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Weakly rational expectations☆

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ABSTRACT

Aumann and Drèze (2008) characterised the set of interim expected payoffs that players may have in rational belief systems, in which there is common knowledge of rationality and a common prior. We show here that common knowledge of rationality is not needed: when rationality is satisfied in the support of an action-consistent distribution (a concept introduced by Barelli (2009)), one obtains exactly the same set of rational expectations, despite the fact that in such 'weakly rational belief systems' there may not be mutual knowledge of rationality, let alone common knowledge of rationality. In the special case of two-player zero-sum games, the only expected payoff is the minmax value, even under these weak assumptions.

1. Introduction

When playing in an arbitrary n-person game situation – meaning a strategic game G along with belief hierarchies of the players with respect to each others' types – what payoffs should players expect? Given the immense range of possible belief hierarchies that may be attached to any game, it might seem that little of substance could be said.

Aumann and Drèze (2008), however, characterised the set of rational expectations that a player may have in game situations. They defined rational belief systems as belief systems satisfying the existence of common knowledge of rationality and common priors (over the type space). A player's set of rational expectations are then the set of interim expected values that the player can have as calculated conditioned on his or her knowing his or her own type in a rational belief system. Aumann and Drèze (2008) then showed that such rational expectations can be exactly characterised as the set of conditional payoffs of the correlated equilibria of a related game, called the doubled game. In particular, they proved that under rational belief systems the expectation of any two-player zero-sum game situation is the value of the underlying game. This significantly strengthened the arguments, going all the way back to von Neumann (1928), in favour of viewing the minmax value as the 'correct' solution concept for zero-sum games.

Here we revisit the definition of a rational belief system, in line with what has come to be called the Wilson doctrine,[1] a programme for weakening the reliance on common knowledge in many models. Common knowledge of rationality is a very demanding requirement. If expressed in terms of belief hierarchies, it requires infinite levels of knowledge recursion. If expressed in terms of type spaces, it assumes that every possible type in the type space is always rational.

The common prior may also be regarded as a conceptually demanding assumption. It is not robust to small perturbations (Hellman and Samet, 2012), and its characterisation by the no betting condition (Samet, 1998; Feinberg, 2000) further indicates that its existence may depend in subtle ways on the higher-order beliefs of the players.

To weaken the conditions in Aumann and Drèze's definition of a rational belief system, we make use of action-consistent distributions, a concept introduced by Barelli (2009) for studying the epistemic conditions for Nash and correlated equilibria.[2]

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[1] 'Game theory has a great advantage in explicitly analysing the consequences of trading rules that presumably are really common knowledge; it is deficient to the extent it assumes other features to be common knowledge, such as one player's probability assessment about another's preferences or information. I foresee the progress of game theory as depending on successive reductions in the base of common knowledge required to conduct useful analysis of practical problems. Only by repeated weakening of common knowledge assumptions will the theory approximate reality', in Wilson (1987).

[2] See also Bach and Tsakas (2013) for a recent paper making use of the same concepts.

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Action-consistent distributions are a generalisation of common priors defined relative to functions that are measurable with respect to the available actions of the players, not the entire σ-algebra of their beliefs. We define a weakly rational belief system to be one in which rationality is satisfied in the support of an action-consistent distribution.

Weakly rational belief systems are a strict superset of Aumann and Drèze's (strongly) rational belief systems. They also represent a significant axiomatic weakening—not only is common knowledge of rationality not required in weakly rational belief systems, even *mutual* knowledge of rationality may not hold. This might lead one to suppose that the set of rational expectations that weakly rational belief systems afford players would be a superset of Aumann–Drèze rational expectations: with fewer restrictions on beliefs, perhaps players may sometimes find reasons to expect greater payoffs?

The surprising answer is that it makes no difference. The set of rational expectations remains exactly the same. Rational expectations do not depend on assuming the full power of common knowledge of rationality. In the special case of two-player zero-sum game situations, the value of the underlying game is the only expectation that players may have, even under our weakened rationality assumptions.

2. Preliminaries and definitions

A strategic n-person *game* G consists of a set I of n players along with n finite sets $A_1, A_2, \ldots, A_n$ (the action sets of the players) and n functions $h_1, h_2, \ldots, h_n$ from $A := A_1 \times A_2 \times \cdots \times A_n$ to $\mathbb{R}$ (the payoff functions).

