Endogenous fertility and intergenerational transfers:
The significance of the sibship size effect[*]

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Abstract

Since the seminal work of Becker, the analysis of endogenous fertility has been based on the trade-off faced by parents between the quantity and the quality of their children. In order to have an interior solution, the model assumes that in case children work, still they get positive income from their parents. However, in some developing countries, child labor is necessary as a source of income. The purpose of this paper is to "adapt" the quantity-quality trade-off of the Beckerian model for the cases where net transfers are in fact from children to parents. The paper shows that by adding a *sibship size effect*, we restore the possibility of the trade-off.

Keywords Endogenous fertility · Intergenerational transfers · Human capital formation

JEL Classification J13

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1   Introduction

Since the seminal work of Becker, the analysis of endogenous fertility has been based on the trade-off faced by parents between the quantity and the quality of their children.[1] The models have taken various formulations, from a simple version directly including the number of children in the parent's utility function to a rather sophisticated version in terms of a dynastic utility function, where the quality of each child, as assessed by the altruistic parent, is identified with the child's expected utility (see Becker and Barro, 1988).

Generally speaking, in all these models the fertility decision is derived from the trade-off between two opposite effects of the number of children on the parents' utility. On one hand, more children are desirable but, on the other hand, they are costly in terms of current consumption, due to rearing costs of all sorts. Also, the child's utility can be increased either through bequests or through investment in human capital, adding to the utility of the altruistic parent, but bequests and investment are both costly in terms of sacrificed current consumption. As a consequence of this trade-off, the optimal number of children may be an interior solution to the parent's problem. Taking child labor into account does not fundamentally change the nature of the trade-off, and therefore does not affect the analysis of fertility decisions, except that the wage earned by each child alleviates the family rearing cost.

The situation is however radically different as soon as child labor becomes so substantial that rearing costs, net of the wage income, become negative. In this case, parents' decisions cease to be constrained by the trade-off between

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[1]The first paper on this theme (Becker, 1960) gave rise to a large demography-oriented literature (Becker and Lewis, 1974, Becker and Tomes, 1976, Becker, 1981, and many others), which eventually shifted to questions of economic growth (see Galor and Weil, 2000).

quantity and quality: increasing the family size may now allow to improve quality, through the higher household income afforded by child labor. In other words, with substantial child labor, the trade-off vanishes, and the desired number of children attains its physiological upper bound.

The purpose of this paper is to "adapt" the quantity-quality Beckerian model to the cases where net transfers are in fact from children to parents. It should be noted that the need for change in the theory of endogenous fertility is not due to the mere existence of child labor, but to its opening the possibility of upstream income transfers. The paper shows that, in order to restore the possibility of a trade-off between quantity and quality, it is sufficient to add a negative *sibship size effect* on the quality of children, working in the process of human capital formation. Such effect has been highlighted in the medical and sociological literatures, which has shown that the family size affects negatively the siblings, mostly through a health effect. This literature, summarized in the following section, has somewhat been ignored in the economic literature. However, a sibship size effect restoring the trade-off between quantity and quality is in fact present although far from emphasized, in the form of resource dilution, in a series of papers beginning with Becker, Murphy and Tamura (1990) and published principally during the last decade.

The main reason for generalizing the Becker model, and allowing the intergenerational transfers to go either downstream or upstream, is that both situations occur in the world economy. It is clear that in developed countries, transfers are from parents to children, and child labor is rare. But, as we will show below, the assumption of transfers flowing from parents to children does not accurately portray what is going on in some poor countries today, especially in Africa, where child labor is a necessity for the family. Nor does it portray what occurred in Western Europe at the onset of industrialization, when the children of the proletariat living in misery started working at an

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early age, as early as age four, generating a positive income for the family.[2]

The paper is divided into five sections. Section 2 presents data on intergenerational transfers and introduces the sibship size effect. Section 3 presents the model, with a minimal amount of changes relative to the Beckerian model for a clearer comparison, and discusses the significance of the sibship size effect. Section 4 analyzes the existence of steady state equilibria, under different regimes of child labor and intergenerational transfers. Section 5 concludes.

2   Demographic evidence for poor countries

Historical evidence suggests dissimilarities between poor and rich countries regarding three elements related to child rearing. The first is that the necessity for child labor differs greatly: while child labor is a necessity for subsistence in some poor countries, this is not the case for rich countries. The second, intimately related to the first, is that the directions of intergenerational transfers are opposite for poor and rich countries: upstream when child labor is a necessity, downstream when it is not. These two elements will be developed in the next subsection. The following one will be devoted to the third element: in big families as they exist in poor countries, family size significantly affects children's health and intellectual development – the *sibship size effect*.

2.1   Intergenerational transfers and the contribution of child labor to family income

We must admit that not all poor countries or poor households present the specificity that intergenerational transfers are from children to parents. As

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[2]See Caldwell (1981), Nardinelli (1990), Schellekens (1993), Horrell and Humphries (1997) for the historical perspective, and Dasgupta (1995), Frankema and van Waijenburg (2012) for contemporaneous evidence.

a matter of fact, there is still an entire debate in the literature on whether parents can survive without child labor, and whether net transfers to children are positive. On the one hand, Basu and Van (1998) claim that child labor is a necessity, and that parents make use of it only because they have no other means of survival. In their own terms, "children's leisure or, more precisely, nonwork is a luxury good in the household consumption" (*op.cit.*, p.415), a situation which they coined the "luxury axiom". On the other hand, some authors pretend that this is not the case, and that child labor is used even when superfluous, a situation which, contrary to the preceding one, is of course compatible with downstream intergenerational transfers.

