Problem Set 7 – Limits – L'Hôpital's Rule – Selected Solutions

1. Calculate the following limits:

A. $\lim\limits_{x\to 1}\frac{x-1}{x^{\alpha}-1}=\frac{1}{\alpha}$

B. $\lim\limits_{x\to 0}\frac{a^{x}-b^{x}}{x}=\ln\frac{a}{b}$

C. $\lim\limits_{x\to 1}\left(\frac{2}{x^{2}-1}-\frac{1}{x-1}\right)=-\frac{1}{2}$

D. $\lim\limits_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)=-\frac{1}{2}$

E. $\lim\limits_{x\to 0}x^{2}\cdot e^{\frac{1}{x^{2}}}=\infty$

F. $\lim\limits_{x\to 1}x^{\frac{1}{1-x}}=\frac{1}{e}$

G. $\lim\limits_{x\to \infty}\sqrt[x]{x^{2}}=1$

H. $\lim\limits_{x\to \infty}x(a^{\frac{1}{x}}-1)=\ln a$

I. $\lim\limits_{x\to \infty}\left(1+\frac{4}{x}\right)^{2x}=e^{8}$

J. $\lim\limits_{x\to \infty}\left(1+\frac{3}{x^{2}}\right)^{2x}=1$

K. $\lim\limits_{x\to 0}\sqrt{x}\cdot(\ln x)^{3}=0$

L. $\lim\limits_{x\to \infty}\left(\frac{1}{x}\right)^{e^{-x}}=1$

M. $\lim\limits_{x\to 0}(e^{x}+3x)^{\frac{1}{x}}=e^{4}$

2. What are the values of a and b such that the function f(x) will be continuous at every point?

$$f(x) = \begin{cases} \dfrac{a \cdot \ln(1+x)}{x} & x < 0 \\ 2 + b & 0 \leq x \leq 1 \\ \dfrac{x-1}{x^2 - 1} & x > 1 \end{cases}$$

Answer: $a = \frac{1}{2}, b = -\frac{3}{2}$