

# **Lotteries vs. All-Pay Auctions in Fair and Biased Contests<sup>1</sup>**

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## **Abstract**

The form of contests for a single fixed prize can be determined by a designer who maximizes the contestants' efforts. This paper establishes that, under common knowledge of the two asymmetric contestants' prize valuations, a fair Tullock-type endogenously determined lottery is always superior to an all-pay-auction; it yields larger expected efforts (revenues) for the contest designer. If the contest can be unfair (structural discrimination is allowed), then the optimal lottery cannot be dominated by an all-pay-auction.

**Keywords:** contest design, efforts (revenue) maximization, discrimination, endogenous lottery, all-pay auction.

**JEL Classification:** D70, D71, D72

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## 1. Introduction

Many economic and political decisions are the outcome of all-pay strategic contests for a given prize. In such contests the single winner depends on the efforts invested by the contestants. Applications of these contests include promotional competitions, litigation, internal labor market tournaments, rent-seeking, R&D races, political and public policy competitions and sports, Epstein and Nitzan (2007), Konrad (2009), Congleton et al. (2008). Endogenous determination of such contests may involve all of their relevant institutional characteristics. These are typically determined by contest designers; economic and political entrepreneurs who wish to maximize the efforts made by the contestants. Most of the literature on optimal contest design has focused on the choice of the contest prize, Glazer and Hassin (1988), Runkel (2006), Singh and Wittman (1998), Epstein and Nitzan (2002), (2006a), (2006b), (2007), the set of contestants, Baye et al. (1993), Amegashie (2000), Moldovanu and Sela (2006), Taylor (1995), both the set of contestants and the prize system, Che and Gale (2003), the structure of multi-stage contests, Gradstein (1998), Gradstein and Konrad (1999), Amegashie (2000), caps on political lobbying, Che and Gale (1998) and the contest success function, CSF, the function that relates the contestants' efforts to their winning probabilities, Che and Gale (1997), Nti (1997), (2004).

In the current study we focus on the design of logit CSF's that include two types of popular mechanisms: Tullock's type lotteries and all-pay-auctions. We show that in asymmetric contests with different prize valuations, the endogenously determined lottery always yields certain efforts that are larger than or equal to the expected efforts in an APA.

We first study fair non-discriminating logit CSF's, as in Alcalde and Dahm (2010) and Nti (2004), allowing control of the exponent determining the particular form of the logit CSF. In this context, Fang (2002) has shown that a fair simple lottery can be superior to an all-pay-auction and induce larger efforts, if the gap between the contestants' prize valuations is sufficiently large. Our main result considerably extends his finding. It establishes that *the optimal* fair (non-discriminating) lottery is *always* superior to the all-pay-auction (APA). That is, it yields larger efforts regardless of the gap between the contestants' stakes. We then extend the setting by allowing discrimination, that is, control of another parameter that determines the preferential treatment received by one of the contestants, as in Lien (1986), (1990), Clark and Riis (2000) and, more recently, Franke (2007), Epstein et al. (2011). Such

discrimination is commonly observed in real political-economic contest environments, and, particularly, in the public sector (For a detailed discussion of the empirical relevance of discrimination and its control by contest designers see Epstein et al. (2011) and Franke et al. (2011)). By our second result, when discrimination is allowed, the optimal discriminating lottery cannot be dominated by the optimal discriminating APA because of the existence of a lottery that yields certain efforts that are equal to the expected efforts under the optimal APA.

Given our assumption of common knowledge of the contestants' prize valuations, the contest designer could resort to a mechanism that completely extracts the higher prize valuation. Focusing on contests with logit CSF's actually implies that we rule out such mechanisms. In other words, the contestants in our setting are in fact assumed to be protected from complete extraction of their surplus because they are ensured that in the contest equilibrium their participation is minimally effective; in equilibrium every contestant makes a positive effort with positive probability and has a positive probability of winning the contested prize.<sup>2</sup> The focus on logit lotteries conforms therefore to a legal constraint that contests induce participation and that the designer cannot induce efforts that exceed, as we shall see, the average value of the contested prize.

It should be noted that the logit CSFs can be justified either axiomatically<sup>3</sup> or on the grounds of common use in practice. In any event, they are the most widely studied functions in the contest literature.<sup>4</sup> Despite their popularity, it must be admitted that it remains an open question whether some other CSFs, while still ensuring minimally effective participation of the contestants, can yield better expected results for the contest designer. In other words although we prove the superiority of Tullock-type lotteries over APAs, we cannot claim that they are the best minimally effective CSFs for the revenue-maximizing contest designer.

The remainder of the paper is laid out as follows. In Section 2, we present the model, the optimal contest design approach that allows discrimination and the two

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<sup>2</sup> The existence of effective incentives of participation precludes the direct abolition of competition by exclusion of contestants or the indirect abolition of competition by application of a Tullock CSF that for some contestants is always unresponsive to their effort, as in Nti (2004).

<sup>3</sup> When discrimination between the contestants is permitted, we can use the axiomatic approach of Clark and Riis (1998). For axiomatization of a symmetric CSF see Skaperdas (1996). For a different axiomatic approach of both a discriminating or a symmetric CSF, see Jia (2008).

<sup>4</sup> Since the contested prize is assumed to be fixed, the possibility of applying a Tullock CSF with reimbursements, as in Cohen and Sela (2005) and Matros and Armanios (2009) is also disregarded.

types of contest success functions, APAs and Tullock's logit lotteries. Section 3 contains the main result which establishes the inferiority of the APA relative to the endogenous lottery, when the contestants have different prize valuations. In Section 4 it is shown that the optimal discriminating lottery cannot be dominated by the optimal discriminating APA. Section 5 contains a brief summary and concluding remarks. All the proofs are relegated to an Appendix.

