

Problem Set 11 – Derivatives and Differentials in Multivariable

Functions – Selected Solutions

1. Calculate all of the partial first and second order derivatives in the following functions:

A. $f(x, y) = \frac{x^2}{y} + \frac{y^2}{x}$

B. $f(x, y) = e^{2x+3y}$
 $f(x, y) = x^y$

C. $f(x, y, z) = 3 \cdot x^2 \cdot y \cdot z + 4 \cdot x \cdot e^{z \cdot y}$

D. $f(x, y) = 3x^{\frac{2}{3}} \cdot y^{\frac{1}{3}}$

E.

2. Prove that the function $z = y \cdot \ln(x^2 - y^2)$ fulfils: $\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{z}{y^2}$

3. Given the function $f(x, y)$, where $\begin{cases} x = s^2 - t^2 \\ y = t^2 - s^2 \end{cases}$, prove that $t \cdot \frac{\partial f}{\partial s} + s \cdot \frac{\partial f}{\partial t} = 0$ exists.

4. Make approximate calculations of the following expressions, using a differential:

a. $\sqrt{3.01^3 + 3 \cdot 2.98} = \mathbf{6.0175}$

b. $(1.01^2 \cdot 0.98)^{1/15} = \mathbf{1}$

5. Given the implicit function $x^2y - xy^2 + x^2 + y^2 = 0$, calculate dy/dx and dx/dy .

$$\frac{dy}{dx} = -\frac{2xy - y^2 + 2x}{x^2 - 2xy + 2y}; \quad \frac{dx}{dy} = -\frac{x^2 - 2xy + 2y}{2xy - y^2 + 2x}$$

6. Given the curve $5x - 2y + y^3 - x^2y = 0$, calculate the equation of the straight line that is tangential to the curve at the origin. $y = \mathbf{2.5x}$

7. Given a box whose base is a square with a side x and height y , calculate dx/dy when the area remains unchanged. $\frac{dx}{dy} = -\frac{x}{x+y}$

8. Calculate $\partial z/\partial x$ and $\partial z/\partial y$ when it is given that $e^{x \cdot y} + e^{y \cdot z} + e^{x \cdot z} = 20$.

$$\frac{\partial z}{\partial x} = -\frac{ye^{xy} + ze^{xz}}{ye^{yz} + xe^{xz}}$$

$$\frac{\partial z}{\partial y} = -\frac{xe^{xy} + ze^{yz}}{ye^{yz} + xe^{xz}}$$

9. Prove that the following holds for every function $\Phi(x,y) = 0$:

$$\frac{d^2 y}{dx^2} = \frac{-\Phi_y^2 \cdot \Phi_{xx} + 2 \cdot \Phi_x \cdot \Phi_y \cdot \Phi_{xy} - \Phi_x^2 \cdot \Phi_{yy}}{\Phi_y^3}$$

10. In the implicit function $z^3 - xy - y = 0$, prove that the following exists:

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{9z^2}$$