The Effect of Limited Search Ability on the Quality of Competitive Rent-Seeking Clubs♦

By

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Abstract

A competitive rent-seeking club (CRSC) offers its members the chance of winning a prize (status, position, privilege) by being selected, typically, by a civil servant or a politician. The selector replaces in our setting the usual contest success function; instead of determining the winner on the basis of the club-members' efforts, he selects the winner on the basis of quality. This paper focuses on the effect of incomplete search of the selector on the efficiency of democratic self-governing and decentralized RSC's that control admittance to the club and its transparency, assuming that quality of their members is fixed. The incomplete search of the selector is assumed to take the simple form of fixed random sampling of the contestants - the members of the CRSC. Our results imply that, even when active rent-seeking expenditures are disregarded, the decisions of CRSC's regarding their composition and transparency tend to reduce quality and are therefore inefficient

Key words: competitive rent-seeking clubs, self governance, decentralized decisions, quality competition, selection by fixed random sampling, simple majority rule, endogenous membership determination, transparency, inefficiency, quality decay.

JEL Classification: D70, D71, D72

♦We are especially indebted to Arthur Fishman, Martin Kolmar, Nava Kahana and Klaus Schmidt for their insightful comments. The first author is grateful to the CES at the University of Munich, the Tinbergen Institute at the University of Amsterdam, the CODE at the Universitat Autonoma de Barcelona and the IAE in Barcelona for their hospitality which enabled the completion of this study.

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1. Introduction

Inefficiency in rent-seeking contests is usually due to the resources spent by the contestants in attempting to win a contested rent. In such contests, the rent seekers behave strategically and expend resources taking into account the contest success function; the function that determines their winning probability on the basis of their rent-seeking efforts, see for example, Baik (1999),(2008), Baik, Kim and Na (2001), Congleton, Hillman and Konrad (2008), Gradstein (1995), Hausken (2005), Konrad (2008), Nitzan (1994) and Nti (1997). The main objective of this study is to show that inefficiency in rent-seeking contests can also be due to quality competition under incomplete search of the agent who determines the winner of the contest – the selector, who is typically a bureaucrat or a politician. The incomplete search of the selector can be due to bounded rationality or to binding constraint on information gathering on the quality of the contestants. In our setting, the contestants are members of a competitive rent-seeking club (CRSC); they compete quality-wise on winning the prize, that is, on being chosen by the selector. Selection is an exclusive privilege of the club members. The quality of the club members is fixed, however, the extent of quality competition is determined by their decisions regarding the composition of the CRSC and regarding the transparency of their quality. The selector who tries to enhance his and the public well being has limited search ability or bounded rationality; he chooses the best (highest-quality) club member from the k members that he samples randomly. To clarify the issues arising in this setting, consider the following illustrative "drinking club" example.

Suppose that a group of n drinking buddies visit a bar. They like to drink beer and meet women. When an attractive woman meets the group, she has time to chat to only k out of the n men chosen at random and without replacement. Following this interaction, the woman will select the most attractive man from the k men she has spoken to. The men are exogenously ordered according to their quality (attractiveness), and they would like to maximize their probability of being selected by the women. A man succeeds in doing this if he has the opportunity to chat with the woman (which happens with probability k/n) and if there is no other man among the sampled group of size k who is more attractive. In this (politically incorrect) story, the n men are members of a competitive club and the woman is the selector. The story raises several questions:

First, is a man's probability of selection increased by the addition of an extra drinking man?

The answer may be positive. For example, assume that $k = n = 10$. In this case the woman samples all the men and chooses the best one, who is ranked 1st. An unattractive entrant, when k remains fixed, reduces the selection probability of the 1st ranked man to (10/11) (there is a (1/11) probability of sampling 10 men that do not include the 1st-ranked man), increases the selection probability of the 2nd ranked man to (1/11) and leaves the selection probability of the remaining men unchanged at 0.

The entry of the $(n+1)^{th}$ agent has two effects on an incumbent: first, it reduces the probability of belonging to the sampled group from k/n to $k/(n+1)$; second, it changes the incumbent's selection probability, provided that he is sampled. Entry of an inferior entrant increases this probability while entry of a superior entrant decreases it. Hence, the group may be divided into three subgroups: the most attractive men who are worse off by the increased competition, the most unattractive men who are indifferent (they face zero selection probability either way) and the remaining men who may be better or worse off due to appearance of the new competitor. The first result in section 2 provides necessary and sufficient conditions for an increase in an existing club member's selection probability due to increased competition (admission of a new club member).