A *belief system* B for G consists of:

1. For each player i a finite set T_i whose members are the *types* of i.
2. For each type t_i of each player i:
   (a) A (pure) *strategy* of i denoted $\sigma_i(t_i) \in A_i$.
   (b) A probability distribution θ_{t_i} on the set of $(n-1)$-tuples of types of the other players, called t_i's *theory*.

Label $T := T_1 \times \cdots \times T_n$. As standard in the literature, we define $T^{-i} := \prod_{j \neq i} T_j$. An element in T is called a *state* and any subset $E \subseteq T$ is called an *event*. In particular, the event

$$[t_i] := \{t \in T \mid \mathrm{proj}_{T_i} t = t_i\} \tag{1}$$

contains all states at which i's type is t_i.

The theory θ_{t_i} associated with player i's type t_i is a probability distribution over T^{-i}. We will find it convenient to extend it to a probability distribution $p(\cdot|t_i) \in \Delta(T)$ over T, which is accomplished by assigning to each $E \subseteq T$ the probability

$$p(E|t_i) := \theta_{t_i}(\{t^{-i} \in T^{-i} \mid (t_i, t^{-i}) \in E\}). \tag{2}$$

Eqs. (1) and (2) imply in particular that $p([t_i] \mid t_i) = 1$, i.e. 't_i knows its own type'.

The action function of player i is a function $\mathbf{a}_i : T \to A_i$ such that $\mathbf{a}_i(t) := \sigma_i(t_i)$ for all $t \in [t_i]$. Setting $\mathbf{a}(t) := (\mathbf{a}_1(t), \ldots, \mathbf{a}_i(t), \ldots, \mathbf{a}_n(t)) \in A$, the payoff functions are then extended by defining $\mathbf{h}_i : A_i \times T^{-i} \to \mathbb{R}$ by

$$\mathbf{h}_i(a_i, t^{-i}) = h_i(\mathbf{a}(t)). \tag{3}$$

The event $[a_i] := \{t \in T \mid \mathbf{a}_i(t) = a_i\}$ contains the states at which player i plays a_i. The event $[a] := [a_1] \cap \cdots \cap [a_n]$ is the event that action profile a is played, and the function $\boldsymbol{\phi}_i : T \to \Delta(A^{-i})$ is i's *conjecture* over actions at each state, defined by

$$\boldsymbol{\phi}_i(t)(a^{-i}) := p([a^{-i}] \mid t_i) \tag{4}$$

for all a^{-i}.

A player i is *rational* at state t if he or she maximises his or her expected payoff at t given his or her conjecture, his or her strategy and the payoff function, that is

$$R_i := \{t \in T \mid \mathbf{a}_i(t) \in BR_i(\boldsymbol{\phi}_i(t))\}$$

is the event that i is rational, where BR_i stands for player i's best-response correspondence. *Rationality*, that is, the event where all players are rational, is $R_1 \cap \cdots \cap R_n$. We say that *rationality is satisfied by* $E \subseteq 2^T$ if E is a subset of the event of rationality.

Knowledge is expressed in terms of events. Given a state $t = (t_1, \ldots, t_n)$, player i *knows* an event E at t if $\mathrm{supp}(p(\cdot|t_i)) \subseteq E$. Hence we can define a knowledge operator $K_i : 2^T \to 2^T$ by defining $K_i(E)$ to be the set of states at which player i knows E. An event E is *mutually known* at t if $t \in K(E)$, where $K(E) = \bigcap_{i \in I} K_i(E)$. E is *commonly known* at t if $t \in K^n(E)$ for all $n > 0$. Since rationality is an event, a belief system specifies in particular whether or not there is common knowledge of rationality at any given state.

A probability measure $P \in \Delta(T)$ is a common prior if for every $i \in I$ and for every $t_i \in T_i$, the theory $p(\cdot|t_i)$ coincides with the conditional distribution of P given $[t_i]$ whenever $P([t_i]) > 0$. A weaker condition, introduced by Barelli (2009), is called action consistency. Denote by $\mathcal{F}_A^T := \{b : T \to \mathbb{R} \mid \mathbf{a}(t) = \mathbf{a}(t') \Rightarrow b(t) = b(t')\}$, the set of A-measurable random variables. A probability measure $\mu \in \Delta(T)$ is *action consistent* if

$$\sum_{t \in T} \mu(t) b(t) = \sum_{t_i \in T_i} \mu([t_i]) \sum_{t' \in [t_i]} p(t' \mid t_i) b(t') \tag{5}$$

for every i and every $b \in \mathcal{F}_A^T$.