The phenomenon of child labor is pervasive, but its intensity is higher in poorer countries. The International Labour Organization (ILO) 2010 report on child labor estimates at 215 million the number of child laborers proper between the ages of 5 and 17, and at 306 million the number of children in the same class of age who are "doing some kind of work." These figures represent 13.6% and 19.3% of the whole world population in the same class of age, respectively. Moreover, the ILO 2006 report indicated that 120 million children between the ages of 5 and 14, in the developing countries alone, were full time workers. In a study covering the global economy, where child labor is presented as a symptom of poverty, Edmonds and Pavcnik (2005) show that there is a strong negative correlation between GDP *per capita* and economic activity rates for children (*op.cit.*, p.210, fig.1). In particular, the importance of child income in alleviating household poverty varies over countries as shown in Table 1, which is based on ILO family surveys.

Insert Table 1 here

It is shown that in most of the reported cases half or more (up to 70%) of the families would see at least a reduction of living standards if children stopped to work. Many families claim that without child labor, the household enterprise would stop operating, which would send them to poverty. In a study

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devoted to child labor in Nigeria, Okpukpara and Odurukwe (2006) report that "the contribution of children's earnings to household income ranges from 3.5% to 38%" (p.25) according to the regions, and note that "many families have no alternative other than to send their children to work because they see their earnings as an input into family survival" (p.27).

Economic historians have shown that this fact, observed nowadays in poor countries, was also prominent in England at the time of the industrialization. Indeed, child labor amounted in the 19th century to a significant part of the workforce in some British industries. Children under 12 years of age constituted 8% of the labor force in the cotton industry, and children in an age between 13 and 18 another 10% (see Evans, 1990, p.250). In the 1830s, in some regions such as Lancashire and Leeds, 36% of the workforce in the textile industry consisted of children under the age of 16 (see Tuttle and Wegge, 2002).

Furthermore, "although small on average relative to men's earnings, the contributions of women and children may have been crucial to most families during certain stages in the family life cycle" (Horrell and Humphries, 1997, p.35). "In only a few occupations were men earning enough to buy their families' sustenance and to provide the roof over their heads; for most households the earnings of women and children were essential" (*ibid.*, p.42). Focusing on the working class, Schellekens (1993) writes that "men's wages among the working class, and among unskilled laborers in particular, were not sufficient to support a family" (p.3). According to Shammas (1984), adult equivalent caloric intakes were only just at minimum subsistence levels in the 1790s. Since real earnings of men fell until the 1830s, an increase in child labor was a necessity to keep people alive, and out of complete misery.

2.2 Effects of family size on education and health: the *sibship size effect*

The medical and sociological literatures point out the negative effects of family size, making up the so-coined "sibship size effect" on the formation of the sibling's human capital, and more specifically on its level attained once the sibling has become an adult. We may distinguish two major components in this effect, one deteriorating health, emphasized by the medical literature, the other retarding intellectual development, emphasized by the sociological literature.

Health externalities constitute an important channel of influence of sibship size. The medical literature points out "the negative consequences for health due to crowding and greater exposure to diseases, such as measles, chicken pox and diarrhoea" (Desai, 1995, p.198; see Aaby, 1988, Aaby *et al.*, 1984). Indeed, "repeated exposure to some organisms that cause infectious disease, which is more likely to occur in crowded households with numerous children, especially of similar ages, appears both to increase the child's risk of contracting the infection and the severity of the infection among those who do become ill" (*ibid.*). As also shown by Desai (1995), in poor countries the addition of a sibling aged less than five years has a statistically negative impact on the child's height-for-age, a good proxy for children's global health. Thus, larger families appear to induce adverse long run effects on health and human capital. Another reason for such negative effects is mothers' sickness, indirectly hindering the development of children. Recent research has shown that ultra-orthodox women in Israel, who have in average more than 7 children, are more often sick, and cannot take care of their children as well as healthy women.

Independently of this particular source of educational deficiency, a negative influence of family size on the emotive and intellectual development of the children has been pointed out by the psychological and sociological literatures, in particular by testing the effects of sibship size on the cognitive

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measures of children. The evidence of reading achievement and mathematics tests suggests that increasing the number of siblings lowers intellectual performance (see Guo and VanWey, 1999). Moreover, there are also empirical economic studies showing directy that the number of siblings adversely affects earned income (see Lampi and Nordblom, 2012).[3] One possible explanation for these observations is what has been termed the *resource dilution theory*, claiming that the sibship size dilutes family resources, in particular parents' time and attention, and affects negatively the children's intellectual development and educational success (see King, 1987, Guo and VanWey, 1999, Phillips, 1999, Downey *et al.*, 1999, Downey, 2001).[4] Sources of the sibship size effect other than simple resource dilution may also be pointed out, for instance scale diseconomies in housekeeping, leaving less time for education as the family size increases.

To conclude, children in large families are *ceteris paribus* less healthy and less developed intellectually. Sibship size affects negatively the human capital of children, through different channels investigated by the medical, psychological, sociological and economic literatures. The modeling of human capital formation should consequently take family size into account, together with time devoted to education and other factors. In the next section, we present a model showing the relationship between intergenerational transfers and a sibship size effect. We show that adding a sibship size effect changes significantly the properties of the model of endogenous fertility.

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[3]See also Birdsall (1982), Psacharopoulos and Arriagada (1989), Knodel *et al.* (1990).

