Problem Set 1: Solutions to Selected Questions

1. Given the following sets:

    N = the set of natural numbers;

    Z = the set of integers;

    $A = \{2n \mid n \in N\}$
    $B = \{2n + 1 \mid n \in N\}$

    Find:

    $$B - A; \quad A - B; \quad A \cap B; \quad A \cap Z; \quad A \cup Z; \quad A \cup N$$

    Answers:

    $$B - A = B; \quad A - B = A; \quad A \cap B = \varnothing; \quad A \cap Z = A; \quad A \cup Z = Z; \quad A \cup N = N$$

2. Given the following sets:

    N = the set of natural numbers;

    Z = the set of integers;

    Q = the set of rational numbers;

    R = the set of real numbers;

    R_+ = the set of non-negative real numbers, $\{x \mid x \geq 0\}$;

    R_{++} = the set of positive real numbers, $\{x \mid x > 0\}$

    For each of the following, state which claim is true or false and explain why:

    A. $\{0\} \subset A$, for every set A. **False**

    B. $\phi \subset A$, for every set A. **True**

    C. $R_{++} \cup \{0\} = R_+$ **True**

    D. $R_{++} \cup \{0\} = R_{++}$ **False**

    E. $R_{++} \subset R_+ \subset R$ **True**

    F. $N \subset Z \subset Q \subset R_+ \subset R$ **False**

    G. $\{x \mid x \text{ is a solution to the equation } x^2 - 5x + 6 = 0\} \subset R_+ \cap Z$ **True**

3. Let A, B and C be sets. For each of the following, state which claim is true or false and explain why (you may use a Venn diagram if it helps):

$(A \cup B) \cap \bar{C} = A \cup (B \cap \bar{C})$ False

$A \cap B \cap C = A \cap B \cap (C \cup B)$ False

$A \cup B = A \cup (B \cap \bar{A})$ True

$A \cap \bar{B} \cap C \subseteq A \cup B$ True

$A - (B \cap C) = (A - B) \cup (A - C)$ True

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ True

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ True

$A - B = \bar{B} - \bar{A}$ True

4. For each of the following, state which claim is true or false and explain why:
   a. $\{x\} \subseteq \{x, \{x\}\}$ **True**
   b. $\{x\} \in \{\{x\}, x\}$ **True**
   c. $x \subseteq \{\{x\}, x\}$ **False**
   d. $x \in \{\{x\}, x\}$ **True**

5. Let A, B and C be sets. Prove that $A \subseteq B \subseteq C$ if and only if $A \cup B = B \cap C$.

6. Prove that the sum of two rational numbers is a rational number.

   What can you say about the sum of two irrational numbers? What about the sum of a rational number and an irrational number?
   And what about products of two rational numbers, two irrational numbers and a rational number and an irrational number?

7. For each of the following sets, establish whether or not it is bounded from above and/or below:

   $\left\{x \mid x = \dfrac{1}{n}, \quad n \in N\right\}$ Bounded from above and below

   $\{x \mid x < 3\}$ Bounded from above, not below

   $\{x \mid x = 3^n, \quad n \in N\}$ Bounded from below, not above

   $\{2^{18}, \quad 3^{-2}, \quad 0, \quad 5, \quad 10^{30}\}$ Bounded from above and below

   $[-3, \ 1) \cup (1, \ 2]$ Bounded from above and below

8. Which of the following is correct?

   a. $\{x\} \subset \{\{x\}, \{y\}, \{\{x\}, \{y\}\}\}$

   b. $\{\{y\}\} \subset \{\{x\}, \{y\}, \{\{x\}, \{y\}\}\}$

   c. $\{\{y\}\} \in \{\{x\}, \{y\}, \{\{x\}, \{y\}\}\}$

   d. None of the other answers are correct.

9. Which of the following is correct?

    a. $\{\{x\}\{y\}\} \subset \{\{y\}, \{x, y, z\}, \{x\}\}$

    b. $\{\{x\}\{y\}\} \in \{\{x\}, \{x, y, z\}, \{y\}\}$

    c. $\{x, y\} \subset \{\{x\}\{y\}\}$

    d. None of the other answers are correct.