

# Cooperation and Effort in Group Contests

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## Abstract

We consider a two group contest over a group specific public good comparing two situations: (i) when all players act independently; and (ii) when the players of each group cooperate. This comparison leads us to the conclusion that it is possible for one group to contribute more (and have a higher expected payoff) in the non-cooperative regime than in the cooperative regime.

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## 1. Introduction

We consider a two-group contest over a group specific public good in which a member of each group can invest efforts so that the group win the contest. Our purpose is to examine the equilibrium efforts invested by individual players in each group. We consider two groups and compare two situations: (i) when all players act independently; and (ii) when each of the group players cooperates. This comparison leads us to the conclusion that, in certain circumstances, players may contribute more in scenario (i) than in scenario (ii).

Economic policy involves a struggle between interest groups: one group that defends the status-quo and another group that challenges it by fighting for an alternative policy. There may be different examples such as taxation, pollution standards, a monopoly facing opposition, capital owners and a workers' union can be engaged in a contest over minimum wages and so on. In Israel there was a public committee headed by Professor Eytan Sheshinski to determine the taxation level on natural resources which had just been found (gas). Different sides tried to affect the outcome. On the one side there was the public and on the other side were the firms leading the extraction of a natural resource which tried to affect the final outcome of the committee. The outcome of the contest depends on the stakes of the contestants, and, in turn, on their exerted efforts. These contests may involve group specific public-goods.

There exists a vast literature dealing with contests with group-specific public-good prizes.<sup>2</sup> In the literature, free-riding is a well known problem in contests and it may overshadow a specific public good. For example, Nitzan (1991) presents a sharing rule to decrease the free-riding problem, while Baik (2008) studies a case of free-riding where only one player invests the effort to win the contest and the other contestants' free-ride.<sup>3</sup> Cheikbossian (2008a) presents a model of endogenous public good provision and group rent-seeking influence. Specifically, two groups with different preferences over public policy and different sizes engage in rent-seeking or lobbying activities to influence policymaking in their preferred direction. When there is within-group cooperation in lobbying, both groups neutralize each other in the

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<sup>2</sup>See for example: Katz, Nitzan and Rosenberg (1990), Ursprung (1990), Riaz, Shogren and Johnson (1995), Konrad (2009, and references within), Baik (2008) and Cheikbossian (2008a).

<sup>3</sup> Baik (2008) considers a model with  $n$  groups competing to win a group-specific public-good prize. The main difference between Baik's paper and ours is that while one can aggregate the total effort invested in the contest, our model can only aggregate effort after using non-linear transformation.

political process. Without within-group cooperation, the free-rider problem in lobbying makes the smaller group politically influential. In both cases, the total level of rent-seeking activities is shown to be increasing in taste heterogeneity while decreasing in group size asymmetry.<sup>4</sup> In a similar type of model Epstein and Mealem (2009) consider a situation in which two groups contest a group-specific public good. They show that the level of free-riding depends on the return on investment and consider the situation in which one group initiates a contest adding different players and/or groups. The question they pose is: what would be the optimal structure of the added groups?

The early literature on coordination of games suggests that coordination failure is common in the laboratory (for example, Cooper et al., 1992). This important finding has been interpreted as relevant for environments ranging from individual organizations to macro-economies, and has led to an active research agenda to investigate possible mechanisms to resolve this coordination failure.

There is a growing literature on experimental economics of group contests with and without cooperation. Riechmann and Weimann (2008) present a means of fostering efficient coordination in minimum effort coordination games and inter-group competition. In a series of laboratory experiments they reveal that the true reason for coordination failure is strategic uncertainty which can be reduced almost completely by introducing an appropriately designed mechanism of (inter-group) competition. In a different experiment Reuben and Tyran (2010) test if cooperation is promoted by rank-order competition between groups in which all groups can be ranked first; i.e., when everyone can be a winner. This type of rank-order competition has the advantage of eliminating the negative externality which a group's performance imposes on other groups. However, its disadvantage is the absence of incentives to out-perform others; therefore, it does not eliminate equilibria where all groups cooperate at an equal but low level. Reuben and Tyran (2010) find that all-can-win competition produces a universal increase in cooperation and benefits a majority of individuals if the incentive to compete is sharp. Costless pre-play communication has been found to effectively facilitate coordination and enhance efficiency in games with Pareto-ranked equilibria. Cason, Sheremeta and Zhang (2010) report an experiment in

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<sup>4</sup> In a similar paper Cheikbossian (2008b) presents a model of endogenous public good provision and group rent-seeking influence. It is shown that when there is group cooperation with lobbying, both groups neutralize each other in the political process. However, without group cooperation, the free-rider problem in lobbying makes the smaller group politically influential.

which two groups compete in a weakest-link contest by expending costly efforts. Allowing intra-group communication leads to more aggressive competition and greater coordination than control treatment without any communication. On the other hand, allowing inter-group communication leads to less destructive competition. As a result, intra-group communication decreases while inter-group communication increases payoffs. This work provides evidence that communication can either reduce or increase efficiency in competitive coordinated games depending on different communication boundaries.