[4]"[...O]ne of the most consistent predictors of educational outcomes is the number of siblings, or sibship size. Across various measures of intellectual skills and educational achievement, individuals with the fewest siblings do the best according to studies that have used multiple data sets collected in the United States [...], Europe [...], and Asia" (Downey, 2001, p.497). The economist concludes in the same way as the sociologist: "The empirical analysis finds that achievement falls systematically with increased family size" (Hanushek, 1992, p.112). See also Li, Zhang and Zhu (2008).

3 The model

We consider an overlapping generations economy with a continuum of identical households. Households are composed of individuals living for two periods, childhood and adulthood. Household consumption is not individualized: it covers consumption by adults and children. A child of generation t participates in household consumption c_t, is reared at a fixed cost σ_t, works during $l_t \in [0,1]$ units of time at a wage rate w_t, and is educated at no extra cost during the remaining time. Education during $1 - l_t$ units of time of a child belonging to a sibship of size n_t leads in adulthood to human capital generating a number $H(l_t, n_t)$ of efficiency units per unit of working time. The function H of human capital formation is assumed continuously differentiable, decreasing and concave.

3.1 The parent's decisions

A representative adult of generation $t - 1$, the parent, takes at period t decisions concerning the number $n_t \in \mathbb{R}_+$ of children, their individual labour supply $l_t \in [0,1]$ and the bequest $b_t \in \mathbb{R}_+$ to be left to each child. For simplicity, we shall assume a constant environment. This means constancy of the wage per efficiency unit ($w_t = w_{t-1} = w$), implicitly resulting from output production by competitive firms endowed with a linear technology. This also means constancy of the cost of rearing a child, assumed smaller than the wage ($\sigma_t = \sigma_{t-1} = \sigma < w$), in order for intergenerational transfers from children to parents, through child labor, to be possible. Given adult income y_t at period t and a degree of altruism $\delta n_t^{-\varepsilon}$ (with $0 < \delta < 1$ and $\varepsilon \geq 0$) toward each child (Becker and Barro, 1988, Barro and Becker, 1989), household decisions are consequently taken so as to solve the program:

$$V(y_t) \equiv \max_{(n_t, l_t, b_t)} \left\{ \begin{array}{c} U(y_t - (\sigma - wl_t + b_t) n_t) + (\delta n_t^{-\varepsilon}) n_t V(y_{t+1}) : \\ y_{t+1} = wH(l_t, n_t) + b_t \end{array} \right\}. \quad (1)$$

The function U represents current utility, which depends exclusively on household consumption $c_t = y_t - (\sigma_t - w_t l_t + b_t) n_t$ and is assumed continuously differentiable, increasing and strictly concave.

The value function V represents the maximum utility an adult can obtain from each given income, including, in addition to current utility $U(c_t)$ derived from household consumption, the sum $n_t \left(\delta n_t^{-\varepsilon} V(y_{t+1})\right)$ of maximum utilities $V(y_{t+1})$ of all (identical) children, weighted by the degree of altruism $\delta n_t^{-\varepsilon}$ toward each one of them. By induction, we see that this formulation is equivalent to a dynastic formulation of the type introduced by Becker and Barro (1988), namely

$$
V(y_0) = \max_{(n_t, l_t, b_t)_{t \in \mathbb{N}}}
\left\{
\begin{array}{c}
\sum_{t=0}^{\infty} \delta^t N_t^{1-\varepsilon} U(c_t) : \\
N_0 = 1, \ c_0 = y_0 - (\sigma - w l_0 + b_0) n_0 \\
\text{and, for } t \geq 1, \ N_t = \Pi_{i=0}^{t-1} n_i, \\
c_t = w H(l_{t-1}, n_{t-1}) + b_{t-1} - (\sigma - w l_t + b_t) n_t
\end{array}
\right\},
$$

(2)

provided the value of the objective function, an infinite sum, remains finite.

Barro and Becker (1989) assume in general that the current utility function is isoelastic ($U(c) = (1/\alpha) c^\alpha$, with $\alpha < 1$, $\alpha \neq 0$), in other words that the elasticity of intertemporal substitution is constant, equal to $1/(1-\alpha)$. This specification covers two cases: the case of intertemporal substitutability, with $\alpha > 0$, and the case of intertemporal complementarity, with $\alpha < 0$. In order to express the idea that parents like having children, we must accordingly assume $(1-\varepsilon)/\alpha > 0$, that is, $\varepsilon < 1$ if $\alpha > 0$ and $\varepsilon > 1$ if $\alpha < 0$ (Jones and Schoonbroodt, 2010).

If we refer to program (1), we see that the first order condition for utility maximization relative to a positive number n_t of children (the equality of the marginal opportunity cost and the marginal utility of children) can be

written as

$$
\begin{aligned}
U'\left(c_t\right)\left(\sigma - wl_t + b_t\right) \quad &\quad (3)\\
= \quad & \delta\left(1 - \varepsilon\right) n_t^{-\varepsilon} V\left(y_{t+1}\right) + \delta n_t^{1-\varepsilon} V'\left(y_{t+1}\right) w H_n'\left(l_t, n_t\right)\\
= \quad & \delta n_t^{-\varepsilon}\left|V\left(y_{t+1}\right)\right|\left(\left|1 - \varepsilon\right| - \left(1 - b_t/y_{t+1}\right)\epsilon_V\left(y_{t+1}\right)\epsilon_{Hn}\left(l_t, n_t\right)\right),
\end{aligned}
$$

where $\epsilon_V\left(y_{t+1}\right) = \left|V'\left(y_{t+1}\right) y_{t+1}/V\left(y_{t+1}\right)\right|$ is the elasticity in absolute value of V at y_{t+1} and $\epsilon_{Hn}\left(l_t, n_t\right) = \left|H_n'\left(l_t, n_t\right) n_t/H\left(l_t, n_t\right)\right|$ is the elasticity in absolute value of H with respect to n at $\left(l_t, n_t\right)$. Notice that, in the second expression of the RHS of this equation (the marginal utility of children), we are still covering both the case of intertemporal substitutability, with $V\left(y_{t+1}\right) > 0$ and $1 - \varepsilon > 0$, and the case of intertemporal complementarity, with $V\left(y_{t+1}\right) < 0$ and $1 - \varepsilon < 0$.[5]