Second, how many extra drinking men maximize an incumbent's probability of selection?

An incumbent's probability of selection may strictly increase due to the entry of a less attractive buddy. However, when too many new drinking buddies enter the bar, the incumbent's probability of selection decreases (when n goes to infinity the probability of selection for each man converges to zero). Hence, there must be a finite number of new and less attractive entrants that maximizes an incumbent's probability of selection. For example, it can be shown that if the sample size is 10, as in the former example, the best group size for the 1st ranked man is 10 (no new club members), the best group size for the 2nd ranked man is 20 (10 new low-quality members) and the best group size for the 3rd ranked man is 30 (20 new low-quality entrants). The second result in section 2 establishes that the selection probability, as a function of the number of club members satisfies the single-peakedness property and , furthermore, for an incumbent ranked r, the optimal group size is kr, that is, his ranking multiplied by the sample size.

Third, assume that the drinking buddies decide who can join them, say, by some voting mechanism. How will this self-governing group evolve quality- wise and number- wise?

Suppose that the group, and in general, a democratic CRSC whose members expect a reward that hinges on their selection, applies a simple majority rule to make marginal entry decisions (about membership approval of single potential entrants) or to make a decision about the desirable size of the club (one time decision is made regarding the total number of entrants to the club). By the first result in section 3, when a democratic CRSC makes a separate decision regarding the admission of every potential entrant with a given quality, the outcome is either stagnation (no one is admitted to the club) or quality decay (only candidates whose quality is lower than the average quality of the club members are admitted). Such quality decay is inevitable when the group is big enough relative to the number of club members sampled by the selector. By the second result in section 3, when the democratic CRSC makes a non-marginal decision regarding membership size, its quality usually deteriorates and is inversely related to the number of club members sampled by the selector.

Fourth, suppose that the number of drinking buddies is fixed, but they can affect the number k of sampled club members by revealing information on their quality. Will they disseminate such information?

Suppose that the amount of information about quality ranking of the club members positively affects k. The third result in section 3 clarifies that a democratic CRSC will not disseminate information on the quality of its members.

Fifth, can the women's situation be improved when admittance decisions or decisions on dissemination of quality information are made democratically by the men at the bar?

The answer is negative. The last result in section 3 establishes that the selector's situation cannot be improved if the decisions are made by a self-governing CRSC that applies a simple majority rule. In the illustrative drinking-men story, the drinking buddies will not reveal information on their quality and if a man is sufficiently attractive for the women (his quality is above the average quality of the man selected), he will not be allowed in to the bar.

Finally, assume that the drinking buddies determine the size and transparency of the club in a decentralized way; each man can admit as many new men as he *desires or disseminate any amount of information on the quality of the club members. What will be the size, transparency and quality of the group?*

By the two results in section 4, the equilibrium group size is increased and quality is reduced relative to the size and quality of a democratic self-governing CRSC and, under a fixed club size, the quality and transparency of a decentralized CRSC are increased.

Our basic observations are that certain members of a CRSC may, on one hand, welcome less able competitors and, on the other hand, object transparency regarding the quality of the club members. Consequently, when decisions on admitting new members and on transparency of the club members' quality are made democratically (applying a simple majority rule) or are decentralized, the decision of a CRSC regarding its membership and transparency tend to reduce its quality and, in turn, adversely affect welfare. As in the "drinking club" example described above, in our setting, the members of the CRSC have fixed quality, they wish to be selected and receive the reward of the quality contest and they control the composition of the CRSC and transparency of its members' quality. The selector who awards the "prize" has a special simple form of limited search ability or bounded rationality; he chooses the best (highest-quality) club member from the k members that he samples randomly. Section 2 presents the CRSC model and two preliminary results. In section 3 we explore the endogenous determination of membership size and of the dissemination of information on the quality of the club members in democratic self-governing CRSCs. Section 4 focuses on size and transparency determination in a decentralized CRSC. Section 5 contains three possible applications of our model. In the first examples the members of the CRSC are TV channels and doctors. In the third IO application, the "club" consists of brands produced by a multi-product firm and the selector is the consumer. The last section 6 contains a summary of our findings regarding the inefficiency of CRSC's and some concluding remarks. The proofs are relegated to an Appendix.