Numerous studies suggest that communication may be a universal means to mitigate collective action problems. Leibbradt and Saaksvouri (2010) challenge this view and show that the communication structure crucially determines whether communication mitigates or intensifies the problem of collective action. They observe the effect of different communication structures on collective action in the context of finitely repeated intergroup conflict and demonstrate that conflict expenditures are significantly higher if communication is restricted to one's own group as compared to a situation without communication. However, expenditures are significantly lower if open communication within one's own group and between rivaling groups is allowed.

In our paper, we consider the generalized logit contest success function. The idea behind this assumption is that one tries to affect the policy outcome at low cost such as writing an e-mail, signing a petition on the internet or sending a text message by phone. This was very common during the sessions of the Sheshinski committee. Many petitions were signed via the internet and many e-mails were sent by different members of each side of the contest. Emails and signing positions are costless. Sending the first e-mail has a stronger effect than sending the second e-mail; signing the first petition has a stronger effect than the second petition etc. Thus these investments have decreasing returns in the contest. Epstein and Mealem (2009) describe this situation in detail and present these types of effort showing them to have a low marginal cost with a decrease returns to scale.

Our main results show that the sufficient condition for one of the groups to “over invest” (invest more than under the situation in which the group cooperates) is that the number of players in this group has to be sufficiently smaller in comparison to the other group. Moreover, in the case where one of the groups invests more effort than the amount invested under cooperation, we would obtain that the expected payoff of this group would be higher than that when there is cooperation.

## 2. The Model

### 2.1. No Cooperation

Consider a contest with two groups competing for a prize as in Epstein and Nitzan (2004) and Epstein and Mealem (2009). Suppose that a status-quo policy is challenged by one interest group and defended by the other. For example, in the contest over monopoly regulations, one firm defends the status-quo, lobbying for the profit-maximizing monopoly price (and against any price regulation) while the consumers challenge the status-quo lobbying preferring a competitive price (a tight price cap).<sup>5</sup>

Assume that in group 1 there are  $N$  players, while in group 2 there are  $M$  players. In group 1, each player has a payoff of  $n$  from winning the contest while in group 2 each player has a payoff  $m$  from winning the contest. Each player from group 1 invests  $x_i$  ( $i = 1, \dots, N$ ) units to change the status-quo to the new policy and each player from group 2 invests  $y_j$  ( $j = 1, \dots, M$ ) units so that the policy will not be changed.

The probability that the new policy will be accepted and the status-quo changed,  $p_x$ , is a function of the resources both groups invest in the contest. It is assumed that the probability is given by the generalized logit contest success function:

$$(1) \quad p_x = \frac{\sum_1^N x_i^\alpha}{\sum_1^N x_i^\alpha + \sum_1^M y_j^\alpha} \quad \text{with } 0 < \alpha < 1$$

We restrict our analysis to the case in which  $0 < \alpha < 1$ .<sup>6</sup> The expected payoff of each player in group 1 will equal:

$$(2) \quad E(U_i) = \frac{\sum_1^N x_i^\alpha}{\sum_1^N x_i^\alpha + \sum_1^M y_j^\alpha} n - x_i \quad \forall i = 1, \dots, N$$

and for each player in group 2:

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<sup>5</sup> See for example Epstein and Nitzan (2003, 2007).

<sup>6</sup> For the other cases where  $\alpha = 1$  see Baik (2008). For  $\alpha > 1$  second order conditions may not hold.

$$(3) \quad E(U_j) = \frac{\sum_1^M y_j^\alpha}{\sum_1^N x_i^\alpha + \sum_1^M y_j^\alpha} m - y_j \quad \forall j = 1, \dots, M.$$

Solving the first order conditions (it can be verified that the second order conditions hold); we obtain that the Nash equilibrium investment of the players of each of the groups equals to:

$$x_i^* = \frac{\alpha n k^\alpha M^{1-\alpha}}{N^\alpha (N^{1-\alpha} k^\alpha + M^{1-\alpha})^2} \quad (i = 1, \dots, N)$$

