

Which Voting Rules Elicit Informative Voting?

by

RUTH BEN-YASHAR^a

and

IGAL MILCHTAICH^b

Bar-Ilan University

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Abstract

When a group of people with identical preferences but different abilities in identifying the best alternative (e.g., a jury) takes a vote to decide between two alternatives, the question of strategic voting arises. That is, depending on the voting rule used to determine the collective decision, it may or may not be rational for group members to always vote for the alternative their private information indicates is better (i.e., vote informatively). In fact, we show in this paper that, if a qualified majority rule is used, informative voting is rational only if the rule is optimal in the class of all qualified majority rules, in the sense that, when everybody votes informatively, none of the other rules in this class would yield a higher expected utility. However, this necessary condition is not sufficient for informative voting to be rational. Specifically, even if the qualified majority rule used is optimal in the above sense, some of those who are least competent in correctly identifying the better alternative may increase the expected utility by sometimes voting for the alternative they believe to be inferior. A sufficient (but not necessary) condition for informative, non-strategic, voting to be rational is that the voting rule is optimal among the class of all qualified weighted majority rules, i.e., rules assigning (potentially) unequal weights to individuals.

^a E-mail: benyasr@biu.ac.il

^b E-mail: milchti@biu.ac.il Personal homepage: <http://faculty.biu.ac.il/~milchti>

Introduction

In the context of decisions by committees, unanimity of preferences among committee members does not guarantee informative voting. That is, it may sometimes be rational for certain members to vote for the alternative they believe to be inferior.¹ This is because, when a rational committee member decides on his vote, he should presume that his vote is pivotal, or decisive, for only in this case does the vote matter. If the committee's optimal decision given the member's private information is different from the optimal decision conditional on the member's vote being pivotal, it would be irrational for him to vote in an informative manner. The problem, of course, is that non-informative, strategic, voting may be better than informative voting only if practiced by a single member only. If everybody in the committee votes strategically, the decision the committee reaches may be suboptimal given the members' private information. Specifically, this is potentially the case if the aggregation rule used for mapping the members' votes to a collective decision is premised on informative voting. Thus, it is not sufficient for a "good" aggregation rule to be optimal assuming that the committee members' votes truly reflect their beliefs as to the best alternative;² it must also be rational for each member to vote in an informative manner when all the others do so.

In this paper, we limit ourselves to a setting in which a group of people must choose between two alternatives. The desirability of each alternative depends on which of two possible states of the world has obtained. All group members share the same views about the costs and benefits of choosing each alternative in each state. However, because of different private information, members may differ in their assessments as to how likely each state is to have obtained. One example of this is a

¹ The growing literature on strategic voting includes the papers of Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996, 1997, 1998), Ladha et al. (1996), McLennan (1998), Myerson (1998), Wit (1998), Dekel and Piccione (2000), Persico (2000), Duggan and Martinelli (2001), and Li et al. (2001).

² Optimal decision-making under the assumption of informative voting has been studied extensively. Earlier studies of two-alternative models include Nitzan and Paroush (1982, 1985), Grofman et al. (1983), and Shapley and Grofman (1984). Other related papers include Ben-Yashar and Nitzan (1997, 1998), Karotkin and Paroush (1995), Sah and Stiglitz (1988), Sah (1990, 1991), and Klevorick et al. (1984). Extensions to the two-alternative model have been suggested by Ben-Yashar and Paroush (2001), Ben-Yashar et al. (2001), and Ben-Yashar and Kraus (2002).

jury deciding on whether to acquit or convict a defendant. The desirability of each alternative depends on whether the defendant is innocent or guilty. Owing to differences in life experience or competence, jurors may reach different conclusions regarding the defendant's innocence or guilt. Another example is a panel of medical experts deciding whether a patient should undergo a particular risky operation. All the panel members agree on the benefits of a successful operation (e.g., the patient will recover completely), the harmful consequences of a failure (e.g., the patient may die), and the consequences of not performing the operation (e.g., the patient will become seriously handicapped). The experts also share similar moral values and attitudes toward risk, and they all know the patient's medical history and get a chance to examine him. Still, their (possibly subconscious) assessments of the probability of success (i.e., that the state of the world is "the operation will, or would, be successful") may well differ. Therefore, some of them may recommend operating the patient while others may not. For both the jury and the panel of medical experts, differences in private information do not necessarily stem from knowledge of different facts pertaining to the case at hand. Rather, they may reflect differences in competence, or level of expertise, in correctly interpreting publicly available data. The levels of expertise may be state-dependent. For example, a doctor relying on a test that tends to over-diagnose a particular medical condition may have a relatively high rate of success in diagnosing the condition in patients who actually suffer from it, but his success in correctly diagnosing healthy patients will be relatively low. Such an interpretation of private information makes it redundant to consider public information explicitly, since such information may be incorporated into the common prior probabilities of the two states. However, other interpretations of the private information or the common prior probabilities are possible. For example, a state's prior probability may be interpreted as the probability that would be assigned to it by someone who is not familiar with the facts of the particular case at hand, e.g., who only knows the average rate of success in the operation under consideration.

As shown by Austen-Smith and Banks (1996), when all committee members are equally competent in identifying the state of the world, the problem of non-informative voting is tightly linked with non-optimality of the aggregation rule. Specifically, an aggregation rule is optimal, assuming informative voting by all committee members, if and only if, when the rule is used, it is rational for each

member to vote informatively if all the others do. However, in this paper we show that this result cannot be extended to the case of heterogeneous committees, in which members differ in their ability to correctly identify the state.

