The Complexity of Choice and Loss Aversion*

Kfir Tshuva

Abstract We show how the complexity of consumption choices may induce loss aversion, and that the induced loss aversion is non-monotonic relative to the size of the loss. To do so, we present a simple model and quantify it by introducing a parsimonious three-parameter utility function for a satisficing consumer with computation costs. We then test our model on existing experimental results. Our model fits well the experimental results, and in some relevant domains, it has better predictive power than prospect theory. Next, we show that lotteries with small (resp., large) losses induce risk aversion (resp., risk loving), and that introducing categorization and mental accounting can induce stronger loss aversion with respect to small losses. Finally, we extend our model to a utility-maximizing consumer and study the induced loss aversion and status quo bias in this setup.

Keywords: loss aversion, consumer choice, status quo bias, risk aversion, mental accounting

JEL Classification: D11, D80, D81

1. **Introduction**

An important phenomenon in the field of decision making under uncertainty is loss aversion. Loss aversion refers to the tendency of people to prefer avoiding losses to acquiring gains; for example, people prefer to avoiding losing $1000 to gaining $1000 (see, e.g., Kahneman & Tversky 1979, 1992). In this paper *we offer a novel rational explanation for loss aversion due to complexity costs, and our results suggest that the induced loss aversion is non-monotonic, and that it can be calibrated to well fit experimental results*.

Following Simon (1957) we present a model where the consumer's goal is to find a feasible bundle of indivisible goods that induces a payoff above a certain threshold (satisficing). We show that this satisficing consumer problem is computationally complex (formally, it is NP-complete, as defined in Section 5.3), therefore, solving the problem entails a significant computation cost.

*Bar Ilan University, Ramat Gan, Israel, kfir.tshuva@biu.ac.il. This work is in partial fulfillment of the requirements for the Ph.D. in Economics at Bar-Ilan University, Israel. I thank Antonio Penta, Yuval Salant, and audiences at seminars and conference in BRIC 2019 (Bounded Rationality in Choice conference), Ariel University and Bar-Ilan University for valuable discussions and suggestions. I would like to express my deep gratitude to Prof. Yuval Heller, Dr. Gabriela Gayer and Prof. Eyal Baharad for their careful supervision, and their continuous help.

We show that this cost induces loss aversion. In the case of a loss, in which the new budget does not allow the consumption of the original bundle, the consumer must search for a new feasible bundle that solves the satisficing problem and may pay high computation cost. By contrast, when there is a gain, the computation is not needed.[1]

We present a simple quantitative model of a satisficing customer with a computation cost with 3 parameters and test it on the experimental data presented in Tversky & Kahneman (1992). Our model assumes that the probability of having to do a new costly computation is increasing in the loss, and that this probability is distributed according to a log-normal distribution with mean μ and standard deviation σ. Lastly, it assumes the cost *C* induced by each new computation. The analysis of the experimental results suggests that our model has empirical validity and that for relevant domains (namely lotteries involving both gains and losses), it has better predictive power than prospect theory.

Next, we present an extension in which the consumer divides the goods into several categories and assigns a separate budget to each category (mental accounting à la Thaler, 1985). We show that under such categorization small losses can induce substantial loss aversion. We study the classic consumer problem of utility maximization and show its implications on loss aversion. Next we present an extended model for utility-maximizing consumer with a reduced computation cost for gain and show that this model has good predictive power relative to the prospect theory model for the entire domain of lotteries.

The paper is organized as follows. Section 2 presents the related literature. Section 3 presents a qualitative analysis of a satisficing consumer with computation cost that induces non-monotonic loss aversion. Section 4 presents a simple quantitative model for determining the utility of a lottery, which is calibrated and tested against existing experimental results. In Section 5 we study the complexity of the consumer choice in a satisficing model and we also analyze complexity with categories of consumption. Section 6 presents an extended model for utility-maximizing consumer with a reduced computation cost for gain. Section 7 concludes.

2 Related Literature

2.1 Loss Aversion

An important phenomenon in the field of decision making under uncertainty is loss aversion (Kahneman & Tversky 1979, 1992). Loss aversion refers to the tendency of people to prefer avoiding losses to acquiring gains; for example, people prefer to avoid losing $1,000 to gaining $1,000. Experimental evidence suggests that, on average, individuals are

[1] We assume that a higher budget raises the required level of satisfaction gradually over time, which implies that the cost of computation induced by a gain is substantially smaller than that induced by a loss (and we neglect this former cost in the baseline model). In section 6 we present an extended model for a utility-maximizing consumer that also pays computation cost when there is a gain.

willing to participate in a 50%–50% lottery only if the possible profit is at least twice as high as than the possible loss (Kahneman & Tversky 1979, 1992). For a textbook introduction and a survey of empirical and experimental evidence see Wakker (2010, Chapters 8–9).

Harnik et al.'s (2007) experimental findings suggest that loss aversion is weaker when a small loss is at stake than when a large loss is at stake. A similar result is presented in Yechiam (forthcoming) who reviews early studies of the utility function of gains and losses, and presents evidence suggesting that these studies show loss aversion mainly in large or medium losses (but not in small losses).

The existing explanations for loss aversion are mainly psychological (see, e.g., Taherdoost & Montazeri,2015). Wang & Johnson (2012) suggest that an individual cannot afford a loss that will make his budget smaller than a minimal required level. Hintze et al. (2015) present an evolutionary explanation for loss aversion that is induced by natural selection as an adaptation to prehistoric life conditions in small bands who face rare, high-risk, high-payoff events (see also the evolutionary explanation of the related bias known as endowment effect in Frenkel et al., 2018). Herold & Netzer (2015) claim that the S-shape of the value function is compensated by an inverted S-shape of the probability weighting.

The present chapter presents a novel rational explanation for loss aversion and shows that the induced loss aversion is non-monotonic relative to the size of the loss.

2.2 Satisficing and Cognitive Limitations

In a seminal paper, Simon (1957) argues that the human brain is limited in its computational ability, and that each computation requires energy and time that are costly to the individual. The limited cognitive capacity of individuals limits the identification of the best alternative within a reasonable time frame (see, e.g., Kahneman & Tversky, 1974, Thaler, 1999, and Birnbaum, 2008, for various empirical evidence for violations of the predictions of the classic consumer choice models).

Simon (1957) suggests that the consumer searches for a satisficing bundle of consumption, instead of maximizing a utility function. Specifically, he assumes that the alternatives are ordered, and the consumer considers each alternative sequentially, and stops when an alternative attains a specific utility threshold (or higher), which is defined as the satisfaction level. This decision process is potentially easier and less time consuming than the process of finding the best alternative. Tyson (2007) and Rubinstein & Salant (2012) discuss the axiomatic foundation and the properties of satisficing and the differences between satisfaction and behavior induced by maximizing utility. Alaoui & Penta (2018) provide a foundation for a cost-benefit analysis involving a tradeoff between cognitive costs and the value of thinking about the decision problem. Ortoleva (2013) present thinking aversion that add a disutility to the individual from the cost of thinking.

2.3 Complexity Levels and NP-complete Problems

The consumer problem is a "hard" problem, according to the definitions of complexity in Computer Science. Complexity is defined as the runtime function on the input length of the problem. For example, a runtime equivalent to multiplying the input by a fixed number is defined as linear complexity. A runtime that is a polynomial function of input length (e.g., a square of the input length) is defined as polynomial complexity.

When the complexity of a decision problem is polynomial, it is not considered a difficult problem and it can be solved (possibly, with the help of computers) relatively quickly. By contrast, a decision problem that cannot be solved in polynomial time (henceforth, an *superpolynomial problem*) is considered to be a very hard problem (see Michael & Johnson 1978, p. 6–11), as the running time quickly increases to very high values.

We note that interpretation complexity level is relevant for a sufficiently large input length. In the analysis, we will assume that the number of goods is sufficiently large, such that showing that the problem has of the superpolynomial complexity implies that the problem has a very long runtime, and hence it is difficult to solve.[2]

In this paper we prove the difficulty of the consumer problem by showing that it is an NP-hard problem, which informally states that the consumer problem is at-least as hard as the hardest problem in, [3] NP. The set NP (non-deterministic Polynomial time) is a complexity class that includes problems for which if one guesses a potential solution to the problem, then one can verify in polynomial time whether it is a valid solution or not. The set NP includes various problems that are widely believed to be hard, i.e., to be superpolynomial. Thus, showing that the consumer problem is NP-hard strongly suggests that it is a hard, above polynomial problem.[4]

3. Satisficing and Loss Aversion

Loss aversion is typically considered an irrational behavior (see Taherdoost & Montazeri, 2015). By contrast, we offer a rational explanation for loss aversion, which suggests that it may be a result of the complexity of the satisficing problem. The satisficing consumer problem with indivisible goods is NP complete problem (which is commonly believed to be a hard problem see Garey & Johnson, 1979) and entails a significant computation cost. We show that this cost can induce loss aversion. We start with a satisficing consumer and in

[2] For example, consider a problem with exponential complexity in which the runtime of the solution is an exponential function of the input (e.g., 2^k, where k represents the length of the input). In this case, a very short input of 2 (k = 2) results in an easy problem, yet for a sufficiently large input, such as an input of 80 (k = 80) the runtime is very long (greater than 10^{24}; assuming that a computer performs 10^{16} actions per second, the actual time will take about 3.17 years).

[3]Examples of applications of complexity analysis in economic theory models can be found in Baron et al, (2004), ,Sung & Dimitrov (2010) and Börgers& Morales (2004).

[4] It is widely believed that all NP hard problems cannot be solved in a polynomial time, though this commonly accepted conjecture has not been proven.

Section 6 we present a utility-maximizing consumer problem. We postpone the formal discussion of the complexity of the satisficing consumer problem to Section 5.

Satisficing consumer problem:

We model a consumer who has to choose a subset of goods from a large set of indivisible goods. Following Simon (1957), we assume that the consumer's goal is to reach a payoff that exceeds a certain utility level (the satisficing level). We refer to this model as the *satisficing consumer problem*.

