Collective Decision Making and Jury Theorems

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Abstract

It is more than two hundred years that issues related to collective decision making and to Condorcet jury theorems are studied and publicly discussed. Recently, there is a burgeoning interest in the topic by academicians as well as practitioners in the fields of Law, Economics, Political Science and Psychology. Typical questions are: what is the optimal size of a panel of decision makers such as a jury, a political committee or a board of directors? which decision rule to utilize? who should be the members of the team, representatives or professionals? what is the effect of strategic behavior, group dynamics, conflict of interests, free riding, social interactions and personal interdependencies on the final collective decision?

The purpose of this article is to present the state of the art in the field, to suggest further research and to allude to possible future developments regarding public choice and collective decision making.

Content

1. Introduction

The Marquis de Condorcet (1743-1794) is considered one of the pioneers of the social sciences. In the English literature, Baker (1976) and Black (1958) were among the first to turn the attention of the scientific community to the importance of Condorcet's writings [see Young (1995)]. No doubt that his seminal work from 1785 has formed the basis for the development of *social choice* and *collective decision making* as modern research fields. In the last forty years, hundreds of articles have been written in these fields and at least five new journals which are mainly devoted to these subjects have been published. This article focuses on Condorcet jury theorems and related collective decision issues which might be of interest to scholars in the areas of Law, Economics and Political Science.

In 1785 no jury existed in France. Condorcet applied probability theory to judicial questions and argued that the English demand for unanimity among jurors was unreasonable and suggested instead a jury of twelve members that can convict with a majority of at least ten. In 1815 the first French juries used Condorcet's rule but later adopted the simple majority rule. At that time the mathematician Laplace argued that simple majority is a dangerous decision rule for juries. Since 1837 juries had been established on several different plans, but the French law has never believed that one could count on twelve people agreeing (see Hacking (1990) ch.11). Since the seventies of the last century, several works analyzed the jury system applying probability theory as well as statistical data. Gelfand and Solomon (1973), (1975), Gerardi (2000), Klevorick and Rothschild (1979) and Lee, Broedersz and Bialek (2013) are a few such studies. The

common English jury system is an extreme form of a qualified majority rule while the French version is less so.

The debate about the size and the composition of the jury and the kind of decision rule it should use has been going on for more than two centuries and will probably continue. This article presents the state of the art in collective decision making and jury theorems as well as some expectations about further developments in the field.

Condorcet (1785) makes the following three-part statement:

1) The probability that a team of decision makers would collectively make the correct decision is higher than the probability that any single member of the team makes such a decision.

2) This advantage of the team over the individual performance monotonically increases with the size of the team.

3) There is a complete certainty that the team's decision is right if the size of the team tends to infinity,i.e., the probability of this event tends to one with the team's size.

A "Condorcet Jury Theorem" (henceforth, CJT) is a formulation of sufficient conditions that validate the above statements. There are many CJT's, but Laplace (1815) was the first to propose such a theorem. Some of the following conditions are explicitly stated by Laplace as sufficient conditions and the others are only implicitly assumed. A major concern of the modern collective decision-making literature is whether or not each of

these conditions is also necessary for the validity of Condrcet's statements. These conditions that are listed below will serve as a road map for the rest of the paper.

Specification of the problem faced by the jury.

1. There are two alternatives.

2. The alternatives are symmetric.

Specification of the decision rule applied by the team:

3. The team applies the simple majority rule.

Specification of the properties of the jury members:

4. All members possess identical competencies.

5. The decisional competencies are fixed.

6. All members share identical preferences.

Specification of the behavior of the jury members:

7. Voting is independent.

8. Voting is sincere.

The core of Laplace's proof is the calculation of the probability of making the right collective decision, (to be denoted by Π), where Π is calculated by using Bernoulli's theorem (1713). Laplace shows, first, that Π is larger than the probability of making the right decision P by any single member of the team, second, that Π is a monotone increasing function of the size of the team and, third, that Π tends to unity with the size of the team. Of course, besides the above conditions there is an additional trivial condition that the decisional capabilities of decision makers are not worse than that of tossing a fair

coin. Namely, the probability P of making the correct decision is not less than one half. Other properties of Π are that it is monotone increasing and concave in the size of the team, n, and in the competence of the individuals, P. Thus, given the costs and benefits of the team members' identical competence, one can find the optimal size of a team and the optimal individual's competence by comparing marginal costs to marginal benefits.

The above eight conditions are quite strong and there is much doubt whether they exist in reality. The present article studies the consequences of the relaxation of these conditions which has been and apparently will continue to be a major topic in the literature of collective decision making related to CJT.

