

8 a d c e

$$f(x, y) = 2\sqrt{x} + 2\sqrt{y} \quad \text{s.t.} \quad x^2 + y^2 = 32$$

$$L(x, y, \lambda) = 2\sqrt{x} + 2\sqrt{y} - \lambda (x^2 + y^2 - 32)$$

$$\begin{cases} L'_x = \frac{1}{\sqrt{x}} - 2\lambda x = 0 \\ L'_y = \frac{1}{\sqrt{y}} - 2\lambda y = 0 \\ x^2 + y^2 = 32 \end{cases} \Rightarrow \lambda = \frac{1}{2x\sqrt{x}} = \frac{1}{2y\sqrt{y}} \Rightarrow 2x\sqrt{x} = 2y\sqrt{y} \Rightarrow x = y$$

$$\begin{aligned} x^2 + x^2 &= 32 \\ x^2 &= 16 \Rightarrow x = 4 \end{aligned}$$

$$x = 4 \Rightarrow y = 4 \Rightarrow \lambda = \frac{1}{2 \cdot 4 \sqrt{4}} = \frac{1}{16} \quad (4, 4, \frac{1}{16})$$

$$L_{xx} = -\frac{1}{2}x^{-\frac{3}{2}} - 2\lambda \quad L_{xy} = 0 \quad L_{yy} = -\frac{1}{2}y^{-\frac{3}{2}} - 2\lambda \quad g'_x = 2x, g'_y = 2y$$

$$|H_2(4, 4, \frac{1}{16})| = \begin{vmatrix} 0 & 8 & 8 \\ 8 & -\frac{3}{16} & 0 \\ 8 & 0 & -\frac{3}{16} \end{vmatrix} \xrightarrow{R_2 \rightarrow R_2 - R_3} \begin{vmatrix} 0 & 8 & 8 \\ 0 & -\frac{3}{16} & \frac{3}{16} \\ 8 & 0 & -\frac{3}{16} \end{vmatrix} = 8 \cdot \begin{vmatrix} 8 & 8 \\ -\frac{3}{16} & \frac{3}{16} \end{vmatrix} > 0$$

פונקציה קמורה בקטע $(4, 4)$ נקודה קיצונית

9 a d c e

$$|A| = \begin{vmatrix} a & 2 \\ 5 & b \end{vmatrix} = ab - 10 = \lambda_1 \lambda_2 = -6 \Rightarrow \boxed{ab = 4}$$

 $\Rightarrow$

$$\text{tr } A = a + b = \lambda_1 + \lambda_2 = 5 \Rightarrow \boxed{a + b = 5}$$

$$a_1 = 1, b_1 = 4$$

$$a_2 = 4, b_2 = 1$$

$$\Rightarrow |a - b| = 3$$

$$A^3 - 2A^2 + A = 2I \Rightarrow$$

$$A(A^2 - 2A + I) = 2I \Rightarrow A \cdot \frac{1}{2}(A^2 - 2A + I) = I \Rightarrow$$

$$A^{-1} = \frac{1}{2}(A^2 - 2A + I)$$

10 a d c e

U של מרחב וקטורי

11 a d c e

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & x_1 \\ -1 & 1 & -3 & x_2 \\ 2 & 2 & 2 & x_3 \\ 0 & 1 & -1 & x_4 \end{array} \right) \xrightarrow{\substack{R_2 \rightarrow R_2 + R_1 \\ R_3 \rightarrow R_3 - 2R_1}} \left(\begin{array}{ccc|c} 1 & 0 & 2 & x_1 \\ 0 & 1 & -1 & x_2 + x_1 \\ 0 & 2 & -2 & x_3 - 2x_1 \\ 0 & 1 & -1 & x_4 \end{array} \right) \xrightarrow{\substack{R_3 \rightarrow R_3 - 2R_2 \\ R_4 \rightarrow R_4 + R_2}} \left(\begin{array}{ccc|c} 1 & 0 & 2 & x_1 \\ 0 & 1 & -1 & x_2 + x_1 \\ 0 & 0 & 0 & -4x_1 - 2x_2 + x_3 \\ 0 & 0 & 0 & x_1 - x_2 + x_4 \end{array} \right)$$

$$\begin{cases} -x_1 - x_2 + x_4 = 0 \\ -4x_1 - 2x_2 + x_3 = 0 \end{cases}$$

U של מרחב וקטורי

מרחב וקטורי של מרחב וקטורי

$$\begin{cases} x_1 - 2x_2 + x_3 + x_4 = 0 \\ -x_1 - x_2 + x_4 = 0 \\ -4x_1 - 2x_2 + x_3 = 0 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ -4 & -2 & 1 & 0 & 0 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 + R_1, R_3 \rightarrow R_3 + 4R_1} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 6 & 5 & 4 & 0 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - 6R_2}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & -1 & -8 & 0 \end{array} \right)$$

לכן $\dim U \cap W = 1$

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$$\|\underline{x} - \underline{y}\| = \sqrt{(\underline{x} - \underline{y}) \cdot (\underline{x} - \underline{y})} = \sqrt{\underline{x} \cdot \underline{x} - \underline{x} \cdot \underline{y} - \underline{y} \cdot \underline{x} + \underline{y} \cdot \underline{y}} = \sqrt{\|\underline{x}\|^2 + \|\underline{y}\|^2}$$

$$\|\underline{x} + \underline{y}\| = \sqrt{(\underline{x} + \underline{y}) \cdot (\underline{x} + \underline{y})} = \sqrt{\underline{x} \cdot \underline{x} + \underline{x} \cdot \underline{y} + \underline{y} \cdot \underline{x} + \underline{y} \cdot \underline{y}} = \sqrt{\|\underline{x}\|^2 + \|\underline{y}\|^2}$$

$$\|\underline{x} - \underline{y}\| = \|\underline{x} + \underline{y}\|$$

לכן $\underline{x} \cdot \underline{y} = 0$

$$\sqrt{\|\underline{x}\|^2 + \|\underline{y}\|^2} \neq \|\underline{x}\| + \|\underline{y}\|$$

לכן $\|\underline{x} - \underline{y}\| \neq \|\underline{x} + \underline{y}\|$

$$|A^*| = |A|^{n-1} = 5^{n-1}$$

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אם C הוא מטריצה $n \times n$ אז $|C^*| = |C|^{n-1}$

$$M^T = (A^T B^T S B A)^T = A^T \cdot B^T \cdot S^T \cdot (A^T)^T \cdot (B^T)^T = A^T \cdot B^T \cdot S \cdot A \cdot B = M$$

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לכן M הוא מטריצה

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$$X = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} -7 & 9 \\ 12 & -14 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} 24 & 13 \\ -34 & -18 \end{pmatrix}$$