Loss aversion:

We assume that the consumer consumes a bundle that is within his budget set and he is offered a lottery in which he may gain or lose. When the consumer incurs a loss, if the loss is more than a spare money[5] that he may have it makes the previous solution to the satisficing problem infeasible, and, as a result, the consumer must pay a computation cost in order to find a "cheaper" solution that achieves his satisficing level. By contrast, when the consumer obtains a gain, he is **not forced** to consume a new bundle immediately because the original bundle is satisfactory.[6] Thus, loss aversion may be the result of the high computation cost that is induced by a loss.

Non-monotonic loss aversion:

We further argue that loss aversion is non-monotonic with respect to the size of the loss: small for both small and large losses and it would be large for intermediate losses. The reason for this is that a sufficiently small loss is likely to allow the consumer to follow the existing (pre-loss) solution to the consumer satisficing problem, and thus it will have a low probability to induce a computation cost. By contrast, a sufficiently large loss is likely to make the existing (pre-loss) solution infeasible, due to the new budget constraint induced by the large loss, and thus it would induce a high computation cost. This high computation cost of the new satisficing problem is independent of the size of the loss, and thus, when the loss is very large, the relative "per-dollar" indirect impact of the additional computation cost would be small relative to the direct cost of the loss. Let the "per-dollar" loss aversion (PDL) be defined for x<0 as $PDL = \frac{|u(x)|-u(|x|)}{|x|}$

[5] We assume below that the expected computation cost is a continuous (monotonic) function of the size of the loss. The continuity assumption might reflect a situation in which the modeler cannot observe the exact amount of spare money the agent have, and thus a slightly larger loss slightly increases the probability of the loss being larger than the unknown amount of spare money (even if we see money that is not use now we don't know if the money intended for future planned consumption or it's excess money for the agent).

[6] We assume that a higher budget will raise the required level of satisfaction over time and as a function of the new budget. Due to the assumption of time and adjustment, we assume that the cost of computation is negligible in this case.

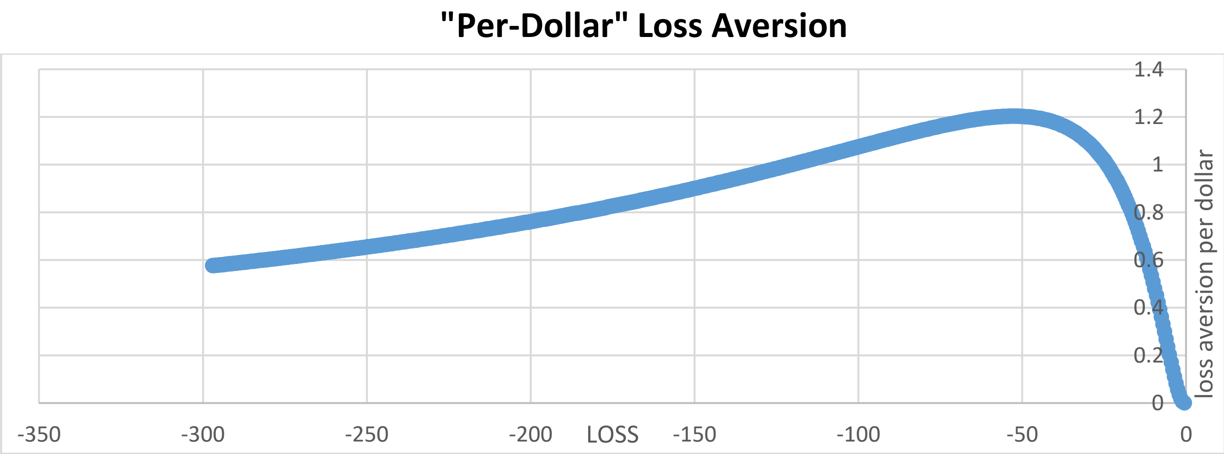

Figure 1. Non-monotonic loss aversion: The "per-dollar" loss aversion (which is based on the functional of the computation cost that is presented in Section 4) is increasing from small to medium losses and then decrease.

Non-monotonic loss aversion leads to different risk attitudes in the loss domain: a lottery over losses with a high expected loss E(x) will be preferred over a degenerate lottery yielding E(x) for sure (risk loving), while a degenerate lottery yielding E(x') will be preferred over a lottery over losses with a low expected loss E(x') (risk aversion).

Harnik et al. (2007) find that loss aversion does not occur in small losses and does occur in large losses, which is consistent with our prediction. Our prediction is also consistent with the inverted S-shape utility that is qualitatively presented (without a functional form) in Markowitz (1952).

4 Log-Normal Computation Cost

In this section we present a simple quantitative model that focuses on lotteries with loss outcomes (mixed lotteries with loss and gain outcomes or lotteries with only loss outcomes). The model includes the computation cost and show one possible functional form for the induced loss aversion. In Section 6 we present an extended model that captures all lotteries (including lotteries with only gain outcomes).

4.1 Description of the Model

In this subsection, we describe a simple quantitative (functional form) model with three parameters that relate to the consumer's utility with respect to lotteries over gains and losses of money (measured, say, in dollar). We assume that a consumer incurring a loss has a positive probability that his existing planned bundle is no longer feasible given his new smaller budget, and that this probability is increasing in the size of the loss. If the planned bundle is no longer feasible, then the consumer has to find a new solution to the satisficing consumer problem (given his new smaller budget), and this induces the consumer to incur a computation cost.

In real life the computation cost is a function of the number of goods, prices, budget, and other constraints that the consumer may be under (e.g., he has to buy some minimum amount of food products, see also the complexity analysis in Section 5). For simplicity the model has a constant computation cost multiplied by a log-normal distribution that depends on the size of the loss. The log-normal distribution can represent two different implications for increasing the size of the loss: (1) it increases the probability to compute a new bundle, and (2) it increases the size of the computation cost. Thus, the result of the multiplication represents the expected computation cost as the function of size of the loss. We assume that a satisficing consumer chooses not to compute a new bundle in a case of a gain because he is already satisfied (i.e., he already has a utility above his satisfaction level). however, he does enjoy the monetary gain (i.e., that is his utility increases, possibly due to his allocating this monetary gain to a petty-cash category budget (Thaler, 1999)). The consumer will either consume this budget spontaneously without incurring a computation cost (as he is already above his satisfaction level), or, for exogenous reasons he will postpones the decision of how to consume this gain till a future time in which he will have to recalculate his consumed bundle. For simplicity, we assume that this utility gain is linear in the monetary gain. Similarly, we assume that, the utility loss from a monetary loss is linear, as we described below. We normalize the linear utility such that one dollar induces one unit of utility.

Our functional form for incorporating computation cost into the utility function includes three parameters that capture the computation cost, and the probability of having to do a new costly computation. Specifically, C>0 is interpreted as the subjective maximal cost (in dollars) that a consumer incurs when computing a new solution to the consumer satisficing problem. We assume that the expected sized of the loss is the product of this parameter C and a log-normal probability distribution (with respect to the size of the loss), with parameters (μ, σ^2)**,** where e^{μ} is the loss inducing an expected computation cost equal to 50% of the maximal cost.

The consumer's total utility, U(x), from obtaining x dollars (in addition to the initial wealth) is equal to the linear money utility minus the expected computation cost. Formally:

$$U(x) = \begin{cases} x & if\ x \geq 0 \\ x - C \cdot \Phi\left(\frac{\ln(-x) - (\mu)}{\sigma}\right) & if\ x < 0, \end{cases} \quad (1)$$

Where Φ is defined as the cumulative distribution function of the standard normal distribution.

4.2 Experimental Calibration

In this subsection, we study the experimental results presented in Tversky and Kahneman (1992), and use them to calibrate the parameters of our model and test its empirical validity. Tversky and Kahneman (1992) present two sets of experimental data (both sets are based on an experiment involving 25 subjects).[7] The first set describes the median cash equivalent of 56 binary lotteries (Table 3 in Tversky & Kahneman, 1992). We included in our analysis 28 of these 56 lotteries, in which at least one of the outcomes is a loss (see Table 2 of Appendix 4). The remaining 28 lotteries involve only gains, and, as utility is limited to being linear, these lotteries were omitted from the analysis.

The second set of relevant outcomes in Tversky and Kahneman (1992, Table 6) includes 8 pairs of binary lotteries in which the median subject is indifferent between the two lotteries. We studied 6 of these 8 pairs of lotteries, as the remaining two lotteries involve only gains (are limited to being linear): see Table 1 below.

We calibrated the parameters of our utility with the goal of minimizing the mean squared error of the relative accuracy of the model. Formally, for $i \leq 28$, let x_i denote the cash equivalent of the *i-th* lottery, and for $29 \leq i \leq 34$ let x_i denote the highest amount of gain in *i-th* pair of equivalent lotteries (also denoted by *x* in Tversky & Kahneman, 1992, Table 6). Let y_i denote the value calculated by the utility function (e.g., the cash equivalent of the lottery according to our utility function, given some parameter values). We define the relative error of the *i-th* lottery as

$$\Delta_i = \frac{x_i - y_i}{x_i}$$

The mean squared error is defined as

$$MSE = \frac{\sum_i^N \Delta_i^2}{N}$$

Based on numeric calibration we chose the following three parameters of our utility function: C = 200 dollars, μ = ln(90), and σ = ln(3.08), which, approximately minimize the MSE (yielding an MSE of 0.0226).

