Problem Set 9 – Differential and Integral

1. Use a differential to estimate 52^2 if you know that $50^2 = 2500$. Test the deviation from the accurate value of 52^2.

2. With the help of a differential, estimate $\sqrt{15}$ and 3.001^3. **Ans: 3.875 and 27.027**

3. Calculate the following integrals:

   A. $\int x^5 dx$

   B. $\int (x + \sqrt{x}) dx$

   C. $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4}\right) dx$

   D. $\int \frac{x^8 + 3x^6 + 5x^5 - 2}{x^4} dx$

A. $\frac{x^5}{6}$ **B.** $\frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}}$ **C.** $6\sqrt{x} - \frac{1}{10}x^{\frac{5}{2}}$ **D.** $\frac{x^5}{5} + x^3 + \frac{5}{2}x^2 + \frac{2}{3}x^{-3}$

4. Use a definite integral to calculate the area bounded by the straight line y = 2x +1, the x axis, the y axis and the straight line x = 2. **Ans: 6**

5. Calculate the area bounded by: $y = 3x^2$, the y axis and the straight line y=3. **Ans: 2**

6. A tangent touches intersection of the curve $y = 2x^3 + 3x^2 + 4$ and the y axis. Calculate the area that is bounded between the tangent and the line of the curve. **Ans:** $\frac{27}{32}$

7. Find the equation of the tangent to the line of the curve $y = x^2 - 6x + 9$ that passes through the point (5,0). Calculate the area that is bounded by the tangent, the parabola and the x axis. **Ans:** $5\frac{1}{3}$

8. Calculate the integral $\int_{-1}^{1} x^3$. What is the geometrical significance of your answer? **Ans: 0**
   What is the area that is bounded by the $y = x^3$ curve and the x axis in over the domain [-1,1]? **Ans: 1/2**

9. What, in your opinion, is the value of the integral $\int_1^\infty \frac{dx}{x^2}$ **Ans: 1**? Explain your answer.

   Do the same for: $\int_1^\infty \frac{dx}{\sqrt{x}}$ **Ans:** ∞

10. Calculate the following integrals:

A. $\int e^{5x} dx = \frac{1}{5}\, e^{5x} + C$

B. $\int \frac{dx}{3x-7} = \frac{1}{3}\ln|3x-7| + C$

C. $\int \frac{dx}{1-x} = -\ln|1-x| + C$

D. $\int (x^2+1)x\, dx = \frac{x^4}{4} + \frac{x^2}{2} + C$

E. $\int \frac{(\sqrt{x}-\sqrt{a})^2\, dx}{\sqrt{x}} = \frac{2}{3}(\sqrt{x}-\sqrt{a})^3 + C$

F. $\int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} + C$

G. $\int \sqrt{x^2+1}\, x dx = \frac{\sqrt{(x^2+1)^3}}{3} + C$

H. $\int \frac{x\ dx}{\sqrt{2x^2+3}} = \frac{\sqrt{2x^2+3}}{2} + C$

I. $\int x^2\sqrt{1+x}\, dx = \frac{2}{7}(1+x)^2 - \frac{4}{5}(1+x)^{\frac{5}{2}} + \frac{2}{3}(1+x)^{\frac{3}{2}} + C$

J. $\int \frac{x\, dx}{\sqrt{a+bx}} = \frac{1}{b^2}\left(\frac{2}{3}\sqrt{(a+bx)^3}\right) - 2a\sqrt{a+bx} + C$

K. $\int (3x^2+2x)e^{5x} dx = (3x^2+2x)\frac{e^{5x}}{5} - (6x+2)\frac{e^{5x}}{5^2} + 6\frac{e^{5x}}{5^3} + C$

L. $\int x^\alpha ln\, x\, dx = (ln\, x)\frac{x^{\alpha+1}}{\alpha+1} - \frac{x^{\alpha+1}}{(\alpha+1)^2} + C$

M. $\int ln(5x+1) dx = x ln(5x+1) - x + \frac{ln(5x+1)}{5} + C$

N. $\int_0^{\sqrt{2}} \frac{3x}{x^2+1} dx = \frac{3}{2} ln\, 3$

O. $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx = 1$

P. $\int_4^8 \frac{x}{\sqrt{x^2-15}} dx = 6$

Q. $\int \frac{2x^2+5}{x+2} dx = x^2 - 4x - 12 + 13 ln|x+2| + C$

R. $\int \frac{6x+4}{2x+5} dx = 3x - \frac{11}{2} ln|2x+5| + C$

S. $\int \frac{5x+9}{x^2+3x+2} dx = 4ln|x+1| + ln|x+2| + C$

T. $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} ln\left|\frac{x-a}{x+a}\right| + C$