

**Endogenous fertility and  
intergenerational transfers:  
The significance of the  
sibship size effect**

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The Becker model:

$$W_p = U(C_p) + n\delta W(C_c)$$

- $W_p$  utility function of parents
- $W$  utility function of children
- $C_p$  own consumption of the parent
- $C_c$  consumption of children
- $n$  number of children,  $\delta$  parameter of altruism

## 2. Budget Constraint with child labor

$$C_p = A + nwl - \sigma n$$

- A income earned by parents
- w salary earned by children
- l number of hours of child labor
- $\sigma n$  expenses on all children

So parents maximize the utility function given the budget constraint, by taking the FOC with respect to  $n$  and  $l$ .

In order to get an interior optimum for  $n$ , it is necessary that:

- $$[\sigma - wl] > 0$$

i.e., transfers are from parents to children.

Intuition:

An increase in the number of children lead to higher utility through the utility of children, but reduces utility through its own consumption (since net income from children is negative).

If it were positive, optimal amount of children would be infinite.

- This assumption on net transfers from parents to children is necessary to get an interior solution for the number of children.
  - This assumption seems perfectly representative for the way parents behave today in rich countries.
  - Since, in the Western world today, transfers are from parents to children.

However, we will present evidence that this assumption is not a good representation of:

- poor countries today and of
- the eighteenth-nineteenth centuries in the Western world.
- Intergenerational transfers today in some poor countries and in the West during the 18th and 19th centuries are from children to parents, through child labor.

So

- What is happening to the model if the intergenerational transfers are in fact from children to parents? (as we will show that it happens in many poor countries)
- 1. What are the changes to be made in the “regular” model compatible with the transfers moving from children to parents?
- 2. Are some main theorems changing when the model is different?

- 2.
- Are some main theorems changing when the model is different?
- Yes.
- When transfers are from parents to children (regular model) then
- An increase in income leads to an increase in fertility rates.
- This is not true when transfers are from children to parents.



- 1.
- What are the changes to be made in the “regular” model compatible with the transfers moving from children to parents?
- We have to change the utility function, when transfers are from children to parents :
- We have to introduce a “sibship size effect” in the utility function.
- $H=H(l,n)$

- A sibship size effect is used by the sociology literature for analyzing the effect of the size of the family on the well being of the family.
- Therefore, following this literature,
- we will introduce a “sibship size effect” in the utility function, when transfers are from children to parents.

- II. FACTS, LITERATURE and DATA on
  - 1. Shibship size effect
  - 2. Intergenerational transfers and child labor
  - 3. Relationship between income and fertility rates
  
- III. THE MODEL
  
- IV. CONCLUSION

- 2. Literature and Data on sibship size Effect
- The medical and sociological literatures stress the negative effects of the family size on:
  - the formation of human capital,
  - and more specifically, on the future human capital of the child when he becomes an adult.

- 1. An increasing number of siblings lowers intellectual performance:
- This was shown by testing the effects of sibship size on cognitive measures of children (on achievement test reading and Math).

- 2. It has been shown that infectious diseases, such as measles, chicken pox and diarrhea, are more likely to occur in crowded households with numerous children.
- In consequence, larger families appear to increase the child's risk of contracting the infection and the severity of the infection among those who do become ill, and to lead to long run effect on health and human capital.

- 3. Desai (1995) has shown that in poor countries, the addition of a sibling aged less than five years has a statistically negative impact on the child's height for age standardized core, which is a good proxy for global health of children

- What are the reasons that bigger families lead to lower human capital of children?
- 1. Dilution theory:
- More children, less time of parents per child,
- Therefore less investment in time,
- therefore lower human capital of children



- 2. Women's sickness
- Research has shown that women who in average has more than 7 children are more often sick.
- Therefore, cannot take care of their children as well as healthy women.
- 3. Size of apartment
- Bigger families, less room per child, therefore more sickness.