Define for each action profile $a \in A$ the indicator function

$$\mathbb{1}_a(t) = \begin{cases} 1 & \text{if } \mathbf{a}(t) = a \\ 0 & \text{otherwise} \end{cases}$$

over states $t \in T$. Then when b is in particular $\mathbb{1}_a$, Eq. (5) yields

$$\mu([a]) = \sum_{t_i \in T_i} p([a] \mid t_i) \mu([t_i]) \tag{6}$$

for every $i \in I$, which gives an intuition: under action consistency, for each player the measure of each action profile is given by a convex weighting of the measure of each $[t_i]$, with the weights determined by the theories that the types have. Alternatively, one may start with Eq. (6) and then, for each set of real numbers $\{b_a\}_{a \in A}$, define a function $b := \sum_{a \in A} b_a \mathbb{1}_a \in \mathcal{F}_A^T$. Eq. (6) then yields Eq. (5).

Definition 1. A belief system for G is *(strongly) rational* if common knowledge of rationality holds at all states $s \in S$ and there exists a common prior. (This concept is simply called *rational* in Aumann and Drèze (2008).)

A belief system for G is *weakly rational* if there exists an action-consistent probability distribution $\mu \in \Delta(T)$ such that rationality is satisfied in $\mathrm{supp}(\mu)$.

Denote the set of strongly rational belief systems for G by $SRBS(G)$ and the set of weakly rational belief systems for G by $WRBS(G)$. ♦

By the definitions, a common prior is always action consistent; hence a strongly rational belief system is weakly rational. The following example, which is derived from Example 2.2 in Barelli (2009), shows that the set of weakly rational belief systems for G is a strict superset of the set of strongly rational belief systems for G.

Example 1. Consider the two-player game

	L	R
U	5, 1	0, 0
D	4, 4	1, 5

with type space

	L_1	R_1
U_1	$1, \frac{1}{4}$	$0, 0$
D_1	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, 1$
U_2	$\frac{1}{4}, \frac{1}{4}$	$\frac{3}{4}, 0$

.

The belief system here is weakly rational: the probability distribution $\mu = (\frac{1}{3}(U_1, L_1), 0(U_1, R_1), \frac{1}{3}(D_1, L_1), \frac{1}{3}(D_1, R_1), 0(U_2, L_1), 0(U_2, R_1))$ is action consistent, and the action of each player in each state with positive support in μ is rational.

But strong rationality does not hold. There can be no common prior in this belief system, because any prior for the row player must assign positive probability to (U_2, R_1), but any prior for the column player must assign zero probability to this entry. Furthermore, there is no mutual knowledge of rationality (let alone common knowledge of rationality) in this belief system, because action U is not rational for type U_2, yet type L_1 ascribes positive probability to type U_2 in his or her theory. ♦

Definition 2. A *strongly rational expectation* in G is an expected payoff of some type t_i in some strongly rational belief system for G.

A *weakly rational expectation* in G is an expected payoff of some type t_i such that $\mu([t_i]) > 0$ in some weakly rational belief system for G, where μ is an action-consistent probability distribution.

Denote as $SRE(G)$ (resp. $WRE(G)$) the set of strongly (resp. weakly) rational expectations. ♦

The following definitions are taken from Aumann and Drèze (2008).

Definition 3. Fix a player i. Let r_i be a particular action of player i in G. The *augmented game* G_{2r_i} is the n-person game in which r_i is replaced by two copies of r_i and the payoff does not depend on which copy is used. ♦

Definition 4. The *doubled game* $2G$ is the game that is the same as G except that each of player i's actions are listed twice. ♦

3. The main results

The first result is the analogue, in weakly rational expectations, of Theorem B′ of Aumann and Drèze (2008).