Austen-Smith and Banks (1996) only consider qualified majority rules, which are anonymous and monotonic aggregation rules. Anonymity means that whether one alternative or the other is chosen only depends on the number of votes for each alternative (abstentions are not allowed). Monotonicity means that a person who did not vote for the alternative chosen could not have changed the collective decision by voting for that alternative. If all committee members are identical, then a qualified majority rule is the natural choice. Indeed, the optimal aggregation rule, under the assumption of informative voting, is a qualified majority rule (Ben-Yashar and Nitzan, 1997). Moreover, a qualified majority rule may still be a plausible choice even if committee members do differ in their innate abilities. This is because differences among the members would only be correctly reflected by an aggregation rule specifically tailored to their case. If, for example, the aggregation rule is determined before the identity of the committee members is determined, then it is likely to be anonymous. Anonymity may also be a necessity stemming from other constraints, for instance, a need to keep the votes confidential. Therefore, this paper considers both qualified majority rules that are optimal among the class of all such rules, and general aggregation rules that are optimal among the class of all aggregation rules. A qualified majority rule that is optimal in the former sense need not be optimal in the latter sense. Indeed, for heterogeneous committees, the optimal aggregation rule need not be anonymous. However, as shown by Ben-Yashar and Nitzan (1997; see also Appendix A below), the optimal aggregation rule is always equivalent to some qualified weighted majority rule. That is, when all the committee members are not equally good at identifying the state of the world, there are weights that can be assigned to them, reflecting the different levels of expertise, and an optimal aggregation rule that specifies the minimum weighted number of votes required for the choice of each alternative. In the special case of homogeneous committees, the weights are equal, and the optimal aggregation rule is, therefore, a qualified majority rule.

The main result of this paper is that, for heterogeneous committees, one, and only one, direction in the result of Austen-Smith and Banks (1996) holds for each class of

aggregation rules. In the class of qualified majority rules, optimality of the aggregation rule is a necessary condition for informative voting to be rational. In the class of all aggregation rules, optimality is a sufficient condition for informative voting to be rational. As the examples in the next section show, in both cases, the other proposition (“necessary” instead of “sufficient,” or vice versa) does not hold. These results immediately imply that of Austen-Smith and Banks (1996). This is because, as mentioned above, for homogeneous committees there is always a qualified majority rule that is optimal among the class of all aggregation rules.

For strategic, non-informative, voting to be a potential problem, some asymmetry between the two states of the world has to exist. The two states may differ in the way they are treated by the aggregation rule (e.g., an asymmetric tie-breaking rule), their prior probabilities, the probability of each committee member being able to correctly identify the state, or the cost of making the wrong collective decision. In this paper, we show that if no such asymmetries exist, informative voting is rational. Thus, under complete symmetry between the two states, informative voting is always rational, and does not depend on the aggregation rule being optimal. The intuition underlying this result is that, under symmetry, the probability of each person being pivotal is the same in both states. Therefore, it is perfectly rational for each committee member to behave as if the collective decision is determined entirely by his vote.

Examples

Example 1. There are two possible states of the world, one of which is slightly more likely than the other. A two-member committee is assigned to identify the state. The cost of misidentification is the same for both states. One member is an expert, who always correctly identifies the state. The other member is slightly less competent: his chance of being right, in both states, is just below unity. Clearly, the optimal qualified majority rule is the one in which the state both members vote for is chosen when the votes are identical, and the more probable state of the world is chosen in the case of a tie. This rule does not elicit informative voting. Specifically, the non-expert can increase the probability of choosing the true state to unity by always voting for the less probable state. This shows that optimality of a qualified majority rule among the class of all such rules does not guarantee that informative voting is rational. (As indicated above, the converse is true: If informative voting is rational, the qualified majority rule used is optimal among the class of all such rules.)

Example 2. In this example, both states have the same prior probability. Misidentifying the state carries the same cost in both states. There are three committee members. One of them is an expert, who always correctly identifies the state. The other two are non-experts, whose probability of being right is greater than one-half but less than unity; this probability is the same for both members and both states. In the class of all anonymous aggregation rules, the optimal rule is the simple majority rule. When this rule is used, informative voting is rational. In particular, if either of the non-experts deviates by voting in a non-informative manner, the probability of choosing the true state decreases. However, if both non-experts deviate, with one of them always voting for one state and the other for the other state, the probability of success increases to unity. This is the same rate of success as with the optimal aggregation rule, which adopts the expert’s opinion. This shows that rationality of informative voting does not guarantee that a qualified majority rule that is optimal among the class of all such rules is also optimal in the class of all aggregation rules. (As indicated above, the converse is true: If the aggregation rule used is optimal in the latter sense, informative voting is rational.)

The model

There are n committee members who must choose between two alternatives, denoted $+1$ and -1 (e.g., whether to acquit or convict a defendant). The outcome of the committee’s decision depends both on the alternative chosen and on the “state of the world,” which is also either $+1$ or -1 (e.g., the defendant is innocent or guilty). Thus, the outcome depends on which of the four state-alternative pairs obtains. Any probability distribution over these four pairs is called a lottery. The n committee members share a common preference relation over lotteries, which satisfies the conditions for the existence of von Neumann-Morgenstern utility. Thus, all members strictly prefer one lottery to the other if and only if the expected utility of the first lottery is higher than the second. In state $+1$, the members’ (common) utility is greater if alternative $+1$ is chosen than if the other alternative is chosen. The difference in utility, c^+ , represents the cost of making the “wrong” decision in state $+1$. Similarly, in state -1 , the utility of choosing -1 is greater from the utility of choosing $+1$ by a positive amount c^- .