## 2. Optimal contest design

### a. The setting

In the basic one-stage contest setting, there are two risk-neutral contestants, the high and low benefit contestants, 1 and 2. The prize valuations of the contestants are denoted by  $n_i$ ,  $n_1 > n_2$  or  $k = \frac{n_1}{n_2} > 1$ . Being chiefly concerned with the conditions under which the contest designer chooses an optimal contest, we assume that he has full knowledge of the contestants' prize valuations. Given the contestants' fixed prize valuations and the CSF, the function that specifies the contestants' winning probability given their efforts,  $\Pr_i(x_1, x_2)$ , the expected net payoff (surplus) of contestant  $i$  is:

$$(1) \quad E(u_i) = \Pr_i(x_1, x_2)n_i - x_i, \quad (i=1,2)$$

where  $x_1$  and  $x_2$  denote the contestants' efforts. In the extended optimal contest design setting, the objective function of a third player, the designer of the contest, is:

$$(2) \quad G = E(x_1 + x_2)$$

The contest designer is assumed to maximize his objective function (2) by setting the CSF, anticipating the Nash equilibrium efforts of the contestants that are obtained in the standard contest where the payoff functions of the contestants are given by (1) and the contest success function (the explicit form of  $\Pr_i(x_1, x_2)$ ) is the one set by the designer. As already mentioned, we are interested in competitive environments that preclude abolition of competition by exclusion of contestants, as explained in the

introduction. We therefore focus on the widely studied contests that are based on logit CSFs that include APAs and Tullock's lotteries.<sup>5</sup>

### b. All-Pay Auctions

In our setting, certain winning means that the designer sets a CSF that leaves no residual winning uncertainty after the revelation of the contestants' efforts. In such a case, the certain winner is the contestant who makes the largest effective effort, where a unit of effort by one contestant is not necessarily equally effective as a unit of effort of his rival, as first suggested in the context of a bribery game by Lien (1986), (1990) and later on by Clark and Riis (2000). That is, the CSF for  $\delta > 0$  is an APA given by:

$$(3) \quad p_1(x_1, x_2) = \begin{cases} 1 & \text{if } x_1 > \delta x_2 \\ 0.5 & \text{if } x_1 = \delta x_2 \\ 0 & \text{if } x_1 < \delta x_2 \end{cases}$$

and for  $\delta = 0$ ,  $p_1(x_1, x_2) = 1$ , where the discrimination variable  $\delta \geq 0$  is selected by the contest designer. By (3), a reduction in  $\delta$  increases the bias in favor of the more motivated contestant 1. Furthermore,  $0 \leq \delta < 1$  implies a bias in favor of contestant 1. When  $\delta = 1$  the contest is fair, there is no bias. When  $\delta > 1$  the bias is in favor of contestant 2.

### c. The logit lotteries

When winning is uncertain, the designer sets a CSF that leaves some residual winning uncertainty after the revelation of the contestants' efforts. In such a case, in equilibrium every contestant has some positive winning probability. Sufficiently large investment of effort can secure a high probability of winning, but not certain winning.

Our optimality and neutrality results are confined to the well studied logit Tullock-type lotteries. For  $\delta > 0$ , these lotteries are given by:<sup>6</sup>

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<sup>5</sup> In Epstein et al. (2011), when the weight assigned to the expected welfare of the contestants is sufficiently high, in equilibrium there is no contest (and so no efforts are made) and the winner is the contestant with the higher valuation. In contrast, in our model, such equilibrium cannot emerge since the weight assigned to the expected welfare of the contestants is zero. Therefore, any equilibrium is an interior one: there is real competition and "meaningful" winning (each contestant makes an effort with a positive winning probability).

$$(4) \quad p_1(x_1, x_2) = \frac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha}$$

$0 < \alpha < \infty$  and for  $\delta = 0$ ,  $p_1(x_1, x_2) = 1$ , where  $\alpha$  and  $\delta$  are selected by the contest designer. The interpretation of  $\delta$  is as in sub-section (b).

### 3. Fair contests

As in Alcalde and Dahm (2010) and Nti (2004), consider environments where the designer cannot or does not want to discriminate between the contestants,  $\delta = 1$ . That is, he cannot grant one contestant an advantage over his rival. This is the standard type of fair (unbiased) contest studied in the literature, see Konrad (2009).

#### a. The unbiased APA

In equilibrium of the unbiased APA, Hillman and Riley (1989), Konrad (2009), the expected efforts are  $x_1^* = 0.5n_2$  and  $x_2^* = \frac{n_2^2}{2n_1}$ . In turn, the expected net payoff of the player with the lower prize valuations is zero, namely, when  $k > 1$ , only the player with the higher prize valuations enjoys some surplus. The expected net payoff of the player with the higher prize valuations is equal to  $E(u_1^*) = (n_1 - n_2)$ . In equilibrium, the value of the designer's objective function (the expected aggregate efforts of the contestants in the mixed-strategy equilibrium) is equal to  $G_A = \frac{n_2(n_1 + n_2)}{2n_1}$ .

#### b. The optimal unbiased lottery

With asymmetric prize valuations,  $k > 1$ , and  $\alpha = 1$ , it is known that in equilibrium  $G_L = \frac{n_1 n_2}{n_1 + n_2}$  is larger than  $G_A$ , provided that the gap between the contestants' stakes is sufficiently large, that is,  $k > 1 + \sqrt{2}$  (see Fang (2002, section 5.2)). Our first result

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<sup>6</sup> As is well known, see Konrad (2009) and references therein, when  $0 < \alpha \leq 2$  the contest has a pure-strategy equilibrium. Since  $p_1(x_1, x_2) = \frac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha} = \frac{1}{1 + (\delta x_2 / x_1)^\alpha}$ , one can easily see that for

$\alpha = \infty$  the logit lottery takes the form (3). The form in (4) is slightly different from the form of the logit lottery in Epstein et al. (2011) because in the present study, as will become clear in the sequel, it is important to make a meaningful comparison between the degree of discrimination under the two types of CSFs. Such comparison requires that discrimination is defined in a similar way in the two cases.

considerably strengthens this finding by establishing that in a fair contest ( $\delta = 1$ ), if  $k > 1$ , then a designer who can select the exponent  $\alpha$ , *always* prefers a logit lottery because it yields larger efforts relative to the APA, even when the sufficient condition  $k > 1 + \sqrt{2}$  is *not* satisfied. In fact, the exponent of the preferred lottery satisfies  $0 < \alpha < 2$ , which means that the corresponding contest game has a unique pure-strategy equilibrium (see footnote 6).

**Proposition 1:**

*If discrimination is not feasible,  $k > 1$  and the designer can select the exponent of the lottery, then there exists a value in the range  $0 < \alpha < 2$  that yields certain contestants' efforts that are larger than the expected efforts obtained under the APA.* (For a proof, see Appendix A).

