Problem Set 5 – The Derivative

1. Differentiate the following functions:

A. $x^4 - 3x^2 - 6$

B. $\frac{x^5}{a+b} - \frac{x^2}{a-b} - x$

C. $(1+4x^3)(1+2x^2)$

D. $\frac{2x^4}{b^2 - x^2}$

E. $(x^2 - \frac{1}{x})^3$

F. $(5x-1)^4(2-3x)^5$

G. $(\frac{2x-1}{x+2})^3$

2. Calculate the derivative of the function f(x) = $|x|$. What can you say about the derivative at the point x = 0 ?

3. Calculate the derivative of the function f(x) = $\lfloor x \rfloor$, where $\lfloor x \rfloor$ stands for the nearest integer smaller than x.

4. Calculate the derivative of the function $y = \sqrt{1+2x}$ in three ways:

   a. Using the definition of the derivative.

   b. Using the inverse function.

   c. As a compound function.

5. Calculate the function of the line tanget to the graph of the function $\frac{x+3}{1-x}$ at the point x = -1.

6. Find tangent lines to the function $y = x^3 + x - 2$ that are parallel to the line $y = 4x + 3$.

7. Find a tanget line to the function $y = x^2 - 2x + 5$ that is parallel to the line segment between the points (3,8) and (1,4).

8. Find the equation of the line passing through the point (5,0) and tangent to the graph of the function $y = x^2 - 6x + 9$ (note that the given point is not on the graph of the function).

9. The total cost function of a firm is given by $TC(q) = q^2 + 27q + 16$.

   a. Find the average cost function AC.

   b. Find the marginal cost function MC.

c. Find the point of intersection of the two functions.

10. The total cost function of a firm is given by $TC(q) = aq^2 + bq$, for $a>0$. Show that for every q in the domain of definition of this function (i.e., for all $q > 0$), MC > AC.