4.3 Comparison of Our Model and Prospect Theory ("Horse-Race")

Recall, that prospect theory (as presented in Tversky and Kahneman, 1992) includes 5 parameters describing loss aversion, probability weighting and curvature of utility. The model assigns to each monetary value *x* (gain or a loss induced by a lottery) utility of:

$$\mathcal{V}(x) = \begin{cases} x^{\alpha} & if\ x \geq 0 \\ -\lambda\,(-x)^{\beta}\ if\ x < 0. \end{cases}$$

[7]We tried to find relevant raw experimental data on lotteries from other experiments (the effort included contacting authors of related experimental papers), but we were not able to obtain additional data that is required for the calibration described in this section (i.e., detailed data about the median (or individual) response to a specific choice between binary lotteries). In the future we plan to do experiment that examine our model with a large sample and check more parameters of the individual like the number of constrains or categories, the income or wealth, the variance of the individual income and more.

and it evaluates each probability according to the following weighting function:

$$w^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}} \quad w^{-}(p) = \frac{p^{\delta}}{(p^{\delta}+(1-p)^{\delta})^{1/\delta}}$$

The utility of a lottery is the expected utility evaluates according to:

$U = v(x_1) * w(p_1) + v(x_2) * w(p_2) \ldots$

The values of the parameters are: α=0.88, β=0.88, λ=2.25, δ=0.69, ϒ=0.61.

We present in Table 1 the experimental results, including the value y_i predicted by our theory (with the above values of the three parameters), the value z_i predicted by prospect theory (taken from Tversky and Kahneman, 1992, Table 6), and the relative error of each model for each of the six lottery pairs involving gains and losses. This data allows us to compare the induced MSE of the two models. Our model yields an MSE of 0.00814 (in this set of 6 lottery pairs), while prospect theory yields a larger MSE of 0.0924 and there for our model reduce the MSE by 91%. The remaining 28 lotteries (taken from Table 3 in Tversky and Kahneman, 1992) are described in Table 2 of Appendix 4. For the entire set of 28 lotteries + 6 lottery pairs, our model yields an MSE of 0.0226, while prospect theory yields MSE of 0.0358. Our model reduces the MSE by **37%** by using only three parameters.

Table 1: 8 Lottery Pairs with 6 Pairs Involving Both Gains and Losses (Adapted from Tversky and Kahneman, 1992, Table 6):

			High outcome for indifference			Relative error	
#	Lottery 1 (50%,50%)	Lottery 2 50%,50%)	**X_i Median experimental result for the high outcome**	**Y_i: Our model's prediction for X_i**	**Z_i : prospect theory's prediction for X_i**	Δ_i of our prediction	Δ_i of Prospect Theory's prediction
1	(0,0)	(-25,X_1)	**61**	**50**	**69**	0.172	0.123
2	(0,0)	(-50,X_2)	**101**	**110**	**137**	0.09	0.357
3	(0,0)	(-100,X_3)	**202**	**207**	**274**	0.027	0.357
4	(0,0)	(-150,X_4)	**280**	**285**	**411**	0.018	0.468
5	(-20,50)	(-50,X_5)	**112**	**122**	**132**	0.089	0.175
6	(-50,112)	(-125,X_6)	**301**	**288**	**357**	0.043	0.186
						RMSE=0.090 (MSE=0.008)	RMSE=0.304 (MSE=0.092)

Table 1. Six pairs of lotteries in Tversky and Kahneman (1992, table 6). The six pairs of lotteries measure loss aversion in lotteries with both gains and losses. Each subject chooses Xi (the high outcome of the second lottery in the pair) that makes her indifferent between the two lotteries. The value of Xi written in the table is the median value reported by the subjects. The parameters Y_i and Z_i are the estimated values of X_i that are predicted by our model and by prospect theory. The relative error

is calculated using Δ_i and the MSE for the all 6 pairs. The RMSE (root mean squared error) reflects a 9% average error compared to 30.4% average error of prospect theory.

Figure 2 shows the utility of gains and losses in our model (with the three calibrated parameters described above), and compares it to the utility induced by prospect theory. Observe that prospect theory has a kinked utility at zero (abrupt change in the utility's derivative), and our model does not have a kink. The prospect theory model show that utility is slightly curved both in the area of gains and in the area of losses. By contrast, our model is linear in the domain of gains, while being convex and concave in the domain of loss that leads to risk loving for high losses and risk aversion for small losses[8].

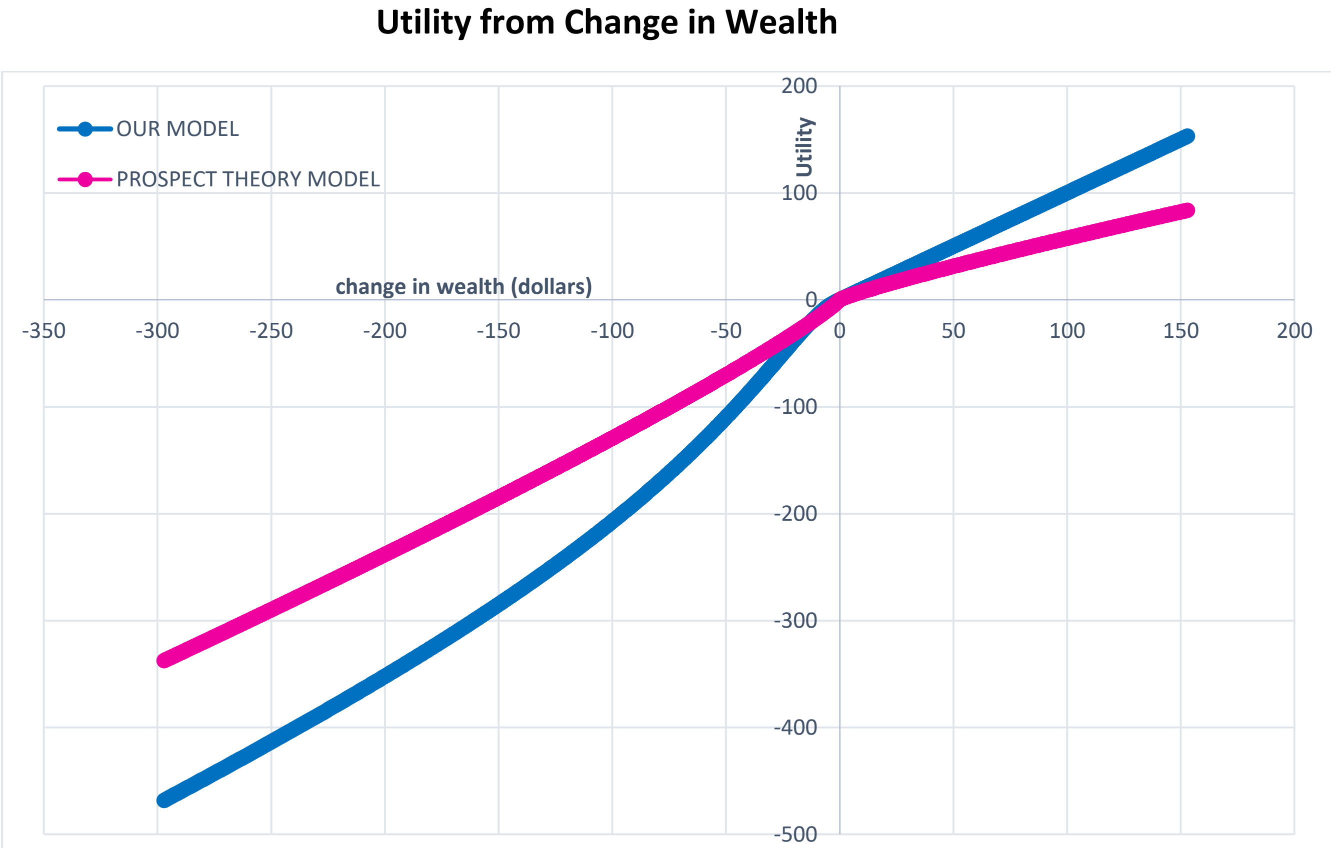

Figure 2. The utility from profit and loss. The blue curve describes our model and the pink curve describes the prospect theory model.

[8] In Section 4.4 we discuss the non–monotonic loss aversion and risk aversion for small losses and risk loving for high losses. One could argue that changing the reference point in the prospect theory model to the point at which the utility transfer from convex to concave in our model (the inflection point) would induce a similar non-monotonicity as in our model. However, doing so would create a loss aversion in the loss domain itself, and create a large difference between different monetary losses around the reference point because of the kink in the prospect theory model (the utility from a dollar loss would be **highly different** between a dollar loss greater than the loss at the reference point and loss less than the reference point). Our model has no kink point (there is an Inflection point) and the difference in the utility of a dollar loss when you move equal distance for the inflection point is small.

4.4 Non-monotonic Loss Aversion

As discussed in Section 3.3, our model induces non-monotonic loss aversion.
We compute the loss aversion for a loss of *X* dollars by calculating the certain equivalent of a 50%–50% lottery that results in a loss *X* dollars or a gain of *X* dollars (the same amount). If there is no loss aversion the certain equivalent (CE) will be zero and if there is loss aversion, it will be negative. The CE is given in absolute value represent a measure for the size of the loss aversion.
Let CE/X denote the certain equivalent relative to the size of the loss, which we use as a unit less measure for the relative loss aversion (henceforth, ***relative loss aversion***). Figure 3 shows the relative loss aversion as a function of the size of the loss. We compare our model with the prospect theory model. Formally, for prospect theory the CE of a 50%-50% lottery with result in a loss of x dollars or a gain of X dollars is equal to (X in absolute value and using probability weighting):

$$0.4206 * X^{.88} - (0.4539) * 2.25X^{.88} = -1 * 2.25 * (0.223X)^{.88}$$
$$CE = 0.233 * X$$

The CE per dollar is defined as the CE divided by the size of the loss. For prospect theory it is equal to:

$$\frac{0.233*X}{X}=0.233$$

In our model the CE is given by (where X is the loss in absolute value):

$$0.50 * X - 0.50 * \left(X - C * \Phi\left(\frac{ln(x) - \ln(\mu)}{\ln(\sigma)}\right)\right) = -0.50 * C * \Phi\left(\frac{\ln(x) - \ln(\mu)}{\ln(\sigma)}\right) = \text{-ce-} \ C * \Phi\left(\frac{\ln(ce) - \ln(\mu)}{\ln(\sigma)}\right)$$