Proposition 1. *Every weakly rational expectation of player i in a game G is a conditional correlated equilibrium payoff of player i over an augmented game G_{2r_i}, for some $r_i \in A_i$.*

Example 1 shows that $SRBS(G) \subsetneq WRBS(G)$. This might lead one to suspect that $SRE(G) \subsetneq WRE(G)$. The main theorem, however, shows that $SRE(G) = WRE(G)$.

Theorem 1. *The set of weakly rational expectations in a game G equals the set of strongly rational expectations in G.*

Since we know that the strongly rational expectation of a two-player zero-sum game is the value, the weakly rational expectation must also be the value.

Corollary 1. *The expectation of any two-player zero-sum game in a weakly rational belief situation is the value of the underlying game.*

4. Proofs

Proof of Proposition 1. Let B be a weakly rational belief system for G, with μ an action-consistent distribution and t_i^1 and t_i^2 two types of player i, satisfying $\mu([t_i^1]) > 0$ and $\mu([t_i^2]) > 0$, that play the same strategy, i.e., $\sigma_i(t_i^1) = \sigma_i(t_i^2) = \widehat{a}_i \in A_i$.

Define B' to be the belief system obtained from B by 'amalgamating the types' t_i^1 and t_i^2 into one type u_i^0, as follows. In B' the types are $T'_1, \ldots, T'_n$, with $T'_j = T_j$ for $j \neq i$ and $T'_i = \{T_i \setminus \{t_i^1, t_i^2\}\} \cup u_i^0$. For each $j \neq i$ and $t^{-j} \in T'^{-j}$, we set $p'(t^{-j} \mid t_j) := p(t^{-j} \mid t_j)$ if $(t^{-j})_i \neq u_i^0$. For the remaining case,

$$p'(t^{-\{j \cup i\}}, u_i^0 \mid t_j) := p(t^{-\{i,j\}}, t_i^1 \mid t_j) + p(t^{-\{j \cup i\}}, t_i^2 \mid t_j). \quad (7)$$

Intuitively, this is exactly what one expects from an amalgamation of two types of player i from the perspective of another player j: the likelihood of u_i^0 is given by summing those of t_i^1 and t_i^2.

The theories of all types $t_i \in T'_i \setminus \{u_i^0\}$ are simply carried through as is: $p'(\cdot|t_i) = p(\cdot|t_i)$. Furthermore, the strategies of all types except for u_i^0 remain the same as in B, that is, $\sigma'_j(t_j) := \sigma_j(t_j)$ for all j such that $t_j \neq u_i^0$, while $\sigma'_i(u_i^0) := \sigma_i(t_i^1) = \sigma_i(t_i^2) = \widehat{a}_i$. Note that this implies that the conjectures over actions of all types other than u_i^0 carry through 'as is' to B'.

Define a probability measure μ' over B' by

$$\mu'(t_j, t^{-j}) := \mu(t_j, t^{-j}) \quad \text{if } (t_j, t^{-j}) \notin [u_i^0], \quad (8)$$

and

$$\mu'(u_i^0, t^{-i}) := \mu(t_i^1, t^{-i}) + \mu(t_i^2, t^{-i}) \quad (9)$$

for all t^{-i}. Note that this implies that for all types $t_j \neq u_i^0$, we have $\mu'([t_j]) = \mu([t_j])$, while $\mu'([u_i^0]) = \mu([t_i^1]) + \mu([t_i^2])$.

Finally, for the type u_i^0 (noting that $\mu'([u_i^0] \neq 0)$), set

$$p'(t^{-i} \mid u_i^0) := \frac{\mu([t_i^1])}{\mu'([u_i^0])} p(t^{-i} \mid t_i^1) + \frac{\mu([t_i^2])}{\mu'([u_i^0])} p(t^{-i} \mid t_i^2), \quad (10)$$

which in words is stating that the p' probability of t^{-1} conditional on u_i^0 is a weighted average of the p probability of t^{-1} conditional on t_i^1 and the p probability of t^{-1} conditional on t_i^2, with the weights determined by the relative measures of $\mu([t_i^1])$ and $\mu([t_i^2])$ with respect to $\mu'([u_i^0])$.