This result implies that, in a fair contest, if  $k > 1$ , then a Tullock-type lottery is always preferred to the APA by a risk neutral or a risk averse designer. Alcalde and Dahm (2010) have shown that for any  $\alpha \geq 2$  there exists an equilibrium in mixed strategies that is equivalent to the equilibrium of the APA. However, so far a characterization of the complete set of mixed-strategy equilibria is not available. Since we show that there exists  $\alpha$ ,  $0 < \alpha < 2$ , that yields efforts that are larger than those obtained under the APA, it is clear that this conclusion remains valid when the parameter  $\alpha$  has to satisfy the requirement  $0 < \alpha < \infty$ .

Let us explain the economic intuition behind the result, namely, why  $k > 1$  gives rise to a pure-strategy equilibrium corresponding to  $0 < \alpha(k) < 2$  which is preferred to the mixed-strategy equilibrium obtained under the APA (the logit CSF where  $\alpha = \infty$ ).

For  $k > 1$ , a designer setting an optimal  $\alpha$  for a pure-strategy equilibrium must choose an exponent  $\alpha$  which is smaller than 2. Furthermore, in a pure-strategy equilibrium the designer may choose an exponent that is smaller than that  $\alpha$  which causes contestant 2's utility to be equal to zero. In other words, in equilibrium contestant 2's utility can be positive. This has been shown by Nti (2004) for a sufficiently large value of  $k$  (see Section 4 in his paper). However, for lower values of

$k$ , but still  $k > 1$ ,  $\alpha$  reduces contestant 2's utility to zero.<sup>7</sup> In any case, as explained above, for  $k > 1$  the optimal  $\alpha$  for a pure-strategy equilibrium is smaller than 2,  $0 < \alpha(k) < 2$ .

Under the APA where  $\alpha = \infty$ , the contest equilibrium is in mixed strategies, the total expected efforts are equal to  $G_L = \frac{n_2(n_1 + n_2)}{2n_1}$  and the surplus of contestant 2 is completely eliminated. It remains to explain why the designer prefers the optimal pure-strategy equilibrium to the mixed-strategy equilibrium. Notice that in the move from equilibrium in pure strategies to equilibrium in mixed strategies  $\alpha$  is increased, however,  $x_1^*/x_2^* = k$ .<sup>8</sup> The increase in  $\alpha$  increases the winning probability of contestant 1 and reduces the winning probability of contestant 2. However, contestant 1 is induced to reduce his efforts in the mixed-strategy equilibrium corresponding to the larger  $\alpha$  because this further increases his expected utility. Such reduction is possible because contestant 2 whose utility is reduced or remains equal to zero in the mixed-strategy equilibrium is also induced to reduce his effort.<sup>9</sup>

Che and Gale (1997) point out that intuition suggests that a society with contest designers who are receptive to rent seeking (a large exponent  $\alpha$ ) would induce greater rent-seeking expenditures than other societies, all else equal. Our result establishes that this intuition is not valid when we move from the range  $0 < \alpha \leq 2$  to  $\alpha = \infty$ . In particular, when  $\delta = 1$  and  $k > 1$ , the parameter  $\alpha$  in the logit CSF on which we focus, that yields the largest efforts is not  $\alpha = \infty$ , since there is some  $\alpha$  which is smaller than 2 that yields larger efforts.

#### 4. Unconstrained contests

In a Nash equilibrium of the contest, every contestant optimally determines his investment, given the parameters of the contest success function. In the unconstrained

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<sup>7</sup> The critical  $k$  is  $k = 3.509$ .

<sup>8</sup> By substituting  $\delta = 1$  in equation (3.B) in Appendix B, we get that the contestants' efforts in a pure-strategy equilibrium are equal to  $x_{1p}^* = \frac{\alpha n_1 k^\alpha}{(k^\alpha + 1)^2}$  and  $x_{2p}^* = \frac{\alpha n_2 k^\alpha}{(k^\alpha + 1)^2}$ . Under the APA their expected efforts are  $x_{1m}^* = 0.5n_2$  and  $x_{2m}^* = \frac{n_2^2}{2n_1}$ . Hence,  $\frac{x_{1p}^*}{x_{2p}^*} = \frac{x_{1m}^*}{x_{2m}^*} = k$ .

<sup>9</sup> Recall that in the pure-strategy equilibrium the utility of contestant 2 can be positive if  $k$  is sufficiently large (see footnote 7).



environment that allows discrimination, anticipating the investments of the contestants, the designer optimally determines the parameters of the contest's success functions. When considering the APA the designer controls the degree of discrimination  $\delta$ . When considering the logit lottery he determines both  $\delta$  and the exponent  $\alpha$ .

Most of the literature studying Tullock's lotteries and the APA disregarded deliberate discrimination between the contestants and control of the exponent  $\alpha$ . However, in the context of an APA, Lien (1986), (1990) and Clark and Riis (2000) studied a bribery game in which an official (designer) exercises discrimination in a multiplicative form. Nti (2004) examined the effect of the exponent  $\alpha$  on the contestants' efforts in Tullock' lotteries disregarding discrimination between the contestants and focusing on pure-strategy equilibria. He thus initiated the study of optimal contest design by control of the exponent  $\alpha$  for Tullock's CSFs. Recently, Epstein et al. (2011) presented a complete analysis of the designer's effect on the contestants' efforts by control of the degree of discrimination both for the APA and the logit lotteries, assuming that in the lotteries' case the exponent  $\alpha$  is given and restricted to the range  $0 < \alpha \leq 1$ . They have thus re-shifted the emphasis in the study of optimal contest design to the control of discrimination. The current section combines the two approaches allowing the designer to control both the exponent  $\alpha$  and the degree of discrimination between the contestants. In addition, it generalizes the analysis by allowing any value of the exponent  $\alpha$  in the logit CSFs that gives rise to a Tullock-type lottery associated with a pure-strategy or a mixed-strategy equilibrium,  $0 < \alpha < \infty$ , or to an APA that is associated with a mixed-strategy equilibria where  $\alpha = \infty$ .<sup>10</sup>

### **a. The optimal APA**

Under the APA, the designer of the contest maximizes his objective function, (2), by determining the optimal value of  $\delta$ . The optimal  $\delta$  is equal to  $k$  (for a proof, see Epstein et al. (2011), Proposition 1, assuming that no weight is assigned to the expected welfare of the contestants). This optimal bias accomplishes the designer's attempt to maximize the extent of competition between the contestants. In fact, such

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<sup>10</sup> Note that under the logit CSF where  $0 < \alpha \leq 2$  the exerted efforts are certain whereas under the APA the meaning of efforts is expected efforts.

bias eliminates the advantage of contestant 1, creates actual equality between the competitors and completely eliminates their surplus. The corresponding value of the designer's objective function (the expected aggregate efforts of the contestants in the mixed-strategy equilibrium of the contest) is  $G_A = 0.5(n_1 + n_2)$ . Thus, in equilibrium, the expected payoff of the designer is equal to the average prize valuation.