(4) and

$$y_j^* = \frac{\alpha m k^\alpha N^{1-\alpha}}{M^\alpha (N^{1-\alpha} k^\alpha + M^{1-\alpha})^2} \quad (j = 1, \dots, M),$$

where  $k = \frac{n}{m}$ . The expected payoff becomes:

$$E(U_i^*) = \frac{n k^\alpha [N^{2-\alpha} k^\alpha + M^{1-\alpha} (N - \alpha)]}{N^\alpha (N^{1-\alpha} k^\alpha + M^{1-\alpha})^2}$$

(5) and

$$E(U_j^*) = \frac{m [M^{2-\alpha} + N^{1-\alpha} k^\alpha (M - \alpha)]}{M^\alpha (N^{1-\alpha} k^\alpha + M^{1-\alpha})^2}.$$

## 2.2. Cooperation

Consider the case of cooperation. Under the scenario in which one of the players (the leading player or a central planner) in each group will determine the optimal investments of each player in his group. The objective function for group 1 would be to maximize:

$$(6) \quad \sum E(U_{ic}) = \frac{\sum_1^N x_{ic}^\alpha}{\sum_1^N x_{ic}^\alpha + \sum_1^M y_{jc}^\alpha} Nn - \sum_1^N x_{ic}$$

and in the case of group 2:

$$(7) \quad \sum E(U_{jc}) = \frac{\sum_1^M y_{jc}^\alpha}{\sum_1^N x_{ic}^\alpha + \sum_1^M y_{jc}^\alpha} Mm - \sum_1^M y_{jc}.$$

The Nash equilibrium investments of each player in both groups will equal:

$$(8) \quad x_{ic}^* = \frac{\alpha nk^\alpha NM}{(Nk^\alpha + M)^2} \quad (i = 1, \dots, N) \quad \text{and} \quad y_{jc}^* = \frac{\alpha mk^\alpha NM}{(Nk^\alpha + M)^2} \quad (j = 1, \dots, M)$$

and the equilibrium expected payoffs becomes:

$$(9) \quad E(U_{ic}^*) = \frac{Nnk^\alpha [Nk^\alpha + M(1-\alpha)]}{(Nk^\alpha + M)^2}$$

$$E(U_{jc}^*) = \frac{Mm[M + Nk^\alpha(1-\alpha)]}{(Nk^\alpha + M)^2}.$$

### 2.3. Comparison

Let us now compare the investments in both of the cases and see if it is possible that, under cooperation, the players will invest less effort than without cooperation. The investment under cooperation is lower than with no cooperation,  $x_i^* > x_{ic}^*$ , if:

$$(10) \quad \frac{\alpha nk^\alpha M^{1-\alpha}}{N^\alpha (N^{1-\alpha} k^\alpha + M^{1-\alpha})^2} > \frac{\alpha nk^\alpha NM}{(Nk^\alpha + M)^2}.$$

Writing (10) differently we obtain:

$$(11) \quad \left( \frac{Nk^\alpha + M}{N^{1-\alpha} k^\alpha + M^{1-\alpha}} \right)^2 > N^{1+\alpha} M^\alpha$$

and after some manipulation (see appendix) we obtain:

$$(12) \quad M(1 - N^{0.5\alpha+0.5} M^{-0.5\alpha}) > Nk^\alpha (N^{0.5-0.5\alpha} M^{0.5\alpha} - 1)$$

Inequality (11) may well hold. For example, if  $\alpha = 0.5$ ,  $N = 2$ ,  $M = 32$  and  $k = 4$ , we obtain that inequality (11) becomes  $18 > 16$ . On the other hand,  $y_j^* < y_{jc}^*$ . This

means that for group 1 we obtain higher levels of investment than under no cooperation, and for group 2 we obtain Easy-riding.<sup>7</sup>

**Proposition 1:**

(a) a necessary condition for  $x_i^* > x_{ic}^*$  is  $M > N^{1+\frac{1}{\alpha}}$ .

(b) a sufficient condition for  $x_i^* > x_{ic}^*$  is that  $M$  is sufficiently large

For proof see appendix.

The question that comes up is why for a sufficiently large number of players in group 2,  $M$ , the effort of each player in group 1 under no cooperation is greater than with cooperation? To answer this question let us consider the following two situations:

1. Under cooperation, if group 2 is sufficiently large ( $M$  is sufficiently large), increasing the size of this group also increases the total effort of the group even though the effort of each player has decreased.<sup>8</sup> Thus, the increase in the size of the group overcompensates for the decrease in the investment of each player. This means that the central planner of group 2 takes advantage of the increase in the size of the group and decreases the investment of each of its players. As a result of the increase in the total effort invested by group 2, and the increase in its size, the central planner of group 1 "substantially" decreases the total effort of his group;<sup>9</sup> therefore, the effort of each player in his group decreases.<sup>10</sup>

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<sup>7</sup> Since investments are not zero, we consider this to be easy-riding, see Cornes and Sandler (1984).

