Advertising and Pricing for Dropout Consumers*

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Abstract

This papers analyzes equilibrium pricing and advertising strategies when consumers incur search costs to observe prices. When prices are unaffordably high, those consumers stop searching and 'drop out' of the market and subsequently only re-enter if low prices are advertised. It is shown that in these markets prices may be sticky if firms face uncertainty about the expected duration of cost changes and advertising is sufficiently costly.

Keywords: search, advertising, asymmetric price adjustment, sticky prices, dropout consumers

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1 Introduction

In many markets consumers consumers are not well informed about prices or product characteristics and must engage in costly search to learn about them. When this is the case, it is generally too costly for consumers to find the lowest priced firm or the product which best matches her taste. This in turn can support price differences between firms which are unsustainable if consumers are better informed. Search models thus typically analyze the role of consumer search costs in determining equilibrium price differences between competing firms, and the average price and match quality consumers obtain in equilibrium. An explicit or implicit assumption of these models is that consumers always buy - the only issue is where they buy and at what price.

In this paper I argue that search costs may not only affect the consumers' match value and the prices consumers pay, but also *when* they search and when they buy. In particular, I show that search frictions may discourage consumers from participating in the market altogether[1]. Specifically, I present a dynamic model with repeat purchasing in which consumers can learn the current price either through search - which is costly for them - or through advertising by firms - which is costly to firms. Because search is costly, consumers search only if they expect prices to be sufficiently low. If they expect prices to be too high, they don't search and then only buy if a low price is advertised. Consumers' expectations about prices are in turn determined by the prices they have observed in the past. As long as prices are low, consumers continue to search and firms don't advertise. However, once prices become unaffordably high, they stop searching and effectively 'drop out' of the market - much as unemployed workers may get discouraged and stop seeking employment after failing to find work. And once they 'drop out', they stay out of the market and don't learn any new information until firms lure them back into the market by advertising low prices. In this setting I analyze how firms adjust their prices and advertising strategies in respose to changes in production costs. In particular, I show that if advertising is sufficiently costly, prices may be slow to adjust to changes in production costs - they may be "sticky" [2].

[1] In static search models this issue is typically ducked by assuming that one search is free. But this is not a satisifactory assumption in a dynamic model with repeat purchases.

[2] Empirical evidence that prices are sticky includes Bils and Klenow (2004), Blinder et al. (1998), Cecchetti

Specifically, suppose that firms' costs unexpectedly decrease when (as a result of high past prices) consumers are "out of the market". If advertising is sufficiently costly, firms would like to advertise a low price only if costs are expected to stay low - in which case they expect their investment to bring dropout consumers back to the market for a 'long' time - but not if the cost reduction is likely to be temporary. Thus if firms are initially uncertain about the expected duration of the cost reduction, they might optimally postpone advertising a new price until better information becomes available, and invest in advertising only once they are more confident that the cost change is likely to persist. In that case, prices will continue to be high and consumers' re-entry into the market may be delayed, even after costs decrease. I also show that the model is consistent with the view that prices rise faster than they decrease (e.g., Peltzman 2000).

Several recent papers (Cabral and Fishman (2012), Lewis (2005), Tappata (2006) and Yang and Ye (2006))) have also analyzed the role of search frictions in delaying price adjustment. In those papers, asymmetric information about firms' costs affect consumers' perceptions about inter firm price differences, and hence which prices they accept without search and which they reject to search for a lower price. For example, in Tappata's model, prices are more dispersed and consumers search more intensely when costs are low than when they are high. Therefore if costs come down unexpectedly, but consumers believe they are still high, they search "too little" and consequently prices remain high longer. In Cabral and Fishman's model, consumers who observe a price cut at one firm may be motivated to search for still lower prices at other firms. Thus, in those models, asymmetric information about changes to production costs affects only the price that consumers pay in equilibrium. By contrast, here, asymmetric information about costs determines *how* consumers become informed - whether through search or through advertising - and thus *when* they buy and when they are out of the market.

(1986), Kashyap (1995), Klenow and Kryvtsov (2008), MacDonald and Aaronson (2001), Peltzman (2000), among many others.

2 Model

Time is discrete and the horizon is infinite. In order to focus on the timing of consumer search in the simplest way, I abstract from competitive effects on prices by assuming a monopoly firm which sells a homogenous product to a continuum of consumers.

