

PRICE LEVEL DETERMINACY AND THE CHOICE OF POLICY TARGETS UNDER RATIONAL EXPECTATIONS *

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We show that in a conventional macro model with rational expectations, the government can use monetary policy to peg real interest rates, and still attain a determinate price level provided it also pegs *some* nominal policy variable, e.g., nominal expenditures.

1. Introduction

In their well-known paper, Sargent and Wallace (1975) established that the following two policy implications are valid in a conventional IS-LM-NRH model with rational expectations. First, if the money supply is the policy control variable, feedback monetary policy will not have any stabilizing effects on output. Second, the price level is indeterminate if the control variable is the nominal interest rate. For literary simplicity, we will henceforth refer to the former as the irrelevance property, and to the latter as the indeterminacy property. A further point, developed in McCallum (1981), is that the indeterminacy property is actually a combination of an *under*-determined price level and *over*-de-

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terminated output, the latter indeterminacy arising because the government attempts to set both the private and public components of real expenditure independently. This results in a level of aggregate demand that is not necessarily equal to aggregate supply with no variables left to equilibrate the system.¹ These results are 'negative' in the sense that they caution against adopting policy objectives that are not attainable in the framework of the given model.² One way of interpreting these results is to attribute a degree of irrationality to a government that in fact tries to pursue such unattainable policy goals. This note discusses the policy options that *are* available to a government (i.e., combined fiscal-monetary authority) that is rational in the sense that it respects the limitations implied by the Sargent-Wallace and McCallum analyses. Specifically, since activist stabilization policy is not feasible, a rational government can be expected to focus attention on other objectives such as the distribution of output between private and public uses and, of course, on the determination of the price level. In a small macro model, the former objective amounts to determining the real interest rate. Within this framework, we show that a policy of pegging the *real* interest rate is *feasible* in the sense that a determinate price level can also be attained by proper policy settings. The question of whether a policy of pegging the real rate is desirable will be shown to depend on whether the government wishes to absorb the effects of stochastic shocks in its own real expenditure or whether it wishes to impose these effects on the private sector. This choice determines whether the real rate *or* the level of real government expenditures is to be the exogenously determined policy variable.

Section 2 of this note develops these results using a model similar to the one in McCallum (1981). Section 3 presents some further speculations and concludes the paper.

¹ In an open economy, one would think that the real exchange rate could be used to equilibrate the system. Recent analysis by Calvo (1982) reveals that this is not a trivial matter if one allows for capital mobility. In any event, the scope of this paper is restricted to closed economies.

² If the information structure of the model is altered to allow room for activist policy, as in Weiss (1980) and King (1982), the options available to the government obviously are expanded but obviously not without limit. Extension of the present analysis to these cases is intended.

2. The model

The equations of the model are

$$Y_t = b_0 + b_1 r_t + b_2 (G_t - P_t) + u_{1t}, \quad \text{IS equation, (1)}$$

$$M_t - P_t = a_0 + a_1 Y_t + a_2 (r_t + {}_{t-1}P_t^* - P_t) + u_{2t}, \quad \text{LM equation, (2)}$$

$$Y_t = c_0 + c_1 (P_t - {}_{t-1}P_t^*) + c_2 Y_{t-1} + u_{3t}, \quad \text{aggregate supply, (3)}$$

where Y_t = real output, G_t = nominal government expenditures (taxes are ignored for simplicity), P_t = price level, M_t = nominal money balances, all in logarithms, and r_t = real interest rate. In general, ${}_{t-j}P_t^* = E(P_t/I_{t-j})$ = expectation of P_t given information at the end of period $t-j$. The u_{it} are white-noise stochastic disturbances. With regard to policy, we maintain throughout the conventional assumption that the government has no informational advantage over the public regarding its own policies. Further details about policy will be specified only as required by the questions to be addressed. In terms of the mechanics of carrying out policy objectives we assume that if the government should wish to establish targets in terms of nominal interest rates, then the stock of bonds available for open market operations consists of nominal bonds while if a real interest rate target is being discussed then the bonds are linked to the price level.³

It is easy to show that the irrelevance property holds in this model; by now, this demonstration is well-known so it is not repeated here. Furthermore, if the real interest rate is replaced by the nominal rate minus expected inflation, then McCallum (1981) has shown that indeterminacy holds when the government tries to fix *both* its real expenditure *and* the nominal rate.

The problems that arise when the government tries to peg the real rate as well as its real expenditure can be seen by direct inspection. From the supply equation, we see that, aside from factors that directly affect the natural rate, the only current period variables that affect output are factors that affect the price shock, $P_t - {}_{t-1}P_t^*$. As will be seen below, this shock is a function only of the disturbances, u_{it} . However, from eq. (1)

³ In other words, in each case we assume that the financial instruments are best suited to the associated policy. Questions associated with transitions from one policy regime to another are not addressed.

we see that if the authorities attempt to fix both r_t and $(G_t - P_t)$, aggregate demand will not necessarily be equal to the supply given by (3), i.e., Y_t is over-determined. Clearly, the authorities must relinquish control over one of these variables.