- So children in big families are, *ceteris paribus*, less developed intellectually, and are less healthy.
- $H(I, n)$  When  $dH/dn < 0$

- 2. Data and Literature on the Intergenerational Transfers and child labor

- Marx:
- “ There is a need for the work of children in order to ensure the family’s survival:
- .. “In order that the family may live, four people must now work... previously, the workman sold his own labor power, which he disposed of nominally as a free agent. Now he sells wife and child.”

- Horrell and Humphries :
- During the nineteenth century, “[workers’] earnings declined, and the man’s relative contribution fell suggesting the necessity of getting other households members into the labour force”
- More specifically, the share of male factory workers’ earnings in the family income went down from 60% in 1800 to 39% in 1835.
- “for most households the earnings of women and children were essential.”

- The Sadler report of 1832 shows that children were supporting their parents.

TABLE 2 ( 3)

EARNINGS AND COST OF LIVING FOR ONE WORKER COUPLE IN BATH FOR THE YEARS 1832-1850

Bath (England)	<u>Earnings</u> Shilling per week	<u>Cost of living</u>
1832	9s 6d	13s 1d
1840	13s 2d	13s 10d
1850	14s 1d	14s 2d

**TABLE 6 (8)****PERCENTAGE OF CHILDREN IN AGE 5-14 BY ACTIVITY**

	<b><u>Economic activity only</u></b>	<b><u>School only</u></b>	<b><u>Combining School and economic activity</u></b>	
<b>Burkina Faso</b>	<b>46.3</b>	<b>26.5</b>	<b>0.7</b>	
<b>Ethiopia</b>	<b>32.9</b>	<b>20.0</b>	<b>16.8</b>	
<b>Gambia</b>	<b>12.3</b>	<b>45.3</b>	<b>12.5</b>	
<b>Guinea Bissau</b>	<b>44.5</b>	<b>17.9</b>	<b>19.9</b>	
<b>Mali</b>	<b>27.4</b>	<b>20.3</b>	<b>16.0</b>	
<b>Namibia</b>	<b>1.6</b>	<b>79.0</b>	<b>12.9</b>	
<b>Niger</b>	<b>45.8</b>	<b>10.8</b>	<b>20.4</b>	
<b>Sao Tome and Principe</b>	<b>4.0</b>	<b>57.4</b>	<b>11.4</b>	
<b>Senegal</b>	<b>19.6</b>	<b>28.3</b>	<b>13.0</b>	



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**TABLE 8 (10)****NIGERIA****CHILD ACTIVITY OPTIONS AND HOUSEHOLD POVERTY STATUS**

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<b><u>Poverty status</u></b>	<b><u>School only</u> <u>(%)</u></b>	<b><u>Work only (%)</u></b>	<b><u>School/Work</u> <u>(%)</u></b>
<b>Core poor</b>	<b>17.6</b>	<b>51.5</b>	<b>30.9</b>
<b>Moderate poor</b>	<b>20.7</b>	<b>61.4</b>	<b>17.9</b>
<b>Non-poor</b>	<b>80.6</b>	<b>1.1</b>	<b>16.7</b>

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**TABLE 12 (16)****CONSEQUENCES TO HOUSEHOLD IF WORKING CHILDREN STOPPED WORK****Attending school/ other educational institution**

	<b>Household living standard declines</b>	<b>Household cannot afford to live</b>	<b>Household enterprise cannot operate</b>	<b>Has no effects</b>	<b>other</b>
<b>Nigeria</b>	<b>30.7</b>	<b>2.1</b>	<b>19.0</b>	<b>N/A</b>	<b>48.3</b>
<b>Sri Lanka</b>	<b>16.3</b>	<b>1.0</b>	<b>31.1</b>	<b>46.9</b>	<b>4.8</b>
<b>Zimbabwe</b>	<b>33.1</b>	<b>7.7</b>	<b>19.0</b>	<b>36.5</b>	<b>3.7</b>
<b>Ghana</b>	<b>43.8</b>	<b>4.9</b>	<b>21.6</b>	<b>28.9</b>	<b>0.6</b>