### **b. The optimal lottery**

Under the logit contest success function, the designer determines the optimal level of  $\alpha$ ,  $0 < \alpha < \infty$ , and  $\delta$ ,  $\delta \geq 0$ . Let us partition the range of the parameter  $\alpha$  into  $0 < \alpha \leq 2$  and  $2 < \alpha < \infty$ . For  $0 < \alpha \leq 2$ , where a unique pure-strategy equilibrium exists, the optimal values of  $\delta$  and  $\alpha$  are equal to  $\delta^* = k$  and  $\alpha^* = 2$  (for a proof, see Appendix B. This proof has to take into account constraints that are not automatically satisfied as in Epstein et al. (2011) where  $0 < \alpha \leq 1$ . For further more specific clarification of this point see footnote 16.<sup>11</sup> The corresponding value of the designer's objective function (the certain aggregate efforts of the contestants in the pure-strategy equilibrium of the contest) is  $G_L = 0.5(n_1 + n_2) = G_A$ .<sup>12</sup> We therefore obtain:

### **Proposition 2:**

*If the contest designer can select both the degree of discrimination  $\delta$ ,  $\delta \geq 0$ , and the exponent  $\alpha$ , then the contestants' expected efforts of the optimal lottery are larger than or equal to those in the mixed-strategy equilibrium of the optimal APA.*

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<sup>11</sup> We suggest the following intuition regarding the equilibrium (in pure strategies) outcome and its sensitivity to  $\alpha$  ( $0 < \alpha \leq 2$ ). The designer tends to support the contestant with the lower prize valuation ( $\delta^* = k$ ) to attain complete “balance” between the contestants' winning probabilities (ensure that  $p_i = 0.5$ ), for any given  $\alpha$ ,  $0 < \alpha \leq 2$ . An increase in  $\alpha$  induces the contestants to increase their effort at the same rate in order to increase their winning probability. Since at the same time the designer favorably discriminates the contestant with the lower prize valuation, the equilibrium winning probability is unchanged,  $p_i = 0.5$ . But by raising  $\alpha$  the designer increases the aggregate efforts, and in turn his payoff. The designer therefore prefers the highest possible  $\alpha$ ,  $\alpha = 2$ , that enables him to extract the maximal possible surplus from the contestants which reduces their net payoff to zero.

<sup>12</sup> Note that this result has the flavor of the neutrality result of Alcalde and Dahm (2010). However, in our setting of optimal contest design, the contestants' maximal efforts are larger than those obtained in the symmetric setting of Alcalde and Dahm (2010). This is due, first, to the allowed discrimination between the contestants and, second, to the allowed control of the designer over the desirable value of the exponent  $\alpha$ .

The combined effect of optimal discrimination  $\delta = k$  and optimal selection of the exponent  $\alpha$  in the logit CSF

$$p_1(x_1, x_2) = \frac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha} = \frac{1}{1 + (\delta x_2 / x_1)^\alpha}$$

that gives rise to a pure-strategy equilibrium when  $0 < \alpha \leq 2$ , results in equal (expected) efforts and elimination of the surplus of both of the contestants. This result implies that the optimal logit lottery (that need not satisfy the constraint  $0 < \alpha \leq 2$ ) cannot be dominated by the APA.

Proposition 2 establishes that the intuition mentioned above that a larger exponent  $\alpha$  would induce greater rent-seeking expenditures is, again, not valid. Under optimal discrimination, even the use of an optimal APA by designers who are maximally receptive to the contestants' efforts, would not induce greater efforts than those obtained by designers who are less receptive to the contestants' efforts,  $\alpha^* = 2$ , allowing random winning. Recall that in both cases bids are optimally discriminated, the optimal degree of discrimination being equal to  $k$ . Nevertheless, the valid part of the above intuition is that a larger exponent  $\alpha$  indeed induces greater efforts under the logit lottery provided that the exponent  $\alpha$  gives rise to a pure-strategy equilibrium.

## 5. Conclusion

### a. Brief summary of the results

Our two results establish that, under common knowledge of the two asymmetric contestants' prize valuations, a fair non-discriminating Tullock-type lottery associated with a pure-strategy equilibrium is always superior to an all-pay-auction. That is, a lottery with an appropriately chosen exponent always yields higher expected revenues for the contest designer than an APA. When discrimination between the contestants is allowed, there exists a lottery which is equivalent to the optimal APA. Hence, the endogenously determined optimal lottery cannot be dominated by the optimal APA.

### b. Extraction of the contestants' surplus and the designer's payoff

In the unconstrained environment that allows discrimination, the designer always captures all the surplus of the two contestants. In the constrained fair environment, this is not the case although under the APA the designer always captures the surplus

of contestant 2, the player with the lower prize valuation. The difference between the two competitive environments is the extent of the contestants' incentives to make efforts. This hinges on the possibility of discrimination between the contestants. The possibility of discrimination increases the intensity of competition enabling the designer to increase his payoff.

### **c. The shadow price of the competitiveness constraint**

The competitiveness constraint means that we deal with interior contest equilibria such that the ability of the contest designer to induce efforts is limited to the average value of the contested prize. Without this constraint, the designer could yield a total effort of  $n_1$  by excluding the rivals of the contestant with the maximal valuation of the contested prize and then exploiting his political power or bargaining advantage to extract (almost) all his surplus, as in Nti (2004). We should point out that several different mechanisms can be used to implement the optimal design. For example, we could use an auction mechanism where the prize is awarded to the highest bidder but the contest designer has a reserve price equal to  $n_1$  (see Glazer, 1993) or, alternatively, use a first-price APA with a reservation price of  $n_1$  (see Hillman and Riley, 1989). The CSFs on which we focus do conform to the competitive environment and the maximal efforts are equal to  $0.5(n_1 + n_2)$ . This result is consistent with the constraint on the ability of the designer to induce efforts being binding. The shadow price of the competitiveness constraint is therefore equal to  $n_1 - 0.5(n_1 + n_2) = 0.5(n_1 - n_2)$ . This is then the price of competition for the designer.