Given the limited space, it is possible to cover only some of the important subjects. Naturally, to some extent this article will be biased towards the interest of the authors.

2. Heterogeneous competencies

Let us start by studying the consequences of relaxing the assumption that the members of the team possess identical competencies. Suppose that there are *n* members in a team facing two symmetric alternatives with decisional competence that can be represented by their fixed probabilities of making the right decision. Denote these probabilities by p = $(p_1,…,p_n)$, where the p_i's are not necessarily identical. By simple numerical examples it is shown that the first two parts of Condorcet's statement are no longer necessarily valid.

Suppose p=(0.95,0.80,0.80). Assuming that the team utilizes a simple majority rule, it is possible to find that the team's competence in this case is Π=0.944. One can see that the competence of the first member of the team is larger. Thus, the first part of the

statement is violated. Add now two more members to the team both with a competence of 0.6 to obtain the enlarged team where p=(0.95,0.80,0.80,0.60,0.60). Utilizing simple majority rule, the team's competence is now reduced to Π=0.910. The second part of the statement is therefore also violated [The accuracy of majority decisions in groups with added members is discussed in Feld and Grofman (1984)].

The numerical examples that are presented here are far from being extreme. For instance, Nitzan and Paroush (1985 p.68) find that if one draws a random sample of individual's competencies from a uniform distribution over the range [$1\!/2$, 1], then within five-member teams, in more than 20% of the cases the first part of Condorcet's statement is violated. To wit, the competence of a random five-member team which utilizes a simple majority rule is smaller in at least 20% of the cases than the probability that the most qualified person of the team would decide correctly.

In contrast to the first two parts of the Condorcet's statement, the survival of the third part is somehow surprising. Many attempts have been made to prove the validity of the third part in case of heterogeneous teams [see Boland (1989), Fey (2003), Kanazawa(1998) and Owen, Grofman and Feld ,(1989)]. In fact, the following is a well known version of CJT: "If a team of decision makers utilizes a simple majority rule, the decision would be perfectly correct in the limit given that the size of the team tends to infinity, even if the individual competencies, the p_i's, are different, provided that $p_i \geq 1\!/2+\varepsilon$, where the value of ε is a positive constant regardless of how small it is". The proof of the theorem relies on the proof of Laplace where P= $1\!/2+\varepsilon$ combined with the fact that Π is an increasing function of the team members' competencies.

One comment is in order. Paroush (1998) shows by means of an example that $\Pi =$ $1/2$ in the limit even if all $p_i > 1/2$. This counter intuitive example occurs when the sequence of the p_i's decreases rapidly from above towards $1/2$. Thus, it looks that regardless of how small it is, the presence of a fixed ε is not only a sufficient but also a necessary condition. But it turns out that existence of a fixed ε is inessential for CJT. Relying on the law of large numbers, Berend and Paroush (1998) found a necessary and sufficient condition for the validity of the third part of the statement, even in the case that competencies are not identical. The condition is: $(A_n - 1/2) \sqrt{n} \rightarrow \infty$, where $A_n = \sum p_i / n$. The meaning of this condition is that if all $p_i > 1/2$, the team's competence Π is still equal to one in the limit, even if the sequence of the p_i 's decreases towards $1/2$ but only in a "moderate pace".

The next section shows that the first part of the statement can be retained if the team utilizes not a simple but a weighted majority rule where the weights are adjusted to the individuals' competencies given that they are known.

3. Optimal decision rules

Condorcet who was a great fan of the wisdom of the crowd was among the first to lay down the philosophical foundations of Democracy. In particular, he strongly believed in the superiority of simple majority over other decision rules [see Grofman (1975)]. But, it is known today that if the team members' competencies are "public knowledge", the simple majority rule may lose its superiority. Many studies suggested alternative criteria for the optimality of a decision rule [see for instance Rae (1969), Straffin (1977) and Fishburn and Gehrlein (1977)]. However, in what follows, the assumption that the team's

competence, Π, is the criterion for the optimality of the decision rule is preserved. Note that given that the team members share homogenous preferences, this criterion is also consistent with (equivalent to) expected utility.

Allowing heterogeneous decisional capabilities, Shapley and Grofman (1981, 1984) and Nitzan and Paroush (1982) find that the optimal decision rule is a weighted majority rule (WMR) rather than a simple majority rule (SMR). By maximization of the likelihood that the team makes the better of the two choices it confronts, they also establish that the optimal weights are proportional to the *log* of the odds of the individuals' competencies, i.e. the weights, w_i ,are proportional to $log[p_i/(1-p_i)]$.