2. The setting and basic results

2(a). The competitive rent-seeking club (CRSC)

Consider an n-member CRSC, $n \geq 3$. The club members are exogenously strictly ranked according to their quality and they are assumed to know their ranking as well as the ranking of potential entrants to the club. The club is thus characterized by its size n and by the strict ranking of its members. One of the club members is selected by an outside agent, henceforth, called the selector. Selection is an exclusive privilege of the club members. The club is competitive because its members compete (quality wise) on being selected. The club-members wish to be selected, in fact, to maximize their probability of selection, because selection results in some reward that is associated with the attainment of a certain position, status, job, prize or the selling of some good or (professional) service. The excludable commodity consumed by the club members is the chance of being selected and receiving the reward[3]. The consumed amount of this commodity may differ across club members because a club member's selection probability depends on his ranking, on n and on the behavior of the selector. Although quality of the club members is fixed, they can affect the extent of quality competition by their decision regarding the composition of the CRSC that determines their number, their ranking and the number of club members sampled by the selector.

The CRSC also offers its members the opportunity to consume at least one other excludable commodity, such as club reputation or, simply, income, which is assumed to be independent of the size of the club and the quality of its members. The existence of this additional commodity is needed in our setting to justify the incentive of prospective (existing) members with zero selection probability to join (stay in) the club. The independence of the benefit associated with the consumption of this additional club commodity enables us to disregard this extra benefit and focus just on a club member's selection probability while examining the effect of the size and quality of the club on the benefit of its members. Note that, even if the club does not offer the privilege of another excludable commodity, an agent with zero selection probability may have two other types of incentives to join or stay in the club. First, his selection probability may become positive due to the admission of lower quality candidates. Second, other club members may subsidize his admission to the club or his staying in it because his admission to (presence in) the club is advantageous to them.[4]

[3] On the notion of clubs, see Cornes and Sandler (1996, Chapter 3).

[4] The admission of a low quality candidate to the club differently affects club members. Some benefit from his admission while the effect on other members may be harmful. In this case, some of the club members may have an incentive to subsidize such a new low quality candidate (like a pool insurance company that offers expensive insurance and is subsidized by the insurance group), while other members may have an incentive to lobby against the admission of such a member.

In our setting, the "reward" of a selected club member is assumed to be independent of the quality of the club. Consequently, the sole concern of a club member competing for selection is his selection probability. In some situations this assumption is not plausible. For example, a less attractive group of drinking buddies may eventually attract less attractive women to the bar. If the group quality and the agents' reward are positively correlated, an incumbent's tendency to prefer low-quality entrants may be offset by the reduced quality of the woman entering the (lower-quality) bar. Or consider the situation in a competitive academic club where selection of a member means that he receives a research grant. In this case, if faculty grants and quality are positively correlated, then current faculty members may prefer 'big shot' professors who may attract higher grants to the faculty, even if they reduce their selection probability.

2(b). The selector

In our stylized model, the selector is assumed to randomly sample k agents without replacement, $1 \leq k \leq n$, and choose the highest-ranked agent from this sample[5]. Hence, the environment of the competitive club and the selector is characterized by the club size n, the strict ranking of the club members and the selector's fixed sample size k, which can be interpreted as effective search constraint or as the extent of bounded rationality of the selector. The outcome of the competition among the club members on the expected reward depends therefore on the quality of the members which determines their ranking, on n and on k. In fact, in our setting the relevant contest success function relates the winning probability of a club member to his and the other members' quality. This function depends therefore on his ranking r, on n and on k. The explicit form of the function is given by equation (1) in the Appendix. Specifically, the probability that a selector chooses club member with ranking r, when k members are sampled, is:

$$\text{(1)} \qquad \frac{(n-r)!(n-k)!k}{(n-r-k+1)!n!}$$

[5] On models of bounded rationality and, in particular, on random choice functions, see Rubinstein (1998) and Rubinstein and Salant (2006).

Recall that in our setting, even-though the quality of the club members is assumed to be exogenously given[6], they can still affect competition on the reward by admitting new members to the club or by reducing the size of the club and by affecting the selector's fixed sample size *k*.

Note that in our simple model the selected subset of club members is not chosen on the grounds of quality signals collected by the selector or offered by the club members. The k-member subset is chosen randomly and *k* is assumed to be fixed and independent of the size or the quality of the club members. Although, for the sake of simplicity, *k* is considered as a primitive, it may be the outcome of a maximization process. The selector could care about the relative and not the absolute expected quality of the chosen alternative. For example, if the selector wants to choose an agent that is ranked, on average, in the top 50%, she has to sample only one agent regardless of the sample size. If the selector wants to choose an agent that is ranked, on average, in the top 66%, she has to sample two agents regardless of the sample size. It is easy to verify[7] that if the selector wishes to choose an agent who is ranked, on average, in the top α%, by sampling a minimal number of agents, she has to sample $\alpha/(1-\alpha)$ agents, regardless of the sample size. The simple assumptions on the specific procedure adopted by the selector enable the derivation of our two basic preliminary results.