The state of the world is determined by a random variable z , which is $+1$ with probability $0 < \mathbf{p}^+ < 1$ and -1 with probability $\mathbf{p}^- = 1 - \mathbf{p}^+$. Individual committee members do not generally know the state of the world. However, in each state, each member i receives a random private signal s_i , which may be $+1$ or -1 . The vector (s_1, s_2, \dots, s_n) of signals is denoted by \mathbf{s} . Conditional on the state, the n signals are independent. The signals represent the members' beliefs as to which of the two alternatives is better. These beliefs may be based on some private information the members have or their individual interpretations of the publicly available data. The probability that member i 's signal coincides with the state is p_i^+ in state $+1$ and p_i^- in state -1 , with $p_i^+, p_i^- > 0$ and $p_i^+ + p_i^- \geq 1$. The latter inequality expresses the assumption that each member's probability of receiving the signal $+1$ is at least as high in state $+1$ as in state -1 , and similarly with $+1$ and -1 interchanged. (For example, the probability that a juror would believe an innocent defendant to be innocent is at least equal to the probability that he would believe a guilty defendant to be innocent.)³

After the signals are received, the committee takes a vote.⁴ Each committee member can only vote $+1$ or -1 ; abstentions are not allowed. A (pure) voting strategy for member i is a rule that determines his vote x_i as a function \mathbf{j}_i of the private signal he received (i.e., $x_i = \mathbf{j}_i(s_i)$). If the signal and the vote are always the same (i.e., $x_i = s_i$), then member i is said to vote informatively. The committee's collective decision is determined by a particular aggregation (or voting) rule that assigns one of the alternatives $+1$ or -1 to each voting vector $\mathbf{x} = (x_1, x_2, \dots, x_n)$. An aggregation rule is anonymous if the collective decision depends only on the number x^+ of members

³ The significance and the justification of the assumption $p_i^+ + p_i^- \geq 1$ are further clarified by the results and discussion in Appendices A and B. Note that, mathematically, this assumption involves minimal loss of generality. Indeed, since $p_i^+ + p_i^- < 1$ would imply $(1 - p_i^+), (1 - p_i^-) > 0$ and $(1 - p_i^+) + (1 - p_i^-) > 1$, it is always possible to make the above assumption hold by reversing the interpretation of i 's signals, if necessary.

⁴ In reality, information aggregation may also involve pre-voting communication among the committee members, in which private information is shared. Since our concern is only with the strategic aspects of voting, we ignore this possibility here. This does not necessarily make our model unrealistic. "Private information" may be equated with intuition, which is based on a person's life experience but may not be easily communicated to others.

whose vote is +1 (or, equivalently, on the number x^- of -1 votes), but not on their identity. A special case of this is a trivial aggregation rule, in which the collective decision is always +1 or always -1 . An aggregation rule is monotonic if, for every pair of voting vectors \mathbf{x} and \mathbf{x}' with $x_i \leq x'_i$ for all i , if the collective decision +1 is assigned to \mathbf{x} , then it is also assigned to \mathbf{x}' . A qualified majority rule is a non-trivial anonymous and monotonic aggregation rule. Such an aggregation rule is completely specified by the quota, which is the integer $1 \leq q \leq n$ such that the collective decision is +1 if and only if $x^+ \geq q$. A weighted majority rule is the simplest kind of non-anonymous monotonic aggregation rule. In such a rule, the collective decision is determined by a particular weighted average of the votes. That is, each member i is assigned a fixed weight $w_i \geq 0$ and the collective decision is +1 if and only if $\sum_i w_i x_i \geq 0$. A qualified weighted majority rule is similar except that the collective decision is +1 if and only if $\sum_i w_i x_i \geq q$, where q is some fixed real number.

Rationality of informative voting and optimality of the aggregation rule

For a given profile of voting strategies, it is possible to compute, for each aggregation rule, the expected utility if this rule is used. An optimal aggregation rule is one that maximizes the expected utility under the assumption that all committee members vote informatively. More generally, an aggregation rule of a particular kind is said to be optimal among the class of all aggregation rules of that kind if, under the assumption of informative voting, none of the other rule in the class would give a higher expected utility. For a given aggregation rule, informative voting is rational if the expected utility when all committee members vote informatively is greater than or equal to the expected utility when all members except one vote informatively, and that member uses some other voting strategy. Equivalently, informative voting is rational if everybody voting informatively is a Nash equilibrium in the game in which the payoff of each committee member is the expected utility.

The following connection between optimality of an aggregation rule under the assumption of informative voting and rationality of informative voting when this rule is used is almost self-evident.

Proposition 1. *When an optimal aggregation rule is used, informative voting is rational.*

Proof. Suppose that the aggregation rule used is optimal. If for some committee member, say the first one, there were a voting strategy \mathbf{j}_1 yielding a higher expected utility than informative voting when all the other members vote informatively, then incorporating that voting strategy into the aggregation rule would give a new aggregation rule that yields a higher expected utility than the original one when all the committee members vote informatively. Specifically, the new aggregation rule is defined by the property that to each voting vector (x_1, x_2, \dots, x_n) it assigns the same collective decision as the original rule assigns to $(\mathbf{j}_1(x_1), x_2, \dots, x_n)$. ■

As already mentioned, Ben-Yashar and Nitzan (1997) showed that every optimal aggregation rule is equivalent to some qualified weighted majority rule in that, to each voting vector, both rules assign the same collective decision. We give our version of the proof of this result in Appendix A. Together with Proposition 1, Ben-Yashar and Nitzan’s result immediately implies the following proposition.

Proposition 2. *Suppose that a qualified weighted majority rule is used. If the rule is optimal among the class of all qualified weighted majority rules, then informative voting is rational.*

As Example 2 above shows, in both Propositions 1 and 2 the converse assertion does not hold. As Example 1 shows, Proposition 2 would not be true if “qualified weighted majority rule(s)” were replaced by “qualified majority rule(s),” i.e., if only rules assigning equal weights to all committee members were considered. On the other hand, for this kind of aggregation rules the converse to the assertion of Proposition 2 does hold. This is shown by the following proposition.