3.2 Intergenerational transfers

In spite of child labor, optimal (utility maximizing) intergenerational transfers are necessarily downstream, from parents to children $\left(\sigma + b_t > wl_t\right)$, as long as the negative *sibship size effect* on human capital formation (measured by the elasticity $\epsilon_{Hn}\left(l_t, n_t\right)$) is kept small enough, that is, as long as $\epsilon_{Hn}\left(l_t, n_t\right) < \left|1 - \varepsilon\right|/\left(\left(1 - b_t/y_{t+1}\right)\epsilon_V\left(y_{t+1}\right)\right)$. If intergenerational transfers are upstream, the marginal opportunity cost of children is negative, so that the number of children is pushed to its biological upper bound, unless the marginal utility of children, diminished by a strong sibship size effect, becomes itself negative. We formally state this result.

Proposition 1 *A solution $\left(n_t, l_t, b_t\right)$ to program (1) can exhibit upstream intergenerational transfers, from children to parents $\left(\sigma + b_t < wl_t\right)$, only under*

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[5]Jones and Schoonbroodt (2010) base their analysis of the demographic transition on the adoption of a dynastic model with intertemporal complementarity ($\alpha < 0$), where the quantity and the quality of children are substitutes (instead of complements, for $\alpha > 0$). However, the opposition between the two cases is irrelevant in the present context.

a strong enough sibship size effect, measured by the elasticity (in absolute value) of the function H with respect to n:

$$\epsilon_{Hn}\left(l_t, n_t\right) > \frac{|1 - \varepsilon|}{\left(1 - b_t/y_{t+1}\right)\epsilon_V\left(y_{t+1}\right)}. \tag{4}$$

A further point should be stressed at this stage. The two regimes of intergenerational transfers may rule in two different economies, but they may also be alternatively viable in the same economy (with unchanged specifications) if the sibship size effect varies in intensity with (l, n). If for instance the elasticity $\epsilon_{Hn}\left(l, n\right)$ is an increasing function of n, we may well obtain existence of two contrasting steady state equilibrium regimes: one with low fertility, a small sibship size effect and downstream intergenerational transfers, the other with high fertility, a large sibship size effect and upstream intergenerational transfers. We illustrate this possibility in the following section.

Before proceeding to the analysis of steady states, it may be useful to compare these results with what we find in the related literature.[6] A sibship size effect is in fact already present, at least implicitly, in Becker, Murphy and Tamura (1990), through the assumption that each child's human capital is a function of the *parent*'s time invested in her/his education. As time availability is limited, this leads to *resource dilution*, one of the possible sources of a sibship size effect[7] This effect is responsible for the possible coexistence

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[6]The models referred to in the following do not necessarily adopt a dynastic specification. This is however immaterial for the point we are discussing. The elasticity of the value function V in equation (4) has simply to be replaced by the elasticity of some other function representing each child's utility, and we obtain the same results. For instance, Baland and Robinson (2000), using a static model, introduce such a function W_c, depending only on child consumption. They ignore any sibship size effect ($\epsilon_{Hn}\left(l, n\right) \equiv 0$ in our notation), so that intergenerational transfers are necessarily downstream in their model.

[7]The parent of generation $t - 1$ supplies l_t units of her time endowment in the labor market, and devotes the remaining time $(1 - l_t)$ to the education of her children. Each one of them, in a sibship of size n_t, will dispose one period ahead of $\widetilde{H}\left((1 - l_t)/n_t\right)$ efficient units of labor, where $\widetilde{H}$ is an increasing function. Hence, there is dilution of the time resource, leading to a negative effect of the sibship size on human capital formation. See also Tamura (1994), where there is human capital dilution: each child's human capital

of two steady states, one (corresponding to a "poverty trap") with high fertility and no investment in human capital, the other with low fertility and possibly endogenous growth (because of human capital accumulation from one generation to the next, which we ignore in our model). As children do not work, intergenerational transfers remain however downstream. The same source of a sibship size effect, namely dilution of the parent's time endowment, has later been used again and again in a series of models with child labor and intergenerational transfers going either way, in order to obtain existence of interior, possibly multiple, steady states:[8] Dessy (2000), Galor and Weil (2000), Wigniolle (2002), Hazan and Berdugo (2002), Blackburn and Cipriani (2005), Chakraborty and Das (2005), Sugawara (2010), Varvarigos and Zakaria (2013). Should we eliminate resource dilution in these models, we would obtain a single regime of extreme upstream intergenerational transfers, corresponding in our framework to a corner solution with $l = 1$ and n equal to some exogenous upper bound which we did not explicitly introduce.[9]

is proportional to the parent's human capital divided by the number of children. As the parent's resources include, in addition to human capital, a goods endowment, there is also dilution of this endowment as the sibship size increases.

[8]The same kind of results is also obtained in a model where fertility remains exogenous by Basu and Van (1998), with sibship size having a wage depressing effect in a context of general equilibrium.