Lemma 1. *μ' is action consistent in B'.*

Proof of Lemma 1. For each $b' \in \mathcal{F}_A^{T'}$ define $b \in \mathcal{F}_A^T$ by

$$b(t) = \begin{cases} b'(u_i^0, t^{-i}) & \text{if } t = (t_i^1, t^{-i}) \text{ or } t = (t_i^2, t^{-i}) \\ b'(t) & \text{otherwise.} \end{cases}$$

Using this definition and Eqs. (8) and (9), it follows that

$$\begin{aligned} \sum_{t' \in T'} \mu'(t') b'(t') &= \sum_{t' \in T' \setminus [u_i^0]} \mu'(t') b'(t') + \sum_{t' \in [u_i^0]} \mu'(t') b'(t') \\ &= \sum_{t \in T \setminus ([t_i^1] \cup [t_i^2])} \mu(t) b(t) + \sum_{t \in [t_i^1]} \mu(t) b(t) \\ &\quad + \sum_{t \in [t_i^2]} \mu(t) b(t) \\ &= \sum_{t \in T} \mu(t) b(t). \end{aligned}$$

Then, using the fact that μ is action consistent yields, for $j \neq i$ and any $b' \in \mathcal{F}_A^{T'}$,

$$\begin{aligned} \sum_{t' \in T'} \mu'(t') b'(t') &= \sum_{t \in T} \mu(t) b(t) \\ &= \sum_{t_j \in T_j} \mu([t_j]) \Bigg(\sum_{t'' \in [t_j] \setminus ([t_i^1] \cup [t_i^2])} p(t'' \mid t_j) b(t'') \\ &\quad + \sum_{t'' \in [t_j] \cap [t_i^1]} p(t'' \mid t_j) b(t'') \end{aligned}$$

$$+ \sum_{t'' \in [t_j] \cap [t_i^2]} p(t'' \mid t_j) b(t'') \Bigg) \quad (11)$$

$$= \sum_{t_j' \in T_j'} \mu'([t_j']) \Bigg(\sum_{t'' \in [t_j'] \setminus [u_i^0]} p'(t'' \mid t_j') b'(t'')$$

$$+ \sum_{t'' \in [t_j'] \cap [u_i^0]} p'(t'' \mid t_j') b'(t'') \Bigg) \quad (12)$$

$$= \sum_{t_j' \in T_j'} \mu'([t_j']) \sum_{t'' \in [t_j']} p'(t'' \mid t_j') b'(t''). \quad (13)$$

On the other hand, for i and any $b' \in \mathcal{F}_A^{T'}$,

$$\sum_{t' \in T'} \mu'(t') b'(t') = \sum_{t \in T} \mu(t) b(t)$$

$$= \Bigg(\sum_{t_i \in T_i \setminus \{t_i^1, t_i^2\}} \mu([t_i]) \sum_{t'' \in [t_i]} p(t'' \mid t_i) b(t'') \Bigg)$$

$$+ \mu([t_i^1]) \sum_{t'' \in [t_i^1]} p(t'' \mid t_i^1) b(t'')$$

$$+ \mu([t_i^2]) \sum_{t'' \in [t_i^2]} p(t'' \mid t_i^2) b(t''). \quad (14)$$

We can rewrite (14) as

$$\Bigg(\sum_{t_i \in T_i \setminus \{t_i^1, t_i^2\}} \mu([t_i]) \sum_{t'' \in [t_i]} p(t'' \mid t_i) b(t'') \Bigg)$$

$$+ \mu'([u_i^0]) \frac{\sum_{t'' \in [t_i^1]} \mu([t_i^1]) p(t'' \mid t_i^1) b(t'')}{\mu'([u_i^0])}$$

$$+ \mu'([u_i^0]) \frac{\sum_{t'' \in [t_i^2]} \mu([t_i^2]) p(t'' \mid t_i^2) b(t'')}{\mu'([u_i^0])}. \quad (15)$$

Appealing now to Eq. (10), Eq. (15) becomes

$$\Bigg(\sum_{t_i' \in T_i' \setminus u_i^0} \mu'([t_i']) \sum_{t'' \in [t_i']} p'(t'' \mid t_i') b'(t'') \Bigg)$$

$$+ \mu'([u_i^0]) \sum_{t'' \in [u_i^0]} p'(t'' \mid u_i^0) b'(t'')$$

which finally yields the conclusion that

$$\sum_{t' \in T'} \mu'(t') b'(t') = \sum_{t_i' \in T_i'} \mu([t_i']) \sum_{t'' \in [t_i']} p(t'' \mid t_i') b(t''),$$

and we conclude that μ' is action consistent. ■

Lemma 2. *Every type is rational at every state $t \in \mathrm{supp}(\mu')$.*

Proof of Lemma 2. Note first that the set of types in the support of μ is the same as the set of types in the support of μ' (except that t_i^1 and t_i^2 are replaced by u_i^0). Furthermore, the conjecture of each type other than t_i^1 and t_i^2 remains the same in the move from B to B'; hence if a strategy is optimal for that type in B it remains so also in B'.

