Problem Set 3 – Special Functions

Linear Functions and the Equation of the Line

1.
   a. Describe the family of linear functions satisfying f(4) = -3
   b. Which of the functions you found above satisfy the equation f(2) = 7?
   c. Which one has a slope of 3/4?
2. Find the equation of the line parallel to the line x+2y = 32 forming a triangle of area 1 with the two axes.
3. Fine the equations of the lines containing the sides of the triangle whose vertice are (-4, 5) ,(0, 3) ,(2, -1) .
4. Given the family of lines defined by $(k^2 - 36)x + (k^2 - 15k + 36)y + (k-1)^2 = 0$, where k is a parameter, find the values of k such that the associated line satisfies the property that:
   a. It is parallel to the x-axis
   b. It passes through the origin
   c. It cuts through the angle between the axes
   d. It is parallel to the line 2y + 5x = 3

5. Given the functions

   $f(x) = x^2 - 2x - 3$

   $g(x) = -2x^2 + 13x - 15$

   a. Describe the domains and ranges of f and g.
   b. Calculate the roots of f and of f – g.
6. Given the function $f(x) = (m-4)x^2 + 10x + m$,

   for what value of m does the range of this function include only numbers that are less than 4?
7. Given the function $x^2 - mx + m + 3 = 0$,

   for what value of m does this function have two solutions, one less than 3 and the other greater than 3?
8. For what values of m is the expression $(m^2 - 1)x^2 + 2(m-1)x + 2$ positive for all x?
9. Production costs for producing Q items at a certain firm depends on the number of machines being used. If 5 machines are run, the total production cost is

$TC_1(Q) = 2Q^2 + 8Q + 10$. If 15 machines are in use, the total production cost is $TC_2(Q) = Q^2 + 4Q + 231$.

For any given production amount Q, the firm will choose the number of machines (either 5 or 15) that reduce production costs to a minimum. For which values of Q will the firm choose to use only 5 machines, and for which values of Q will the firm choose to use 15 machines?

10. Suppose the following game is given: the player chooses a number x such that $0 \le x \le 3$, and then receives a payoff the lesser of $2x+3$ and x^2-x+5.
    a. Find the function that gives the payoff that the player receives as a function of x.
    b. Which x should the player choose in order to receive the maximal payoff?

Sequences

11. Determine whether the sequence $a_n = \frac{n-1}{3n+5}$ is monotonic and whether it is bounded.
12. Given the sequence

$$a_n = \begin{cases} \frac{2n}{n+3} & \textit{if} \quad n \quad \textit{even} \\ 2 & \textit{otherwise} \end{cases}$$

write down the first five elements of the sequence. Determine whether this sequence is monotonic. Is it bounded? Is it injective?

13. Given the sequence 1, 1.1, 1.11, 1.111 ...
    a. Write this sequence as a function of n.
    b. Determine whether this sequence is monotonic. Is it bounded?
14. Given the sequence $\frac{(-1)^{n+1}}{n+1}$
    a. Write down the first five elements of this sequence.
    b. Determine whether this sequence is monotonic. Is it bounded? If it is bounded, find its upper bound and its lower bound.
15. Given the function $f(n) = \frac{4-2n}{3n+2}$ $\qquad f: N \to R$
    a. Determine this is a surjective function.
    b. Is it injective?

c. Is it monotonic? If yes, is it monotonically increasing or monotonically decreasing?
d. Is the function bounded? If yes, find its bounds.