Problem Set 4 – Limits and Continuity

Calculate the following limits

A. $\lim_{x \to 2} \quad x^2 - 4 = 0$

B. $\lim_{x \to \infty} \quad x^3 - x^2 + x - 5 = +\infty$

C. $\lim_{x \to -\infty} \quad x^3 + x^2 + x - 5 = -\infty$

D. $\lim_{x \to -\infty} \quad 3x^4 + 2x^3 + x - 1 = +\infty$

E. $\lim_{x \to 3} \quad \frac{x^2 - 4}{x^2 - 5x + 6}$

F. $\lim_{x \to \infty} \quad \frac{2x+3}{4x-5} = \frac{1}{2}$

G. $\lim_{n \to \infty} \quad \frac{\sum_{i=1}^{n} i}{n^2} = \frac{1}{2}$

H. $\lim_{x \to \infty} \quad \frac{x^3 - x^2 + x - 5}{-(x^2-1)^2} = 0$

I. $\lim_{x \to -\infty} \quad \frac{(x^2-1)^2}{-x^3 - x^2 + x - 5} = +\infty$

J. $\lim_{x \to 2} \quad \frac{x^2 - 4}{x^2 - 5x + 6} = -4$

K. $\lim_{x \to a} \quad \frac{x^3 - a^3}{x^2 - a^2} = \frac{3a}{2}$

L. $\lim_{h \to 0} \quad \frac{(x+h)^3 - x^3}{h} = 3x^2$

M. $\lim_{x \to 1} \quad \frac{1}{1-x} - \frac{1}{1-x^2}$

N. $\lim_{n \to \infty} a_n = \begin{cases} \frac{3n}{n+3} & \text{if } n \text{ even} \\ 2 & \text{otherwise} \end{cases}$

O. $\lim_{x \to \infty} \frac{\sqrt{x+2} - \sqrt{x+1}}{\sqrt{x}}$

P. $\lim_{x \to 4} \frac{\sqrt{2x+1} - 3}{\sqrt{x-2} - \sqrt{2}}$

Q. $\lim_{x \to \infty} x(\sqrt{x^2+6} - x)$

R. $\lim_{x \to \infty} \sqrt{x}(\sqrt{x+1} - \sqrt{x})$

N. no defined limit

O. =0

P. = $\frac{2\sqrt{2}}{3}$

Q. = 3

R. = $\frac{1}{2}$

2. Find the points of discontinuity in each of the following functions, and categorise which type of discontinuity you have found at each such point:

$f(x) = |x-2|$ __ *Always _ Continuous*

$f(x) = \dfrac{x^3-8}{x-2}$ _ $x=2$ _ *removable*

$f(x) = \begin{cases} x^2 & x \geq 1 \\ x & 0 < x < 1 \\ 5 & x \leq 0 \end{cases}$ __ $x=0$ _ *jump*

$f(x) = \begin{cases} \dfrac{1}{x}\{(1+x)^2-1\} & x \neq 0 \\ 3 & x = 0 \end{cases}$ __ $x=0$ _ *removable*

$f(x) = \dfrac{1}{x-2} - \dfrac{6}{x^2+2x-8}$ ___ $x=2, removable$ __ $x=-4, essential$

$f(x) = \begin{cases} \dfrac{1}{x+3} & x < -1 \\ \dfrac{x^2+|x|}{|x|} & x > -1 \end{cases}$ ___ $x=0, removable$ __ $x=-1$, *jump* __ $x=-3, ess$

$f(x) = \left\lfloor x + \dfrac{1}{2} \right\rfloor$

$f(x) = \begin{cases} \dfrac{1}{x+4} & x < 1 \\ -1 & x = 1 \\ \dfrac{x^2-9x+14}{x^2+3x-10} & x > 1 \end{cases}$ __ $x=-4, essential$ __ $x=1$, *jump* __ $x=2, remov$

$f(x) = \dfrac{|x+2|}{x^2-4}$ _____ $x=-2$ __ *jump*, ____ $x=2$ ___ *essential*

$f(x) = \begin{cases} x^2-2 & x \leq 0 \\ \dfrac{\sqrt{x}+1}{3\sqrt{x}-2} & x > 0 \end{cases}$ _____ $x=0$, *jump* ____ $x=4/9, essential$