### **d. The shadow price of the equalitarian constraint**

Equalitarianism results in reduced payoff for the designer relative to the unconstrained environment when  $k > 1$ . In this case the shadow price of this constraint is positive, the exact value depending on the gap between the contestants' prize valuations that gives a Tullock's lottery a clear-cut advantage over the APA, as implied by Proposition 1. This may certainly support an ideology protecting the equalitarian 'welfare state'. Simply, in our setting, such equalitarianism limits the ability of the political-economic entrepreneur to extract resources from the contestants.

### **e. Generalization to n-player contests**

A potential interesting extension of our study is the analysis of the multiple-player case. Results for the all-pay auction should be robust with respect to the number of players because only two players will actively participate in equilibrium. The case of optimal discrimination by control of the bias scheme has been recently analyzed in Franke et al. (2011) assuming the simple lottery case where  $\alpha = 1$ ; the contestants' winning probabilities are equal to their relative exerted efforts. Even the analysis of this relatively simple case is rather complex. Its extension to our more general setting of Tullock lotteries, where  $0 < \alpha \leq 2$ , seems an especially demanding challenge.

### **References**

- Alcalde, J. and Dahm, M., (2010), "Rent Seeking and Rent Dissipation: A Neutrality Result", *Journal of Public Economics*, 94, 1-7.,
- Amegashie, J.A., (2000), "Some Results on Rent-Seeking Contests with Shortlisting," *Public Choice*, 105 (3-4), 245-253.
- Baye, M.R., Kovenock, D. and de Vries, C., (1993), "Rigging the Lobbying Process: An Application of the All-Pay Auction", *American Economic Review*, 83, 289-294.
- Che, Y.K and Gale, I., (1997), "Rent Dissipation When Rent Seekers Are Budget Constrained", *Public Choice*, 92, 109-126.
- Che, Y.K and Gale, I., (1998), "Caps on Political Lobbying", *American Economic Review*, 88, 643-651.
- Che, Y.K and Gale, I., (2003), "Optimal Design of Research Contests", *American Economic Review*, 93(3), 646-671.
- Clark, D.J., Riis, C., (1998), Contest success functions: an extension, *Economic Theory*, 11, 201-204.
- Clark, D.J., Riis, C., (2000), "Allocation Efficiency in a Competitive Bribery Game", *Journal of Economic Behavior and Organization*, 42, 109-124.
- Cohen, C. and Sela, A., (2005), Manipulations in Contests, *Economics Letters*, 86, 135-139.
- Congleton, R.D., Hillman A.L. and Konrad K.A., (2008), editors, *The Theory of Rent Seeking: Forty Years of Research*, Springer, Heidelberg and Berlin.
- Epstein G.S., Mealem Y. and Nitzan S., (2011), "Political Culture and Discrimination in Contests", *Journal of Public Economics*, 95(1-2), 88-93.

- Epstein G.S. and Nitzan S., (2002), "Endogenous Public Policy, Politicization and Welfare", *Journal of Public Economic Theory*, 4(4), 661-677.
- Epstein, G.S. and Nitzan, S., (2006a), "Effort and Performance in Public-Policy Contests", *Journal of Public Economic Theory*, 8(2), 265-282.
- Epstein, G.S. and Nitzan, S., (2006b), "The Politics of Randomness", *Social Choice and Welfare*, 27(2), 423-433.
- Epstein, G.S. and Nitzan, S., (2007), *Endogenous Public Policy and Contests*. Springer.
- Fang, H., (2002), Lottery versus All-Pay Auction Models of Lobbying", *Public Choice*, 112, 351-371.
- Franke, J., (2007), "Does Affirmative Action Reduce Effort Incentives? A Contest Game Analysis", Ruhr Economic Papers #185, TU Dortmund.
- Franke J. , Kanzow C. Leininger W. and Schwartz A. , (2011), "Effort Maximization in Asymmetric Contest Games with Heterogeneous Contestants", *Economic Theory*, forthcoming.
- Glazer, A. and Hassin, R., (1988), "Optimal Contests", *Economic Inquiry*, 26(1), 133-143.
- Glazer, A., (1993), "On the Incentives to Establish and Play Political Rent-Seeking Games", *Public Choice*, 75, 139-148.
- Gradstein, M., (1998), "Optimal Contest Design: Volume and Timing of Rent Seeking in Contests", *European Journal of Political Economy*, 14(4), 575-585.
- Gradstein, M. and Konrad, K., (1999), "Orchestrating Rent Seeking Contests", *Economic Journal*, 109, 536-545.
- Hillman, A.L. and Riley, J.G., (1989), "Politically Contestable Rents and Transfers", *Economics and Politics*, 1, 17-39.
- Jia, H., (2008), "A Stochastic Derivation of the Ratio Form of Contest Success Functions", *Public Choice*, 135, 125–130.
- Konrad, K., (2009), *Strategy and Dynamics in Contests* (London School of Economic Perspectives in Economic Analysis), Oxford University Press, USA.
- Lien, D., (1986), "A Note on Competitive Bribery Games", *Economics Letters*, 22, 337-341.
- Lien, D., (1990), "Corruption and Allocation Efficiency", *Journal of Development Economics*, 33, 153-164.

- Matros, A. and Armanios, D., (2009), "Tullock's contest with reimbursements", *Public Choice*, 141, 49-63.
- Moldovanu, M. and Sela, A., (2006), "Contest Architecture", *Journal of Economic Theory*, 126(1), 70-96
- Nti, K.O., (1997), "Comparative Statics of Contests and Rent-Seeking Games", *International Economic Review*, 38(1), 43-59.
- Nti, K.O., (2004), "Maximum Efforts in Contests with Asymmetric Valuations", *European Journal of Political Economy*, 20(4), 1059-1066.
- Runkel, M., (2006), "Optimal Contest Design, Closeness and the Contest Success Function", *Public Choice*, 129, 217-231.
- Singh, N. and Wittman, D., (1998), "Contest Design and the Objective of the Contest Designer: Sales, Promotion, Sports Events and Patent Races", in *Baye's (ed.) Advances in Microeconomics*, Vol. 7, JAI Press, 139-167.
- Skaperdas, S., (1996), "Contest success functions". *Economic Theory*, 7, 283-290.
- Taylor, C.R., (1995), "Digging for Golden Carrots: An Analysis of Research Tournaments", *American Economic Review*, 85(4), 872-90.
- Tullock, G., (1980), Efficient Rent-Seeking. In: Buchanan J.M, Tollison R.D, Tullock G., (ed) *Toward a Theory of the Rent-Seeking Society*, College Station, TX, Texas A. and M., University Press, 97-112