Proposition 3. *Suppose that a qualified majority rule is used. If informative voting is rational, then the rule is optimal among the class of all anonymous aggregation rules.*

Proof. Suppose that informative voting is rational. In particular, each committee member i who receives the signal +1 should vote +1, assuming that everyone else also votes in accordance with his private signals. Equivalently, if i changed his voting strategy by voting -1 whenever his signal is +1, the expected utility would not

increase. This assertion is equivalent to the following inequality:

$$c^+ \mathbf{P}(z = +1, s_i = +1, s^+ = q) \geq c^- \mathbf{P}(z = -1, s_i = +1, s^+ = q), \quad (1)$$

where the additional condition that the number s^+ of committee members whose signal is +1 equals the quota q comes from the fact that only in this case, i.e., when member i is pivotal, does changing his vote matter. The probability on the left-hand side of (1) is equal to $\mathbf{p}^+ \mathbf{P}(s_i = +1, s^+ = q | z = +1)$, i.e., the prior probability that the state of the world is +1 times the conditional probability that i 's signal is +1 and s^+ is equal to q , given that the state is +1. This conditional probability is equal to the sum of the conditional probabilities of all the vectors of signals with these two properties. The right-hand side of (1) can be changed in a similar way. Therefore, (1) is equivalent to

$$c^+ \mathbf{p}^+ \sum_{\substack{\mathbf{x} \\ x_i = +1 \\ x^+ = q}} \mathbf{P}(\mathbf{s} = \mathbf{x} | z = +1) \geq c^- \mathbf{p}^- \sum_{\substack{\mathbf{x} \\ x_i = +1 \\ x^+ = q}} \mathbf{P}(\mathbf{s} = \mathbf{x} | z = -1). \quad (2)$$

Summing over $i = 1, 2, \dots, n$ gives

$$c^+ \mathbf{p}^+ \sum_{\substack{\mathbf{x} \\ x^+ = q}} q \mathbf{P}(\mathbf{s} = \mathbf{x} | z = +1) \geq c^- \mathbf{p}^- \sum_{\substack{\mathbf{x} \\ x^+ = q}} q \mathbf{P}(\mathbf{s} = \mathbf{x} | z = -1), \quad (3)$$

where the factor q comes from the fact that each voting vector \mathbf{x} that appears in the summation does so exactly q times (since this is the number of committee members i with $x_i = +1$). As the q 's in (3) cancel out, this inequality is equivalent to

$$c^+ \mathbf{P}(z = +1, s^+ = q) \geq c^- \mathbf{P}(z = -1, s^+ = q). \quad (4)$$

This proves that changing the aggregation rule by choosing -1 when there are exactly q members whose vote is +1 would not have increased the conditional expected utility. A similar proof shows that the assumption that it is rational for committee members receiving the signal -1 to vote -1 implies that an inequality opposite to that in (4) holds when q is replaced by $q - 1$. Thus, when $q - 1$ members receive the signal -1 , the collective decision -1 is optimal. To complete the proof of this proposition, we need the following general lemma, the proof of which is given in Appendix B.

Lemma. For every \mathbf{a} and \mathbf{b} such that $\mathbf{P}(s^+ = \mathbf{a}) > 0$ and $\mathbf{P}(s^+ = \mathbf{b}) > 0$, if $\mathbf{b} > \mathbf{a}$, then

$$\mathbf{P}(z = +1 \mid s^+ = \mathbf{b}) \geq \mathbf{P}(z = +1 \mid s^+ = \mathbf{a}).$$

According to the lemma, the conditional probability that the state is +1 is a nondecreasing function of the number of committee members receiving the signal +1. Equivalently, the conditional probability that the state is -1 is a nonincreasing function of that number. Now, suppose that $\mathbf{P}(s^+ = q) > 0$. The inequality (4) is then equivalent to $c^+ \mathbf{P}(z = +1 \mid s^+ = q) \geq c^- \mathbf{P}(z = -1 \mid s^+ = q)$. By the lemma, a similar inequality holds when q is replaced by any bigger number \mathbf{b} with $\mathbf{P}(s^+ = \mathbf{b}) > 0$. Therefore, when \mathbf{b} members receive the signal +1, the collective decision +1 is optimal. Similarly, if $\mathbf{P}(s^+ = q - 1) > 0$, then an inequality opposite to that in (4) holds when q is replaced by any smaller number \mathbf{a} with $\mathbf{P}(s^+ = \mathbf{a}) > 0$. This proves that, if $\mathbf{P}(s^+ = q) > 0$ and $\mathbf{P}(s^+ = q - 1) > 0$, then the qualified majority rule used is optimal among the class of all anonymous aggregation rules. A similar conclusion is also true if one or both of these inequalities do not hold. Suppose, for example, that $\mathbf{P}(s^+ = q) = 0$. Then, in both states of the world, s^+ is always different from q . Since, by assumption, $p_i^+, p_i^- > 0$ for all members i , it follows that s^+ is always greater than q in state +1 and less than q in state -1. (If, for example, there were some $\mathbf{a} < q$ such that $\mathbf{P}(s^+ = \mathbf{a} \mid z = +1) > 0$, then a similar inequality would be true also for $\mathbf{a} + 1, \mathbf{a} + 2, \dots, n$, and in particular for q , which contradicts the assumption.) Hence, $s^+ \geq q$ if and only if the state of the world is +1. It follows that the qualified majority rule used is, in fact, optimal among the class of all aggregation rules. Indeed, if all committee members vote informatively, the collective decision always coincides with the true state of the world. ■

Who should vote non-informatively?