2(c). The effect of marginal admittance on the probability of selection

Suppose that the *n* incumbents, the existing club members, are augmented by a single entrant to the club. Note that in this simple setting, increased competition, that is, admission of a new member to the club, may strictly increase the selection probability of some incumbents. For example, suppose that $k = n$. In this case the selector samples all the club members and chooses the best member, who is ranked 1st. A new inferior member, when *k* remains fixed, reduces the selection probability of the 1st ranked member to $(n/(n+1))$ (there is a $(1/(n+1))$ probability of sampling *n* agents that

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[6] In our setting quality externalities are disregarded. Therefore, the quality of existing members is not affected by the admittance of new members.

[7] To formally prove this assertion, note that the expected ranking of the agent the selector chooses is equal to $((n+1)/(k+1))$. The expected relative ranking is therefore equal to $((n+1)/(k+1)n)$. When the selector insists on a relative ranking that does not exceed α, the minimal *k* that secures this α is equal to $k^*=(\alpha/(1-\alpha))+(1/n(1-\alpha))$. And since $(1/n(1-\alpha)) <1$, if the selector wants to secure α and do it by sampling the smallest number of agents, then indeed his optimal sample size is robust, that is, k^* is independent of *n*.

do not include the 1st-ranked agent), increases the selection probability of the 2nd ranked agent to (1/(n+1)) and leaves the selection probability of the remaining agents unchanged at 0.

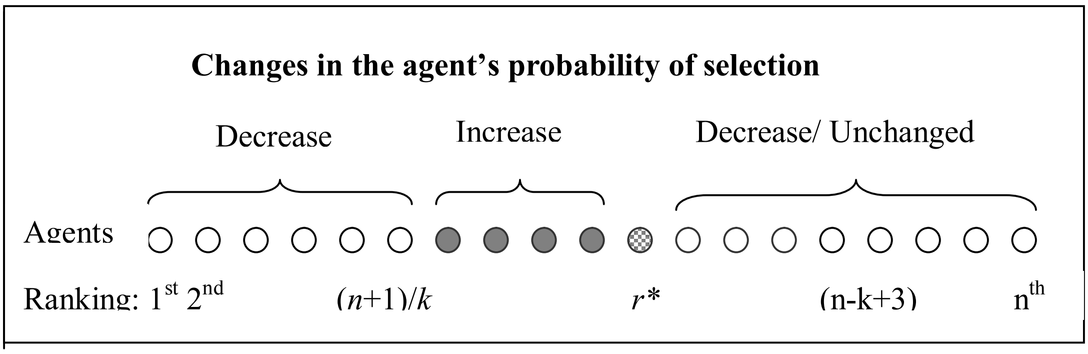
More formally, let r^* be the ranking from the top of the new entrant, $r^* \in \{1,2,\ldots,n+1\}$. Note that the admittance of this agent positions him in the r^* place while decreasing (adversely affecting) the ranking of all the agents of lower quality by one position. The following proposition provides the necessary and sufficient conditions for an increase in an existing agent's selection probability due to increased competition (admission of a new member to the club).

Proposition 1: Let r be the ranking of member x. A new entrant, ranked r^*, increases the selection probability of member x iff:

$$\min\left[(n-k+3), r^*\right] > r > \frac{n+1}{k}.$$

Proof: See Appendix.

A club member's probability of selection is equal to the probability that this member is sampled, multiplied by his conditional probability of selection (the probability that the member is chosen if he is sampled). A new entrant decreases the probability that the club member will be sampled. The conditional probability of selection depends on the quality of the new entrant: the admittance of an inferior agent increases this probability while the admittance of a superior agent decreases it. Hence, a new entrant can only increase the probability of members who are superior to the new member. By Proposition 1, a new inferior entrant increases the selection probability of other club members who are not ranked very high (they do not belong to the top (n+1)/k)-quality members) or very low (such that their selection probability is 0 before and after admittance). The following graph illustrates Proposition 1 (for the case where r^*<(n-k+3)):