Let us apply this result to the case of a team of three members with known competencies of $p_1 > p_2 > p_3$, where p_i is the probability that member i decides correctly. It is obvious that the simple majority is the superior decision rule if and only if $\{log\ [p_2/(1-p_2)] +log\ [p_3/(1-p_3)]\} > log\ [p_1/(1-p_1)]$ or $[p_2/(1-p_2)]\bullet[p_3/(1-p_3)] >p_1/(1-p_1)$. Thus, the simple majority rule looses its superiority to the expert rule if either p_1 is close enough to 1 or p_3 is close enough to $1/2$.

For the general case of a team of *n* members, one can make the following statement: A sufficient condition for the violation of the first part of Condorcet's statement is the inequality: $p_1/(1-p_1) > \Pi\ [p_i/(1-p_i)]$ where p_1 is the competence of the most competent person in the team and Π indicates here the multiplication of the odds of the rest of the members.

In the case of a team with more than three members, besides the simple majority rule (SMR) and the expert rule (ER), there are other efficient rules that are potentially

optimal. For instance, in a five-member team there exist seven different efficient potentially optimal rules. These rules include "the almost expert rule" , "the almost majority rule" and "the tie-breaking chairman rule". The number of efficient rules increases very rapidly with the team size. For instance, in a team of nine members the number of efficient rules is already 172,958 [see Isbell (1959)]. Now the following question is raised: is there a mathematical relation between the size of the team and the exact number of efficient rules? This simple question is still an open one.

Note that a choice of the optimal size of a team may be considered as a special case of a choice of the optimal decision rule. For instance, the optimal rule within a team of seven members could be a restricted majority rule where simple majority rule is utilized only by the three or by the five most qualified members where the rest of the members are inessential (their decisions are disregarded, that is, the weight assigned to an inessential member is zero).

Paroush and Karotkin (1989) investigate the robustness of optimal restricted majority rules over teams with changing size. One of the useful findings is that optimal restricted majority rules are robust over reduction of the size of the team. For instance, suppose that a SMR is the optimal rule in a team of seven members, then a SMR will stay optimal even in a team of five where the two least qualified members of the original team are discarded. The clarification of the conditions for the robustness of optimal decision rules when the team members' competencies are allowed to vary is an important topic for future research.

4. Order of decision rules

The existence of order relations among decision rules is firstly noted in Nitzan and Paroush (1985, p.35). Existence of such an order means, first, that the number of rankings of m efficient rules is significantly smaller than the theoretical number $m!$ of all possible rankings of these rules and, secondly, that its existence is independent of the team's competence.

Beyond the theoretical interest in studying order among efficient decision rules, the information about the order has useful applications. Since the order relations are independent of the specific competencies of the decision makers, the knowledge about the order of the rules is important in cases where the competencies are unknown or only partially known. For instance, if for some reason(e.g., excessive costs) the optimal rule cannot be used, then even in the absence of knowledge about the decisional competencies, the team can identify by the known order of the decision rules the second best rule, the third best and so on.

Karotkin, Nitzan and Paroush (1988) show that six out of the seven efficient rules available to a team of five members generate a complete scale or an essential ranking. Thus, only less than 200 rankings out of $7! = 5,040$ possibilities are feasible.

Paroush (1997) finds that the SMR and the ER are always polar rules in the sense that if one of them is optimal and thus the most efficient, the other one is the least efficient. He also identified second best rules and penultimate rules in cases that majority rules (simple or restricted) are optimal or the most inferior.

Karotkin (1998) exposes an interesting analogy between the network yielding the ranking of all weighted majority rules by efficiency and the weighted majority games in terms of winning and losing coalitions.

Paroush (1990) shows how to apply the knowledge of essential ranking of decision rules when the team of decision makers faces multi-choice problems. Karotkin and Paroush (1994) apply the essential order in cases where the team members' competencies may be subject to some fluctuations. For instance, if the optimal rule loses its optimality status due to fluctuations, then one of its neighbors on the ranking will become the second best.

However, the complete order relations among efficient decision rules as well as the coiled network among them are still to be discovered; the task of studying these issues is far from being exhausted.

5. Competencies are not fixed

Assume that the individual's decisional competence p_i is not constant anymore but monotonically increasing (in a decreasing rate) with some invested resources, c_i , which are intended to improve the individual's competence p_i. Such an investment in human capital might be a direct cost in the form of pecuniary outlay or an indirect cost in the form of money equivalent of the time and efforts necessary to collect data or evidence, to elaborate, process and transfer the relevant information into a decision. It also includes the costs necessary to shorten the delay of reaching a decision. Note that in most cases such an investment is very specific and depreciates completely immediately after the voting.