There are two production costs, c_L (the low cost) and $c_H > c_L$ (the high cost). The cost at period t is denoted c_t.c_t is determined jointly by the *cost state,* which describes general conditions about costs in the economy and is denoted by s_t, and by firm - specific cost shocks. Specifically, at any period, s_t assumes one of two values, L and H. If $s_t = L$, then $c_t = c_L$ with probability η, and $c_t = c_H$ with probability 1-η, where 1-η is the probability of an firm-specific shock which causes its cost to temporarily diverge from the cost state. I assume that $1 > \eta > 0.5$. Thus if $s_t = L$, then $c_t = c_L$ "most of the time". Similarly, if $s_t = H$, then $c_t = c_H$ with probability η and $c_t = c_L$ with probability 1-η.[3] The cost state itself evolves as a Markov process with the persistence probability γ: If $s_t = L$, then $s_{t+1} = L$ with probability γ and $s_{t+1} = H$ with probability 1- γ, where $1 > \gamma > 0.5$, and, similarly, if $s_t = H$, then $s_{t+1} = H$ with probability γ.

At every period t the firm learns c_t and s_{t-1} (the state at the preceding period) but does not observe s_t. The idea is that the firm observes its own cost in real time but information which is not firm specific spreads is only learned with delay. Thus, if $s_{t-1} = L$, $c_{t-1} = c_L$ and $c_t = c_H$, the cost increase at period t is 'unexpected'. In this case, the firm does not know if the cost increase reflects a 'long term' change in the cost state from L to H, and hence costs are expected to remain high, or only a temporary firm-specific shock without a change in the state. Similarly in the case of an unexpected cost decrease, the firm is uncertain about its expected duration.

Consumers are infinitely lived, have unit demand each period, and are of two types, 'shoppers' and 'search consumers'. A shopper has zero search costs, visits the firm each period and buys one unit if and only if the price $\leq V$, where $V > c_H$. The proportion of shoppers in the population is $\alpha < 1$. The analysis will focus on the case where α is small (i.e., most consumers incur search costs).

Search consumers incur a search cost $\phi > 0$ to learn the current price at any period (for

[3] The firm specific shock is i.i.d at successive periods, t and $t + 1$, at which $s_t = s_{t+1}$.

example, by visiting the firm or consulting its website), unless it is advertised. Specifically, at the beginning of a period, the firm may advertise its current price at a fixed advertising cost A, [4] in which case it is commited to that price at that period [5]. For simplicity I assume that an advertised price is learned by all searchers[6] before they decide whether or not to search that period. In any case consumers are uninformed about production costs, past or present. At any period, a searcher gets utility U from a consuming a unit, where $c_L < U < c_H$. If she searches before buying at the price p, her utility that period is $U - p - \phi$ and if she searches without buying her utility is $-\phi$. Thus a search consumer only searches if she expects the price to be $\leq U - \phi$. Search consumers remember prices they have previously observed since (and including) the last time the price was advertised.

A strategy for a firm at each period is whether or not to advertise, and which price to set. A strategy for a search consumer at each period is whether or not to search.

The preceding assumptions imply that if $\phi = 0$ or if $A = 0$, then, if α (the proportion of random buyers) is sufficiently small, the unique equilibrium prices are $p_t = V$ when $c_t = c_H$ and $p_t = U$ when $c_t = c_l$.[7] In these 'benchmark' cases, search consumers buy whenever the cost is c_L and only shoppers buy when the cost is c_H.

Things are less clear cut if $\phi > 0$ and A is large, because consumers' search decision then depends on the price they expect to find. In fact, given the infinite horizon, a plethora of equilibria then exist if the discount factor is sufficiently near 1 and A is sufficiently large, including equilibria in which search consumers never buy. [8]

I shall focus on equilibria in which, in line with the benchmark cases $\phi = 0$ or $A = 0$,

[4] Models in which consumers are informed by advertising and/or search include Butters (1977), Robert and Stahl (1993), Janssen and Non (2008, 2009).

[5] I assume that the seller cannot commit to prices at future periods. If it could, price rigidity could result simply from the fact that the seller is unable to change its price while still committed to a previously fixed price.