And by our finding:

$$= -0.50 * 200 * \Phi\left(\frac{ln(x) - ln(90)}{ln(3.08)}\right) = \text{-=-}ce\text{-} \ 200 * \Phi\left(\frac{ln(ce) - ln(90)}{ln(3.08)}\right)$$

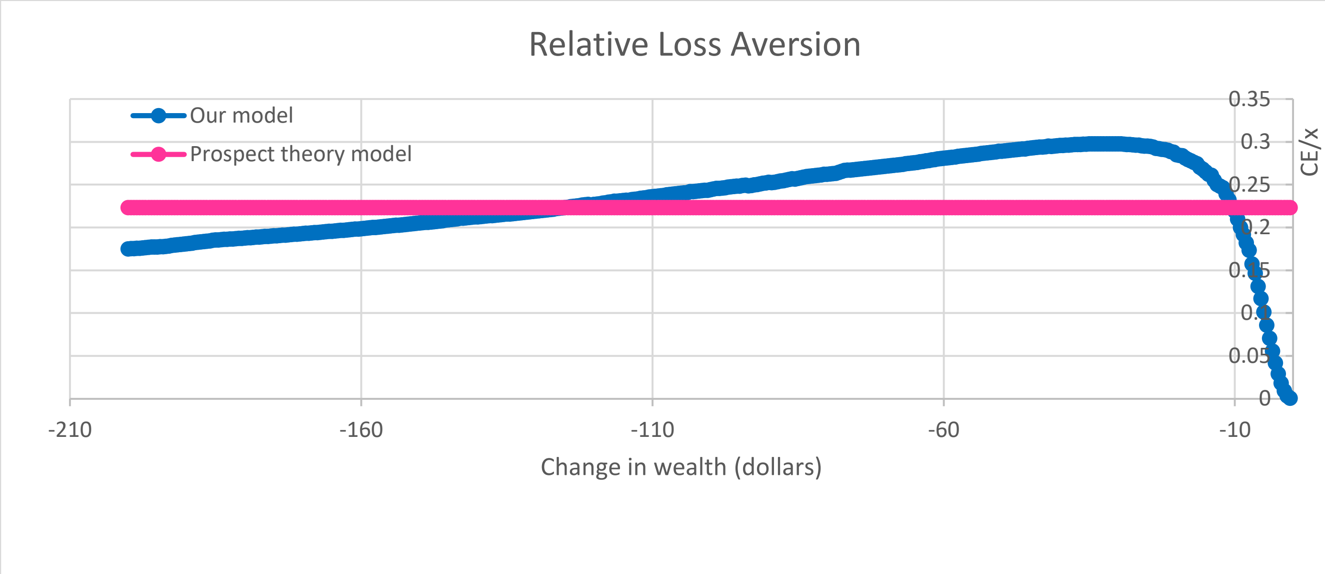

Figure 3. The non-monotonic loss aversion: the relative loss aversion (CE/X) for our model (blue) increase from small to medium losses and then decrease. In the prospect theory model (pink) the loss aversion per dollar is constant number for all X.

4.5 Risk Attitude in the Loss Domain

Our analysis leads to different risk attitude in the loss domain. Observe that all binary lotteries with either a zero outcome or a negative outcome induce risk loving according to the prospect theory model.[9] By contrast, our model induces **risk aversion** for these lotteries when the absolute value of the negative expected payoff is sufficiently small (i.e., in these lotteries, the utility induced by obtaining the expected monetary value for sure is higher than the expected utility of the lottery), whereas it induces **risk loving** for these lotteries when the absolute value of the negative expected payoff is sufficiently large. Recall that the expected utility of a binary lottery $(x_1, 0\ , p_1, p_2)$, where x_1 and x_2=0 are the outcomes of state 1 and state 2 and p_1 and p_2 are the probabilities of state 1 and state 2 is equal to $u(x_1) \cdot p_1$. Formally $U(x_1, 0\ , p_1, p_2) = u(x_1) \cdot p_1$

Definition 1: We say that a binary lottery $(x_1, x_2\ , p_1, p_2)$ induces risk aversion if

$$U(x_1, x_2\ , p_1, p_2)) < u(E(x_1, x_2\ , p_1, p_2)) = u(x_1 p_1 + x_2 p_2)$$

and that it induces risk loving if the converse inequality holds, i.e., if

$$U(\mathrm{x}_1, \mathrm{x}_2\ , p_1, p_2)) > u(E(x_1, x_2\ , p_1, p_2)) = u(x_1 p_1 + x_2 p_2).$$

Specifically, the lottery $(x_1, x_2\ , p_1, p_2)$ induces risk aversion if $U(x_1, 0\ , p_1, p_2) = u(x_1) \cdot p_1 < u(x_1 \cdot p_1)$ and it induces risk loving if the converse inequality holds.

[9] The utility function for x<0 (loss) in the prospect theory model is convex and leads to risk loving in the domain of loss.

<u>Proposition 1:</u> For every probability 0<*p*<1 there is *X(p)*>0 such that the binary lottery $(-x, 0, p, 1-p)$ induces risk aversion if *x*< *X(p*) and induces risk loving if *x*> *X(p).*

The proof of Proposition 1 is presented in Appendix 1. The intuition of the proof is that our utility function (Eq. (1)) is concave in the domain of small losses (which implies risk aversion for small losses), while it is convex in the domain of large losses (which implies risk loving with respect to large losses).

5 Complexity Analysis

5.1 The Consumer Problem is NP-Complete

In Section 5 we present a consumer under a budget constraint who has to choose a subset of goods from a large set of indivisible goods. Following Simon (1957), we assume that the consumer`s goal is to reach a payoff that exceeds a certain utility (satisficing level). We also assume (in addition to the assumptions in Section 3) that the utility level can be attained by a consumer who is under certain set of linear constrains on consumption, the consumer attained the utility level under these constraints, we say that he has attained the aspiration utility level. We will refer to this as - *the consumer problem*

Our latter assumption is formalized as follows. The consumer problem is an integer linear problem, stated as *Ax* ≥ *c*, where $x = (x_1, ..., x_n)$ is a vector of non-negative integer-valued decision variables, where *A* represents the constraint matrix of *w* linear constraints (with real coefficients) and *n* goods. The vector *c* contains real numbers representing the constraints that the system must satisfy (i.e., each linear constraint must be greater than or equal to the relevant number in the vector *c*). If the consumer has a constraint of ≤ (less than or equal to), this constraint can be presented as a constraint of "≥" by multiplying the inequality by -1.

We assume that the constraint matrix includes for each good the constraint that the quantity of a good is non-negative. To simplify the representation of the problem, we can reconstruct the constraint system by adding the budget constraint in a constraint matrix with n columns and *w*+1 rows. Hence the input of the problem is $(w + 1) * (n + 1)$ (the number of rows multiplied by the number of columns including column C) and the output of the problem is a vector of the quantities of the products that meet the constraints.

In what follows, we show that the consumer problem is NP complete. As stated above, it is widely believed that a problem that is a NP-complete belongs to the level of exponential complexity; that is, even when the size of input is relatively small, the problem has a very long runtime, and is considered to be very difficult (Garey & Johnson, 1979).

<u>Proposition 2:</u> *The consumer problem is NP-complete*

Gilboa et al. (2010, Claim 1) proves a closely related result for a simple satisficing problem in which the quantity of each good and the value of each coefficient of the matrix A can be either 0 or 1. For completeness we present in Appendix 2 an alternative proof that shows that the consumer problem can be reduced to an NP-complete problem.

5.2 Complexity Analysis with Categorization

Thaler (1985, 1990, 1999) demonstrates that consumers tend to divide their total budget into small budgets, and apply each smaller "mental" budget to a different consumption category, where money that is assigned to one budget is not fungible for another. Thaler (1999) and Gilboa, Postlewaite & Schmeidler (2010) suggest that the division into categories is a heuristic that helps the consumer to transform the original hard consumer problem to a simpler one.

Two main questions arise when considering this kind of categorization:

1. Does categorization substantially simplify the consumer problem?
2. What is the implication of categorization on loss aversion?

We discuss the first question in Section 5.2.1, and the second question in Section 5.2.2.

5.2.1 Categorization Simplifies the Consumer Problem

Proposition 3 below shows under reasonable assumptions on the categorization process, the division of the goods into different categories substantially simplifies the consumer problem.

A categorization scheme is a function describing how each consumer problem with n goods is transformed into a problem with k categories, in which the n goods are divided into k disjoint subsets (categories). Each category includes n_j goods (where $n=n_1+\ldots+n_k$), and for each category $j\leq k$ there is a set of w_j linear constraints, and a budget assigned to that category. We further assume that the coefficients in all constraints are non-negative rational numbers.

A categorization scheme satisfies logarithmic bounds if, for each category j, either the number of goods in the category, or the number of constraints is bounded by the logarithm of n; i.e., if one of the following inequalities hold for each category j: (1) $n_j < O(log(n))$, or (2) $w_j < O(log(n))$.

The following result shows that categorizations that satisfy the logarithmic bounds (i.e., each category is bounded by $O(log(n))$) induce polynomial ("easy") decision problem.

<u>Proposition 3:</u> Assume that a categorization scheme satisfies the logarithmic bounds. Then, if a solution to the consumer problem with categories exists, it can be found in $O(n^c)$ steps.[10]

The proof of Proposition 3, which relies on Lenstra (1983), is presented in Appendix 3.

Remark 1: Proposition 3 implies that computation costs are small if the either number of the number of goods or the number of constraints the consumer is small. This suggest a

[10] $O(n^c)$ is defined as runtime that is a polynomial function of n where c represents a constant number.

prediction of our model that consumers with either a larger number of goods or more consumption constraints, are likely to induce higher computation costs, and, as a result, present higher levels of loss aversion.

5.2.2 Loss Aversion with Categorization

As mentioned above, our baseline model (i.e., with no categorization) suggests that small (large) losses are less (more) likely to induce cognitive costs due to a smaller (larger) likelihood that the consumer will have to solve a new satisficing problem after inducing a small (large) loss.