## Appendix A: Proof of Proposition 1

When discrimination is not allowed,  $\delta = 1$ , and the exponent  $\alpha$  is chosen optimally taking the form  $\alpha(k)$ , by Nti (2004), in the range  $0 < \alpha \leq 2$  the pure-strategy equilibrium results in<sup>13</sup>

$$G_L^{ps} = \frac{\alpha k^\alpha (n_1 + n_2)}{(k^\alpha + 1)^2}$$

and under the APA, where  $\alpha = \infty$ , the mixed-strategy equilibrium results in

$$G_A = \frac{n_2(n_1 + n_2)}{2n_1} = \frac{n_1 + n_2}{2k}$$

Denote by  $\bar{\alpha}$ , which is a function of the parameter  $k$ , the solution of

$$(1.A) \quad (\alpha - 1)k^\alpha = 1$$

Under such equality the expected utility of contestant 2, the contestant with the low value of the prize, is equal to zero. Equality (1.A) implies that  $\bar{\alpha} > 1$ ,  $\bar{\alpha}k^{\bar{\alpha}} = k^{\bar{\alpha}} + 1$ ,  $\frac{\partial \bar{\alpha}}{\partial k} < 0$ ,  $\bar{\alpha}(k=1) = 2$  and  $(k \rightarrow \infty) \Rightarrow (\bar{\alpha} \rightarrow 1)$ , Hence, for every  $k > 1$ ,  $1 < \bar{\alpha} < 2$ .

Nti (2004, Theorem 3) has shown that for  $k \leq 3.509$ , the solution (in implicit form) of equation (1.A),  $\bar{\alpha}$ , is the optimal solution that maximizes  $G_L^{ps}$ . Otherwise the solution differs from  $\bar{\alpha}$ . This means that

$$\text{Max } G_L^{ps} \geq \frac{\bar{\alpha}k^{\bar{\alpha}}(n_1 + n_2)}{(k^{\bar{\alpha}} + 1)^2} = \frac{\bar{\alpha}k^{\bar{\alpha}}(n_1 + n_2)}{(\bar{\alpha}k^{\bar{\alpha}})^2} = \frac{n_1 + n_2}{\bar{\alpha}k^{\bar{\alpha}}}$$

Therefore, when  $\frac{n_1 + n_2}{\bar{\alpha}k^{\bar{\alpha}}} > G_A$  we get that  $\text{Max } G_L^{ps} > G_A$ . That is,

$$\frac{n_1 + n_2}{\bar{\alpha}k^{\bar{\alpha}}} > \frac{n_1 + n_2}{2k}$$

or

$$2k^{1-\bar{\alpha}} > \bar{\alpha}$$

By (1.A),  $k = (\bar{\alpha} - 1)^{-\frac{1}{\bar{\alpha}}}$ . Substituting  $k$  in the latter inequality we get:

$$2(\bar{\alpha} - 1)^{\frac{\bar{\alpha}-1}{\bar{\alpha}}} > \bar{\alpha}$$

---

<sup>13</sup> Notice that Nti defines  $k$  as the inverse of our  $k$ , that is, in his case  $k = n_2/n_1$ . Consequently, his result is presented with the appropriate modification.



or  $(\bar{\alpha}-1)^{\frac{\bar{\alpha}-1}{\bar{\alpha}}} > 0.5\bar{\alpha}$  or  $(\bar{\alpha}-1)^{\bar{\alpha}-1} > (0.5\bar{\alpha})^{\bar{\alpha}}$  or:

$$(2.A) \quad (\bar{\alpha}-1)^{\bar{\alpha}-1} - (0.5\bar{\alpha})^{\bar{\alpha}} > 0$$

Notice that if inequality (2.A) holds for  $1 < \bar{\alpha} < 2$ , then it is satisfied for every  $k > 1$ .<sup>14</sup>

The reason is that an increase in  $k$  reduces  $\bar{\alpha}$  and for  $k=1$ ,  $\bar{\alpha}=2$  whereas for  $k \rightarrow \infty$ ,  $\bar{\alpha}$  converges to 1.

Let  $f(\bar{\alpha}) = (\bar{\alpha}-1)^{\bar{\alpha}-1}$  and  $g(\bar{\alpha}) = (0.5\bar{\alpha})^{\bar{\alpha}}$ . To prove inequality (2.A), we have to show that for  $1 < \bar{\alpha} < 2$ ,  $f(\bar{\alpha}) > g(\bar{\alpha})$ .<sup>15</sup>

Consider first the two functions  $f$  and  $g$  at  $1 < \bar{\alpha} \leq 2$ . It can be verified that  $\lim_{\bar{\alpha} \rightarrow 1} f(\bar{\alpha}) = 1$ ,  $f(2) = 1$  and  $\frac{df(\bar{\alpha})}{d\bar{\alpha}} = (\bar{\alpha}-1)^{\bar{\alpha}-1} [1 + \ln(\bar{\alpha}-1)]$ . Therefore, for  $1 < \bar{\alpha} \leq 2$ ,  $f$  is minimized at  $\bar{\alpha} = 1 + e^{-1}$ , where  $\min f(\bar{\alpha}) = (e^{-1})^{e^{-1}} = 0.6922$ . In contrast, for  $1 < \bar{\alpha} \leq 2$ , we get that  $\frac{dg(\bar{\alpha})}{d\bar{\alpha}} = (0.5\bar{\alpha})^{\bar{\alpha}} [1 + \ln(0.5\bar{\alpha})] > 0$ . That is, the function  $g$  is increasing in the domain  $1 < \bar{\alpha} \leq 2$ .

Partitioning this domain,  $1 < \bar{\alpha} \leq 2$ , into two subsets:  $1 < \bar{\alpha} \leq 1 + e^{-1}$  and  $1 + e^{-1} < \bar{\alpha} \leq 2$ , let us show that in each of these sub-domains inequality (2.A) is satisfied.