[6] A more realistice assumption is that advertising costs are convex in the number of consumers reached. This would complicate the analysis but should not change the main results.

[7] Also, if $A > 0$ but sufficiently small, the price must be sufficiently near U at low cost periods or else the firm would advertise U at those periods.

[8] In this equilibrium search consumers always expect unadvertised prices to be greater than U, therefore never search and thus if A is sufficiently large, the firm never sells to search consumers.

search consumers buy when costs are low and are expected to stay low, which is when it is most profitable for the firm to sell to those consumers. More specifically, let $t \in [i, c_i]$ denote a period t such that $s_{t-1} = i$, $c_t = c_j$, $i, j = L, H$ (only s_{t-1} is relevant because s_t is unobserved). For example, $t \in [H, c_L]$ denotes a period such that $s_{t-1} = L$ and $c_t = c_L$ and $t \in [H, c_L]$ denotes a period such that $s_{t-1} = H$ and $c_t = c_L$. I restrict attention to equilibria in which search consumers buy at least at $t \in [L, c_L]$, when the cost is is most likely to stay low. These equilibria are termed 'search equilibria'. It will be argued that search equilibria are characterized by price rigidity if costs are sufficiently persistent.

I procede as follows. First, in proposition 1 below, I construct a simple and intuitive equilibrium in which prices generally change more slowly than costs if costs are sufficiently persistent, i.e. if γ and η are sufficiently large. This equilibrium features two prices, V and $p_L < U$. Search consumers search as long as the price is p_L but stop searching if the price increases above p_L. After that they only re-enter the market if p_L is again advertised. Thus, if the firm increases its price, it will have to invest in costly advertising to bring search consumers back. Therefore, if the firm is initially uncertain whether the cost increase is temporary or long term - which is the case if $t \in [L, c_H]$ - and if advertising is sufficiently costly, it delays a price increase to keep search consumers in the market until it can ascertain whether or not the cost increase is long term (i.e. reflects a state change). Similarly, if the cost comes down after search consumers have already stopped searching, then, since it is necessary to advertise to bring search consumers back, a price change is delayed until better information becomes available.

Then, in proposition 2, I explore the robustness of this property with respect to the class of search equilibria in general and show that it is robust with respect to downward price rigidity (when the cost comes down) but not with respect to upward rigidity (when the cost increases).

Let the discount factor be $\delta < 1$ and let p_t be the price at period t.

Proposition 1 *If A, γ, η and δ are sufficiently large, and ϕ and α are sufficiently small, there exists a search equilibrium characterized by the two prices, V and p_L, where $c_L < p_L < U - \phi$, such that:*

(1.i) If $t \in [H, c_H]$, $p_t = V$

(1.ii) if $t \in (L, c_L)$, $p_t = p_L$; advertise p_L if consumers don't search that period.

$$(1.\text{iii})\ \text{if } t \in (H, c_L),\ p_t = \left\{ \begin{array}{c} V \text{ if } \ p_{t-1} = V \\ \\ p_L \text{ if } p_{t-1} = \ p_L \end{array} \right\}$$

$$(1.\text{iv})\ \text{If } t \in [L, c_H] :\ p_t = \left\{ \begin{array}{cc} p_L & \text{if } p_{t-1} = \ p_L \\ V & \text{if } \ p_{t-1} = V \end{array} \right\}$$

Proof: In the appendix

To see why the equilibrium described above is characterized by price rigidity, suppose $t-1 \in [H, c_H]$, and $t \in [H, c_L]$. Then, by (1.i) and (1.iii), $p_{t-1} = p_t = V$. Thus the price doesn't change when the cost comes down at period t, and only changes at period $t+1$ if it turns out that $s_t = L$ and the cost stays low. Similarly, if $t-1 \in [L, c_L]$, and $t \in [L, c_H]$, then, by (1.ii) and (1.iv), $p_{t-1} = p_t = p_L$; the price only goes up with delay at the following period if if it turns out that $s_t = H$ and the cost stays high.

To what extent are these properties an artifact of a specific (and possibly arbitrary) equilibrium construction, and to what extent do they characterize search equilibria more generally? Proposition 2 below addresses this issue and establishes that search equilibria are characterized by *downward* price inertia (prices decrease more slowly than costs) if costs are sufficiently persistent (γ and η are sufficiently large) and advertising is sufficiently costly. First, we need the following lemma.