**TABLE 13 (17)**

**NIGERIA**

**AMOUNT CONTRIBUTED BY CHILDREN AND ITS PERCENTAGE OF THE HOUSE HOLD INCOME**

<b><u>Zone</u></b>	<b><u>Child's income per hour (N)</u></b>	<b><u>Child's income to household income (%)</u></b>
<b>NC</b>	<b>13</b>	<b>3.5</b>
<b>NE</b>	<b>5</b>	<b>9.7</b>
<b>NW</b>	<b>8</b>	<b>17.7</b>
<b>SE</b>	<b>10</b>	<b>38.0</b>
<b>SS</b>	<b>13</b>	<b>12.8</b>
<b>SW</b>	<b>29</b>	<b>10.7</b>
<b>Male</b>	<b>19</b>	<b>14.5</b>
<b>Female</b>	<b>14</b>	<b>19.1</b>
<b>Rural</b>	<b>19</b>	<b>16.0</b>
<b>Urban</b>	<b>17</b>	<b>9.3</b>

- 3. **Correlation between Income and Fertility Rates**

- Regular models predict that the correlation is positive ( $y \downarrow$  then  $n \downarrow$ )
- Our model predicts that it is negative
- ( $y \downarrow$  then  $n \uparrow$ )
- What are the data showing?

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TABLE (2)

CORRELATIONS BETWEEN FERTILITY RATE and INCOME IN ENGLAND DURING  
THE 19TH CENTURY

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	(1)	(2)	(3)	(4)
1800-1850	-.36	.90	-.19	-.49
1800-1840	-.30	.77	-.76	-.72
1800-1900	-.69	-.43	-.52	-.43

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**TABLE ( 4 )**

**COUNTRIES WITH DECREASING Y AND INCREASING N**

	<u>Period</u> <u>(t,00)</u>	$Y_t$	$Y_{2000}$	<u>Annual</u> <u>growth</u> <u>rate</u>	<u>Fertility Rates</u>	
		(In \$)	(In \$)		t	2000
<b>Burundi</b>	<b>1970-00</b>	<b>806</b>	<b>699</b>	<b>-0.47</b>	<b>6.55</b>	<b>6.95</b>
<b>Chad</b>	<b>1965-00</b>	<b>1165</b>	<b>830</b>	<b>-0.96</b>	<b>6.23</b>	<b>6.61</b>
<b>Congo Republic</b>	<b>1975-00</b>	<b>1514</b>	<b>1286</b>	<b>-0.65</b>	<b>6.37</b>	<b>6.92</b>
<b>Gabon</b>	<b>1975-00</b>	<b>2053</b>	<b>1043</b>	<b>--2.67</b>	<b>4.65</b>	<b>4.92</b>
<b>Gabon</b>	<b>1970-00</b>	<b>1443</b>	<b>1043</b>	<b>-1.07</b>	<b>4.14</b>	<b>4.92</b>
<b>Liberia</b>	<b>1970-00</b>	<b>1982</b>	<b>472</b>	<b>-4.67</b>	<b>6.38</b>	<b>6.43</b>

- We present a model showing the relationship between intergenerational transfers and a sibship size effect.
- We show that when transfers are from children to parents then, the utility function necessarily has to introduce a sibship size effect in the formation of human capital.



The model



- OLG economy in discrete time.
- Continuum of identical households; individuals living for 2 periods: *childhood* and *adulthood*.
- Household consumption:  $c_t = y_t - (\sigma - wl_t + b_t)n_t$ 
  - Parent's income:  $y_t$
  - Cost of rearing a child:  $\sigma$
  - Wage per efficiency unit:  $w > \sigma$
  - Labour time supplied by each child:  $l_t \in [0,1]$
  - Bequest to be left to each child:  $b_t$
  - Sibship size:  $n_t$
- Human capital formation (in efficiency units):  $H(l_t, n_t)$

Parent's program at time  $t$ :

$$V(y_t) \equiv \max_{(n_t, l_t, b_t)} \left\{ \begin{array}{l} U(y_t - (\sigma - wl_t + b_t)n_t) + (\delta n_t^{-\varepsilon}) n_t V(y_{t+1}) : \\ y_{t+1} = wH(l_t, n_t) + b_t \end{array} \right\}$$