Consider first the sub-domain  $1 < \bar{\alpha} \leq 1 + e^{-1}$ . Since at  $\bar{\alpha} = 1 + e^{-1}$ ,  $g(1 + e^{-1}) = 0.5947 < 0.6922 = \min f(\bar{\alpha})$ , and since, as noted above,  $g$  is increasing in the domain  $1 < \bar{\alpha} \leq 2$ , we get that for  $1 < \bar{\alpha} \leq 1 + e^{-1}$ ,  $f(\bar{\alpha}) > g(\bar{\alpha})$ .

To complete the proof, let us show that the latter inequality is also satisfied for  $1 + e^{-1} < \bar{\alpha} < 2$ . In this subset of the domain of  $f$  and  $g$ ,  $0.5\bar{\alpha} > \bar{\alpha} - 1 > e^{-1}$  or  $\ln(0.5\bar{\alpha}) > \ln(\bar{\alpha} - 1) > \ln e^{-1}$  or  $\ln(0.5\bar{\alpha}) > \ln(\bar{\alpha} - 1) > -1$  or  $1 + \ln(0.5\bar{\alpha}) > 1 + \ln(\bar{\alpha} - 1) > 0$  or  $\frac{1 + \ln(0.5\bar{\alpha})}{1 + \ln(\bar{\alpha} - 1)} > 1$ . Note that when  $\bar{\alpha} = 2$ ,

<sup>14</sup> When  $k=1$ ,  $\bar{\alpha}=2$  and we get that  $\text{Max } G_L^{ps} = G_A = 0.5(n_1 + n_2) = n_1$ .

<sup>15</sup> We are indebted to Arkadi Koziashvili for his help in establishing this part of the proof.

$\frac{1+\ln(0.5\bar{\alpha})}{1+\ln(\bar{\alpha}-1)}=1$ . To sum up, when  $1+e^{-1} < \bar{\alpha} < 2$ , we get that  $\frac{1+\ln(0.5\bar{\alpha})}{1+\ln(\bar{\alpha}-1)} > 1$  and

when  $\bar{\alpha} = 2$ ,  $\frac{1+\ln(0.5\bar{\alpha})}{1+\ln(\bar{\alpha}-1)} = 1$ .

Let  $h(\bar{\alpha}) = f(\bar{\alpha}) - g(\bar{\alpha})$ . This function is continuous and differentiable in the interval  $1+e^{-1} \leq \bar{\alpha} \leq 2$  and  $\frac{dh(\bar{\alpha})}{d\bar{\alpha}} = f'(\bar{\alpha})[1+\ln(\bar{\alpha}-1)] - g'(\bar{\alpha})[1+\ln(0.5\bar{\alpha})]$ . When the

function  $h(\bar{\alpha})$  has an extremum,  $\frac{dh(\bar{\alpha})}{d\bar{\alpha}} = 0$  or

$f'(\bar{\alpha})[1+\ln(\bar{\alpha}-1)] - g'(\bar{\alpha})[1+\ln(0.5\bar{\alpha})] = 0$  or:

$$(3.A) \quad \frac{f'(\bar{\alpha})}{g'(\bar{\alpha})} = \frac{1+\ln(0.5\bar{\alpha})}{1+\ln(\bar{\alpha}-1)}$$

We therefore get that:

1. The latter equality is satisfied for  $\bar{\alpha} = 2$  which means that this value is an extremum value. In addition, for this extremum value,  $f'(\bar{\alpha}) = g'(\bar{\alpha})$  or  $h'(\bar{\alpha}) = 0$ .

2. As noted above, when  $1+e^{-1} < \bar{\alpha} < 2$ , we get that  $\frac{1+\ln(0.5\bar{\alpha})}{1+\ln(\bar{\alpha}-1)} > 1$  and, in

addition, by (3.A),  $\frac{f'(\bar{\alpha})}{g'(\bar{\alpha})} = \frac{1+\ln(0.5\bar{\alpha})}{1+\ln(\bar{\alpha}-1)}$ . Therefore, every extremum value in

this sub-domain,  $1+e^{-1} < \bar{\alpha} < 2$ , satisfies the inequality

$\frac{f'(\bar{\alpha})}{g'(\bar{\alpha})} = \frac{1+\ln(0.5\bar{\alpha})}{1+\ln(\bar{\alpha}-1)} > 1$  or  $\frac{f'(\bar{\alpha})}{g'(\bar{\alpha})} > 1$ . Hence, every extremum value in this

sub-domain satisfies  $f'(\bar{\alpha}) > g'(\bar{\alpha})$  or  $h'(\bar{\alpha}) > 0$ .

3. Also note that  $h'(\bar{\alpha} = 1+e^{-1}) = 0.0975$ .

By 1 - 3, we get that in the interval  $1+e^{-1} \leq \bar{\alpha} \leq 2$ , the point  $(\bar{\alpha}, h(\bar{\alpha})) = (2, 0)$  is an absolute minimum point. In other words, in the interval  $1+e^{-1} < \bar{\alpha} < 2$ ,  $h(\bar{\alpha}) > 0$  or  $f(\bar{\alpha}) > g(\bar{\alpha})$ .

**Q.E.D**

## Appendix B: The optimal logit CSF under discrimination

The designer controls the parameters  $\delta$  and  $\alpha$ . For  $0 < \alpha \leq 2$ ,<sup>16</sup> which ensures the existence of an interior equilibrium, see Konrad (2009), and for  $\delta > 0$ , the expected payoff of the contestants are:

$$(1.B) \quad E(u_1) = \frac{n_1 x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha} - x_1 \text{ and } E(u_2) = \frac{n_2 (\delta x_2)^\alpha}{x_1^\alpha + (\delta x_2)^\alpha} - x_2$$