Lemma 1 *In any equilibrium, if γ and η are sufficiently large, then $p_t = V$ at $t \in [H, c_H]$.*

Proof: Search consumers will only buy at a price $\leq U < c_H$. The only possible reason for selling to those consumers below cost can be to keep them searching, so as to save on future advertising costs. But if costs are sufficiently persistent, i.e., γ and η are sufficiently high, so that the cost is expected to remain high with sufficiently high probability, it is more profitable to sell only to shoppers at the price V until the cost goes down again. **End proof**.

Based on the lemma, Proposition 2 below shows that if costs are sufficiently persistent and advertising is sufficiently costly, search equilibria are necessarily characterized by downward price inertia. The intuition is as follows. If prices are high for several consecutive periods, search consumers should realize that the state is high, high prices are therefore likely to persist and they should stop searching. But, as they don't receive any new information once they've stopped searching, those consumers will only return to the market if a low price is advertised. Thus, suppose $t \in [H, c_L]$ and search consumers have already stopped searching prior to period t. Since the firm doesn't observe s_t, the 'mismatch' between s_{t-1} and c_t could have resulted either from a change in the state from H to L at period t, in which case the cost is expected to remain low, or reflect only a temporary change. If advertising is sufficiently costly, relative to period profits, the firm would like to advertise in the former case (when the investment is expected to bear fruit for a long time) but not in the latter case. Since the firm's uncertainty about s_t is (at least partially) resolved at period $t+1$, it optimally postpones a price change until better information becomes available.

Proposition 2 *In any search equilibrium, if* $t-2 \in [H, c_H]$, $t-1 \in [H, c_H]$, *and* $t \in [H, c_L]$, *then if* A *and* γ *and* η *are sufficiently large,* $p_{t-2} = p_{t-1} = p_t = V$.

Proof: In the Appendix.

The preceding proposition thus establishes that downward price rigidity is a general characteristic of the class of equilibria under consideration. But a paralel logic does not imply *upward* stickiness. Specifically, prices are *downwardly* sticky because once search consumers have stopped searching, they are unaware of a cost change until the firm advertises. By contrast, in the case of a cost increase, consumers *are* in the market when the price increases. Thus, whether or not they stop searching in response to a price increase depends only on their expectations about future prices: They stop searching if they believe that a price increase portends high future prices as well (the belief which underpins the equilibrium in proposition 1). However, if they consider that a price increase may be only temporary, they may rationally keep searching until it becomes evident that the price increase is long term. In the latter case, the firm can "safely" increase its price without incurring additional advertising costs even if the cost increase is only temporary. This logic is demonstrated formally in the following proposition, which modifies the equilibrium of proposition 1 so that prices are sticky downwards, but not upwards.

Proposition 3 *If* A, γ, η *and* δ *are sufficiently large, and* ϕ *and* α *are sufficiently small, there exist search equilibria characterized by two prices,* V *and* p_L , *where* $c_L < p_L < U - 2\phi$ *such* *that:*

(III.i) If $t \in [H, c_H]$, $p_t = V$

(III.ii) if $t \in (L, c_L)$, $p_t = p_L$; advertise p_L if consumers dont search that period

(III.iii) If $t \in [L, c_H]$: $p_t = \left\{ \begin{array}{c} (a)\ V \text{ if } p_{t-1} = p_L \text{ or if } p_{t-1} = p_{t-2} = V \\ (b)\ p_L \text{ if } p_{t-1} = V \text{ and } p_{t-2} = p_L \end{array} \right\}$.

(III.iv) if $t \in (H, c_L)$: $p_t = \left\{ \begin{array}{ccc} (a) & p_L \text{ if } p_{t-1} \leq p_L \text{ or if } p_{t-2} \leq p_L & \\ (b) & V & \text{otherwise} \end{array} \right\}$

Proof: In the Appendix.

In this equilibrium, prices are sticky with respect to cost reductions, as in the equilibium of proposition 1, but respond to cost increases without delay. Thus, search equilibria are more generally characterized by downward price rigidity than upward price rigidity. In this sense, the model is consistent with the view that prices tend to respond more quickly to cost increases than to cost decreases (e.g., Peltzman (2000).)