The difference between qualified majority rules and qualified weighted majority rules can also be looked at from another point of view. As Example 2 shows, when a non-optimal qualified weighted majority rule is used, informative voting may be rational. Thus, it may not be possible for a single committee member to offset the non-optimality of the aggregation rule by voting in a non-informative manner; it may take more than one member to make a positive change. By contrast, Proposition 3 shows

that, if a qualified majority rule that is not optimal among the class of all such rules is used, then informative voting is irrational. Thus, there is always some committee member who, by voting non-informatively, can at least partially offset the non-optimality of the aggregation rule. This raises the following question: If a qualified majority rule that is not optimal among the class of all such rules is used, who should vote non-informatively? As the next proposition shows, the answer is “one of the members with minimal competence in identifying the state of the world.” This result simplifies the test for rationality of informative voting under a qualified majority rule, since it implies that it suffices to check the voting strategies of members with minimal competence. A committee member i will be said to be more competent than another member j if $p_i^+ \geq p_j^+$ and $p_i^- \geq p_j^-$, and at least one of the inequalities is strict. A committee member with minimal competence is one who is not more competent than any other member.

Proposition 4. *Suppose that a qualified majority rule is used. A necessary and sufficient condition for informative voting to be rational is that none of the committee members with minimal competence can increase the expected utility by adopting a non-informative voting strategy when all the other members vote informatively.*

Proof. We will show that, if it is rational for committee member i to vote +1 whenever his signal is +1, then the same is true for any member j more competent than i . (The proof for -1 signals is similar.) Since the inequalities (1) and (2) are equivalent, it suffices to show that, for every $1 \leq q \leq n$, if (2) holds, then a similar inequality holds with i replaced by j . Consider how this replacement changes the sum on the left-hand side of (2). An equivalent way of implementing this change is replacing the voting vector \mathbf{x} in the summand with the vector $\mathbf{x}^{(ij)}$, in which the i th and j th coordinates are exchanged. The new sum is

$$\sum_{\substack{\mathbf{x} \\ x_i = +1 \\ x^+ = q}} \mathbf{P}(\mathbf{s} = \mathbf{x}^{(ij)} \mid z = +1).$$

For each voting vector \mathbf{x} with $x_i = +1$, either $\mathbf{x}^{(ij)}$ is equal to \mathbf{x} , or it differs from \mathbf{x} only in its i th coordinate, which is -1 rather than $+1$, and in its j th coordinate, which is $+1$ rather than -1 . In the latter case, $p_i^+ (1 - p_j^+) \mathbf{P}(\mathbf{s} = \mathbf{x}^{(ij)} \mid z = +1) = (1 - p_i^+) p_j^+ \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1)$ by the conditional independence of the members' signals. Since $p_j^+ \geq p_i^+ > 0$,

this equality implies $P(\mathbf{s} = \mathbf{x}^{(ij)} | z = +1) \geq P(\mathbf{s} = \mathbf{x} | z = +1)$. (Intuitively, this says that it is more likely that i gets the state wrong and j gets it right than vice versa.) By similar considerations, $P(\mathbf{s} = \mathbf{x} | z = -1) \geq P(\mathbf{s} = \mathbf{x}^{(ij)} | z = -1)$. It follows that replacing i with j in (2) does not decrease the left-hand side and does not increase the right-hand side. Therefore, if this inequality holds for i , it also holds for j . ■

A corollary of Propositions 3 and 4 is that, if a qualified majority rule that is not optimal among the class of all such rules is used, then informative voting is irrational for at least one committee member with minimal competence. This need not be the case for all such members, and not even for all members i for whom the sum $p_i^+ + p_i^-$ is minimal. The following example demonstrates this.

Example 3. The prior probabilities of the two states of the world are given by $\mathbf{p}^+ = 2/3$ and $\mathbf{p}^- = 1/3$. The cost of making the wrong decision is the same in both states (i.e., $c^+ = c^-$). There are two committee members, one with $p_1^+ = 1$ and $p_1^- = 2/3$ and the other with $p_2^+ = 3/4$ and $p_2^- = 1$. When both members receive the same signal, it clearly corresponds to the true state. When the signals are conflicting, i.e., 1 receives the signal +1 and 2 the signal -1, the state +1 is more probable than -1 (because $2/3 p_1^+ (1 - p_2^+) > 1/3 (1 - p_1^-) p_2^-$). Therefore, the optimal aggregation rule is the qualified majority rule with quota $q = 1$. Consider the non-optimal rule obtained by setting $q = 2$. Since neither committee member is more competent than the other, Proposition 4 does not give us any information as to which of them should vote non-informatively. However, since $p_1^+ + p_1^- < p_2^+ + p_2^-$, it would seem logical to look at member 1 first. Whenever that member receives the signal -1, so does member 2. Hence, in this case, changing 1's vote would have no effect on the collective decision. When 1 receives the signal +1, his vote may affect the decision only if 2 also receives the signal +1. However, in this case, the state is +1, and therefore 1 should vote +1. Thus, for committee member 1, informative voting is rational. It follows, by Proposition 3, that informative voting is irrational for member 2. Indeed, it is not difficult to see that the expected utility is increased if that member votes +1 regardless of his signal, while member 1 continues to vote informatively.

Symmetry

As already mentioned, rationality of informative voting does not imply that the aggregation rule used is optimal. The counterexample given above (Example 2) involves a simple majority rules. This counterexample is, in fact, just one of an array of possible examples. Indeed, as the following proposition shows, informative, non-strategic, voting is rational in any setting in which both “nature” and the aggregation rule treat the two states of the world symmetrically.

Proposition 5. *Suppose that the two states of the world are similar in terms of their prior probabilities, the cost of making the wrong collective decision, and the probability that the private signal each committee member receives coincides with the state. Then, informative voting is rational under any aggregation rule (anonymous or not, monotonic or not) that is neutral in the sense that, if all committee members reverse their votes, the committee’s collective decision is reversed as well.*

Proof. By assumption, $\mathbf{p}^+ = \mathbf{p}^-$, $c^+ = c^-$, and $p_i^+ = p_i^- (\geq 1/2)$ for all members i . Suppose that a neutral aggregation rule is used. The key to proving that, under these assumptions, informative voting is rational is showing that, for each committee member i , the probability that i is pivotal is the same in both states of the world. Hence, when considering how to vote, each member may simply ignore the others.