Typically,  $U(c) = (1/\alpha) c^\alpha$ , with  $\alpha < 1$ ,  $\alpha \neq 0$ ,

and  $(1 - \varepsilon)/\alpha > 0$

or, equivalently, dynastic utility maximization:

$$V(y_0) = \max_{(n_t, l_t, b_t)_{t \in \mathbb{N}}} \left\{ \begin{array}{l} \sum_{t=0}^{\infty} \delta^t N_t^{1-\varepsilon} U(c_t) : \\ N_0 = 1, c_0 = y_0 - (\sigma - wl_0 + b_0)n_0 \\ \text{and, for } t \geq 1, N_t = \prod_{i=0}^{t-1} n_i, \\ c_t = wH(l_{t-1}, n_{t-1}) + b_{t-1} - (\sigma - wl_t + b_t)n_t \end{array} \right\}$$

First order condition for utility maximization w.r.t.  $n_t$ :

Marginal opportunity cost of having a child

$$U'(c_t) (\sigma - wl_t + b_t) \\ = \delta n_t^{-\varepsilon} |V(y_{t+1})| (|1 - \varepsilon| - (1 - b_t/y_{t+1}) \epsilon_V(y_{t+1}) \epsilon_{Hn}(l_t, n_t))$$

Marginal utility of having a child

With child labour, intergenerational transfers may be upstream:

$$\sigma + b_t < wl_t$$

An interior solution is then only possible with a high enough *sibship size effect*:

$$\epsilon_{Hn}(l_t, n_t) > |1 - \varepsilon| / ((1 - b_t/y_{t+1}) \epsilon_V(y_{t+1}))$$

Otherwise,  $n_t$  will be pushed to its maximum biological value.

## Proposition 1

A solution  $(n_t, l_t, b_t)$  to the parent's program can exhibit *upstream intergenerational transfers*, from children to parents ( $\sigma + b_t < w l_t$ ), only under a strong enough *sibship size effect*, measured by the elasticity (in absolute value)  $\varepsilon_{Hn}(l_t, n_t)$  of the function  $H$  with respect to  $n$ .

- Variability of the sibship size effect  $\varepsilon_{Hn}(l_v, n_t)$  may lead to multiplicity of steady states with different regimes of intergenerational transfers.
- This is what we find in fact in the literature where the sibship size effect is usually introduced in the form of *resource dilution*: the parents have access to a scarce resource (typically time) required to raise their children, and this resource is diluted as the number of children increases.
- Examples: Becker, Murphy and Tamura (1990), Dessy (2000), Galor and Weil (2000), Wigniolle (2002), Hazan and Berdugo (2002), Blackburn and Cipriani (2005), Chakraborty and Das (2005), Sugawara (2010).



Steady states

We introduce the specifications:

$$U(c) = (1/\alpha) c^\alpha, \text{ with } \alpha < 1, \alpha \neq 0, \text{ and } (1 - \varepsilon)/\alpha > 0$$

$$H(l, n) = h(1 - l/\lambda)(1 - n/\nu), \text{ with } h > 1, \lambda > 1 \text{ and } \nu > 1$$

$$\delta \nu^{1-\varepsilon} < 1$$

By the f.o.c. relative to  $l$ , an interior solution is compatible with a single value  $\tilde{n}$  for the sibship size.

The f.o.c. relative to  $n$  is an equation:  $h F(n) = G(l)$ .

