Mathematics for Economists (66110)

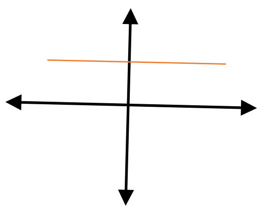
Lecture Notes 6

Linear Functions

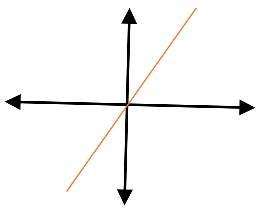
<u>Linear Function = Straight Line</u>

The general form of such a function is $y = ax + b$.

If $a = 0$ the function is a constant function.



If $b = 0$ and $a \neq 0$ one gets a line going through the origin.



A linear function is sometimes written as $Ax + By + C = 0$. In that case, if $B = 0$, we don't have a well-defined function, but if $B \neq 0$ then the equation cam be rewritten as $y = -\frac{A}{B}x - \frac{C}{B}$ recapitulating the more standard form.

Returning to the form $y = ax + b$:

b is the point of intersection of the line with the y axis (to see this, set $x = 0$).

a is the slope of the line, that is, given an pair of points on the line, (x_2, y_2) , (x_1, y_1), one has

$$\frac{y_2 - y_1}{x_2 - x_1} = a$$

The entire line can be deduced from a single point and the slope: if a is the slope and (x_1, y_1) is a point on the line, then the equation of the line is $y - y_1 = a(x - x_1)$.

Alternatively, the entire line can be deduced from any pair of points on the line: given the points (x_2, y_2) , (x_1, y_1), the equation of the line is

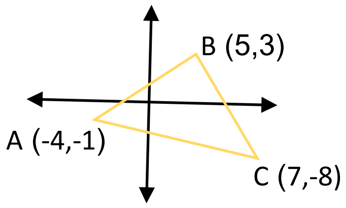
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

When are two lines $y_1 = a_1x + b_1$, $y_2 = a_2x + b_2$ parallel? When are they perpendicular?

They are parallel when they have the same slope, i.e., $a_1 = a_2$.

They are perpendicular when their slopes are negative inverses of each other, i.e., $a_1 \cdot a_2 = -1$.

<u>Exercise</u>: Given the triangle



find the point of intersection of the interior perpendicular bisectors of the triangle (i.e., the circumcentre of the triangle).

Solution:

First, work out the equations of two of the triangles perpendicular bisectors.

For example, start with the perpendicular bisector to BC. The slope of BC is $a_{BC} = \frac{3-(-8)}{5-7} = -\frac{11}{2}$, which implies that the slope of the perpendicular bisector to it is = $2/_{11}$. The midpoint of BC is given by the pair

$$(\text{BC centre})y = \frac{3 + (-8)}{2} = -2.5$$

$$(\text{BC centre})x = \frac{5 + 7}{2} = 6$$

Putting it together, using the point-slope formula, yields, for the perpendicular bisector,

$$y + 2.5 = \frac{2}{11}(x - 6)$$

$$y = \frac{2}{11}x - \frac{12}{11} - 2.5$$

For the perpendicular bisector of AC, note that the slope of AC is $a_{AC} \frac{-1-(-8)}{-4-7} = -\frac{7}{11}$ and therefore the slope of the perpendicular bisector is = $\frac{11}{7}$.

$$(\text{AC centre})y = \frac{\left(-1 + (-8)\right)}{2} = -4.5$$

$$(\text{AC centre})x = \frac{(-4 - 7)}{2} = 1.5$$

Putting it together, using the point-slope formula, yields, for the perpendicular bisector,

$$y + 4.5 = \frac{11}{7}(x - 1.5)$$

$$y = \frac{11}{7}x - \frac{33}{14} - 4.5$$

Setting them equal to find the point of intersection,

$$\frac{11}{7}x - 4.5 - \frac{33}{14} = \frac{2}{11}x - \frac{12}{11} - 2.5$$

$$.\ x\left(\frac{11}{7} - \frac{2}{11}\right) = 2 + \frac{33}{14} - \frac{12}{11}$$