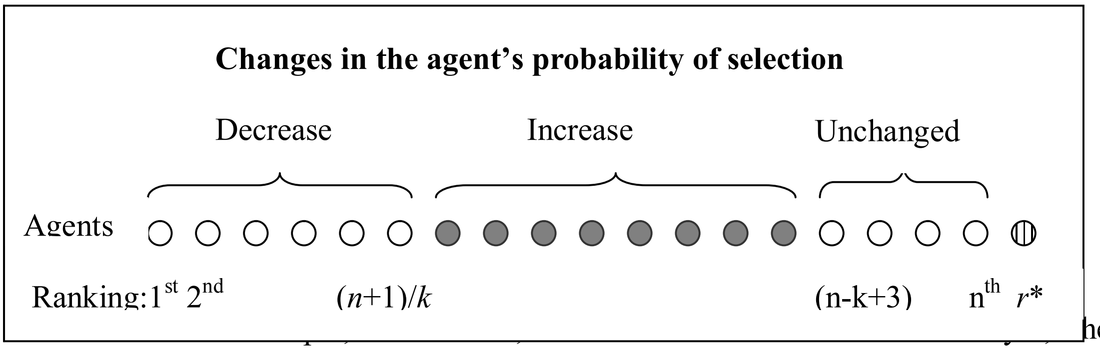


Note that in the special case where the quality of the prospective entrant is inferior to the quality of all the existing club members, that is, $r^* = n+1$, by Proposition 1, we obtain:

Corollary 1: Let r be the ranking of club member x. A new inferior entrant increases the selection probability of member x iff:

$$(n - k + 3) > r > \frac{n+1}{k}.$$

The following graph illustrates Corollary 1:



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The former example, where $k = n$, is of course consistent with Corollary 1, according to which, in such a case only the selection probability of the second-best member, the individual ranked in the second position ($r = 2$), increases, because

$3 > r > (1+(1/n))$.

To sum up, by Proposition 1 (and its corollary that focuses on the special case where $r^* = n+1$), increased competition adversely affects club members who are inferior to the new entrant whose ranking is lower than r^*, the ranking of the new entrant, as well as some high-quality members whose ranking is higher than r^*, and positively affects the remaining club members (if there are any). This latter effect is essentially due to k being smaller than $(n+1)$.

2(d). Club members' optimal membership size

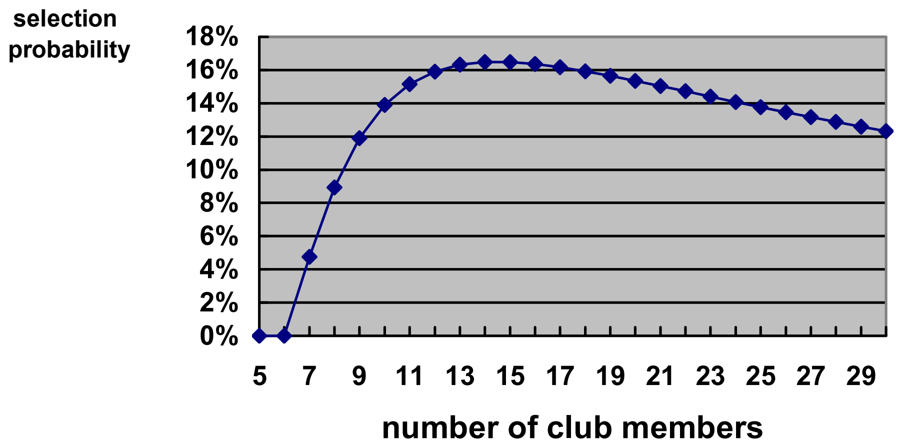
An incumbent's probability of selection may strictly increase due to the admission of a low-quality member to the club. The admission of too many low-quality members to the club decreases an incumbent's probability of selection (when n goes to infinity the probability of selection of each agent converges to zero). Hence, there must be a finite number of new and less attractive entrants that maximizes an incumbent's probability of selection. The following result specifies the most preferred number of club members from the viewpoint of a member ranked r, given that the selector samples k agents.

Proposition 2: Suppose that k club members are sampled. The maximal selection probability of a member ranked r is obtained when the number of club members is equal to $n_r = kr$.

Proof: See Appendix.

For example, assume that $k = 5$. Figure I illustrates the selection probability (in %) of a member ranked 3rd, $r = 3$, as a function of the number of club members.

Figure I



Note that when 5 members are sampled and there are 5 or 6 members in the club, the 3rd ranked member cannot be chosen (the 3rd ranked member cannot be the best member in the sample). The maximal selection probability of a member ranked 3rd is 16.5% and it is achieved when the number of club members is equal to 15 (by Proposition 3, $n_r = kr = 3{*}5$).