The question of optimal investment in human capital is important and still open for future research. Only few results are available and they have been obtained under restrictive assumptions.

Assume that all the team members possess an identical learning function. To wit, for each one of the members $p_i(c_i) = p(c_i)$. It is worth noting that this assumption is consistent with a liberal viewpoint that the differences among individuals' competencies are due to reasons such as unequal opportunities and are not due to genetic, gender, race or origin differences.

Based on this model, Nitzan and Paroush (1980), Karotkin and Paroush (1995) and Ben-Yashar and Paroush (2003) obtain the following results.

1) Given a standard learning function such as the logistic function, which is commonly used in psychology and education, the optimal social policy is first to invest in the least competent member of the team and only then in the more competent ones and only finally in the most competent member.

2) Denote by $c^* = (c^*_1, c^*_2, \ldots, c^*_n)$ the socially optimal investment in each member where the investment in each person is decided in a centralized system. Denote by $c^{**} = (c^{**}_1, c^{**}_2, \ldots, c^{**}_n)$ the optimal investment that is determined by each individual in a decentralized system. Assume that individual's reward is independent of her own vote but depends only on the success of the whole team. Under this assumption, the motivation to free-ride emerges and the necessary result is that $c^* > c^{**}$.

3) An incentive-scheme which compensates not only for the team's success but also for the individual's success is a possible remedy against free riding . However, such a remedy has two deficiencies as by-products. First, it encourages individuals to vote insincerely (see Section 9). Second, it encourages individuals to violate the assumption of independence (see Section 8). For instance, even if the simple majority is the optimal decision rule and even if individuals share identical preferences, each of them will try in this case to copy the vote of the most competent member in order to increase the likelihood of her personal reward.

4) Modern technology offers ways to overcome the above deficiency. Secret voting together with automatic recording of votes will guarantee both the independence as well as the possibility to compensate individuals for correct voting.

It is worth noting that free-riding behavior in information acquisition affects not only the optimal investment in human capital, but also the optimal size of the team. Gradstein and Nitzan (1987) and Mukhopadhaya (2003) find, for instance, that in contrast to Condorcet's second statement, a larger jury may reach worse decisions.

In an extended setting of endogenous competencies, Ben Yashar and Nitzan (2001c) demonstrate that it is no longer the case that the SMR is superior to the ER, even if individuals share identical skills. Since now the objective of the team takes into account both the probability of making a correct collective decision as well as the aggregate costs associated with investment in human capital of the team members, it may very well be the case that it is better to invest in a single member and apply the ER rather

than in all the members. Thus, Condorcet's statement may not be valid. Further discussion on variability of decisional skills and of information acquisition within teams appears in Gerling et al. (2005), Gradstein and Nitzan (1988) and Nitzan and Paroush (1984c).

In concluding this section, we wish to point out that the area of the economics of decision making that takes into account various elements of cost and benefit is only in its initial stage and further developments are expected.

6. Diverse Preferences

Consider a group of individuals with different preferences (note that such a group can no longer be called "a team"). A typical such group is a political committee, a parliament, a randomly selected jury or any representative decision panel whose members have different utilities or social values. In a binary setting, it is clear that the alternative which is considered "correct" by one of the members might be considered "incorrect" by another. There is no agreed upon "truth" to seek; each member seeks her own subjective truth. Social choice theory struggles to find a decision rule that aggregates the diverse individual preferences and aims to come up with some "desirable" social preference relation or some common social objective. The classical normative approach establishes such objectives or social welfare functions on the basis of philosophical considerations or principles of justice, Rawls (1971). But the more modern approach in the social choice literature is axiomatic. The axiomatic approach assumes a set of "reasonable" axioms or "desirable" properties that are deemed as necessary and sufficient conditions for the derivation of an aggregation rule that enables society to take a collective action even in

situations where conflicts of interests prevail among its members. We shall mention here a few studies that use the axiomatic approach in a binary setting. May (1952) derives SMR, Fishburn (1973) and Nitzan and Paroush (1981) characterize WMR and Houy (2007) and Quesada (2010) come up with qualified majority rules.

The following dilemma still remains. Which of the two juristic systems is socially preferable: a randomly selected group of jurors who possess diverse preferences as well as a variety of social norms or a team of professional judges who share a common goal of seeking the truth? Obviously, this dilemma has important applications, but as far as we know, there is neither an answer to this question nor even a theoretical framework within which the question can be analyzed.