## Proposition 2

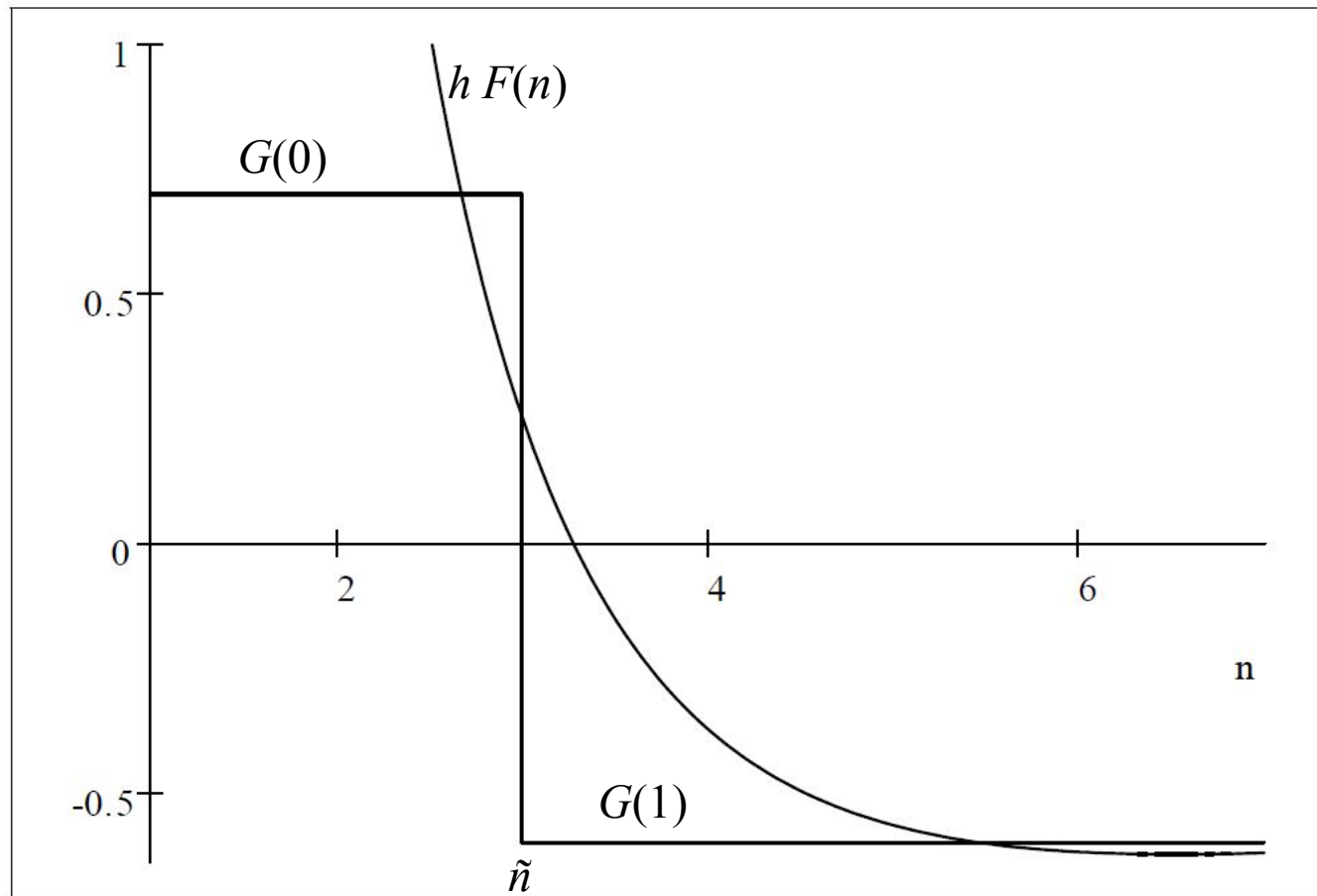
Each steady state equilibrium belongs to one of three possible regimes:

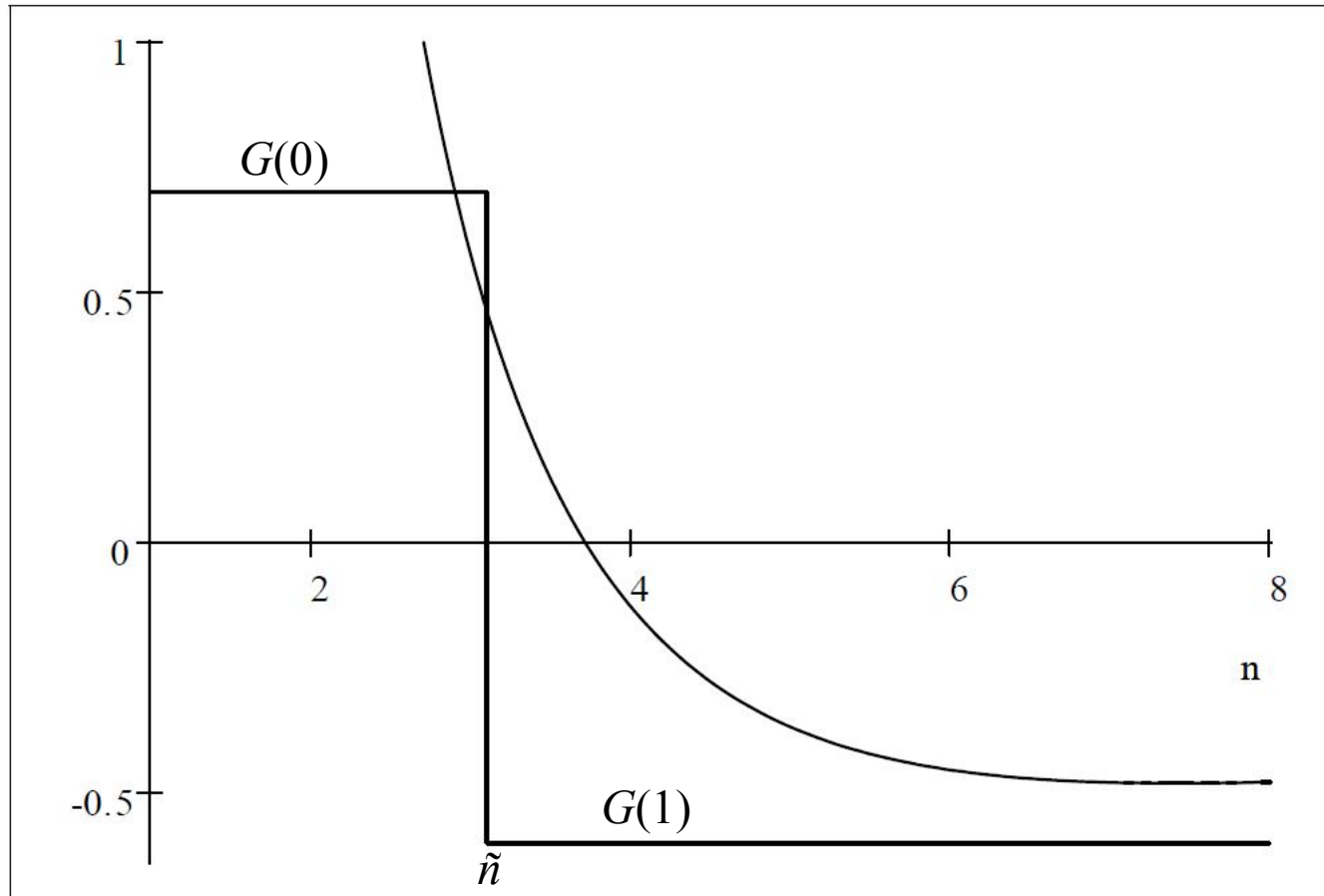
- the regime of *full child labor* ( $l = 1$ ), with high fertility ( $\tilde{n} < n < v$ ,  $n$  given by  $hF(n) = G(1)$ ), a strong sibship size effect and transfers from children to parents;
- the regime of *full schooling* ( $l = 0$ ), with low fertility ( $n < \tilde{n}$ ,  $n$  given by  $hF(n) = G(0)$ ), a weak sibship size effect and transfers from parents to children;
- the intermediate regime, with  $n = \tilde{n}$ ,  $l$  given by  $hF(\tilde{n}) = G(l)$  s.t.  $0 < l < 1$ , and intergenerational transfers going *a priori* either way.



## Proposition 3

There exists at least a steady state equilibrium if the (maximum) productivity  $h$  in human capital formation is high enough.





Same parameter values, except for a higher  $\nu$   
(weaker sibship size effect)