[9]To illustrate, consider the last five cited papers. In order to eliminate the phenomenon of the parent's time endowment dilution, we take: $z = 0$ in Hazan and Berdugo (2002, p.814); $s = 0$ in Blackburn and Cipriani (2005, p.197 *in fine*), and Sugawara (2010, eq.(3)); $\tau = 0$ in Chakraborty and Das (2005, p.274 *in fine*); $q = 0$ in Varvarigos and Zakaria (2013, eq.(2)). The switch values of human capital which separate the different regimes become then infinite, namely: $\widetilde{h}$ in eq. (12a) of Hazan and Berdugo; z^c in Prop. 3 and (z_1^c, z_2^c) in Prop. 7 of Blackburn and Cipriani; $\underline{e}$ in eq. (4) of Chakraborty and Das; $\widetilde{h}$ in eq. (12), $\widehat{h}$ in eq. (16) and $\overline{h}$ in eq. (20) of Sugawara. In Varvarigos and Zakaria, where the regime switches are not explicitly detailed, the equilibrium number of children becomes anyway infinite (eq.(13)). A single regime remains, the one with maximum possible transfers from children to parents: no bequests in Blackburn and Cipriani; child labor and fertility at their upper extreme values in Hazan and Berdugo, Chakraborty and Das, Sugawara, Varvarigos and Zakaria.

4 Steady states

We assume isoelasticity of the current utility function: $U(c) = (1/\alpha) c^\alpha$, with $\alpha < 1$, $\alpha \neq 0$, and $(1 - \varepsilon)/\alpha \geq 1$ (for a degree of altruism $\delta n^{-\varepsilon}$, $\varepsilon \geq 0$).[10] For simplicity, we further assume that the function describing human capital formation is multiplicatively separable and linear affine in both its arguments $l \in [0, 1]$ and $n \in [0, \nu]$:

$$H(l, n) = h(1 - l/\lambda)(1 - n/\nu), \text{ with } h > 1, \lambda > 1 \text{ and } \nu > 2. \quad (5)$$

We look for steady state equilibria, along which all the variables are stationary. As the value of the objective function in the dynastic program (2) must remain finite, we have to impose the condition $\delta n^{1-\varepsilon} < 1$ on any admissible steady state value n.

4.1 First order conditions

We begin our analysis with first order conditions. Referring to the dynastic program (2), we see that the condition relative to b_t (for any $t \geq 0$) can be expressed as

$$\delta^t N_{t+1}^{1-\varepsilon} \left(\delta c_{t+1}^{\alpha-1} - c_t^{\alpha-1} n_t^\varepsilon\right) \leq 0, \quad (6)$$

with equality if $b_t > 0$. It is easy to check that the corresponding second order condition is satisfied (the LHS of this inequality is decreasing in b_t). Inequality (6) implies

$$\frac{n_t^\varepsilon}{\delta} \left(\frac{c_{t+1}}{c_t}\right)^{1-\alpha} \geq 1, \quad (7)$$

so that $\delta n^{-\varepsilon} \leq 1$ in a steady state equilibrium, a condition which is satisfied strictly as soon as $n \geq 1$. Thus, bequests are always zero in the steady state equilibria we are going to consider (where the representative family has at

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[10]The reinforcement of the initial assumption $(1 - \varepsilon)/\alpha > 0$ is introduced to ensure concavity of the objective function with respect to n_t.

least one child).

The first order condition relative to l_t (for any $t \geq 0$) can be expressed as

$$
\begin{aligned}
-\delta^t N_{t+1}^{1-\varepsilon} w \left(\delta c_{t+1}^{\alpha-1} (h/\lambda) (1 - n_t/\nu) - c_t^{\alpha-1} n_t^\varepsilon \right) &= 0 \text{ for } l_t \in (0,1) \qquad (8) \\
&\leq 0 \text{ if } l_t = 0, \; \geq 0 \text{ if } l_t = 1.
\end{aligned}
$$

It is easily checked that the second order condition is again satisfied in this case (the LHS of (8) is decreasing in l_t). The preceding equation applying to an interior steady state solution $l \in (0,1)$ determines a value $\widehat{n}$ such that

$$
-\left(\delta \widehat{n}^{1-\varepsilon} (1/\widehat{n} - 1/\nu) - \lambda/h \right) = 0. \qquad (9)
$$

As $\delta n^{1-\varepsilon} (1/n - 1/\nu)$ is decreasing in n, tending to ∞ as $n \to 0$ and to 0 as $n \to \nu$, the value $\widehat{n} \in (0, \nu)$ is uniquely determined. Also, $l = 0$ for $n < \widehat{n}$ and $l = 1$ for $n > \widehat{n}$: full schooling is thus associated with low fertility, full child labor with high fertility.

Finally, the first order conditions relative to $n_0, ..., n_t, ...$ can be expressed in terms of $N_1, ..., N_{t+1}, ...$, by taking $n_t = N_{t+1}/N_t$, in order to get rid of the infinite sum in (2). The condition relative to N_t (for $t \geq 1$) is then

$$
\begin{aligned}
& ((1-\varepsilon)/\alpha) \delta^t N_t^{-\varepsilon} c_t^\alpha \\
+ \; & \delta^t N_t^{1-\varepsilon} c_t^{\alpha-1} \left((\sigma - wl_t + b_t) N_{t+1} N_t^{-2} - w (h/\nu) (1 - l_{t-1}/\lambda) N_{t-1}^{-1} \right) \\
+ \; & \delta^{t+1} N_{t+1}^{1-\varepsilon} c_{t+1}^{\alpha-1} w (h/\nu) (1 - l_t/\lambda) N_{t+1} N_t^{-2} \\
- \; & \delta^{t-1} N_{t-1}^{1-\varepsilon} c_{t-1}^{\alpha-1} (\sigma - wl_{t-1} + b_{t-1}) N_{t-1}^{-1} = 0, \qquad (10)
\end{aligned}
$$