In the context of Condorcet's setting, given the individuals' common objective and diverse information which yields their decisions, the optimal collective decision rule can be identified, as we have seen in Section 3. However, in a binary setting and diverse preferences, one can reach these same optimal collective decision rules by their unique axiomatic characterization. In a more general multi-alternative setting, however, the potential success of the axiomatic approach is clouded by the classical Impossibility Theorems of Arrow (1951) and his followers. As is well known, if few reasonable axioms have to be satisfied by the aggregation rule, a social welfare function does not exist. This type of finding, which has become one of the cornerstones of social choice theory, has raised a wave of works that attempted (mostly in vain) to disperse the pessimistic atmosphere implied by Arrow's "Impossibility Theorem".

7. Asymmetric alternatives

Suppose now that asymmetry exists between the two alternatives. Asymmetry may stem from three different sources. First, the priors of the two states of nature (the defendant being innocent and the defendant being guilty) may be different. For example, assume that most people are not criminals so in order to convict a defendant the state "innocent" is considered as a status-quo that has to be refuted and the state "guilty" is deemed as the alternative that has to be proved. Thus, a collective decision to convict is expected to be "beyond any doubt", whereas collective acquittal may remain doubtful. Second, the net benefits of a correct decision under the two states of nature can be different. The two types of errors, acquittal of a guilty defendant and conviction of an innocent defendant, may have different costs. Third, an individual's decisional competency may depend on the state of nature. In particular, the probability to decide correctly if the state is "innocent" can be different from the probability to decide correctly if the state is "guilty". In any case, the decisional competency of an individual is not parameterized by a single probability of making a correct choice but by two probabilities of deciding correctly in the two states of nature.

Under asymmetry it is required that one of the two alternatives, say conviction, will be the collective choice only when it receives the support of a special majority with a quota larger than one half . Thus, under asymmetry the decision rule should be a qualified majority rule (QMR).

QMR are discussed in several works in the political context of constitutions and fundamental laws as well as in relation to juries [see, for instance, Buchanan and Tullock (1962) and Rae (1969)]. Nitzan and Paroush (1984d) were the first to derive the exact

quota necessary for the optimal QMR. However, their quota is derived under the restrictive conditions of identical competencies that are invariant to the state of nature. The special case of identical competencies that depend on the state of nature was extensively analyzed by Sah and Stiglitz(1988) and Sah (1990,1991).

Allowing heterogeneous and state-dependent competencies, Ben-Yashar and Nitzan (1997) specify the expression for both the weight that has to be assigned to each member of the team under the optimal rule as well as the desirable quota of votes necessary for the rejection of the status-quo. The optimal rule in this more general case therefore becomes a weighted qualified majority rule, WQMR. The optimal weight is now proportional to the average of the *logs* of the odds of the two probabilities of making a correct choice and the optimal quota is a function of four parameters: the *log* of the two probabilities of making a correct decision, the *log* of the odd of the prior probability and the *l*og of the ratio of the two net benefits.

The QMR may take two extreme forms. In the first one, the quota is maximal and equal to 1 and in the second form the quota is minimal and equal to 1/n where n is the number of members in the team. In the former case the choice of the alternative with the unfavorable bias requires unanimity (as in the case of the English and American jury) and in the latter case it requires the minimal support of a single member. Ben-Yashar and Nitzan (1997) present necessary and sufficient conditions for the optimality of an extreme quota and Sah and Stiglitz (1985, 1986, 1988) compare the two extreme QMRs in an organizational context. Further discussion of these extreme QMRs appear in Ben-Yashar and Nitzan (2001b) and Koh (2005).

Recently, Ben-Yashar and Nitzan (2013) present sufficient conditions for the superiority of a QMR that is solely based on the prior and entirely disregards the members' decisions. This degenerate "prior based rule" emerges as the most efficient rule in cases where the prior probability is large enough in comparison to the individuals' ability to decide correctly.

8. On the violation of independence

Before taking a vote there is, in most cases, a preliminary process of deliberation and elaboration. Such an early interaction among the members of the panel has two major important contrasting effects on the likelihood to reach collectively a correct decision. One is positive (it increases Π) and the other is negative (it decreases Π). The positive effect is the learning factor which improves the competencies, exactly as investment in human capital does (see Section 5). For instance, in many cases, the process includes witnessing evidences, hearing testimony, listening to other opinions, collecting bits of information, observing signals about states of nature and being exposed to different viewpoints where all these factors elevate the decisional competencies to a higher level.

The negative effect is entirely due to factors of social psychology that distort the independence. For instance, in many cases, social pressures, false persuasive arguments, threats, influential power or leadership charisma enhance conformity, affecting the weights of the optimal WMR and thus reducing the collective competency. Some enlightening examples of negative impacts of interdependencies on the correct decision are presented in the following two best sellers: Surowiecki (2004, ch.3) and Kahneman (2011, ch.7).