3. Democratic self - governing CRSC's

3(a). Membership determination [8]

[8] In the standard rent-seeking literature, the issue of admission of contestants is also examined, albeit usually from the point of view of the contest designer and not that of the group of contestants. A useful survey of this literature can be found in Konrad (2008, section 3.3).

In a self-governing CRSC, the current members are free to determine, one way or another, whether or not to admit a potential candidate to the club. Suppose that approval of admittance of a new agent is determined democratically by applying a simple majority rule; that is, the decision to approve admission of a new member to the club is made only if it gets the support (vote) of a simple majority. Recall that, by assumption, the incumbents know their ranking as well as the ranking of the potential entrant. An existing member votes in favor of admission of a new member provided that such admission strictly increases his selection probability. The new member is admitted to the club if at least half of the existing club members approve its admission.

A CRSC is called *stable* if the existing members prevent any admittance, regardless of the prospective entrant's quality. A *decaying CRSC* is an unstable club that is vulnerable to decay; that is, it is vulnerable to an endogenous decline in the expected quality of its members. In other words, the existing members in a decaying club approve only prospective entrants that reduce the average quality of the club members. The following proposition describes the outcome of marginal admittance decisions in a self-governing CRSC that applies a simple majority rule and provides a sufficient condition for decay.

Proposition 3: A self-governing CRSC that applies a simple majority rule to admit members is either stable or decaying. Assuming that the selector samples $k \geq 3$ agents, a sufficient condition for decay is that $k \leq 0.5n$.

Proof: See Appendix.

That is, self-governance based on the use of a simple majority rule results either in stability (stagnation) or in quality decay. By Proposition 1, a potential entrant of above-average quality is rejected by all club members of less than average quality and by some top-quality members; hence a majority of the members never approves the admission of such a candidate. Only potential members of less than average quality can be approved and this happens when the sufficient condition is satisfied. Notice that if k is sufficiently high relative to n, then the admission of a prospective entrant, regardless of quality, is never approved and the club is stable. For example, suppose that $k = n$. By Proposition 1, the majority of the club members, that is, the 1^{st} ranked member as well as the n-2 = k-2 low-quality members reject the entrant, and only the 2nd ranked member approves it. If k is sufficiently small relative to n, then the admission of an inferior prospective entrant is approved. For example, suppose that $k = 3$ and $n = 100$. By Proposition 1, the 33 top-quality club members as well as the 100th ranked member reject the entrant, while the majority of the club members (66 of them) approve it. By Proposition 3, given n, the threshold sampling size is $k = 0.5n$. That is, if half or fewer of the existing members are sampled[9], then the club decays. Put differently, given k, if the existing number of club members exceeds a certain threshold, $n \geq 2k$, then self-governance based on the simple majority rule necessarily results in quality decay. Suppose, for example, that $k = 3$ and that a new inferior agent proposes his candidacy to the club. Table I specifies the number of club members that vote in favor of his admission. In this case, if the number of the existing members is equal to or exceeds 6, then admission of the inferior agent is approved and the club decays. When the number of the existing club members is smaller than 6, the club is stable or decays.

[9] This observation is not valid in the extreme cases where $k = 1$ or 2 and n is odd. By proposition 1, in the former case all the existing members reject entry. In the latter case, a majority of the members reject entry.

Table I

Size of the club	$\frac{n+1}{k}$	Members who vote against admission of the new candidate			Members who vote in favor of the new candidate		Admittance approved
		Top-quality		Low-quality			
		Decreased selection probability if $r < \frac{n+1}{k}$	Unchanged selection probability if $r = \frac{n+1}{k}$	Unchanged selection probability	Number[10]	%	
3	1.3	1		1	1	33%	No
4	1.7	1		1	2	50%	Yes
5	2	1	1	1	2	40%	No
6	2.3	2		1	3	50%	Yes
7	2.7	2		1	4	57%	Yes
8	3	2	1	1	4	50%	Yes
9	3.3	3		1	5	56%	Yes
10	3.7	3		1	6	60%	Yes
11	4	3	1	1	6	55%	Yes
12	4.3	4		1	7	58%	Yes
13	4.7	4		1	8	62%	Yes
14	5	4	1	1	8	57%	Yes
15	5.3	5		1	9	60%	Yes

Note that the basic model can easily be extended to a multi-period model. Suppose that there are j periods such that in each period t_i, $i \in \{1,2,\ldots,j\}$, there is a prospective entrant of ranking r_i. This ranking is known to the other club members. Furthermore, suppose that all the club members have the same discount factor δ, $0 \le \delta \le 1$, and that each member maximizes his discounted selection probability. For a small enough δ

[10] Note that the number of members who vote in favor of the new entrant (column 6), is derived by subtracting the number of members who vote against the new entrant (columns 3-5) from the total number of club members (column 1).