3 Discussion

Note that in the equilibria discussed above, prices come down (with delay) only if the cost change turns out to be long term. If the cost change is temporary, the price does not change. Thus a testable implication of the model is that prices change less frequently in response to short term cost changes than to long term ones.

Let us consider the role of the various assumptions in driving these results. First, the results have been derived under the assumption that costs are "sufficiently persistent". To see why the persistence probability must be sufficiently large - at least greater than 0.5 - consider the extreme case in which costs are distributed i.i.d. Then past prices are irrelevant and thus either consumers always search or prices are always advertised - in either case prices adjust without delay.

The assumption that the firm doesn't observe s_t - and hence is uncertain whether or not an unexpected cost change is likely to be long term - is crucial; If the firm observes the current state without delay, then it optimally advertises a low price as soon as the state changes. Also, the analysis should be qualitatively similar even if the firm only observes its own cost but *never* directly observes the state, current or past. Specifically, if the cost has been uninterruptedly c_i for several sequential periods, and if γ and η are sufficiently large, it can infer that the cost state was probably i at those periods and the above results should continue to obtain, although the analysis would be considerably more complicated.

One may also formulate an alternative version of the model in which the firm can not advertise but consumers are better informed. Specifically, suppose that as in the base model, search consumers incur search costs to learn the *current* price, p_t, but, in contrast to the base model, costlessly learn *past* prices p_{t-1}, p_{t-2},...whether or not they searched at those periods. Then, by similar reasoning to the above, it continues to be true that search equilibria are characterized by short term price rigidity. Again, the reason is that if costs change infrequently, consumers stop searching once costs and prices have been high long enough. Thus, when the price goes down, at period t, consumers only learn this with delay, at period $t+1$. In that case, reducing the cost at period t lowers profit at period t and only increases profit after period t if it turns out the state has turned L. Hence, just as in the base model, the firm will delay a price change until it becomes clear whether the cost change is likely to persist.

4 Appendix

Proof of Proposition 1: Let $p_L = U - \phi - \xi$, where $\xi > 0$ is an (arbitrarily) small number bounded away from zero. Let search consumers' search strategy be: Search at period t if and only if $p_{t-1} \leq p_L$. We now show that the firm's pricing strategy is a best response to this search strategy.

Proof of (I.i): Same as the proof of lemma 1.

Proof of (I.ii): Search consumers keep searching as long as the price is p_L. Thus, if search consumers search at period t, $c_t = c_L$ and $p_t = p_L$, the period t profit is $(p_L - c_L) > \alpha(V - c_L)$ (where $\alpha(V - c_L)$ is the period profit from selling to random buyers only) if α is sufficiently small. A price p', $p_L < p' < U$, might give higher profit than p_L at period t (since search consumers get positive surplus at this price, and since their search cost is already sunk, they should accept it) but causes search consumers to stop searching after period t, which requires the investment of additional advertising costs after period t in order to sell to search customers. Thus if γ and η are sufficiently large (so that the cost is expected to continue to be c_L) and if A and δ are sufficiently large, then p_L is more profitable than $p' > p_L$. An analagous argument implies that if search consumers don't search at period t, then it is optimal to advertise p_L in order to get consumers to start searching if γ and η are sufficiently large[9].

Proof of (I.iii): Suppose $p_{t-1} = V$. Then search consumers don't search at period t. Consider $t \in (H, c_L)$ and let V_t^A be the firm's expected discounted profit, evaluated at period t, if it advertises p_L at period t, and let $V_{t+1}(L)$ and $V_{t+1}(H)$ be the discounted continuation profits evaluated at period $t+1$ if it turns out that $s_t = L$, and $s_t = H$ respectively. Then

$$V_t^A = -A + p_L - c_l + \delta[\theta V_{t+1}(L) + (1-\theta)V_{t+1}(H)] \tag{1}$$

where $\theta = \frac{\gamma(1-\eta)}{\gamma(1-\eta)\ +\ (1-\gamma)\eta}$ is the posterior probability[10] that $s_t = L$ given that $t \in [H, c_L]$.