For the case of u_i^0, it suffices to recall that Eq. (10) constructs $p'(\cdot \mid u_i^0)$ as a weighted average of $p(\cdot \mid t_i^1)$ and $p(\cdot \mid t_i^2)$, while the strategies of all three types are the same, namely to play $\widehat{a}_i$. It follows that if an action $r_i \in A_i$ is better than $\widehat{a}_i$ for type u_i^0 in B' then it must also be optimal for either t_i^1 or t_i^2 in B, contradicting the assumption that B is weakly rational. ■

An immediate corollary of what has been shown so far is that amalgamation does not affect the expectation of any type of any player except for the types that have been amalgamated. Furthermore, using the same reasoning, it is straightforward to show that starting with a weakly rational belief system B and any type u_i^0, a new system B' can be derived in which u_i^0 is 'split' into two types u_i^1 and u_i^2 having the same type and theory as u_i. A new action-consistent distribution is obtained from the original one by halving the probability of all states affected by the split.

Let α be a weakly rational expectation in G, that is, there is a weakly rational belief system B and a type u_i of some player i whose expectation is α. Let $r_i := \sigma_i(u_i)$ be the strategy of type u_i. Without loss of generality there is another type, different from u_i, that plays r_i (splitting u_i if necessary).

By repeatedly amalgamating types, one then arrives at a weakly rational belief system B'' with an action-consistent probability distribution μ'' such that: for each strategy of each player, except for r_i, at most one type plays that strategy; there is in B'' a type u_i^1 that plays the strategy r_1 and has expectation α; there is one other type u_i^2 that also plays r_i (if necessary, amalgamating all other types of i that play r_i).

As defined above, the game G_{2r_i} is like G except that the action r_i is doubled; the duplicates could be named r_1^1 and r_1^2. Each type in B'' corresponds to precisely one action in G_{2r_i}. We now appeal to Proposition 6.2 in Barelli (2009), which states that if the strategy of each player in the support of μ' is rational then $\mathrm{marg}_A(\mu')$ is a correlated equilibrium of the game. But by construction, since each type corresponds to one action, this is stating that μ' itself is a correlated equilibrium of G_{2r_i}. ■

Proof of Theorem 1. Denoting by $CCEP(G)$ the set of conditional correlated equilibrium payoffs of a game G, we first claim that $WRE(G) \subseteq CCEP(2G)$.

Let α be a weakly rational expectation of player i in G. By the proposition, it is a conditional payoff to a correlated equilibrium ρ' in some augmented game G_{2r_i}, given an action a' of player i in that game. The action profiles in G_{2r_i} are in one-to-one correspondence with those action profiles in $2G$ whose first component either is one of the two copies of r_i in $2G$ or is the first copy of a strategy that differs from r_i. If we assign any such action profile the ρ' probability of the corresponding profile in G_{2r_i} while assigning 0 to all other profiles, we get a correlated equilibrium in $2G$. Similarly, the action x' in G_{2r_i} corresponds directly to an action x in $2G$ and then α is the conditional payoff to ρ in $2G$ given x, completing the claim.

By Theorem B of Aumann and Drèze (2008), $CCEP(2G) = SRE(G)$, while $SRBS(G) \subsetneq WRBS(G)$ implies that $SRE(G) \subseteq WRE(G)$. Putting these set containments together,

$$SRE(G) \subseteq WRE(G) \subseteq CCEP(2G) = SRE(G)$$

completes the proof that $WRE(G) = SRE(G)$. ■

Proof of Corollary 1. This follows from the main theorem here and Theorem A of Aumann and Drèze (2008), which states that the only strongly rational expectation in a two-player zero-sum game is the value. ■

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