the corresponding second order condition being satisfied under the assumption $(1-\varepsilon)/\alpha \geq 1$ (see Appendix). For steady state values $N_t = n^t$, $l_t = l$ and $b_t = 0$, we obtain

$$
hF(n) \equiv h\delta n^{1-\varepsilon} \frac{(1/n - 1/\nu)(1-\varepsilon)/\alpha - (1/\nu)(1 - \delta n^{1-\varepsilon})}{\delta n^{1-\varepsilon}(1-\varepsilon)/\alpha + 1 - \delta n^{1-\varepsilon}} = \frac{\sigma/w - l}{1 - l/\lambda} \equiv G(l).
$$

$$(11)$$

Since the numerator of the fraction on the LHS of equation (11) has the derivative

$$\frac{1-\varepsilon}{\alpha\nu n}\left(\alpha\delta n^{1-\varepsilon}-\frac{\nu}{n}\right)<0, \tag{12}$$

and since $\delta n^{1-\varepsilon}<1$ for the objective function of the dynastic program to remain finite, the function F can change its sign at most once, becoming negative for higher values of n, such that

$$\epsilon_{Hn}\left(l,n\right)=\frac{n/\nu}{1-n/\nu}>\frac{\left(1-\varepsilon\right)/\alpha}{1-\delta n^{1-\varepsilon}}, \tag{13}$$

that is, for a high enough sibship size effect as measured by the elasticity in absolute value of H with respect to n (cf. equation (3)). Steady state intergenerational transfers become then upstream ($\sigma < wl$).

4.2 The regimes of steady state equilibrium

As stated in the following proposition, which recapitulates our preceding results, three regimes of steady state equilibrium are possible:

Proposition 2 *Each steady state equilibrium belongs to one of three possible regimes: (i) the regime of* full child labor *(l = 1), with high fertility ($\widehat{n} < n < \nu$, n being determined by the equation $hF\left(n\right)=G\left(1\right)$), with a strong sibship size effect and with transfers from children to parents; (ii) the regime of* full schooling *(l = 0), with low fertility ($\delta^{1/\varepsilon}<n<\widehat{n}$, n being determined by the equation $hF\left(n\right)=G\left(0\right)$), with a weak sibship size effect and with transfers from parents to children; (iii) the intermediate regime, with $n=\widehat{n}$ and $l\in\left(0,1\right)$ determined by the equation $hF\left(\widehat{n}\right)=G\left(l\right)$, where intergenerational transfers may a priori go either way.*

These three regimes can alternatively characterize a unique steady state, but they can also coexist in the same economy in the case of multiplicity of steady states. By the first order condition relative to l, the dependence on

n of the RHS of equation (11) can be represented in the space (n, G) by a decreasing staircase curve, with an upper stair $G(0) \in \mathbb{R}_+$ for $n < \widehat{n}$, a lower stair $G(1) \in \mathbb{R}_-$ for $n > \widehat{n}$, and a vertical segment linking the two stairs at $n = \widehat{n}$. A steady state equilibrium is determined by the intersection of this curve with the graph of $hF(n)$, representing the LHS of equation (11). This is illustrated in Figure 1, in a case where the three steady state regimes coexist.[11]

<center>**Insert Figure 1 here**</center>

Of course, the existence of three steady states will not necessarily survive perturbations of the parameter values. However, the following proposition ensures existence of at least one steady state equilibrium for any configuration of parameter values, such that h is high enough.

Proposition 3 *Assume that $\delta \nu^{1-\varepsilon} < 1$. Then there exists a steady state equilibrium with $n \geq 1$ for a high enough (maximum) productivity h in human capital formation.*

Proof. Refer to Figure 1. By continuity, a steady state exists if the graph of $hF(n)$ is (i) above or coinciding with $G(0)$ at $n = 1$, and (ii) below $G(1)$ for n close enough to ν. Condition (i) can be written as

$$h \frac{\delta}{\nu} \frac{(\nu - 1)(1 - \varepsilon)/\alpha - (1 - \delta)}{\delta(1 - \varepsilon)/\alpha + 1 - \delta} \geq \frac{\sigma}{w}.$$

As $\nu > 2$ and $(1 - \varepsilon)/\alpha \geq 1$ by assumption, the LHS of this inequality is positive, so that the inequality is satisfied for h large enough. Condition (ii)

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[11]The two curves are computed according to the following parameter values: $\alpha = -0.99$, $\delta = 0.3$, $\varepsilon = 2$, $\lambda = 2$, $\nu = 7$, $h = 105$ and $\sigma/w = 0.9$. The steady state with full schooling ($l = 0$) has low fertility ($n = 2.855$), high human capital ($H(0, 2.855) = 62.175$) and high family consumption in wage units ($c/w = 59.606$). These characteristics are reversed in the steady state with full child labor: $l = 1$, $n = 4.025$, $H(1, 4.025) = 22.31$ and $c/w = 22.715$. In intermediate regime, we have: $l = 0.346$, $n = \widehat{n} = 3$, $H(0.346, 3) = 49.607$, $c/w = 47.958$, and transfers from parents to children.