For a given voting vector \mathbf{x} , committee member i is pivotal if and only if the collective decision reached when the votes are given by \mathbf{x} is different from the decision reached when the votes are given by the voting vector $\mathbf{x}^{(i)}$ defined by $x_i^{(i)} = -x_i$ and $x_j^{(i)} = x_j$ for all $j \neq i$. By the assumed neutrality of the aggregation rule, the former collective decision is different from the decision when the votes are given by $-\mathbf{x}$ (the components of which are the negative of the respective components of \mathbf{x}) and the latter is different from the decision when the votes are given by $-\mathbf{x}^{(i)}$. It follows, since $-\mathbf{x}^{(i)} = (-\mathbf{x})^{(i)}$, that i is pivotal in \mathbf{x} if and only if he is pivotal in $-\mathbf{x}$. This proves the left equality in

$$\sum_{\substack{\mathbf{x} \\ i \text{ is pivotal in } \mathbf{x}}} \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) = \sum_{\substack{\mathbf{x} \\ i \text{ is pivotal in } \mathbf{x}}} \mathbf{P}(\mathbf{s} = -\mathbf{x} \mid z = +1) = \sum_{\substack{\mathbf{x} \\ i \text{ is pivotal in } \mathbf{x}}} \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = -1), \quad (5)$$

and the right equality follows from the assumption that $p_j^+ = p_j^-$ for all j . The leftmost sum in (5) is equal to $\mathbf{P}(i \text{ is pivotal} \mid z = +1)$, i.e., the probability that member i is

pivotal in the state +1. The rightmost sum equals $P(i \text{ is pivotal} \mid z = -1)$. The equality between these two probabilities, together with the above assumptions, implies that

$$c^+ \mathbf{p}^+ p_i^+ P(i \text{ is pivotal} \mid z = +1) \geq c^- \mathbf{p}^- (1 - p_i^-) P(i \text{ is pivotal} \mid z = -1).$$

Whether i is pivotal only depends on the votes of the other committee members. Therefore, conditional on the state, it is independent of the signal i himself receives. The above inequality is therefore equivalent to

$$c^+ \mathbf{p}^+ P(s_i = +1, i \text{ is pivotal} \mid z = +1) \geq c^- \mathbf{p}^- P(s_i = +1, i \text{ is pivotal} \mid z = -1),$$

and hence to

$$c^+ P(z = +1, s_i = +1, i \text{ is pivotal}) \geq c^- P(z = -1, s_i = +1, i \text{ is pivotal}),$$

which is the condition for +1 to be an optimal vote for i when his signal is +1 and all the other members vote informatively (cf. (1)). A very similar argument shows that it is optimal for i to vote -1 when his signal is -1. Thus, it is rational for i to vote informatively. ■

How can Proposition 5 be reconciled with Proposition 3? The latter proposition asserts that, when a qualified majority rule is used, informative voting is rational only if the rule is optimal in the class of all anonymous decision rules. Hence, it follows from the two propositions that, under complete symmetry between the two states of the world, any neutral qualified majority rule is optimal among the above class. In fact, the only neutral qualified majority rule (which exists only if the number of committee members is odd) is the simple majority rule. Therefore, as a corollary of Propositions 3 and 5, we get the result that, in case of symmetry between the two states, the optimal anonymous decision rule is the simple majority rule. (This result is also true if the number of committee members is even. In case of a tie in votes, both collective decisions yield the same expected payoff.)

Appendix A

In this appendix, we prove the result that an optimal aggregation rule can always be expressed as a qualified weighted majority rule. We start by assuming that $0 < p_i^+, p_i^- < 1$ for all i .

Suppose that all committee members vote informatively. For every voting vector \mathbf{x} that occurs with positive probability, the conditional expected utility of choosing +1, given that the committee members' signals are given by \mathbf{x} , is less than the utility of choosing -1 if and only if

$$c^+ \mathbf{P}(z = +1 \mid \mathbf{s} = \mathbf{x}) < c^- \mathbf{P}(z = -1 \mid \mathbf{s} = \mathbf{x}).$$

By Bayes' rule, this inequality is equivalent to

$$c^+ \mathbf{p}^+ \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) < c^- \mathbf{p}^- \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = -1). \quad (6)$$

The conditional probability on the left-hand side of (6) equals the product

$$\prod_{\substack{i \\ x_i = +1}} p_i^+ \prod_{\substack{i \\ x_i = -1}} (1 - p_i^+). \quad (7)$$

The logarithm of this product can be written as

$$\sum_i \log \sqrt{p_i^+ (1 - p_i^+)} + \sum_i x_i \log \sqrt{p_i^+ / (1 - p_i^+)}.$$

The logarithm of the conditional probability on the right-hand side of (6) is given by a similar expression, in which p_i^+ is replaced by p_i^- and x_i is replaced by $-x_i$. Therefore, taking the logarithm of both sides of (6) and multiplying by 2 gives the equivalent inequality

$$\sum_i w_i x_i < q,$$

where

$$w_i = \log \frac{p_i^+ p_i^-}{(1 - p_i^+)(1 - p_i^-)} \quad \text{and} \quad q = 2 \log \frac{p_i^- c^-}{p_i^+ c^+} + \sum_i \log \frac{p_i^- (1 - p_i^-)}{p_i^+ (1 - p_i^+)}.$$

This explicitly gives a qualified weighted majority rule that, for each voting vector \mathbf{x} , chooses +1 if and only if choosing +1 maximizes the conditional expected utility, given that the vector of signals is \mathbf{x} . Our assumption that $p_i^+ + p_i^- \geq 1$ implies that the weight w_i assigned to committee member i is nonnegative. The weight is zero if and only if $p_i^+ + p_i^- = 1$.