(such as in the myopic case where $\delta = 0$), the results for each period are identical to those obtained in the one-period model. In such a case, the self-governing club would demonstrate either stability or continuous growth in membership, accompanied by an endogenous decline in the club's expected quality.

Suppose now that admittance decisions are not made on the margin; that is, the issue is no longer whether to admit or reject a potential entrant to the CRSC, but to determine the number of club members. The equilibrium club membership naturally hinges on the preferences of the existing members regarding the size of the club. Under the commonly used simple majority rule, the chosen club membership size is the Condorcet winner; namely, the club-size proposal that defeats by a simple majority any other proposal. If such a winner does not exist, then the status quo n is the chosen club size. To apply Proposition 2 in determining the preferred club size for every voter, we preserve the ranking r of every club member by assuming that potential entrants are inferior low-quality agents (the ranking of a potential entrant r^* is lower than n). By Proposition 2, we obtain the following result:

Proposition 4: Suppose that the number of existing club members n is odd. Then the equilibrium size of a self-governing CRSC that applies a simple majority rule is $k(n+1)/2$. That is, when $k \geq 3$ the n existing club members approve the admission of $\{k(n+1)/2 - n\}$ new inferior low-quality members.

Proof: See Appendix

Thus, in a democratic self-governing CRSC that applies a simple majority rule, the most preferred club size, $((k(n+1))/2)$, of the median-quality member, the member ranked $(n+1)/2)$), is the equilibrium club membership (the Condorcet winning alternative), see Gans and Smart (1996). This means that the equilibrium club membership increases with the existing fixed k. In the extreme case of limited search ability or of the selector's bounded rationality, k=1, the median-quality club member actually prefers to reduce the club size to $(n+1)/2$, dismissing all members whose quality is lower than his quality. If such dismissal is feasible, the median voter becomes the worst-quality club member and, in turn, the quality of the club is increased. In the extreme case of unconstrained search ability o r unconstrained rationality, $k=n$, the equilibrium size is $(n(n+1))/2$) and the resulting deterioration in the club quality is maximal. Notice that club quality is inversely related to the fixed sample size *k*. In the previous case of marginal admission decisions, unbounded search ability of the consumer results in preservation of the quality of the club. In contrast, in the current case of a non-marginal decision on the size of the club, the initial unbounded search of the selector results in bounded search because the size of the club changes from n to $n(n+1))/2$ and this maximal club size results in the largest reduction in quality.

Finally note that the self-governing CRSC may apply alternative democratic rules to determine its size, marginally or non-marginally. One extreme such rule is the hierarchy rule (see Sah & Stiglitz, 1986). Under hierarchy, a potential entrant is approved if all the existing club members vote for his admission. Otherwise he is rejected. Clearly, by Proposition 1, under hierarchy, no candidate is admitted to the club and therefore the existing quality is maintained. In a non-dichotomous setting, under hierarchy, a particular club size is the choice of the self-governing club, if it is supported by all the existing club members who take part in the decision. If such unanimous support cannot be secured, then the status quo n is the chosen club size. Propositions 1 and 2 imply that under hierarchy, the self-governing CRSC chooses to prohibit entry. That is, again, under hierarchy n is the chosen size of the club because the highest-quality member who objects entry to the club is decisive. Hence, hierarchy is superior to the simple majority rule in terms of its effect on club quality.

3(b). Information disclosure affecting the number of sampled club members

Suppose that the competitive CRSC has a data base that can be useful in revealing the relative quality of its members (C.V's of club members, information on their past performance, etc.). The control of such information is typically carried out by decisions on the nature of the data base of the club and the accessibility of that data base. Assume that dissemination of information on the quality of the club members positively affects the actual fixed sample size k, that is, the more information the competitive club reveals, the easier it becomes to sample more members and, consequently, the selector increases k[11]. In the democratic self-governing CRSC, the decision on how much information to disseminate is also reached by applying the simple majority rule. In such a case the equilibrium decision of the club is to avoid any dissemination of information on the quality of its members. That is,

[11] In such a case, the "contest success function" in our model is affected by the decision of the CRSC on information disclosure.