Consider the alternative strategy of setting $p_t = V$, and advertising p_L at period $t+1$ only if it turns out that $s_t = L$. Denoting the profit from this strategy as V_t^{NA}, we have:

$$V_t^{NA} = \alpha(V - c_l) + \delta\theta[V_{t+1}(L) - A] + (1-\theta)V_{t+1}(H)] \tag{2}$$

Thus, $V_t^{NA} > V_t^A$ if $A > A^* = \dfrac{p_L - c_L - \alpha(V - c_L)}{1 - \theta\ \delta}$. Thus, if γ and η are sufficiently large and $A > A^*$, $p_t = V$ is optimal[11].

By contrast, if $p_{t-1} = p_L$, then search consumers search at period t, therefore it is unnecessary to advertise in order to sell to search consumers and thus p_L is more profitable than V.

Proof of (I.iv): If search consumers search at period t, then they will stop searching at t+1 if $p_t = V$ and will continue searching if $p_t = p_L$. Thus a completely analagous argument to the proof of (iii) implies that $p_t = p_L$ is more profitable than $p_t = V$. If search consumers don't search at period t than obviously V is more profitable than p_L.

[9] Because consumers start searching once the price is advertised, the investment will bear fruit for a sufficiently long horizon.

[10] Given that $s_{t-1} = H$, the probability that $s_t = H$ and $c_t = c_L$ is $\gamma(1-\eta)$ and the probability that $s_t = L$ and $c_t = c_L$ is $(1-\gamma)\eta$. Thus by Bayes' rule, the posterior probability that $s_t = L$ is given by θ.

[11] Note that as γ and $\eta \to 1$, $\theta \to 0.5$ and $A^* \to 2(p_L - c_L - \alpha(V - c_L))$.

Consumers: The preceding proves that the firm's strategy is optimal. Consider consumers. If $p_{t-1} = p_L$, then, given the firm's strategy, if γ and $\eta \rightarrow 1$, then the probability (as perceived by consumers) that $t \in (L, c_L) \mid p_{t-1} = p_L) \rightarrow 1$ and thus the expected utility from searching at period $t \rightarrow U - p_L - \phi = \xi > 0$. Thus, the consumer's strategy is optimal if γ and η are sufficiently large. If $p_{t-1} = V$, then either $p_t = V$ or the price will be advertised and thus it's optimal not to search. End proof

Proof of Proposition 2: Suppose to the contrary that search consumers buy whenever $c_t = c_L$. Then whenever $c_t = c_L$, $p_t \leq U$. Suppose $p_{t'-2} = p_{t'-1} = V$. Then if a consumer searched at periods, $t' - 2$ and $t' - 1$, and , she should conclude that $c_{t'-2} = c_{t'-1} = c_H$, and therefore, if $\eta \rightarrow 1$, should believe that the probability that $s_{t'-2} = s_{t'-1} = H \rightarrow 1$.Therefore, if $\gamma \rightarrow 1$, she should believe that the probability that $c_{t'} = c_H \rightarrow 1$. Hence (by lemma 1), she believes that the probability that $p_t = V \rightarrow 1$ for $t \geq t'$ as well and hence optimally stops searching at period t' if γ and η are sufficiently large. If she stopped searching before period t', then unless she receives new information at period t' she has no reason to search at period t'. Thus in either case, search consumers will not search from period t' and onwards until a price $\leq U$ is advertised. Thus, if $t' - 2 \in [H, c_H]$, $t' - 1 \in [H, c_H]$, and $t' \in [H, c_L]$, then the firm advertises a price $p_L \leq U$ at period t'.

Let $V_{t'}^A$ be the firm's expected discounted profit, evaluated at period t', from advertising p_L at period t', and let $V_{t'+1}(L)$ and $V_{t'+1}(H)$ be the discounted continuation profits evaluated at period $t' + 1$ if it turns out that $s_{t'} = L$, and if $s_{t'} = H$ respectively. Then

$$V_{t'}^A = -A + p_L - c_l + \delta[\theta V_{t'+1}(L) + (1 - \theta)V_{t'+1}(H)] \quad (3)$$

where $\theta = \frac{\gamma(1-\eta)}{\gamma(1-\eta)\ +\ (1-\gamma)\eta}$ is the posterior probability that $s_{t'} = L$ given that $t' \in [H, c_L]$[12].

Consider the alternative strategy of setting $p_{t'} = V$, and advertising p_L at period $t'+1$ if and only if it turns out that $s_{t'} = L$. [13] Denoting the profit from this strategy as $V_{t'}^{NA}$, we have:

[12] See footnote 10.

