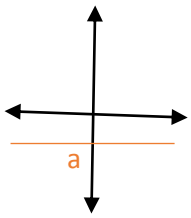
Mathematics for Economists (66110)

Lecture Notes 5

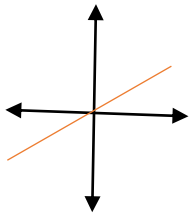
Special Functions

Constant function: $f(x) = a$



A constant function is an even function and is never injective and never surjective.

Identity function: $f(x) = x$



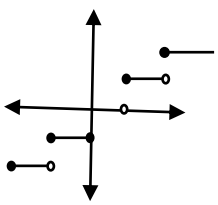
The identity function is odd, injective and surjective. It is its own inverse.

Integer Value function: $f(x) = \lfloor x \rfloor$

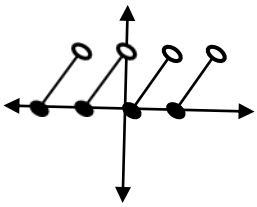
This is more commonly called rounding down, i.e. $\lfloor 3.9 \rfloor = 3$.

This function is neither injective nor surjective. It is non-decreasing and is an odd function.

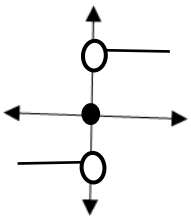
Its domain is $\mathbb{R}$ and its range is $\mathbb{Z}$.



Consider the function $g(x) = x - \lfloor x \rfloor$. This function always has values in the interval $[0,1)$. It is neither injective nor surjective.



Sign function:



$$sign(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

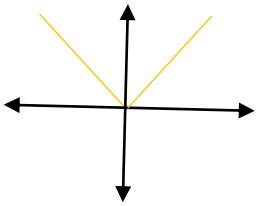
This function is odd and is neither injective nor surjective.

Absolute Value function:

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

The range of this function is $\mathbb{R}_+$, since its values are always non-negative.

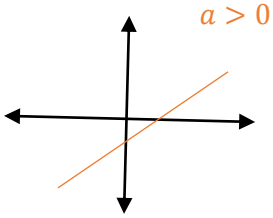
It is often convenient to regard this function as composed of two separate functions, for $x < 0, x > 0$.



Linear function: $f(x) = ax + b$

For this function, if $a > 0$ the function is strictly increasing, if $a = 0$ this reduces to a constant function and if $a < 0$ the function is strictly decreasing. If $a \neq 0$ then the function is injective and has an inverse, which is also linear and given by

$$x = \frac{y}{a} - \frac{b}{a}$$



If f, g are both linear, then their composition is also linear:

$$f\big(g(x)\big) = f(cx+d) = a(cx+d)+b = acx+ad+b \Rightarrow$$

No matter how many times we compose linear functions with each other, the result is a linear function.

Polynomial of n-th order:

$$a_n \neq 0, f(x) = a_n x^n + a_{n-1}x^{n-1} + .. + a_1 x + a_0$$

Example: $f(x) = 8x^7 + 2x^6 + 4x + 2$ is a 7-th order (a.k.a. 7-th degree) polynomial.

Note: a polynomial of 1-st order is a linear function.

Rational function: $f(x) = \frac{p(x)}{q(x)}$, where $q(x)$ and $p(x)$ are polynomials.

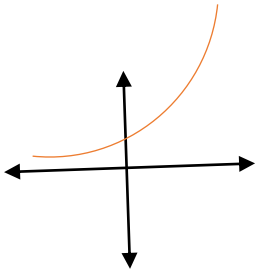
The domain of this function is everywhere that $q(x) \neq 0$.

For example, for $f(x) = \frac{2x^5-9x^3+3x^2-4}{2x^2+3x-2}$, the range is $x \neq \frac{1}{2}, -2$.

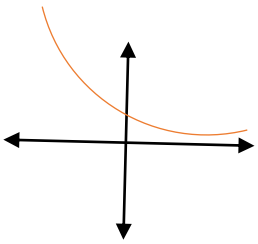
Exponential function: $f(x) = a^x$, where $a > 0$.

For $a > 1$, this function is strictly increasing.

For $0 < a < 1$, this function is strictly decreasing.



$a > 1$



$0 < a < 1$

This function is injective. Its domain is $\mathbb{R}$ and its range is $\mathbb{R}_{++}$.