It remains to remove the restriction that $p_i^+, p_i^- < 1$ for all committee members i . In order to do this, we approximate each pair (p_i^+, p_i^-) by a pair $(\hat{p}_i^+, \hat{p}_i^-)$ that satisfies

$0 < \hat{p}_i^+, \hat{p}_i^- < 1$ and $\hat{p}_i^+ + \hat{p}_i^- \geq 1$. (Since $p_i^+ + p_i^- \geq 1$, this can always be done.) The continuity of (7) in each of the p_i^+ 's, and the continuity of the analog expression with the p_i^- 's, guarantee that these approximations can be chosen in such a way that the corresponding random vector of signals $\hat{\mathbf{s}}$ satisfies the following condition: For each voting vector \mathbf{x} that satisfies (6),

$$c^+ \mathbf{p}^+ \mathbf{P}(\hat{\mathbf{s}} = \mathbf{x} \mid z = +1) < c^- \mathbf{p}^- \mathbf{P}(\hat{\mathbf{s}} = \mathbf{x} \mid z = -1), \quad (8)$$

and similarly with the inequalities in (6) and (8) both reversed. As shown above, for the approximate probabilities there is some qualified weighted majority rule that is optimal. The same rule is also optimal for the original probabilities. This is because, if \mathbf{x} is such that -1 is a (strictly) better collective decision than $+1$ when the vector of signals is \mathbf{x} , then, by virtue of (8), the aggregation rule assigns the collective decision -1 to \mathbf{x} . Similarly, this rule chooses $+1$ whenever $+1$ is a better collective decision than -1 .

Appendix B

In this appendix, we discuss our assumption that, for all committee members, the probability of receiving the signal $+1$ in state $+1$ is at least equal to that in state -1 , and similarly with $+1$ and -1 interchanged. We also derive one important implication of this assumption.

Mathematically, this assumption is equivalent to $p_i^+ + p_i^- \geq 1$, for all members i . We claim that this represents a minimal rationality assumption. Indeed, if $p_i^+ + p_i^-$ were less than unity, then the following would be true: Conditional on member i believing that $+1$ is a better alternative than -1 , the expected utility if $+1$ is chosen is, in fact, less than the expected utility if -1 is chosen; or a similar statement is true with $+1$ and -1 interchanged. (If $c^+ \mathbf{p}^+ \leq c^- \mathbf{p}^-$, the first statement would be true; if $c^+ \mathbf{p}^+ \geq c^- \mathbf{p}^-$, the second would be true.) The assumption $p_i^+ + p_i^- \geq 1$ guarantees that at least one of these statements is false. (If $c^+ \mathbf{p}^+ = c^- \mathbf{p}^-$, then it guarantees that both statements are false, since in this case the two statements are equivalent.) Note that either statement implies that, from a person's own private information, it is sometimes possible to conclude that his belief is wrong as to which of the two alternatives is better. Therefore, it would be quite reasonable to require that, not only one but both

statements are false (also in the case in which $c^+ \mathbf{p}^+ \neq c^- \mathbf{p}^-$). However, this stronger rationality requirement is not required for our results.

In the proof of Proposition 3, we used a lemma saying that, the greater the number of committee members receiving the signal +1, the greater the probability that the state is +1. We now prove this result, and show how it follows from our assumption that, for all committee members i , $p_i^+ + p_i^- \geq 1$. (Without this assumption, the lemma would not be true.)

Lemma. *For every \mathbf{a} and \mathbf{b} such that $\mathbf{P}(s^+ = \mathbf{a}) > 0$ and $\mathbf{P}(s^+ = \mathbf{b}) > 0$, if $\mathbf{b} > \mathbf{a}$, then*

$$\mathbf{P}(z = +1 \mid s^+ = \mathbf{b}) \geq \mathbf{P}(z = +1 \mid s^+ = \mathbf{a}). \quad (9)$$

Proof. By Bayes' theorem, for every \mathbf{a} such that $\mathbf{P}(s^+ = \mathbf{a}) > 0$,

$$\mathbf{P}(z = +1 \mid s^+ = \mathbf{a}) = \frac{\mathbf{p}^+ \mathbf{P}(s^+ = \mathbf{a} \mid z = +1)}{\mathbf{p}^+ \mathbf{P}(s^+ = \mathbf{a} \mid z = +1) + \mathbf{p}^- \mathbf{P}(s^+ = \mathbf{a} \mid z = -1)}. \quad (10)$$

The right-hand side of this equation depends continuously on the various p_i^+ 's and p_i^- 's. Therefore, it suffices to prove the lemma under the additional assumption that $0 < p_i^+, p_i^- < 1$ for all i . Under this additional assumption, $\mathbf{P}(s^+ = \mathbf{a}) > 0$ for all $0 \leq \mathbf{a} \leq n$, and it therefore suffices to prove (9) in the special case $\mathbf{b} = \mathbf{a} + 1$.

