

## Problem Set 6 – Limits and Derivatives of Logarithmic Functions

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1. Calculate the limits of the following functions:

$$\lim_{x \rightarrow +\infty} \frac{3^x - 3^{-x}}{3^x + 3^{-x}} \quad \text{A.}$$

$$\lim_{x \rightarrow -\infty} \frac{3^x - 3^{-x}}{3^x + 3^{-x}} \quad \text{B.}$$

$$\lim_{x \rightarrow 0} \frac{2^{\frac{1}{x}} - 1}{2^{\frac{1}{x}} + 1} \quad \text{C.}$$

$$\lim_{x \rightarrow \infty} \sqrt[n]{2^n + 3^n + 4^n} \quad \text{D.}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x \quad \text{E.}$$

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x \quad \text{F.}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{x+5} \quad \text{G.}$$

$$\lim_{x \rightarrow \infty} \left(\frac{2x+3}{2x+1}\right)^{x+1} \quad \text{H.}$$

2. Calculate the derivatives of the following functions:

$$\sqrt{3x} + \sqrt[3]{x} + \frac{1}{x} \quad \text{A.}$$

$$\frac{ax^2}{\sqrt[3]{x}} + \frac{b}{x \cdot \sqrt{x}} - \frac{\sqrt[3]{x}}{\sqrt{x}} \quad \text{B.}$$

$$\sqrt{x + \sqrt{x + \sqrt{x}}} \quad \text{C.}$$

$$e^{ax} \quad \text{D.}$$

$$e^{4x+5} \quad \text{E.}$$

$$7^{x^2+2x} \quad \text{F.}$$

$$a \cdot e^{\sqrt{x}} \quad \text{G.}$$

$$e^{\ln x} \quad \text{H.}$$

$$\frac{1}{x^x} \quad \text{I.}$$

$$\frac{1}{x + \sqrt{x^2 - 1}} \quad \text{J.}$$

$$\ln(x^2 + x - 1)^3 \quad \text{K.}$$

$$\ln \frac{x^2}{\sqrt{1-x^4}} \quad \text{L.}$$

$$\log_5 \sqrt[3]{x} \quad \text{M.}$$

$$x^{\ln x} \quad \text{N.}$$

$$\frac{(x-2)^2 \sqrt{x^2-5}}{\sqrt[3]{(x-1)^4(3x-2)}} \quad \text{O.}$$

3. For each natural number  $n$ , find the  $n$ -th derivative of each of the following functions (and of course, show your work):

$$\frac{1}{x^2} \quad \text{A.}$$

$$\frac{1}{3x+2} \quad \text{B.}$$

$$\ln x \quad \text{C.}$$

$$e^{ax} \quad \text{D.}$$