We argue that the threshold for which a loss will not allow a consumer to keep the original budget allocation between the various categories is likely to be smaller than the threshold according to which a satisficing consumer (without categories) will have to find a new solution. This is due to the process of categorizing goods and allocating a budget to each category that reduces the budget set (all possible consumption bundles) by adding **new restrictions**. The loss of potential bundles makes it difficult to solve the problem and can lead to a smaller spare money. If the consumer only uses the leftover of a category budget, that the loss if associated with, then the threshold will be smaller than the spare money in the overall budget. As a result of a "large" loss, the consumer will have to re-allocate the budget between the different categories (or significant revise in the categories), and that entails a computationally hard problem. Proposition 3 shows that the high complexity of the consumer problem is a function of the number of different goods (n) and a function of the number of constraints (w). This implies that the higher the number of goods or constraints on the consumer, the stronger the loss aversion.

6 Utility-Maximizing Consumer and Reduced Computation Costs for Gains

6.1 Classic consumer choice:

Consider a consumer who solves the classic consumer choice, i.e., by finding the bundle that maximizes the consumer’s payoff (rather than satisficing as in our baseline model). It is well known that the classic consumer problem with indivisible goods is an NP-complete (“hard”) problem (see, e.g., Gilboa, Postlewaite & Schmeidler 2010), and that, in general, it can be more difficult than our satisficing problem.

When a classic consumer faces a loss or a gain, he will have to find a new optimal solution following the loss or the gain. As a result, the consumer is likely to have a status quo bias: he will reject a 50%–50% lottery with equal values of loss and gain, due to the induced computation cost in both cases. We can also argue that in the case of a small gain or a small loss the cost of computation of a new optimal bundle is higher than the benefit induced by this new optimal bundle (relative to the existing bundle). In such a case there might be asymmetry in the minimal value of the loss or gain in which the individual chooses to compute a new optimal bundle.

6.2 Computation cost for gains:

In what follows we present a model for classic consumer choice that yields good fit for all the lotteries in the experimental data of Tversky and Kahneman 1992. We extend our baseline model (presented in Section 4) by allowing computation costs after the agent obtains a large gain. There are two plausible cases in which one needs to add computation costs for gains. The first case is when dealing with a utility-maximizing consumer who has to compute a new bundle also in the case of a gain. The second case is for a satisficing consumer who adjusts his satisficing level and computes a new bundle after obtaining a large gain. We argue that the mean gain that induces a computation of a new bundle is likely to be higher than the mean loss that induces a computation. The reasons for this are as follows:

1. The value of a loss that is taken from a petty-cash budget is limited to the money deposited in the petty-cash budget but the value of a gain has no limit. It is plausible that the value of a gain that make computation worthwhile is higher than the money kept in the petty-cash budget on average. The consumer keeps a modest amount of money in a petty cash category (Thaler, 1999, p. 194), and that he adds intermediate gains and losses to this category.
2. If we assume that the utility function for money in the petty-cash budget is strictly concave (i.e., the consumer has decreasing marginal utility from keeping money in this petty-cash budget) and the first derivative of the utility function is strictly convex, it will lead to a shadow price (the difference in the utility between the optimal bundle and the non-optimal bundle) that is asymmetric between loss and gain.[11] The shadow price is higher in the case of a loss and hence the loss that induces a computation cost (which will be worthwhile) is low compere to the gain that induces a computation (in Appendix 6 we sketch a simple model that describes a suboptimal utility from the petty-cash budget).

6.3 Functional Form with a Reduced Computation Cost for Gains:

We extend our model by allowing a computation cost following a large gain. The extended model introduces a single new parameter capturing how large this gain need to be in order to induce a new, costly computation.

The first three parameters are identical to the basic model: (1) C>0 is interpreted as the subjective maximal cost (in dollars) that a consumer incurs when computing a new bundle.

[11] The three conditions (convexity of the first derivative, concavity and monotonicity) are the minimum requirements for the asymmetric and higher loss shadow price. These conditions are met by known utility functions for risk aversion like CARA, DARA, and CRRA.

We assume that the expected sized of the loss is the product of this parameter C and a log-normal probability distribution (with respect to the size of the change) with variance σ^2 and with an expectation that depends on whether the change is a loss or a gain. Specifically, the parameters of the a log-normal distribution are: (2) (μ_L, σ^2) for a loss, where e^{μ_L} is the loss inducing expected cost of 50% of the maximal cost, and (3) parameters (μ_G, σ^2), where e^{μ_G} is the gain inducing expected cost of 50% of the maximal cost.

The functional form of the utility in the extended model is as follows:

$$U(x) = \begin{cases} x \quad - C \cdot \Phi\left(\dfrac{ln(x) - \mu_G}{\sigma}\right) & if\ x > 0 \\ 0 & if\ x = 0 \\ x \quad - C \cdot \Phi\left(\dfrac{ln(-x) - \mu_L}{\sigma}\right) & if\ x < 0. \end{cases}$$

We also add probability weighting **for gain** (but not for loss) using the same function from prospect theory. We use probability weighting for gain to improve the fit of our model to the empirical data on lotteries out empirical fix (some lotteries with only gains induce risk loving that cannot be explained by the utility function for gain). We will leave the theoretical foundation for using probability weighting only for gains for future research.

The probability weighting is given by

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

Where *P* is the probability for gain and Υ is the power of the distortion (smaller Υ = higher distortion).

The results for all 64 lotteries in Tversky and Kahneman 1992, (see Table 3 of Appendix 5) achieved MSE = 0.0217 in our extended model compared to MSE=0.0287 achieved by the prospect theory model and there for our model reduce the MSE by **24%** with the same number of parameters as the prospect theory.

The parameters values are $C = 89$, $\sigma = \ln(2.39)$, $\mu_L = \ln(55)$, $\mu_G = \ln(277)$, $\Upsilon = 0.68$.

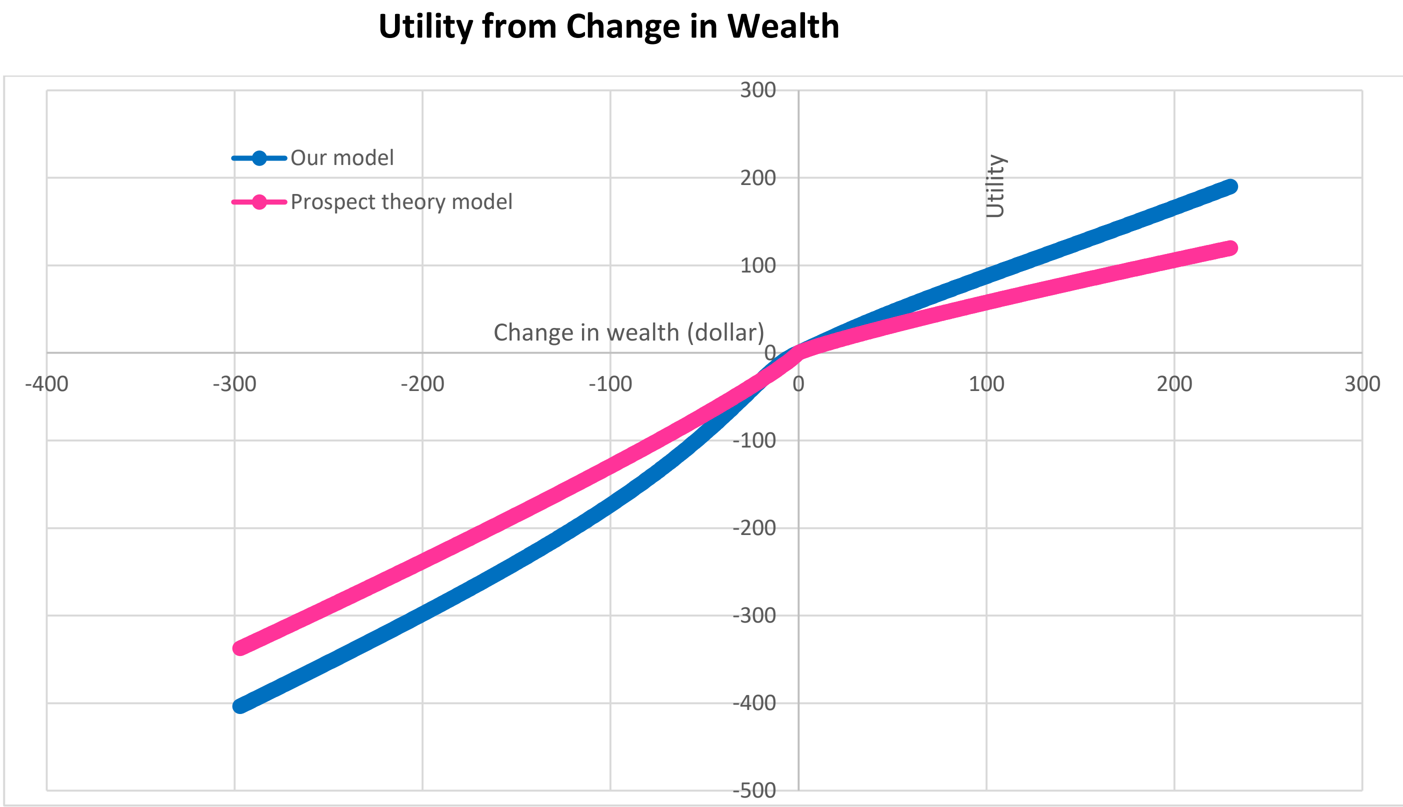

Figure 4. The utility from profit and loss in the extended model. The blue curve describes our model and the pink curve describes the prospect theory model. The utility function for the loss domain in the extended model is similar to that in the simple model. The utility function for gain is different due to the computation cost. The utility function in the domain of gains is concave for small values and convex for large values (converging to a linear function for very high gains).