Since $\mathbf{p}^+, \mathbf{p}^- > 0$, it follows from (10) that the difference $\mathbf{P}(z = +1 \mid s^+ = \mathbf{a} + 1) - \mathbf{P}(z = +1 \mid s^+ = \mathbf{a})$ has the same sign (positive, negative, or zero) as

$$\mathbf{P}(s^+ = \mathbf{a} + 1 \mid z = +1) \mathbf{P}(s^+ = \mathbf{a} \mid z = -1) - \mathbf{P}(s^+ = \mathbf{a} \mid z = +1) \mathbf{P}(s^+ = \mathbf{a} + 1 \mid z = -1). \quad (11)$$

The first of the two terms in (11) can be written as a double sum:

$$\mathbf{P}(s^+ = \mathbf{a} + 1 \mid z = +1) \mathbf{P}(s^+ = \mathbf{a} \mid z = -1) = \sum_{\substack{\mathbf{x}, \mathbf{y} \\ x^+ = \mathbf{a} + 1 \\ y^+ = \mathbf{a}}} \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1). \quad (12)$$

For every voting vector \mathbf{x} and every committee member i , let $\mathbf{x}^{(i)}$ be the voting vector defined by $x_i^{(i)} = -x_i$ and $x_j^{(i)} = x_j$ for all $j \neq i$. For every pair of voting vectors \mathbf{x} and \mathbf{y} , let $n_{\mathbf{x}\mathbf{y}}$ be the number of committee members i with $x_i = +1$ and $y_i = -1$. For each of these members i ,

$$\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1) \geq \mathbf{P}(\mathbf{s} = \mathbf{x}^{(i)} \mid z = +1) \mathbf{P}(\mathbf{s} = \mathbf{y}^{(i)} \mid z = -1). \quad (13)$$

The reason for this is that, since the only difference between \mathbf{x} and $\mathbf{x}^{(i)}$ is that in the

former i 's signal is $+1$ and in the latter -1 , the only difference between $\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1)$ and $\mathbf{P}(\mathbf{s} = \mathbf{x}^{(i)} \mid z = +1)$ is that the term p_i^+ , which appears in the former when it is explicitly expressed as the product of probabilities (i.e., in the form (7)), is replaced in the latter by $1 - p_i^+$. Similarly, to get $\mathbf{P}(\mathbf{s} = \mathbf{y}^{(i)} \mid z = -1)$ from $\mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1)$, the term p_i^- has to be replaced by $1 - p_i^-$. Thus, if both sides of (13) are explicitly expressed as the products of probabilities, the only difference between them is that the product $p_i^+ p_i^-$, which appears on the left, is replaced on the right by $(1 - p_i^+)(1 - p_i^-)$. Since $p_i^+ p_i^- \geq p_i^+ p_i^- - (p_i^+ + p_i^- - 1) = (1 - p_i^+)(1 - p_i^-)$, the inequality (13) holds. Summing over all the committee members i with $x_i = +1$ and $y_i = -1$ and dividing by their number $n_{\mathbf{x}\mathbf{y}}$ (assuming this number is not zero) gives

$$\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1) \geq \frac{1}{n_{\mathbf{x}\mathbf{y}}} \sum_{\substack{i \\ x_i = +1 \\ y_i = -1}} \mathbf{P}(\mathbf{s} = \mathbf{x}^{(i)} \mid z = +1) \mathbf{P}(\mathbf{s} = \mathbf{y}^{(i)} \mid z = -1).$$

Now, if $x^+ = \mathbf{a} + 1$ and $y^+ = \mathbf{a}$, then, clearly, $n_{\mathbf{x}\mathbf{y}} \geq 1$. In addition, for every i with $x_i = +1$ and $y_i = -1$, $n_{\mathbf{x}\mathbf{y}} = n_{\mathbf{y}\mathbf{x}} + 1 = n_{\mathbf{y}^{(i)}\mathbf{x}^{(i)}}$, where the first equality follows from the fact that $x^+ = y^+ + 1$. Therefore, by (12),

$$\begin{aligned} \mathbf{P}(s^+ = \mathbf{a} + 1 \mid z = +1) \mathbf{P}(s^+ = \mathbf{a} \mid z = -1) &\geq \sum_{\substack{\mathbf{x}, \mathbf{y} \\ x^+ = \mathbf{a} + 1 \\ y^+ = \mathbf{a}}} \sum_i \frac{1}{n_{\mathbf{y}^{(i)}\mathbf{x}^{(i)}}} \mathbf{P}(\mathbf{s} = \mathbf{x}^{(i)} \mid z = +1) \mathbf{P}(\mathbf{s} = \mathbf{y}^{(i)} \mid z = -1) \\ &= \sum_i \sum_{\substack{\mathbf{x}, \mathbf{y} \\ x^+ = \mathbf{a} + 1, x_i = +1 \\ y^+ = \mathbf{a}, y_i = -1}} \frac{1}{n_{\mathbf{y}^{(i)}\mathbf{x}^{(i)}}} \mathbf{P}(\mathbf{s} = \mathbf{x}^{(i)} \mid z = +1) \mathbf{P}(\mathbf{s} = \mathbf{y}^{(i)} \mid z = -1) \quad (14) \\ &= \sum_{\substack{\mathbf{x}, \mathbf{y} \\ x^+ = \mathbf{a} \\ y^+ = \mathbf{a} + 1}} \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1), \end{aligned}$$

where the last equality follows from the observation that, in each term in the preceding triple sum, the pair $(\mathbf{x}^{(i)}, \mathbf{y}^{(i)})$ satisfies $(x^{(i)})^+ = \mathbf{a}$ and $(y^{(i)})^+ = \mathbf{a} + 1$, and each such pair appears in this sum exactly $n_{\mathbf{y}^{(i)}\mathbf{x}^{(i)}}$ times. By an equation similar to (12), the last sum in (14) is equal to $\mathbf{P}(s^+ = \mathbf{a} \mid z = +1) \mathbf{P}(s^+ = \mathbf{a} + 1 \mid z = -1)$. Hence, (14) shows that the expression in (11) is nonnegative, which proves that $\mathbf{P}(z = +1 \mid s^+ = \mathbf{a} + 1) - \mathbf{P}(z = +1 \mid s^+ = \mathbf{a}) \geq 0$, as had to be shown. ■

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