<sup>8</sup>  $\frac{\partial \left( \sum_1^M y_{jc}^* \right)}{\partial M} > 0$ .  $\frac{\partial y_{jc}^*}{\partial M} < 0$  if and only if  $k^\alpha N < M$ .

<sup>9</sup>  $\frac{\partial \left( \sum_1^N x_{ic}^* \right)}{\partial M} < 0$  and  $\frac{\partial x_{ic}^*}{\partial M} < 0$  if and only if  $k^\alpha N < M$ .

<sup>10</sup> The "larger" group can take advantage of its position by increasing its investment, and, as a result, the "smaller" group decreases its investment. This result has the same type of flavor as the result presented in Epstein and Nitzan (2006) where increasing both players' stakes may increase the effort invested by the players.



2. In the case of no cooperation, and when the number of players in group 2 increases ( $M$  increases), each player in the group decreases effort (easy riding) - the intensity of this reduction depends on the size of the group. If  $M$  is sufficiently large, then increases in the size of the group will also raise the level of the free riding and thus decrease the total investment made by the group.<sup>11</sup> This means that the affect of the decrease in the effort of each player dominates the increase in the size of the group. The increase in  $M$  results in a decrease in the effort of each player in group 1 and therefore in the total effort of group 1.<sup>12</sup> However, since there is no coordination in group 1, and each player easy-rides, the decrease in the investments of this group (and therefore by each player) will be "moderate" in comparison to the first case because group 2 has decreased its efforts. This is also reflected in a "moderate" decrease in the winning probability.

A lower boundary to the expression  $\left(1 + \frac{1}{\alpha}\right)$  is 2; thus, from Proposition 1 we

may conclude the following Corollaries:

**Corollary 1:** If  $x_i^* > x_{ic}^*$  then  $M > N^2$ .

**Corollary 2:** If  $M \leq N^{1+\frac{1}{\alpha}}$  then  $x_i^* \leq x_{ic}^*$  independent on the values of  $m$  and  $n$ .

**Corollary 3:** If  $x_i^* > x_{ic}^*$  then  $y_j^* \leq y_{jc}^*$ .

Corollary 3 is a direct outcome of Corollary 1. Let us consider the following proof using a contradicting argument: Assume that when  $x_i^* > x_{ic}^*$  it holds that  $y_j^* > y_{jc}^*$ . In order for this to hold true, by Corollary 1, since  $x_i^* > x_{ic}^*$  and  $y_j^* > y_{jc}^*$ , it holds that  $M > N^2$  and  $N > M^2$ , respectively. It is clear that both inequalities cannot be true at

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$$^{11} \frac{\partial y_j^*}{\partial M} < 0. \quad \frac{\partial \left( \sum_{j=1}^M y_j^* \right)}{\partial M} < 0 \text{ if and only if } k^{\frac{\alpha}{1-\alpha}} N < M .$$

$$^{12} \frac{\partial x_i^*}{\partial M} < 0 \text{ and } \frac{\partial \left( \sum_{i=1}^N x_i^* \right)}{\partial M} < 0 \text{ if and only if } k^{\frac{\alpha}{1-\alpha}} N < M .$$

the same time. Thus, if the investment of a group under non-cooperative is higher than under cooperative, then the opposite would hold for the other group.

**Proposition 2:** *If  $x_i^* > x_{ic}^*$  then  $E(U_i^*) > E(U_{ic}^*)$ .*

For proof see appendix.

Proposition 2 states that in the case where group 1 invests more effort than the amount that would have been invested under cooperation, we would obtain that the expected payoff of each player in group 1 would be higher than that of cooperation ( $E(U_i^*) > E(U_{ic}^*)$ ). Let us explain this result. From proposition 1 and corollary 3 we obtain that when we have cooperation, and group 2 is sufficiently large, the central planner of group 2 uses its advantage with regard to the group's size increasing the effort of each player in the group relatively to what they would have invested under no cooperation. Therefore, moving from cooperation to no cooperation, the winning probability of group 2 decreases, and the winning probability of group 1 increases.<sup>13</sup> Indeed, the effort of each player in group 1 has increased (proposition 1); however, the increase in the probability dominates the increase in efforts and thus the expected payoff is also increased.