We now show that relinquishing control over either r_t or $(G_t - P_t)$ is sufficient to achieve determinacy. In either case, there is a policy variable, G_t or M_t , respectively, available for determining P_t .

Consider first the case where the government sets real expenditure, $(G_t - P_t)$; r_t is endogenous and M_t is set exogenously to determine P_t . This case is the standard, classical story which is so well-known that explicit solution of the model is not required. In this case, the interest-sensitive components of private expenditure absorb all the effects of shocks to the *IS* curve, u_{1t} , and to the aggregate supply curve, u_{3t} , while the price level absorbs all money demand shocks, u_{2t} .

In the alternative case, the government pegs r_t via open market operations in linked bonds. Thus, the money supply is endogenous leaving nominal government expenditures, G_t , to determine P_t . Solution of the model in this unconventional case reveals that all variables are determined. To see this, start by eliminating Y_t from eqs. (1) and (3) yielding an equation in P_t and ${}_{t-1}P_t^*$,

$$c_1(P_t - {}_{t-1}P_t^*) + b_2P_t + A_t + v_t = 0, \quad (4)$$

where

$$A_t = c_0 + c_2Y_{t-1} - b_0 - b_1\bar{r} - b_2\bar{G}, \quad v_t = u_{3t} - u_{1t},$$

and \bar{r} and \bar{G} are the policy-determined values of r_t and G_t . Applying the method of undetermined coefficients, we write proposed solution for P_t and ${}_{t-1}P_t^*$ as

$$P_t = \Pi_0 A_t + \Pi_1 v_t, \quad {}_{t-1}P_t^* = \Pi_0 A_t. \quad (5)$$

Substituting (5) into (4) yields the required identities in the unknown Π coefficients,

$$(b_2 + c_1)\Pi_1 v_t + b_2\Pi_0 A_t + A_t + v_t = 0, \quad (6)$$

$$(b_2 + c_1)\Pi_1 + 1 = 0 \Rightarrow \Pi_1 = -1/(b_2 + c_1),$$

$$b_2\Pi_0 + 1 = 0 \Rightarrow \Pi_0 = -1/b_2.$$

The solutions for the Π coefficients determine P_t and ${}_{t-1}P_t^*$. By substituting these into (1) or (3) the solution for Y_t is obtained and then substituting for Y_t , P_t and ${}_{t-1}P_t^*$ in (2) yields M_t . Thus, all three endogenous variables for this case P_t , Y_t , and M_t are determined. Note that in this case, the policy setting for r_t can be determined so as to provide the government with a given *ex ante* level of real expenditures, just as setting $(G_t - P_t)$ did in the previous case. In this sense, the government does not lose control of its real budget even though the value of \bar{G} has no intrinsic effect on real expenditures since P_t will adjust to achieve the level of $(\bar{G} - P_t)$ implied by Y_t and \bar{r} .⁴ In this case the real interest rate is the fiscal policy variable while, surprisingly, \bar{G} is the 'monetary' policy variable. The difference between setting fiscal policy via \bar{r} as opposed to $(G_t - P_t)$ is that in the case of \bar{r} the *ex post* level of $(\bar{G} - P_t)$ is not known. The real level of government expenditures absorbs all of the effects on P_t of the stochastic disturbances. Thus, the meaningful difference between the two cases is in terms of which type of expenditures, public or private, acts as the shock absorber.

3. Conclusion

We have shown that in a Sargent-Wallace model, where activist policy is irrelevant for determining output, a rational government will not try to set its own real expenditures and at the same time peg an interest rate. In fact, setting either real government expenditures or the real interest rate is sufficient to determine *ex ante* the allocation of output between the public and private sectors. The choice between the two policy variables hinges on whether private or public expenditures are to absorb the effects of real shocks. Possible extensions of this analysis include consideration of an open economy, analysis of a case where some form of stabilization policy is possible and consideration of 'combination' policies where, for example, the real interest rate on linked bonds would be allowed to drift freely within set bands.

⁴ Another conceivable policy would be to let the government determine a desired level of real expenditures, say g_t , and determine G_t according to $G_t = {}_{t-1}P_t g_t$, i.e., fix nominal spending according to g_t times a rational expectation of the price level. It is easy to show that indeterminacy would result in this case - simply try to solve the model to find that there is no unique solution for Π coefficients.

References

- Calvo, Guillermo, 1982, Pegged interest rates and indeterminacy of equilibrium, unpublished Mimeo.
- King, Robert, Monetary policy and the information content of prices, *Journal of Political Economy* 90, 247-279.
- McCallum, Bennet T., 1981, Price level determinacy with an interest rate policy rule and rational expectations, *Journal of Monetary Economics* 8, 319-329.
- Sargent, Thomas J. and Neil Wallace, 1975, 'Rational' expectations, the optimal monetary instrument, and the optimal money supply rule, *Journal of Political Economy* 83, 241-254.
- Weiss, Lawrence, The role for active monetary policy in a rational expectations model, *Journal of Political Economy* 88, 221-233.