<center>17</center>

can be written as

$$-h\frac{\delta\nu^{-\varepsilon}\left(1-\delta\nu^{1-\varepsilon}\right)}{\delta\nu^{1-\varepsilon}\left(1-\varepsilon\right)/\alpha+1-\delta\nu^{1-\varepsilon}} < \frac{\sigma/w-1}{1-1/\lambda}.$$

The LHS of this inequality is now negative by the assumption $\delta\nu^{1-\varepsilon} < 1$, so that the inequality is again satisfied for h large enough. To conclude, observe that the inequality $\delta n^{-\varepsilon} \leq 1$ resulting from the first order condition (7) with respect to b, is satisfied for any $n \geq 1$. Also, the inequality $\delta n^{1-\varepsilon} < 1$, which ensures that the value of the objective function in the dynastic program (2) remains finite, is always satisfied for $n \geq 1$ if $\varepsilon > 1$, and for $n \leq \nu$ if $\delta\nu^{1-\varepsilon} < 1$. ∎

4.3 The importance of the sibship size effect

Our argument to prove existence of a steady state supposed that the LHS of equation (11) eventually becomes smaller than its RHS, as n becomes closer and closer to ν, hence as the sibship size effect becomes stronger and stronger (see equation (13)). Although this is not a necessary condition for existence, we see in Figure 2, where the black curves are computed on the basis of the same configuration of parameter values as before, except for a larger ν, that existence has been lost. Existence can only be restored by changing other parameter values, for instance, in accordance with Proposition 2, by increasing the productivity of human capital h (see the gray curve $h'F(n)$, computed for $h' > h$, now intersecting the stair $G(1)$).[12]

Insert Figure 2 here

What if we completely suppress the sibship size effect, by taking $\nu = \infty$? Proposition 3 does not apply anymore, since the proof is based on the

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[12]The parameter ν is now equal to 14; the parameter h equal, as before, to 105 for the black curves, and to 150 for the gray curves. All other parameter values have been kept unchanged.

possibility of taking n close enough to (the finite value of ν). As a matter of fact, it is easy to show that we lose existence of a steady state when $\nu = \infty$, according to the following proposition.

Proposition 4 *In the absence of a sibship size effect, if $\nu = \infty$, no steady state equilibrium exists in our economy.*

Proof. For $\nu = \infty$, equation (11) becomes:

$$h\,F\left(n\right)|_{\nu=\infty} = h\frac{\delta n^{-\varepsilon}\left(1-\varepsilon\right)/\alpha}{\delta n^{1-\varepsilon}\left(\left(1-\varepsilon\right)/\alpha - 1\right) + 1} = \frac{\sigma/w - l}{1 - l/\lambda}. \qquad (14)$$

The LHS is then always positive, so that a steady state under full child labor is excluded. Also, at $n = \widehat{n} = \left(\delta h/\lambda\right)^{1/\varepsilon}$ (by equation (9)) this LHS is equal to:

$$h\,F\left(\widehat{n}\right)|_{\nu=\infty} = \lambda\frac{\left(1-\varepsilon\right)/\alpha}{\delta\widehat{n}^{1-\varepsilon}\left(1-\varepsilon\right)/\alpha + 1 - \delta\widehat{n}^{1-\varepsilon}} \geq \lambda > \frac{\sigma}{w} = G\left(0\right), \qquad (15)$$

provided $\delta\widehat{n}^{1-\varepsilon} < 1$, which is always true if $\varepsilon > 1$. Otherwise, if $\varepsilon < 1$ and $\delta\widehat{n}^{1-\varepsilon} \geq 1$, $\widehat{n}$ is outside the admissible range $\left[1, \delta^{-1/(1-\varepsilon)}\right)$ of n. We can then take $n = \delta^{-1/(1-\varepsilon)}$ and compute:

$$h\,F\left(\delta^{-1/(1-\varepsilon)}\right)\Big|_{\nu=\infty} = h\delta^{1/(1-\varepsilon)} \geq \lambda > \frac{\sigma}{w} = G\left(0\right), \qquad (16)$$

obtaining the same kind of result. As $G\left(0\right)$ corresponds to the upper stair of the staircase curve in Figures 1 and 2, and since $F\left(n\right)|_{\nu=\infty}$ is decreasing in n, the two curves can never intersect ($F\left(n\right)|_{\nu=\infty} > G\left(l\right)$ for any n and any l), whatever the (admissible) parameter values. ∎

Notice that the assumption of some exogenous biological upper bound $\overline{n}$ on fertility would restore existence of a steady state equilibrium with the corner solution $n = \overline{n}$ and $h\,F\left(\overline{n}\right)|_{\nu=\infty} > G\left(1\right)$ (at this upper bound $\overline{n}$, the marginal opportunity cost of children would remain smaller than the corresponding marginal utility).

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5   Conclusion

Intergenerational transfers are part of a long-running debate on child labor and standards of living. On one hand, there are those who believe that during the industrial revolution parents sent children to work to prevent idleness, who perceive child labor today as a natural way to employ children, and for whom the standard assumption of downstream intergenerational transfers is perfectly adequate. However, continuing the line of research of Caldwell and Basu, there are those who perceive child labor to be an economic necessity imposed by poverty, and for whom parents send children to work today because they have no choice, no more than those parents who sent their children to work in the mines during the European industrial revolution. If one adopts this view, the standard assumption cannot apply.