**To conclude:** Our main results show that for one of the groups, the sufficient condition to invest more under cooperation than under no cooperation is that the number of players in the other group has to be sufficiently large. Moreover, in the case where each player in one of the groups invests more effort than the amount that would have been invested under cooperation, we would obtain that the expected payoff of each player in this group would be higher than in the case of cooperation.

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<sup>13</sup> The winning probability of group 1 increases from moving from cooperation to no cooperation if and only if  $M > N$ .

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## Appendix

### Proof of Proposition 1:

Part (a): From (11) we take the square root from both sides and obtain

$$\frac{Nk^\alpha + M}{N^{1-\alpha}k^\alpha + M^{1-\alpha}} > N^{0.5+0.5\alpha}M^{0.5\alpha}$$

Multiply both sides by  $(N^{1-\alpha}k^\alpha + M^{1-\alpha})$  we obtain

$$Nk^\alpha + M > N^{0.5+0.5\alpha}M^{0.5\alpha}(N^{1-\alpha}k^\alpha + M^{1-\alpha})$$

Thus:

$$Nk^\alpha + M > N^{1.5-0.5\alpha}M^{0.5\alpha}k^\alpha + N^{0.5\alpha+0.5}M^{1-0.5\alpha}$$

Rewriting the inequality

$$M - N^{0.5\alpha+0.5}M^{1-0.5\alpha} > N^{1.5-0.5\alpha}M^{0.5\alpha}k^\alpha - Nk^\alpha$$

Thus

$$M(1 - N^{0.5\alpha+0.5}M^{-0.5\alpha}) > Nk^\alpha(N^{0.5-0.5\alpha}M^{0.5\alpha} - 1)$$

Since  $0 < \alpha < 1$  then  $N^{0.5-0.5\alpha}M^{0.5\alpha} > 1$  therefore the right hand side of the above inequality is positive. A necessary condition for (11) is that the left hand side is also positive; thus,  $1 > N^{0.5\alpha+0.5}M^{-0.5\alpha}$  which is identical to  $M > N^{1+\frac{1}{\alpha}}$ .

Part (b) dividing (11) by  $M^{2\alpha}$  we obtain

$$\frac{1}{M^{2\alpha}} \left( \frac{Nk^\alpha + M}{N^{1-\alpha}k^\alpha + M^{1-\alpha}} \right)^2 > \frac{N^{1+\alpha}}{M^\alpha}$$

Rewriting the inequality we obtain

$$\left[ \frac{Nk^\alpha + M}{M^\alpha(N^{1-\alpha}k^\alpha + M^{1-\alpha})} \right]^2 > \frac{N^{1+\alpha}}{M^\alpha}$$

Thus

$$\left( \frac{Nk^\alpha + M}{M^\alpha N^{1-\alpha}k^\alpha + M} \right)^2 > \frac{N^{1+\alpha}}{M^\alpha}$$

Divide the nominator and dominator in the brackets of the LHS by  $M$ :

$$\left( \frac{\frac{Nk^\alpha}{M} + 1}{\frac{N^{1-\alpha}k^\alpha}{M^{1-\alpha}} + 1} \right)^2 > \frac{N^{1+\alpha}}{M^\alpha}$$

As we can see for  $M \rightarrow \infty$  the LHS converges to 1 and the RHS to 0. ■

**Proof of Proposition 2:**

$E(U_i^*) > E(U_{ic}^*)$  is identical to  $\left(\frac{Nk^\alpha + M}{N^{1-\alpha}k^\alpha + M^{1-\alpha}}\right)^2 > \frac{N^{1+\alpha}[Nk^\alpha + M(1-\alpha)]}{[N^{2-\alpha}k^\alpha + M^{1-\alpha}(N-\alpha)]}$ . Also

$x_i^* > x_{ic}^*$  is identical to inequality (11). We will show that the right hand side of inequality (11) is larger than the right hand side of

$\left(\frac{Nk^\alpha + M}{N^{1-\alpha}k^\alpha + M^{1-\alpha}}\right)^2 > \frac{N^{1+\alpha}[Nk^\alpha + M(1-\alpha)]}{[N^{2-\alpha}k^\alpha + M^{1-\alpha}(N-\alpha)]}$ , and by that we have proven our

proposition.  $N^{\alpha+1}M^\alpha > \frac{N^{1+\alpha}[Nk^\alpha + M(1-\alpha)]}{[N^{2-\alpha}k^\alpha + M^{1-\alpha}(N-\alpha)]}$  is identical to

$Nk^\alpha(N^{1-\alpha}M^\alpha - 1) > M(1-N)$ . The left hand side of this inequality is positive while the right hand side is negative. ■

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