Problem Set 10 – Multivariable Functions: Basic Concepts and Limits

1. For each of the following functions, determine its domain of definition and sketch the domain:

   A. $$f(x,y) = \frac{1}{x^2 + y^2 - 1}$$

   $$f(x,y) = \ln(x+y)$$

   B. $$f(x,y) = \sqrt{\frac{x+y-1}{x^2+y^2-1}}$$

   C. $$f(x,y) = \sqrt{x \cdot y - x^3 + x}$$

   D.

2. For each of the following functions, sketch the equivariant curves for the integers from -3 to +3 (except in the last function, J, for which the values should be 1 and e^3 :
3.
   A. $$f(x,y) = 5x + 2y$$

   B. $$f(x,y) = x^2 + y^2$$

   C. $$f(x,y) = x \cdot y$$

   D. $$f(x,y) = x^2 \cdot y$$

   E. $$f(x,y) = x^2 + 2x + y$$

   F. $$f(x,y) = \begin{cases} \sqrt{x^2 + y^2} & x \le 0 \\ |y| & x > 0 \end{cases}$$

   $$f(x,y) = \min\{x, y\}$$

   G.

   H. $$f(x,y) = \max\{x^2 + y^2, x \cdot y\}$$

   I. $$f(x,y) = \min\{x^2 + y^2, x \cdot y\}$$

   J. $$f(x,y) = \frac{e^{(x^2)}}{e^{2x+y}}$$

4. For each of the following limits (except for E), determine whether the limit exists, and calculate it:

A. $$\lim_{(x,y)\to(1,2)} 3\cdot x\cdot y$$

B. $$\lim_{(x,y)\to(1,2)} f(x,y)=\begin{cases} 3\cdot x\cdot y & (x,y)\neq(1,2) \\ 0 & (x,y)=(1,2) \end{cases}$$

C. $$\lim_{(x,y)\to(0,0)} \frac{3x-2y}{2x-3y}$$

D. $$\lim_{(x,y)\to(0,0)} f(x,y)=\begin{cases} \dfrac{x^2-y^2}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$

E. $$\lim_{(x,y)\to(0,1)} \frac{x+y-1}{\sqrt{x}+\sqrt{1-y}}$$

F. $$\lim_{(x,y)\to(0,1)} e^{\left(\frac{-1}{x^2(y-1)^2}\right)}$$

Answers: A. Limit exists and is 6; B. Limit exists and is 6 C. Limit does not exist; D. Limit does not exist. E. Function is not well-defined in neighbourhood of the limit point. F. Limit exists and is 0

5. For each of the following functions, determine whether it is continuous at the point (0,0):

A. $$f(x,y)=\begin{cases} \dfrac{x\cdot y^2}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$

B. $$f(x,y)=\begin{cases} \dfrac{x\cdot y}{|x|+|y|} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$

Answers: A. Limit exists and is 0, hence function continuous. B. Same as A