Within the dichotomous choice framework, Nitzan and Paroush (1984e) establish that independent voting is always superior to almost any pattern of dependent voting, given that the team utilizes the optimal decision rule. In case of violation of the independency their study also shows how it is possible to adjust the individuals' weights in order to derive a new optimal WMR.

To illustrate the argument, consider a team with the competencies p= (0.70, 0.65, 0.65, 0.65, 0.65). The optimal rule in this case is SMR which yields under independence Π=0.78. Suppose now that the most qualified member exercises her leadership or personal charisma on the rest of the team so that the other members become followers who try to mimic her vote with the mistaken idea that they can increase Π by increasing their own ability from 0.65 to 0.70. Obviously under such dependency the value of Π is reduced to 0.70. Expecting this result, the weights can be adjusted by turning the leader together with another member to be inessentials (with zero weights). After this adjustment the SMR yields Π=0.7182. Let us add that the negative effects of social psychology have stronger impact when the members of the panel possess diverse rather than identical preferences. The reason is that, in the absence of conflicts of interests, the members' self consciousness help to overcome the potential negative psychological effects. This result magnifies the necessity and the importance of **secret voting**. Secret voting may eliminate, at least in part, the negative effects of dependencies. Also note that it is very difficult to impose secret voting on a representative group of decision makers, like a political committee, because the conspicuous vote serves as a report to the public about the members' actual opinion or about the loyalty to their declared attitude. However it is evident that governments allocate a lot of resources to carry out secret elections.

A discussion of the general case where independence of decisional skills does not exist appears in several studies, for example, Berg (1993), Ladha (1992), (1993) who prove a CJT under correlated votes by imposing some restrictions on the correlation coefficients. See also Peleg and Zamir (2012). Their results display again the robustness of Condorcet's third statement.

Besides the available results in the psychological literature, it is expected that further studies of the impact of group dynamics or social behavior in general on collective decision making will yield more results. Such studies will certainly contribute to the debate about the necessity of secret voting.

9. Strategic behavior

It seems that no individual has an incentive to manipulate her vote and act insincerely. It turns out, however, that this may not be a valid assumption; that is, it is possible that individuals have an incentive to vote insincerely.

Using a game theoretic analysis, Austen-Smith and Banks (1996) show that non-strategic voting may be inconsistent with Nash equilibrium, even when all members have identical preferences and beliefs. More precisely, if the number of voters is sufficiently large, then voting based on private information only (informative voting) is generically not a Nash equilibrium of a Bayesian game that formally represents a CJT. The general idea is that an individual's vote affects the collective decision only when it is pivotal. But if all the others vote informatively, the fact that they are tied may be very informative in the sense that this reveals more precise information than that privately held by the

individual. Following this newly revealed information, it may be rational not to vote according to one's own private information.

The following example illustrates the argument. Consider three individuals with identical preferences over two alternatives, A and B. There is uncertainty about the true state of the world, which may be either state A or state B. In each state, individuals receive a payoff of 1 if the alternative of the specific state is chosen and a payoff of 0 otherwise. There is a common prior probability that the true state is A. Individuals have private information about the true state of the world. Majority rule is used to select an alternative. There are two additional assumptions on individuals' beliefs: (1) sincere voting is informative; and (2) the common prior belief that the true state is A is sufficiently strong, such that if any individual were to observe all the three individual signals, then that individual believes B is the true state only if all the available evidence supports the true state being B.

In this example sincere voting is not rational. Suppose that you are playing this game and the two other individuals vote sincerely. So you must be in one of the three following situations: (1) The others have observed that the state is A and accordingly vote for A; (2) The others have both observed B and vote for B; or (3) The others have observed different signals and one votes for A and the other for B. In the first two scenarios the aggregated outcome is independent of your own vote, and in the third scenario your vote is decisive. However, if you are in the third scenario your best decision is to vote for A. Therefore, voting sincerely is not a Nash equilibrium.

In response to the above finding, McLennan (1998) and Wit (1998) found conditions under which Nash equilibrium behavior, although it may be inconsistent with

non-strategic voting, still predicts that groups are more likely to make correct decisions than individuals. Feddersen and Pesendorfer (1997), (1998) adapt the general framework of Austen-Smith and Banks (1996) to the specific case of jury procedures in criminal trials. In their model it is never a Nash equilibrium for all jurors to vote non-strategically under the unanimity rule. Moreover, Nash equilibrium behavior leads to higher probabilities of both convicting the innocent and acquitting the guilty under the unanimity rule than under alternatives rules, including the simple majority rule.

