# Problem Set 2 – Introduction to Functions

1. Let A be the set of all possible sentences in the English language. Associate each element of A with the number of letters it contains. Does this define a function, and if yes, is it injective (one-to-one)?

2. Let a function f: [0, 20] → R be defined by f(p) = 10,000 - 500p. Suppose that this function is the total amount of apples (in kilograms) demanded by consumers of a particular market when the price of apples is p shekels per kilogram.
    a. What is the amount demanded when the price p is 10 shekels per kilogram?
    b. Denote demand by Q=f(p). Calculate the price when the demand is Q=2000.
    c. What is the range of the function f?
    d. Is f an injective function?
    e. Find f^{-1} .
    f. What are the functions $f(f^{-1})$ and $f^{-1}(f)$?

3.
    a. In each of the following diagrams, determine whether or not the diagram depicts a function from A to B.
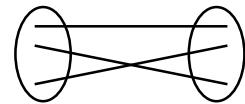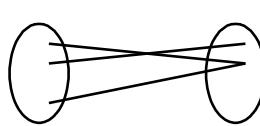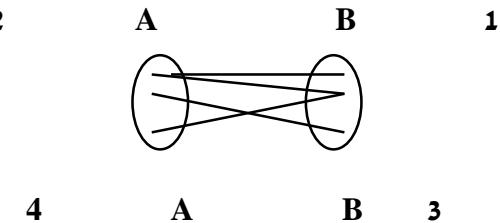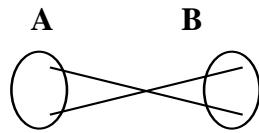
    b. Which of the functions you identified in part a) have inverse functions?

4. Which of the following equations define Y as a function of x?
    a. $\sqrt{Y+1} = x - 5$
    b. $(Y+1)^4 = x^3 - 2$

5. Determine the domain of definition of each of the following functions:

.b $y = \sqrt{\dfrac{x-2}{x+1}}$ .a $y = x^3 + 2x^2 - x + 1$

.d $y = \dfrac{\sqrt{x-2}}{\sqrt{x+1}}$ .c $y = \dfrac{x^2 - x + 1}{2x^2 + 3x - 5}$

.f $y = \dfrac{1}{\sqrt{|x-4| - 3}}$ .e $y = \dfrac{2x - 1}{x^2 - x + 2}$

       .g $y = \sqrt{-x^2 - 2}$

6. Which of the following functions have inverse functions? For those that do, explicitly write the inverse functions:

$y = 3x - 9$

$y = \dfrac{5}{x - 10}$

$y = x^2 + 1$

$y = \dfrac{5}{(2x - 1)^3}$

$y = (x - 1)^4 + 3$

$y = \dfrac{1}{\dfrac{1}{x} + 1}$

7. Let $f(x)$ and $g(x)$ be injective (one to one) functions.

     a. Prove that f^{-1} is also injective.

     b. Prove that if the function $g[f(x)]$ is well-defined then it, too, is injective.

     c. Use what you showed above to determine whether or not the following function is injective: $f(x) = \sqrt{3x + 4}$

8. Determine the domain and range of the first two functions in Question 6. Furthermore, determine the domain and range of the composition of the first function with the second function and the composition of the second function with the first function.

9. Given the following functions:

$$f(x) = \frac{2x+2}{x+3}$$

$$g(x) = \frac{1}{4x+1}$$

$$h(x) = x+1$$

$$t(x) = \frac{1}{x}$$

Prove that $f[h(x)] = g[t(x)] + 1$ for all x not equal to 0.

10. Given the functions:

$$f(x) = \begin{cases} x+2 & x < 2 \\ 3x & x \geq 2 \end{cases}$$

$$g(x) = \begin{cases} 2x & x < 1 \\ x^2 & x \geq 1 \end{cases}$$

   a. For each of the above functions, determine whether it is monotonic, and whether it is injective, and find its inverse function if such exists.
   b. Determine f(g(x)), g(f(x)) and f(f(x)).

11. What is the domain and range of the function $y = |3x - 6|$? Is it injective?

12. Which of the following functions are even/odd/neither?

$$f(x) = x^2$$

$$f(x) = \frac{1}{2}(a^{2x} + a^{-2x})$$

$$f(x) = \frac{x^2 - 4}{x^2 + x} + x \cdot 10^{-x^2}$$

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