7 Discussion

Our paper presents a novel rational explanation for loss aversion, which is induced by the computation cost of the satisficing consumer problem. We show that due to the computation cost, a satisficing consumer will be loss averse, and the induced relative loss aversion will be non-monotonic: (1) loss aversion will be mild when the loss is small because in such a case there is a relatively low probability to induce a new costly computation, (2) loss aversion will be strong for an intermediate loss that is likely to induce a new costly computation, and (3) loss aversion will be mild for a large loss, as the cost of the required new computation will be negligible with respect to the large loss itself. We offer a simple quantitative model of a functional form for the utility function that captures the continuous non-monotonic loss aversion. As discussed in Remark 1 our model suggests that agents who have more constraints on their consumption are likely to have higher levels of loss aversion. It will be interesting to experimentally test this prediction in future research.

In addition, we present an extended model for utility-maximizing consumer with a reduced expected computation cost for gains that induce a status quo bias because any change (gain or loss) can induce a computation cost, and this reduces the attractiveness of the change. By adding computation costs to gains we increase the loss aversion (the asymmetry between gain and loss) as it reduces the utility from gain. Both models fit the experimental data presented in Tversky and Kahneman (1992) and yield better predictions than prospect theory in the domain of loss and mix lotteries (the simple model) and in the entire domain of lotteries (the extended model). Since the parameters of the model are in U.S. dollars and calibrated to Tversky and Kahneman`s (1992) experimental data the values of the parameters have to be adjusted to changes in purchasing power.

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Appendix

Appendix 1: Proof of Proposition 1

Recall that the utility of a loss x<0 (see Eq. (1)) is $u(x)= x - C \cdot \Phi\left(\frac{ln(-x) - \mu}{\sigma}\right)$.

Fix any parameters μ>0 σ>0 and C>0 and let

$F(x) \equiv \Phi\left(\frac{ln(-x) - \mu}{\sigma}\right)$.

observe that F is a CDF of a log-normal distribution with parameters μ and σ. Observe that the utility of a loss can be given by

$u(x) = x - C * F(x)$.

The density function for the log-normal distribution is

$f(x) \equiv F`(x)= \frac{1}{x\sigma 2\pi} e^{-\frac{1}{2}\left(\frac{ln(x)-\mu}{\sigma}\right)^2}$.

Observe that $\lim_{x\to\infty} f(x) = 0$.

The derivative of the *f(x)* is

$f`(x) = -\frac{1}{x^2\sigma 2\pi} e^{-\frac{1}{2}\left(\frac{ln(x)-\mu}{\sigma}\right)^2} * \left(1 + \frac{\ln(x)-\mu}{\sigma^2}\right)$.

In the range of $0 < |x| < e^{\mu-\sigma^2}$ the derivative is positive and in the range of $|x| > e^{\mu-\sigma^2}$ the derivative is negative.

Observe that the CDF *F*(*x*) is convex in the range $0 < |x| < e^{\mu-\sigma^2}$ and it is concave in the range $|x| > e^{\mu-\sigma^2}$, and that the maximum of *F*(*x*) is equal to 1 (because *F*(*x*) is a CDF function) and$lim_{x\to\infty} \frac{F(x)}{|x|} = 0$.

The slope of a straight line from zero to *F(x)* is *F(x)/x*. Let *g(x)= F(x)/x*. Because of the properties of *F(x),* the function *g(x)* is increasing and then decreases to zero as X goes to infinity. In the range $0 < |x| < e^{\mu-\sigma^2}$, the function g(x) is higher than the function *g(x*p).*

For each P we can find X such that *g(x)* is lower than *g(xp)* in the range when *g(x)* decreases because as X goes to infinity *g(x)* goes to zero, and for every P we can increase X enough such that xp and x are in the range where *g(x)* is decreasing.

We define function *d(x)* $= g(\mathrm{x}) - g(\mathrm{x} \cdot p)$. We know that for low enough $|x|$ ($|x| < e^{\mu-\sigma^2}$) this function gives a positive value and for large enough|x| it gives a negative value. By the intermediate value theorem we have that there is a value X(p) that gives a value of zero for d(x).

Recall that risk aversion implies that $u(x \cdot p) > u(x) \cdot p$. In words, the utility of the expected monetary value is higher than the expected utility of the lottery). In such a case the slope of a straight line from$|u(0)|$ to $|u(x)|$ (denoted by *S*(*x*)). Is lower than the slope of a straight line from u(0) to $|u(x \cdot p)|$ (denoted by (*S*(*xp*)). By contrast in risk loving the slope of a straight line from $u(0)$*)* to $|u(x)|$ will be higher than the slope of a straight line from u(0) to $|u(x \cdot p)|$.

The absolute value of the slope of a straight line from zero to X in the utility function is

$$|S(x)| = 1 + c * g(x) .$$

We see that when $0 < |x| < e^{\mu - \sigma^2}$ then $g(x) > g(x \cdot p)$ and therefore $|S(x)| > |S(x \cdot p)|$). In contrast, for high enough values of *x* the opposite inequalities hold. Thus, by continuity, there is an intermediate value X(p) for which $g(x) = g(x \cdot p)$ and $|S(x)| = |S(x \cdot p)|$). □

Appendix 2: Proof of Proposition 2

The Satisfiability Problem

A *Boolean variable* is a variable that can take only the two truth values: TRUE and FALSE.

We usually denote the truth values by 1 and 0, respectively.

The *Boolean operations* AND, OR, and NOT are represented by the symbols ∧, ∨, and ¬, respectively.

A *Boolean formula* is an expression involving Boolean variables and operations. Here is an example of a Boolean formula over 3 Boolean variables x_1, x_2, x_3:

θ = $(\neg x_1 \wedge x_2) \vee (x_1 \wedge x_3)$

A *satisficing assignment* is an assignment of 0s and 1s to the Boolean variables which makes the formula evaluate to 1.

A Boolean formula is satisfied if it has at least one satisficing assignment. For example, the formula θ above is satisfied, as the assignment x_1 = 1, x_2 =0, x_3 =1 is a satisficing assignment.

The satisfiability problem, also known as the SAT problem, is defined as follows. Given a Boolean formula θ, decide whether θ can be satisfied or not.

The SAT problem is known to be NP-complete.

Sometimes it is helpful to work with a well-known variant of the SAT problem, called the 3SAT problem. The 3SAT problem is helpful in reducing the SAT problem to the consumer problem. In order to define the 3SAT Problem, we need some more definitions.

A *literal* is a Boolean variable or a negated Boolean variable. For example: x_1 , $\neg x_1$, $\neg x_2$, x_3 , etc.

A *clause* is several literals connected with the OR operators. For example: $(\neg x_1 \vee x_2 \vee \neg x_3 \vee x_4)$.

A Boolean formula is in conjunctive normal form, usually abbreviated as CNF form, if it is composed of several clauses connected by AND operators. For example:

$\theta = (\neg x_1 \lor x_2 \lor \neg x_3 \lor x_4) \land (\neg x_2 \lor x_4) \land (\neg x_5 \lor x_2 \lor \neg x_3)$.

A Boolean formula is in 3CNF form, if each clause contains three literals. For example:

$\theta = (\neg x_1 \lor x_2 \lor \neg x_4) \land (\neg x_1 \lor x_4 \lor x_6) \land (\neg x_5 \lor x_2 \lor x_3) \land (\neg x_7 \lor \neg x_2 \lor x_3)$.

Note that in a satisfies assignment for a CNF formula, each clause must have at least one literal that evaluates to 1.

The 3SAT problem defined as follows. Given a Boolean formula θ in 3CNF form, decide whether θ can be satisfied or not.

The 3SAT problem is also known to be NP-complete.

Claim:

The consumer problem is NP-Complete.

Proof:

In order to prove that a problem is NP-complete, two conditions must hold:

1. The problem belongs to the class NP.
2. The problem is NP-hard.

We show below that the consumer problem satisfies the above conditions.

The consumer problem belongs to NP, as it is possible to verify in polynomial time that a given solution is feasible.

In order to prove that the problem is NP-hard, we describe a polynomial time reduction from the 3SAT problem, which is known to be NP-complete, to the consumer problem.

Let θ be a Boolean formula in 3CNF-form over n Boolean variables $x_1, x_2, \dots, x_n$.

We define an instance for the consumer problem as follows.

- We use n products: $x_1, x_2, \dots, x_n$.
- We set the budget limit B to be equal to n.
- We set all prices to 1.
- We define the following linear constraints on our variables $x_1, x_2, \dots, x_n$:
    - For each variable $x_{i,1}$ we define two constraints:
        - $x_i \geq 0$
        - $x_i \leq 1$
    - For each clause of the form $(l_i \lor l_j \lor l_k)$, where l_i, l_j, l_k are literals corresponding to the variables x_i, x_j, x_k, we define one constraint as follows.
      Let:
      $$L_i = \begin{cases} x_{i,1} & l_i = x_i \\ 1 - x_{i,1} & l_i = \neg x_i \end{cases} \qquad L_j = \begin{cases} x_{j,1} & l_j = x_j \\ 1 - x_{j,1} & l_j = \neg x_j \end{cases} \qquad L_k = \begin{cases} x_{k,1} & l_k = x_k \\ 1 - x_{k,1} & l_k = \neg x_{k.} \end{cases}$$

      Then define
        - $L_i + L_j + L_k \geq 1$.

For example: the clause (¬ x_1 ∨ x_2 ∨ ¬x_4) induces the constraint (1-$x_{1,1}$) + $x_{2,1}$ + (1-$x_{4,1}$) ≥ 1, which is equal to -$x_{1,1}$ + $x_{2,1}$ + -$x_{4,1}$ ≥ - 1.

The reduction can be computed in polynomial time, and it holds that the formula θ is satisfied if and only if there is a feasible solution for the consumer problem induced by θ.

Thus, we have proved that the consumer problem is NP-complete.

Appendix 3: Proof of Proposition 3

In order to test a proposed solution, multiply the matrix in the vector of solution *x* and compare the resulting product to the constraint vector *c*. The running time of the multiplication and comparison operations is equal to the multiplication of the input by a finite number. The maximum number of units that can be purchased from any product is bounded from above by $\left\lfloor\frac{B}{P}\right\rfloor$ where p is the price of the cheapest product. It should be noted that the constraint matrix coefficients are also finite.