Ben-Yashar and Milchtaich (2007) examine the question of strategic voting, when voters are solely concerned by the common collective interest. They find that under the optimal WMR, individuals do not have an incentive to vote strategically and non-informatively. Such strategy-proofness does not hold under second-best anonymous voting rules. Thus, assigning the proper weight to each voter achieves the optimal team performance and guarantees sincere voting.

10. Latent competencies

In section 3 we present the optimal WMR where the weight assigned to each team's member is proportional to the *log* of the odd of her competency. In reality, individuals' competencies are mostly latent and far from being public knowledge. But, in some cases there are observed signals. It is evident that, in order to demonstrate their expertise and display their excellence, professional consultants such as lawyers and medical doctors decorate the walls of their offices with the records of their education, experience, seniority etc. With the help of "word of mouth" such signals might become even public knowledge. The public should also be aware of false signals and thus, in general,

attempts of subjective evaluation of decisional skills might be unreliable so that the optimal WMR cannot be applied.

However, in certain cases it is possible to use objective statistical data for estimating personal competencies. For instance, Nitzan and Paroush (1985 p.107) use 362 medical records to estimate competencies of cardiologists and Karotkin(1994) uses data of successful appeals for estimation of the competencies (in fact lack of competencies) of professional judges.

Grofman and Feld (1983) and Nurmi (2002) among others also suggest the estimation of voters' decisional capability by the extent that their observed past decisions align with those of the majority. In other words, the majority decision is used as a plausible endogenous proxy for the true alternative. Recently, Baharad et al. (2011), (2012) make use of this idea along with the algorithm suggested by Dempser, Laird and Rubin (1977) to generalize and optimize the method by constructing a specific algorithm which iteratively updates the weights assigned to each of the team members. The procedure converges to consistent and stable evaluation of the voters' skill and, in turn, it enables the identification of the optimal aggregation rule.

Voter equality as implicit in *SMR* is justified in situations where decisional skills are all of sufficient quality and homogeneity. In this case, a sufficiently large number of voters results in convergence to the maximal collective performance. In contrast, optimal decision-making that is based on the voting procedure proposed in Baharad et al. (2011), (2012) is *always* warranted, deriving its superiority from its ability to identify skills through learning from experience. The merit of this procedure and its advantage over

SMR are especially high when skills are sufficiently spread out and the track record of individual decisions is sufficiently abundant.

Many studies attempt to draw conclusions regarding the desirable collective decision rule when competencies are only partially known. For instance, consider the case where the statistical distribution from which competencies are randomly drawn can be specified [see, for instance, Nitzan and Paroush (1985, Ch 5), Nitzan and Paroush (1984a) and the studies by Ben Yashar and Paroush (2000) and Berend and Harmse (1993)].

Let us conclude with the following comments.

Even in the absence of any information on individuals' decisional ability, if a team utilizes SMR the first and the third Condorcet's statements are valid as long as the voters stay away from being fair coins.

1) The value of the unknown probability of correct decision of a team that utilizes SMR exceeds the expected value of the competency of a random representative of the team, i.e., $\Pi > \sum p_i / n$ [see Section 2 above]. Thus, the first statement is valid. Nitzan and Paroush (1985 p.69) offer a simple proof of this result for the case of a three-member team.

2) The third statement is also valid and apparently several Democracies use this fact to run public opinion polls, especially where binary critical questions are at stake. Some restrictions, like age limitation, are imposed to guarantee that voters' competencies are larger than one half.

Is it a coincidence that the custom of using public opinion polls is more popular in the French speaking Democracies ?

11. Miscellaneous

11.1 *The hung-jury problem*

In many states juries must reach a unanimous decision. While a larger jury may be more likely to make the correct decision (Condorcet's second statement), it may not reach any decision at all. Therefore, the balance between making the correct decision and reaching a decision whatsoever must be found. Several papers researched jury decision-making under the unanimity method.

The intuition that larger juries may be more likely to end in hung juries is demonstrated by Grofman (1976). He shows that if the unanimity rule is used then juries of twelve are more likely to be hung juries compared to juries of six.

Alternatively, Luppi and Parisi (2013) show that when information cascades exist the effect of jury size on hung juries can be reduced or even reversed. Information cascades occur when people make a decision based on the observation of others while ignoring or discounting their private information. The intuition here is that with an increase in the size of the jury, the potential for different opinions to result in hung juries increases but, at the same time, if the opinions are expressed in succession, the jurors may adjust their views in light of what others have expressed and an information cascade can be initiated. Therefore, the probability of a unanimous outcome may rise with the size of the jury. Consequently, consensus will be more likely to occur in larger groups if enough jurors are open to opinions of other jurors.

