

## Problem Set 7 – Limits – L'Hôpital's Rule

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1. Calculate the following limits:

- A.  $\lim_{x \rightarrow 1} \frac{x-1}{x^a - 1}$
- B.  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$
- C.  $\lim_{x \rightarrow 1} \left( \frac{2}{x^2 - 1} - \frac{1}{x-1} \right)$
- D.  $\lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\ln x} \right)$
- E.  $\lim_{x \rightarrow 0} x^2 \cdot e^{\frac{1}{x^2}}$
- F.  $\lim_{x \rightarrow 1} x^{1-x}$
- G.  $\lim_{x \rightarrow \infty} \sqrt[x]{x^2}$
- H.  $\lim_{x \rightarrow \infty} x(a^{\frac{1}{x}} - 1)$
- I.  $\lim_{x \rightarrow \infty} \left(1 + \frac{4}{x}\right)^{2x}$
- J.  $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x^2}\right)^{2x}$
- K.  $\lim_{x \rightarrow 0} \sqrt{x} \cdot (\ln x)^3$
- L.  $\lim_{x \rightarrow \infty} \left(\frac{1}{x}\right) e^{-x}$
- M.  $\lim_{x \rightarrow 0} (e^x + 3x)^{\frac{1}{x}}$

2. What are the values of  $a$  and  $b$  such that the function  $f(x)$  will be continuous at every point?

$$f(x) = \begin{cases} \frac{a \cdot \ln(1+x)}{x} & x < 0 \\ 2+b & 0 \leq x \leq 1 \\ \frac{x-1}{x^2-1} & x > 1 \end{cases}$$