

Problem Set 1: Sets and Numbers

1. Given the following sets:

N = the set of natural numbers;

Z = the set of integers;

$$A = \{2n \mid n \in N\}$$

$$B = \{2n + 1 \mid n \in N\}$$

Find:

$$B - A; \quad A - B; \quad A \cap B; \quad A \cap Z; \quad A \cup Z; \quad A \cup N$$

2. Given the following sets:

N = the set of natural numbers;

Z = the set of integers;

Q = the set of rational numbers;

R = the set of real numbers;

R_+ = the set of non-negative real numbers, $\{x \mid x \geq 0\}$;

R_{++} = the set of positive real numbers, $\{x \mid x > 0\}$

For each of the following, state which claim is true or false and explain why:

A. $\{0\} \subset A$, for every set A .

B. $\emptyset \subset A$, for every set A .

C. $R_{++} \cup \{0\} = R_+$

D. $R_{++} \cup \{0\} = R_{++}$

E. $R_{++} \subset R_+ \subset R$

F. $N \subset Z \subset Q \subset R_+ \subset R$

G. $\{x \mid x \text{ is a solution to the equation } x^2 - 5x + 6 = 0\} \subset R_+ \cap Z$

3. Let A , B and C be sets. For each of the following, state which claim is true or false and explain why (you may use a Venn diagram if it helps):

$$(A \cup B) \cap \bar{C} = A \cup (B \cap \bar{C})$$

$$A \cap B \cap C = A \cap B \cap (C \cup B)$$

$$A \cup B = A \cup (B \cap \bar{A})$$

$$A \cap \bar{B} \cap C \subseteq A \cup B$$

$$A - (B \cap C) = (A - B) \cup (A - C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A - B = \bar{B} - \bar{A}$$

4. For each of the following, state which claim is true or false and explain why:

a. $\{x\} \subseteq \{x, \{x\}\}$

b. $\{x\} \in \{\{x\}, x\}$

c. $x \subseteq \{\{x\}, x\}$

d. $x \in \{\{x\}, x\}$

5. Let A , B and C be sets. Prove that $A \subseteq B \subseteq C$ if and only if $A \cup B = B \cap C$.

6. Prove that the sum of two rational numbers is a rational number.

What can you say about the sum of two irrational numbers? What about the sum of a rational number and an irrational number?

And what about products of two rational numbers, two irrational numbers and a rational number and an irrational number?

7. For each of the following sets, establish whether or not it is bounded from above and/or below:

$$\{x \mid x = \frac{1}{n}, \quad n \in \mathbb{N}\}$$

$$\{x \mid x < 3\}$$

$$\{x \mid x = 3^n, \quad n \in \mathbb{N}\}$$

$$\{2^{18}, \quad 3^{-2}, \quad 0, \quad 5, \quad 10^{30}\}$$

$$[-3, 1) \cup (1, 2]$$

8. Which of the following is correct?

a. $\{x\} \subset \{\{x\}, \{y\}, \{\{x\}, \{y\}\}\}$

- b. $\{\{y\}\} \subset \{x, \{y\}, \{\{x\}, \{y\}\}\}$
- c. $\{\{y\}\} \in \{x, \{y\}, \{\{x\}, \{y\}\}\}$
- d. None of the above are correct.

9. Which of the following is correct?

- a. $\{\{x\}\{y\}\} \subset \{\{y\}, \{x, y, z\}, \{x\}\}$
- b. $\{\{x\}\{y\}\} \in \{\{x\}, \{x, y, z\}, \{y\}\}$
- c. $\{x, y\} \subset \{\{x\}\{y\}\}$
- d. None of the other answers are correct.