However, as soon as we weaken the assumption on intergenerational transfers, and allow child labor to generate a positive net income, we are in trouble to theorize endogenous fertility on the basis of the Beckerian trade-off between children's quantity and quality. We showed in this paper that the introduction of a negative sibship size effect in the child's human capital formation, hence ultimately in the parent's utility function, is a possible answer to this difficulty, since it restores the trade-off. As long as that effect remains weak, a situation which seems appropriate in developed economies, the model works as under the usual assumptions that sibship size does not affect siblings' future income and that intergenerational transfers are necessarily downstream. In poor economies, the effect may however be strong enough for a regime with reverse intergenerational transfers to obtain.

Allowing the sibship size effect to increase with sibship size favors the emergence of multiple steady states with contrasting regimes of child labor, high fertility, low incomes and transfers from children to parents vs. child schooling, low fertility, high incomes and transfers from parents to children. This kind of equilibrium multiplicity has been very much present in recent models designed to analyze the demographic transition, where the multi-

plicity can indeed be ascribed to the sibship size effect. As this effect was generally introduced in the silent form of the dilution of parents' time required to raise their children, its significance remained however essentially unnoticed.

Appendix

Second order condition relative to n_t

By differentiating the LHS of equation (10), we obtain the second derivative with respect to N_t of the objective function in program (2):

$$-\delta^{t+1}N_{t+1}^{2-\varepsilon}N_t^{-3}c_{t+1}^{\alpha-2}w\frac{h}{\nu}\left(1-\frac{l_t}{\lambda}\right)\left((1-\alpha)\,w\frac{h}{\nu}\left(1-\frac{l_t}{\lambda}\right)N_{t+1}N_t^{-1}+2c_{t+1}\right)$$

$$-(1-\alpha)\,\delta^{t-1}N_{t-1}^{-(1+\varepsilon)}c_{t-1}^{\alpha-2}\left(\sigma-wl_{t-1}+b_{t-1}\right)^2$$

$$-2\delta^t N_t^{-\varepsilon}N_{t-1}^{-1}c_t^{\alpha-1}w\frac{h}{\nu}\left(1-\frac{l_{t-1}}{\lambda}\right)$$

$$-(1-\alpha)\,\delta^t N_t^{1-\varepsilon}c_t^{\alpha-2}\left(\begin{array}{c}\left(\sigma-wl_t+b_t\right)N_{t+1}N_t^{-2}\\-w\frac{h}{\nu}\left(1-\frac{l_{t-1}}{\lambda}\right)N_{t-1}^{-1}+N_t^{-1}c_t\frac{\varepsilon}{1-\alpha}\end{array}\right)^2$$

$$-\delta^t N_t^{-(1+\varepsilon)}c_t^{\alpha}\frac{\varepsilon}{1-\alpha}\left(\frac{1-\varepsilon}{\alpha}-1\right),$$

which should be negative. All the terms of this sum but the last one (nonpositive under the assumption $(1-\varepsilon)/\alpha>1$) are indeed negative.

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	Attending school					Not attending school				
	Household living standard declines	Household cannot afford to live	Household enterprise cannot operate	Has no effects	Other	Household living standard declines	Household cannot afford to live	Household enterprise cannot operate	Has no effects	Other
Nigeria	30.7	2.1	19.0	N/A	48.3	23.9	1.7	22.1	N/A	52.3
Sri Lanka	16.3	1.0	31.1	46.9	4.8	45.8	3.6	15.6	30.2	4.8
Zimbabwe*	33.1	7.7	19.0	36.5	3.7	-	-	-	-	-
Ghana*	43.8	4.9	21.6	28.9	0.6	-	-	-	-	-
Malawi	N/A	N/A	N/A	N/A	N/A	15.5	5.0	7.2	25.1	47.2
Notes: (*) including both attending and not attending school						Source: ILO Surveys on child labor (2006)				

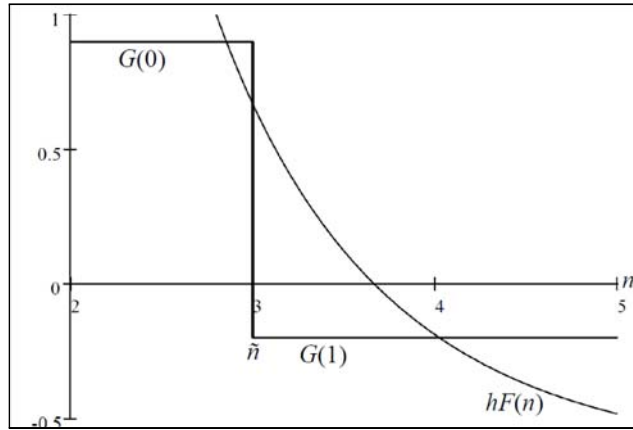Table 1: Consequences to household if working children stopped work
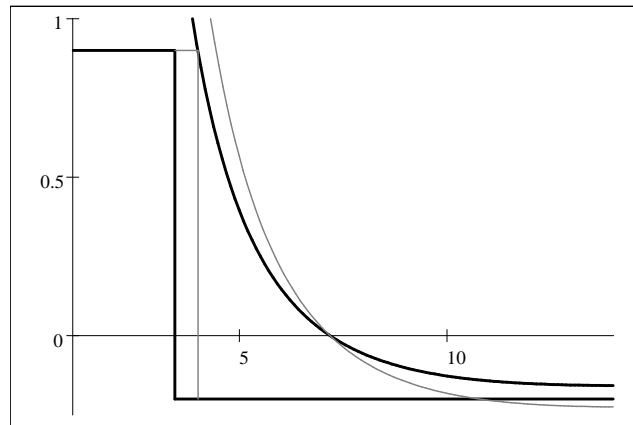
Figure 1: Multiplicity of steady states

Figure 2: Steady state existence and the sibship size effect

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