The first order conditions are:

$$(2.B) \quad \frac{\partial E(u_1)}{\partial x_1} = \frac{\alpha x_1^{\alpha-1} n_1 (\delta x_2)^\alpha}{[x_1^\alpha + (\delta x_2)^\alpha]^2} - 1 = 0 \text{ and } \frac{\partial E(u_2)}{\partial x_2} = \frac{\alpha \delta^\alpha x_2^{\alpha-1} n_2 x_1^\alpha}{[x_1^\alpha + (\delta x_2)^\alpha]^2} - 1 = 0$$

and, after rearranging, we get that in equilibrium:

$$(3.B) \quad x_1^* = \frac{\alpha n_1 (\delta k)^\alpha}{(k^\alpha + \delta^\alpha)^2}, x_2^* = \frac{\alpha n_2 (\delta k)^\alpha}{(k^\alpha + \delta^\alpha)^2} \text{ and } x_1^* + x_2^* = \frac{\alpha (\delta k)^\alpha (n_1 + n_2)}{(k^\alpha + \delta^\alpha)^2}$$

$$(4.B) \quad p_1 = \frac{k^\alpha}{k^\alpha + \delta^\alpha} \text{ and } p_2 = \frac{\delta^\alpha}{k^\alpha + \delta^\alpha}$$

$$(5.B) \quad E(u_1^*) = \frac{n_1 k^\alpha [(1-\alpha)\delta^\alpha + k^\alpha]}{(k^\alpha + \delta^\alpha)^2} \text{ and } E(u_2^*) = \frac{n_2 \delta^\alpha [(1-\alpha)k^\alpha + \delta^\alpha]}{(k^\alpha + \delta^\alpha)^2}$$

The second order equilibrium conditions (SOC) are:

$$\frac{\partial^2 E(u_1)}{\partial x_1^2} = \frac{\alpha n_1 (\delta x_2)^\alpha x_1^{\alpha-2} \{(\alpha-1)(\delta x_2)^\alpha - (\alpha+1)x_1^\alpha\}}{[x_1^\alpha + (\delta x_2)^\alpha]^3} \leq 0$$

$$\frac{\partial^2 E(u_2)}{\partial x_2^2} = \frac{\alpha n_2 \delta^\alpha x_1^\alpha x_2^{\alpha-2} \{(\alpha-1)x_1^\alpha - (\alpha+1)(\delta x_2)^\alpha\}}{[x_1^\alpha + (\delta x_2)^\alpha]^3} \leq 0$$

In equilibrium we obtain that  $\frac{x_1^*}{x_2^*} = k$ , therefore the SOC can be written as:

$$(6.B) \quad (1-\alpha)\delta^\alpha + (\alpha+1)k^\alpha \geq 0 \text{ and } (1-\alpha)k^\alpha + (\alpha+1)\delta^\alpha \geq 0$$

Also, in equilibrium, the expected contestants' payoffs must be non-negative, that is,

$E(u_1^*) \geq 0$  and  $E(u_2^*) \geq 0$ , which requires (see (5.B)) that

$$(7.B) \quad (1-\alpha)\delta^\alpha + k^\alpha \geq 0 \text{ and } (1-\alpha)k^\alpha + \delta^\alpha \geq 0$$

Note that the conditions in (7.B) ensure that the SOC in (6.B) are also satisfied.

<sup>16</sup> Notice that the proof in our case where  $0 < \alpha \leq 2$  is not a straightforward extension of the proof of Proposition 1 in Epstein et al. (2011) where  $0 < \alpha \leq 1$ . When  $0 < \alpha \leq 1$ , (6B) and (7B) are automatically satisfied and, in turn, the second order conditions are satisfied and the utility of every contestant is not negative. However, when  $1 < \alpha \leq 2$ , (6B) and (7B) are not automatically satisfied. Hence, in the current proof of Proposition 2, we have to take into account constraints that were not relevant when  $0 < \alpha \leq 1$ , as pointed out in the sequel, see problem (8B).

Now, anticipating the equilibrium efforts of the contestants, the designer's objective is to maximize  $x_1^* + x_2^* = \frac{\alpha(\delta k)^\alpha (n_1 + n_2)}{(k^\alpha + \delta^\alpha)^2}$ . For  $0 < \alpha \leq 2$ , the designer's problem is:

$$\begin{aligned}
 (8.B) \quad & \text{Max } G_L = \frac{\alpha(\delta k)^\alpha (n_1 + n_2)}{(k^\alpha + \delta^\alpha)^2} \\
 & \text{s.t.} \\
 & (1 - \alpha)\delta^\alpha + k^\alpha \geq 0 \\
 & (1 - \alpha)k^\alpha + \delta^\alpha \geq 0 \\
 & \delta \geq 0
 \end{aligned}$$

The first two constraints ensure that the SOC for the equilibrium strategies of the two contestants are satisfied and that their expected payoffs are non-negative. Let us temporarily ignore these conditions and solve the designer's unconstrained problem in two stages. In the first stage,  $\alpha$  is considered as given and  $\delta$  is computed from the first order condition for the (unconstrained) maximization of  $G_L$ :

$$(9.B) \quad \frac{\partial G_L}{\partial \delta} = \frac{\alpha^2 \delta^{\alpha-1} k^\alpha (n_1 + n_2) (k^\alpha - \delta^\alpha)}{(k^\alpha + \delta^\alpha)^3} = 0$$

which yields

$$(10.B) \quad \delta^*(\alpha) = k$$

Substituting this value into  $\frac{\partial^2 G_L}{\partial \delta^2}$ , we get that the SOC of the designer's problem is satisfied. That is,

$$\frac{\partial^2 G_L}{\partial \delta^2} = -\frac{\alpha^3 (n_1 + n_2)}{32k^{3\alpha+2}} < 0$$

Furthermore, the optimal bias in (10.B) also satisfies all the constraints of problem (8.B).  $\delta^*(\alpha) = k$  is therefore the optimal bias for any  $\alpha$ ,  $0 < \alpha \leq 2$ . Substituting  $\delta^*(\alpha) = k$  in equations (3.B)-(5.B), we get that  $x_1^* = 0.25\alpha n_1$ ,  $x_2^* = 0.25\alpha n_2$ ,  $p_1 = p_2 = 0.5$ ,  $E(u_1^*) = 0.25n_1(2 - \alpha)$ ,  $E(u_2^*) = 0.25n_2(2 - \alpha)$  and

$G_L = 0.25\alpha(n_1 + n_2)$ . This implies that  $\alpha = 2$ <sup>17</sup> is optimal for the designer yielding the maximal certain aggregate efforts of  $G_L = 0.5(n_1 + n_2)$ .

**Q.E.D**

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<sup>17</sup> Notice that Nti (2004) has established that for the existence of a unique pure-strategy equilibrium when  $k > 1$  it must be the case that  $\alpha < 2$ . In our case, we allow discrimination between the contestants. Therefore we obtain that even though  $k > 1$ , in a unique pure-strategy equilibrium  $\alpha = 2$ .

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