In a different strategic setting, Feddersen and Pesendorfer (1998) study the intuition that the unanimous decision requirement reduces the probability of convicting

an innocent defendant, while increasing the probability for acquitting the guilty defendant. They show that under strategic voting the unanimity rule can lead to a high probability of both types of errors and the probability of convicting an innocent defendant might actually increase with the number of jurors.

11.2 *Multi-Criteria Decisions*

The problem of aggregating individual judgments may take the form of a multi-issue (criteria) problem where individual judgments need to be the basis of the collective outcome. In this generalized setting the simple majority rule can be applied sequentially but in different forms.

Consider a three-member jury that has to come to a conclusion on whether a defendant is guilty or not. Let the available evidence consist of finger prints found at the scene of the crime, an eye witness and the motive for the crime. Each juror decides to convict the defendant if she believes that at least two of the evidence criteria justify this decision, and conviction requires the support of a majority of the jury. For example, let us look at the following table:

	Fingerprints	Witness	Motive	Majority
Juror 1	Guilty	Innocent	Guilty	Guilty
Juror 2	Innocent	Guilty	Guilty	Guilty
Juror 3	Innocent	Innocent	Innocent	Innocent
Majority	Innocent	Innocent	Guilty	Innocent/Guilty

In this situation the defendant is found guilty because according to the judgments of Juror 1 and Juror 2 she is guilty. Notice, however, that the collective decision could have been based not on the majority of the jury members' judgments, but on the majority of the criteria where the signal of each criterion is based on the majority of the jurors' judgments regarding that criterion. In the above example a majority of the jury believes that according to the fingerprints criterion and according to the eye witness criterion the accused is innocent so the defendant is declared innocent according to a majority of the criteria. The different collective outcomes obtained under the two alternative majority-based decision rules raise the so called doctrinal paradox.

Kornhauser and Sager (1986) illustrate the doctrinal paradox using a three-member court where a defendant is liable according to the legal doctrine, if and only if she committed some action which, by contract, she was obliged not to commit. The paradox arises in this context if a majority of the judges find that the defendant is not liable, whereas a majority may still find that she committed some action that, by contract, she was obliged not to perform.

List (2006) discusses possible procedures to escape the doctrinal paradox and obtain a reasonable final decision. One procedure is the minimally liberal one, where each of the judges reaches her own decision and then a majority rule is used. An alternative procedure is the comprehensive deliberate procedure, where each judge gives her judgment on each of the specific issues and these are aggregated by majority rule.

More recently, Hartmann et al. (2010) shift the focus to the search for the procedure that, as in our setting, maximizes the probability of selecting the correct result. In pursuit of the most desirable judgment aggregation procedure, they use probabilistic

methods, looking at the reliability of the judges involved focusing on two approaches: distance-based methods and Bayesian analysis.

Let us conclude by noting that the existence of the doctrinal paradox fortunately implies that, for a particular profile of judgments, at least one of the sequential majority-based decision rules presented above yields the appropriate outcome. A more troubling possibility is the existence of a "truth-tracking paradox" whereby, for a particular profile of judgments, the doctrinal paradox does not exist, nevertheless, the appropriate collective decision is missed. In other words, the two majority-based rules may result in the same collective decision, however, the optimal majority rule surprisingly supports the alternative possible outcome, see Baharad *et al*. (2013).

11.3. *Applications*

The study of uncertain dichotomous choices has numerous applications in a variety of fields where collective decision making is of major significance. This is true in particular in law, medicine, economics, business administration and political science. The relevance to law is clear in court and jury decision making, Gelfand and Solomon (1973), (1975), Gerardi (2000), Karotkin (1994), Romeijn and Atkinson (2011),), Klevorick, Rothschild, and Winship (1984). The same is true in the area of medical diagnostics, Nitzan and Paroush (1985). Within economics (public economics, industrial organization) and business administration, the insights we have discussed can shed light on strategies for an organization's investment in human capital under conditions of uncertainty, Ben Yashar and Nitzan (1998), (2001a), on the size of decision-making bodies such as boards of directors, Kahana and Paroush (1995), Gradstein, Nitzan and Paroush (1990), on the

decision rules applied by firms, Nitzan and Procaccia (1986), on the organizational form of industries, e.g., partnership, Paroush (1985), and on methods of personnel selection in organizations, Ben Yashar et al. (2006). In political science it is central, for instance, to debates on the "epistemic" conception of democracy, List and Goodin (2001). And it can even shed light on decision making by editors of academic journals and, in particular, on whether referees are sufficiently informed about a journal's editorial practices, Ben Yashar and Nitzan (2001d).

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