[13] Since by lemma 1, on the equilibrium path p$[H, c_H] = V$ and since we're assuming that in equilibrium the price is $\leq U$ whenever the cost is c_L, and since consumers don't observe costs, this deviation can't change consumers' future search behavior after period t'.

$$V_{t'}^{NA} = \alpha(p_h - c_l) + \delta\theta[V_{t'+1}(L) - A] + (1-\theta)V_{t'+1}(H)] \qquad (4)$$

Thus, $V_{t'}^{A} < V_{t'}^{NA}$ if $A > \dfrac{p_L - c_L - \alpha(V - c_L)}{1 - \theta\,\delta}$. The RHS of the preceding inequality attains its maximal value if $p_L = U$, and $\theta \to 0.5$ as γ and $\eta \to 1$. Thus, if γ and η are sufficiently large and $A > \dfrac{U - c_L - \alpha(V - c_L)}{0.5}$, then $p_{t'} = V$, contradicting the assumption that search consumers buy whenever $c_t = c_L$. *End proof.*

Proof of Proposition 3: Let $p_L = U - \phi - \xi$, where $\xi > 0$ is an (arbitrarily) small number bounded away from zero and let search consumers search strategy be:

Search at period $t > 1$ if and only if either: $p_{t-2} = V$ and $p_{t-1} = p_L$ or if $p_{t-2} = p_L$ and $p_{t-1} = V$ or if $p_{t-1} = p_{t-2} = p_L$; Otherwise search only if a price $p_{t-1} \leq p_L$ was advertised at period $t-1$.

We now show that the firms's pricing strategy is an optimal response to this search strategy.

(III.i) and (III.ii). The proof is completely analagous to proof of (I.i) and (I.ii) in propostion 1.

(III.iii (a)): If $p_{t-1} = p_L$, consumers search at period t+1 even if $p_t = V$. Conversely, if $p_{t-1} = p_{t-2} = V$, search consumers don't search at period t or afterwards unless the price p_L is advertised. Thus, in this case as well, charging V at period t doesn't lead to any additional future advertising costs and thus, since $c_H > U$, V is optimal if $c_t = c_H$.

Proof of (III.iii (b)): If $p_{t-1} = V$ and $p_{t-2} = p_L$ search consumers will search at period $t+1$ if and only if $p_t = p_L$. The firm will want to sell to search consumers at period t+1 if $t+1 \in (L, c_L)$. Given that $t \in (L, c_H)$, the posterior probability that $s_t = L$ is $\theta = \frac{\gamma(1-\eta)}{\gamma(1-\eta) + (1-\gamma)\eta}$[14] and hence the probability that $t+1 \in (L, c_L)$ is given by $\theta\gamma\eta$. Thus if $p_t = V$, the firm will incur an additional advertising cost at period t+1 with probability $\theta\gamma\eta \to 0.5$ as $\gamma, \eta \to 1$. Thus if γ, η and A are sufficiently large, $p_t = p_L$ is optimal.

Proof of (III.iv) In case (a), consumers search at period t, therefore selling to search consumers at the low price doesn't entail additional advertising costs and hence p_L is optimal if $c_t = c_L$. In case (b) consumers don't search and thus the proof is analagous to the proof of (I.iv) of proposition 1.

[14] see footnote 10.

Now we show that consumers' search strategy is an optimal response to the firm's pricing strategy. Once search customers have stopped searching, they have no reason to search until p_L is advertised. Supppose they searched at periods $t-1$ and $t-2$. Then given the firms strategy, and given that $p_{t-1} = V$ and $p_{t-2} = p_L$, it is easy to check that the consumers' posterior probability that $c_{t-2} = c_L$ and $c_{t-1} = c_H \to 1$. Therefore as $\gamma, \eta \to 1$, the probability that $(s_{t-1} = H) \to 0.5$ and the probability that $(s_{t-1} = L) \to 0.5$ and thus the consumers' probability that $p_{t+1} = p_L \to 0.5$ and thus her expected utility from searching at $t \to -\phi + 0.5(U - p_L) \geq 0$ (as $p_L < U - 2\phi$). And a fortiori the expected utility ≥ 0 if $p_{t-1} = p_L$. End proof.

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