Hence the maximum runtime of the multiplication operation will be

$(\log(a)+1)^2$

The overall runtime for each line is: n-1 (connection operations) + n (multiples) + comparison action = 2n

The runtime as a function of input length (K) is blocked up as follows:

Runtime: $2n * (n + m + 1) * (\log(a) + 1)^2$

Block up by $2K * (K + K + 1) * (\log K + 1)^2$

This is a polynomial expression of the input length.

An algorithm that solves the general problem:

The algorithm calculates all possible solutions, checks them one after the other, and stops when a solution is found that meets the constraints. If no such solution is found, a negative result is obtained at the end of the run.

The maximum runtime of this algorithm is equal to the number of possible solutions multiplied by the runtime of a solution.

The number of possible solutions is blocked up by the budget constraint.

Each product can have complete values between zero and $\left\lfloor \frac{B}{Pi} \right\rfloor$ (the nearest integer rounded down).

The maximum number of possible solutions is blocked from above by:

$$\prod_{i=1}^{n}(1 + \left\lfloor \frac{B}{Pi} \right\rfloor)$$

In general, the runtime is blocked from above by:

$$((m+n+1) * (2n)(loga+1)^2) * \prod_{i=1}^{n}(1 + \left\lfloor \frac{B}{Pi} \right\rfloor)$$

An algorithm that solves the problem under categorization:

We assume four assumptions:

The number of products is limited in each category: For simplicity we assume that the consumer belongs to each category up to $\log_{G+1}(n)$ products.

A product can only belong to one category.

Each category J has its own consumption constraints. Let us denote the number of constraints in category J as m_j.

The number of units that can be bought from any product is limited to G units.

Under these assumptions the running time of the algorithm to a category will be blocked from above:

$$((m_j+logn+1)*(2logn)(loga+1)^2) * \prod_{i=1}^{logn}(1 + G).$$

Because the number of categories is blocked by n (number of products) and the number of consumption constraints is restricted to m, we have

$$n((m+logn+1)*(2logn)(loga+1)^2) * \prod_{i=1}^{logn}(1 + G)$$

$$\prod_{i=1}^{logn}(1 + G) = (1+G)^{logn} = n$$

$$n((m+logn+1)*(2logn)(loga+1)^2)*n$$

The runtime as a function of the input size (K) is blocked up as follows:

$$K((K+logK+1)*(2logK)(logK+1)^2)*K)$$

This is a polynomial runtime of the input length.

The consumer problem is: $Ax \geq c$ and could be transfer to a problem of $Ax \leq c$

by multiplying by -1. Lenstra (1983) proves that this problem can in turn be reformulated as a problem with min(n,w) variables. if the following three conditions hold:

1. All coefficients are integers.
2. The X vector coefficients are non-negative.
3. The X vector coefficients are bounded by $(n + 1)n^{n/2}a^{n}$ value, where **a** is defined as the maximum of the absolute values of the coefficients of A and c.

We assume that all coefficients are integers (if a constraint has rational numbers, it can be transformed to an integer constrain by multiplying it by the common denominator).

The X vector coefficients are non-negative in our model and they are bounded by the budget constraint to a value of B/(minimum price) and they can be rounded down to the nearest integer. We define B= budget* minimum price and accordingly multiply the budget constrain left side by the minimum price.

The X vector coefficients are bounded by the new B. By definition B≤a thus condition three is achieved.

We conclude that if n or w are limited to $\log_{G+1}(n)$, then the consumer problem has a polynomial runtime algorithm.

Appendix 4: Experimental Data for the Baseline Model

Table 2: In the table below there is the data of the 28 lotteries pairs with only losses or zero outcomes we examine from Tversky and Kahneman (1992). Each subject chooses *Xi* (the high outcome of the second lottery in the pair) that makes her indifferent between the two lotteries. The value of *Xi* written in the table is the median value reported by the subjects. The parameters Y_i and Z_i are the estimated valued of X_i that are predicted by our model and by prospect theory. Relative error is calculating by Δ_i.

#	Lottery 1 states	Lottery 1 probability	Lottery 2 states	Lottery 2 probability	**X_i Median experimental result for the high outcome**	**Y_i: Our model's prediction for X_i**	**Z_i : prospect theory's prediction for X_i**	Δ_i of our prediction	Δ_i of Prospect Theory's prediction
1	(0,-50)	(0.9,0.1)	(0,X_1)	(0,1)	-8	-7.938	-6.681	0.008	0.164
2	(0,-50)	(0.5,0.5)	(0,X_2)	(0,1)	-21	-26.84	-20.38	0.278	0.029
3	(0,-50)	(0.1,0.9)	(0,X_3)	(0,1)	-39	-45.15	-37.42	0.158	0.040
4	(0,-100)	(0.95,0.05)	(0,X_4)	(0,1)	-8	-7.594	-8.261	0.051	0.032
5	(0,-100)	(0.75,0.25)	(0,X_5)	(0,1)	-23.5	-25.55	-24.83	0.087	0.056
6	(0,-100)	(0.5,0.5)	(0,X_6)	(0,1)	-42	-47.16	-40.76	0.123	0.029
7	(0,-100)	(0.25,0.75)	(0,X_7)	(0,1)	-63	-71.65	-58.76	0.137	0.067
8	(0,-100)	(0.05,0.95)	(0,X_8)	(0,1)	-84	-94.00	-83.12	0.119	0.010
9	(0,-200)	(0.99,0.01)	(0,X_9)	(0,1)	-3	-3.222	-5.109	0.074	0.703
10	(0,-200)	(0.9,0.1)	(0,X_{10})	(0,1)	-23	-18.81	-26.72	0.182	0.162
11	(0,-200)	(0.5,0.5)	(0,X_{11})	(0,1)	-89	-82.38	-81.52	0.074	0.083
12	(0,-200)	(0.1,0.9)	(0,X_{12})	(0,1)	-155	-173.10	-149.6	0.117	0.034
13	(0,-200)	(0.01,0.99)	(0,X_{13})	(0,1)	-190	-197.26	-187.5	0.038	0.012
14	(0,-400)	(0.99,0.01)	(0,X_{14})	(0,1)	-14	-4.8753	-10.21	0.652	0.270
15	(0,-400)	(0.01,0.99)	(0,X_{15})	(0,1)	-380	-394.6	-375.1	0.038	0.012
16	(-50,-100)	(0.9,0.1)	(0,X_{16})	(0,1)	-59	-54.41	-55.03	0.078	0.067
17	(-50,-100)	(0.5,0.5)	(0,X_{17})	(0,1)	-71	-73.29	-66.74	0.032	0.059
18	(-50,-100)	(0.1,0.9)	(0,X_{18})	(0,1)	-85	-94.37	-85.06	0.110	0.001
19	(-50,-150)	(0.95,0.05)	(0,X_{19})	(0,1)	-60	-53.96	-58.19	0.101	0.030
20	(-50,-150)	(0.75,0.25)	(0,X_{20})	(0,1)	-71	-70.77	-73.17	0.003	0.030
21	(-50,-150)	(0.5,0.5)	(0,X_{21})	(0,1)	-92	-94.28	-88.19	0.005	0.041
22	(-50,-150)	(0.25,0.75)	(0,X_{22})	(0,1)	-113	-120.71	-106.2	0.068	0.060
23	(-50,-150)	(0.05,0.95)	(0,X_{23})	(0,1)	-132	-143.9	-131.7	0.090	0.001
24	(-100,-200)	(0.95,0.05)	(0,X_{24})	(0,1)	-112	-104.3	-106.2	0.069	0.051
25	(-100,-200)	(0.75,0.25)	(0,X_{25})	(0,1)	-121	-122	-119.1	0.008	0.015
26	(-100,-200)	(0.5,0.5)	(0,X_{26})	(0,1)	-142	-146.3	-133.4	0.031	0.059
27	(-100,-200)	(0.25,0.75)	(0,X_{27})	(0,1)	-158	-172.3	-152.1	0.091	0.037
28	(-100,-200)	(0.05,0.95)	(0,X_{28})	(0,1)	-179	-194.3	-179.7	0.086	0.004

Appendix 5: Experimental Data for the Extended Model

Table 3: In the table below there is the data of the 64 lotteries pairs from Tversky and Kahneman (1992). Each subject chooses *Xi* (the high outcome of the second lottery in the pair) that makes her indifferent between the two lotteries. The value of *Xi* written in the table is the median value reported by the subjects. The parameters Y_i and Z_i are the estimated valued of X_i that are predicted by the extended model and by prospect theory model. Relative error is calculating by Δ_i.

