 1}$. Assuming no change in the density function $f_E(h)$, the elite self-reproduction rate moves from:

$$\int_{h_{\alpha 1}}^{\bar{h}_E^B} f_E(h)dh \text{ up to } \int_{h_{\alpha 2}}^{\bar{h}_E^B} f_E(h)dh > \int_{h_{\alpha 1}}^{\bar{h}_E^B} f_E(h)dh \qquad \text{since } h_{\alpha 2} < h_{\alpha 1}.$$

Hence, the elite self-reproduction rate increases.

Appendix E. Increase in γ

We firstly discuss the conditions for a decrease in $\hat{h}$ to entail an increase in the weight of the middle class γ (and thus a decline in the weight of the lower class). This is because, as noted above, the existence of the lower class at the steady stratification supposes that $\hat{h} \geq \overline{h}_B$. Hence, starting from a steady stratification: (i) if the decrease in $\hat{h}$ is insufficient to make $\hat{h}$ move beneath $\overline{h}_B$, then there is no change in the size of each social group; (ii) if the decrease makes $\hat{h}$ move below $\overline{h}_B$, then the new steady stratification is characterised by the vanishing of the lower class and $\gamma = 1-\alpha$. Thus, for changes in $\hat{h}$ to induce an increase in γ without vanishing of the lower class, the move must be twofold. Firstly, the decrease in $\hat{h}$ must make it move below $\overline{h}_B$ so that a number of lower class children can accede to the university. In a second stage, $\hat{h}$ must move back above $\overline{h}_B$ to avoid the total vanishing of the lower class. If only the first shift occurs, then the size of the middle class increases from $\gamma < 1-\alpha$ to $\gamma = 1-\alpha$.

Let us assume an increase in the size of the middle class γ without change in the weight of the elite α. We also suppose no change in the density functions $f_S(h)$ and $f_E(h)$.

Let $h_{\alpha 1}$ be the thresholds to accede to the elite university before the increase in γ. The increase in γ without change in $f_S(h)$ makes the number of middle class offspring to increase at each point of segment $\left[\underline{h}_S^B, \overline{h}_S^B\right]$. Thus, the number of offspring from the middle class above $h_{\alpha 1}$ at the end of basic education increases. This induces:

1) An upward move of threshold h_α from $h_{\alpha 1}$ to $h_{\alpha 2} > h_{\alpha 1}$.

2) A decrease in the number of offspring from the elite entering the elite university, and hence a decrease in the elite self-reproduction rate.

3) A decrease in the middle class upward mobility rate $\mu = \int_{h_\alpha}^{\overline{h}_S^B} f_S(h)dh$ for a given density function $f_S(h)$ since h_α increases.

Appendix F. Results of the Simulations

Table A1. Results of the Scenario '*Equal*'

Generations	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Size of the Lower class	500	152	37	11	4	2	0	0	0	0	0	0	0	0
Size of the Middle class	0	323	438	464	471	473	475	475	475	475	475	475	475	475
Size of the Elite	0	25	25	25	25	25	25	25	25	25	25	25	25	25
moves from LC to MC		323	115	26	7	2	2	0	0	0	0	0	0	0
Moves from MC to E		25*	21	20	22	19	23	21	19	23	23	21	16	21
μ			6.5	4.56	4.74	4.03	4.86	4.42	4	4.84	4.84	4.42	3.37	4.42
ρ			84.0	80.0	88.0	76.0	92.0	84.0	76.0	92.0	92.0	84.0	64.0	84.0
β			16.0	20.0	12.0	24.0	8.0	16.0	24.0	8.0	8.0	16.0	36.0	16.0

* From the lower class to the elite.

Table A2. Results of the Scenario '*In-between*'

Generations	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Size of the Lower class	500	152	61	20	4	2	0	0	0	0	0	0	0	0
Size of the Middle class	0	323	414	455	471	473	475	475	475	475	475	475	475	475
Size of the Elite	0	25	25	25	25	25	25	25	25	25	25	25	25	25
moves from LC to MC		323	91	41	16	2	2	0	0	0	0	0	0	0
Moves from MC to E		25*	9	8	9	9	11	11	9	7	9	7	8	7
μ			2.79	1.93	1.98	1.91	2.3	2.32	1.89	1.47	1.89	1.47	1.68	1.47
ρ			36.0	32.0	36.0	36.0	44.0	44.0	36.0	28.0	36.0	28.0	32.0	28.0
β			64.0	68.0	64.0	64.0	56.0	56.0	64.0	72.0	64.0	72.0	68.0	72.0

* From the lower class to the elite.

Table A3. Results of the Scenario '*Elite*'

Generations	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Size of the Lower class	500	152	44	14	5	3	2	0	0	0	0	0	0	0
Size of the Middle class	0	323	431	461	0.47	472	473	475	475	475	475	475	475	475
Size of the Elite	0	25	25	25	25	25	25	25	25	25	25	25	25	25
moves from LC to MC		323	108	30	9	2	2	2	0	0	0	0	0	0
Moves from MC to E			3	0	1	0	2	1	1	0	1	1	1	1
μ			0.93	0.00	0.22	0.00	0.42	0.21	0.21	0.00	0.21	0.21	0.21	0.21
ρ			12.0	0.0	4.0	0.0	8.0	4.0	4.0	0.0	4.0	4.0	4.0	4.0
β			88.0	100.0	96.0	100.0	92.0	96.0	96.0	100.0	96.0	96.0	96.0	96.0