#	Lottery 1 states	Lottery 1 probability	Lottery 2 states	Lottery 2 probability	X_i Median experimental result for the high outcome	Yi: Our model's prediction for Xi	Z_i : prospect theory's prediction for X_i	Δ_i of our prediction	Δ_i of Prospect Theory's prediction
1	(0,50)	(0.9,0.1)	$(0,X_1)$	(0, 1)	9	8.23	7.408	0.086	0.177
2	(0,50)	(0.5,0.5)	$(0,X_2)$	(0, 1)	21	21.7	18.689	0.033	0.110
3	(0,50)	(0.1,0.9)	$(0,X_3)$	(0, 1)	37	37.68	33.973	0.018	0.082
4	(0,-50)	(0.9,0.1)	$(0,X_4)$	(0, 1)	-8	-7.91	-6.682	0.011	0.165
5	(0,-50)	(0.5,0.5)	$(0,X_5)$	(0, 1)	-21	-26.95	-20.382	0.283	0.029
6	(0,-50)	(0.1,0.9)	$(0,X_6)$	(0, 1)	-39	-45.1	-37.421	0.156	0.040
7	(0,100)	(0.95,0.05)	$(0,X_7)$	(0, 1)	14	10.16	9.983	0.274	0.287
8	(0,100)	(0.75,0.25)	$(0,X_8)$	(0, 1)	25	26.52	24.567	0.061	0.017
9	(0,100)	(0.5,0.5)	$(0,X_9)$	(0, 1)	36	41.48	37.379	0.152	0.038
10	(0,100)	(0.25,0.75)	$(0,X_{10})$	(0, 1)	52	58.64	52.612	0.128	0.012
11	(0,100)	(0.05,0.95)	$(0,X_{11})$	(0, 1)	78	82.62	76.853	0.059	0.015
12	(0,-100)	(0.95,0.05)	$(0,X_{12})$	(0, 1)	-8	-7.41	-8.262	0.074	0.033
13	(0,-100)	(0.75,0.25)	$(0,X_{13})$	(0, 1)	-23.5	-25.25	-24.834	0.074	0.057
14	(0,-100)	(0.5,0.5)	$(0,X_{14})$	(0, 1)	-42	-46.15	-40.764	0.099	0.029
15	(0,-100)	(0.25,0.75)	$(0,X_{15})$	(0, 1)	-63	-70.72	-58.769	0.123	0.067
16	(0,-100)	(0.05,0.95)	$(0,X_{16})$	(0, 1)	-84	-93.77	-83.124	0.116	0.010
17	(0,200)	(0.99,0.01)	$(0,X_{17})$	(0, 1)	10	6.98	7.447	0.302	0.255
18	(0,200)	(0.9,0.1)	$(0,X_{18})$	(0, 1)	20	29.46	29.630	0.473	0.482
19	(0,200)	(0.5,0.5)	$(0,X_{19})$	(0, 1)	76	83.34	74.758	0.097	0.016
20	(0,200)	(0.1,0.9)	$(0,X_{20})$	(0, 1)	131	150.95	135.893	0.152	0.037
21	(0,200)	(0.01,0.99)	$(0,X_{21})$	(0, 1)	188	187.85	180.030	0.001	0.042
22	(0,-200)	(0.99,0.01)	$(0,X_{22})$	(0, 1)	-3	-2.8	-5.110	0.067	0.703
23	(0,-200)	(0.9,0.1)	$(0,X_{23})$	(0, 1)	-23	-18.695	-26.728	0.187	0.162
24	(0,-200)	(0.5,0.5)	$(0,X_{24})$	(0, 1)	-89	-81.46	-81.528	0.085	0.084
25	(0,-200)	(0.1,0.9)	$(0,X_{25})$	(0, 1)	-155	-173.86	-149.684	0.122	0.034
26	(0,-200)	(0.01,0.99)	$(0,X_{26})$	(0, 1)	-190	-197.36	-187.567	0.039	0.013
27	(0,400)	(0.99,0.01)	$(0,X_{27})$	(0, 1)	12	14.12	14.895	0.177	0.241
28	(0,400)	(0.01,0.99)	$(0,X_{28})$	(0, 1)	377	378	360.060	0.003	0.045
29	(0,-400)	(0.99,0.01)	$(0,X_{29})$	(0, 1)	-14	-4.67	-10.219	0.666	0.270

#	Lottery 1 states	Lottery 1 probability	Lottery 2 states	Lottery 2 probability	X_i Median experimental result for the high outcome	Yi: Our model's prediction for Xi	Z_i : prospect theory's prediction for X_i	Δ_i of our prediction	Δ_i of Prospect Theory's prediction
30	(0,-400)	(0.01,0.99)	$(0,X_{30})$	(0, 1)	-380	-395.15	-375.134	0.040	0.013
31	(50,100)	(0.9,0.1)	$(0,X_{31})$	(0, 1)	59	55.45	53.113	0.060	0.100
32	(50,100)	(0.5,0.5)	$(0,X_{32})$	(0, 1)	71	66.95	61.206	0.057	0.138
33	(50,100)	(0.1,0.9)	$(0,X_{33})$	(0, 1)	83	84.2	79.031	0.014	0.048
34	(-50,-100)	(0.9,0.1)	$(0,X_{34})$	(0, 1)	-59	-54.3	-55.031	0.080	0.067
35	(-50,-100)	(0.5,0.5)	$(0,X_{35})$	(0, 1)	-71	-73	-66.749	0.028	0.060
36	(-50,-100)	(0.1,0.9)	$(0,X_{36})$	(0, 1)	-85	-94.27	-85.069	0.109	0.001
37	(50,150)	(0.95,0.05)	$(0,X_{37})$	(0, 1)	64	58.25	57.987	0.090	0.094
38	(50,150)	(0.75,0.25)	$(0,X_{38})$	(0, 1)	72.5	72.986	69.300	0.007	0.044
39	(50,150)	(0.5,0.5)	$(0,X_{39})$	(0, 1)	86	87.8	80.868	0.021	0.060
40	(50,150)	(0.25,0.75)	$(0,X_{40})$	(0, 1)	102	105.7	96.587	0.036	0.053
41	(50,150)	(0.05,0.95)	$(0,X_{41})$	(0, 1)	128	131.37	123.581	0.026	0.035
42	(-50,-150)	(0.95,0.05)	$(0,X_{42})$	(0, 1)	-60	-53.86	-58.195	0.102	0.030
43	(-50,-150)	(0.75,0.25)	$(0,X_{43})$	(0, 1)	-71	-70.48	-73.179	0.007	0.031
44	(-50,-150)	(0.5,0.5)	$(0,X_{44})$	(0, 1)	-92	-94.16	-88.192	0.023	0.041
45	(-50,-150)	(0.25,0.75)	$(0,X_{45})$	(0, 1)	-113	-120.9	-106.211	0.070	0.060
46	(-50,-150)	(0.05,0.95)	$(0,X_{46})$	(0, 1)	-132	-144.02	-131.775	0.091	0.002
47	(100,200)	(0.95,0.05)	$(0,X_{47})$	(0, 1)	118	106.6	104.037	0.097	0.118
48	(100,200)	(0.75,0.25)	$(0,X_{48})$	(0, 1)	130	119.61	111.824	0.080	0.140
49	(100,200)	(0.5,0.5)	$(0,X_{49})$	(0, 1)	141	134.1	122.412	0.049	0.132
50	(100,200)	(0.25,0.75)	$(0,X_{50})$	(0, 1)	162	152.6	139.050	0.058	0.142
51	(100,200)	(0.05,0.95)	$(0,X_{51})$	(0, 1)	178	179.92	169.549	0.011	0.047
52	(-100,-200)	(0.95,0.05)	$(0,X_{52})$	(0, 1)	-112	-104.38	-106.269	0.068	0.051
53	(-100,-200)	(0.75,0.25)	$(0,X_{53})$	(0, 1)	-121	-122.88	-119.135	0.016	0.015
54	(-100,-200)	(0.5,0.5)	$(0,X_{54})$	(0, 1)	-142	-147.3	-133.499	0.037	0.060
55	(-100,-200)	(0.25,0.75)	$(0,X_{55})$	(0, 1)	-158	-173.24	-152.097	0.096	0.037
56	(-100,-200)	(0.05,0.95)	$(0,X_{56})$	(0, 1)	-179	-194.58	-179.771	0.087	0.004
57	(0,0)	(0.5,0.5)	$(-25,X_{57})$	(0.5,0.5)	61	47.71	68.517	0.218	0.123
58	(0,0)	(0.5,0.5)	$(-50,X_{58})$	(0.5,0.5)	101	114.4	137.034	0.133	0.357
59	(0,0)	(0.5,0.5)	$(-100,X_{59})$	(0.5,0.5)	202	220.8	274.069	0.093	0.357
60	(0,0)	(0.5,0.5)	$(-150,X_{60})$	(0.5,0.5)	280	300.81	411.103	0.074	0.468
61	(-20,50)	(0.5,0.5)	$(-50,X_{61})$	(0.5,0.5)	112	131.56	131.645	0.175	0.175
62	(-50,150)	(0.5,0.5)	$(-125,X_{62})$	(0.5,0.5)	301	295.6	357.015	0.018	0.186
63	(50,120)	(0.5,0.5)	$(20,X_{63})$	(0.5,0.5)	149	155.4	155.503	0.043	0.044
64	(100,300)	(0.5,0.5)	$(25,X_{64})$	(0.5,0.5)	401	373.12	392.926	0.070	0.020

Appendix 6: Petty Cash Money and Computation Costs for Gains

In Section 6.2 we briefly mention two reasons for having computation costs for gains, which are reduced with respect to costs induced by losses. In what follows, we describe a model that explains these reduced costs due suboptimal allocation of money in a petty-cash category.

Let assume the following assumptions for this model:

There are two consumption categories: a general-consumption category (C) and a petty-cash category (P), which is used to allow buying some goods due to unexpected needs. Given an optimal allocation in the general-consumption category the utility from money is assumed to be linear in the relevant domain. By contrast, the utility from money in the petty-cash category is strictly concave (i.e., the consumer has decreasing marginal utility from keeping money in this petty-cash budget), and the first derivative of the utility function is strictly convex. The budget B is allocated between these two categories such that $B=b_C+b_P$. We further assume that the constant marginal utility in the general-consumption category is lower than the maximal marginal utility in the petty-cash category.

It is clear that in optimal solution satisfy $u_C`_{(bC)} = u_P`_{(bP)}$ (i.e., the marginal utility in each category is the same). We assume that money that is not optimally allocated is placed in (i.e., added to or taken from) the petty cash category, it induces a suboptimal solution where $u_C`_{(bC)} \neq u_P`_{(bP)}$.

We add computation cost to this model for every change in the allocation of money in the general-consumption category. Given a non-optimal allocation, we define a shadow price to be the difference in the utility between the optimal allocation and the non-optimal allocation. The assumptions made above imply that this shadow price is asymmetric between loss and gain: the shadow price is higher in the case of a loss (relative to a gain), which implies that the agent will have a smaller threshold for incurring a computation cost to find the optimal solution in case of a loss. The log-normal functional form presented in Section 6.3 is one possible functional form to capture the above model of petty cash category and computation cost.