Proposition 5: Suppose that the dissemination of information on the quality of the club members positively affects the selector's fixed sample size k. Then a self-governing CRSC that applies a simple majority rule will not disseminate information on its quality.

Proof: See Appendix.

When $k = 1$, each member has a selection probability of $1/n$. When $k = 2$, the selection probability of the median member is $1/n$, whereas the selection probability of higher (lower) quality members is higher (lower) than $1/n$. When $k \geq 3$, the selection probability of a majority of the members is less than $1/n$, hence the majority of the club members strictly prefer a small sample size k (k =1 or k =2) and given that dissemination of information on the quality of the club members positively affects the actual fixed sample size k, the equilibrium decision of the CRSC is to avoid any dissemination of information on the quality of its members.

3(c). *The selector's welfare*

The selector chooses the best sampled club member. In our probabilistic setting, the selector's preferences are represented by the expected quality of the selected club member. Note that this is an ordinal measure because, given the quality of the existing club members, we assume that a member's quality is represented by his ranking. In particular, the ranking of a potential entrant is represented by his ranking relative to the ranking of the existing club members. The use of such an ordinal measure implies that the selector is only concerned about how well he can do, given the existing quality of the club.[12] As we have seen, the democratic decisions of a competitive club about admittance of new members adversely affect its quality.[13] Our last result establishes that the expected welfare of the selector cannot be improved when the self-governing club resorts to a simple majority rule.

[12] The application of the measure is therefore not plausible for inter-club welfare comparisons of the selector because, obviously, selection of the best-quality member in one club, is not necessarily better than the selection of a medium-quality member of another different club.

[13] This conclusion is valid for any voting rule like hierarchy that is positively responsive to the club members' votes.

Proposition 6: Suppose that a competitive self-governing CRSC applies a simple majority rule when making marginal or non-marginal admittance decisions or when making a decision on disclosure of information about the quality of its members. Then the CRSC never makes a decision that is advantageous to the selector.

Proof: See Appendix.

Basically, if a new potential entrant has a sufficiently high quality (higher than $(n+1)/(k+1)$), so that the selector benefits from his entry, all the members of higher quality vote against his admission (they belong to the $(n+1)/k$ top-quality members of the club) and so do the members of lower quality. Since all the existing members reject such a potential entrant, no democratic voting rule applied by the self-governing club can benefit the selector.

Disclosure of information on the quality of the club members may be advantageous to the selector. However, the democratic club prefers not to disseminate such information. Proposition 6 thus implies that the selector can be "saved" only by external regulation of club membership and of transparency of club-members' quality.

4. Decentralized CRSC's

In a decentralized CRSC, every member can approve a candidate to the club. In other words, a candidate ranked r^* is admitted to the group iff there exists r, such that $\min\left[(n-k+3), r^*\right] > r > \frac{n+1}{k}$. In such a club, admittance of a high-quality candidate (possibly, with above-average quality) with ranking $r^* > [(n+1)/k]+1$, is therefore approved. Although a candidate with above-average quality can be admitted to the club, in general, the effect of marginal entry on quality is ambiguous. Sufficient conditions for quality decay of a decentralized CRSC can, however, be provided. For example, it can be verified that if the ranking r^* of a prospective entrant is uniformly distributed over the possible rankings 1 to n, then the quality of an approved entrant is lower than the average quality, i.e., in such a case the decentralized CRSC decays. Notice that, by Proposition 3, in the dichotomous setting where admittance of a candidate is approved or rejected, the application of the simple majority rule, that can never improve quality, is more harmful to the club's quality.

Consider now the non-dichotomous setting, assuming that the club makes decisions in a decentralized way. In this case, every member i, $i = 1,\ldots,n$, makes a proposal e_i regarding the number of candidates that should be admitted to the club, and every proposal is approved, which means that the decentralized CRSC approves admittance of $e=e_1+e_2+\ldots+e_n$ new members. The chosen club size is therefore $(n+e)$. Note that we now allow strategic decisions by the existing club members. That is, a club member is not assumed to behave sincerely (make a sincere proposal). To apply Proposition 2 in determining the preferred club size for every member, we preserve the ranking r of every club member by assuming that potential entrants are inferior low-quality agents (the ranking of a potential entrant r^* is lower than n). By Proposition 2, we obtain the last two results:

Proposition 7: In a decentralized CRSC, the (Nash) equilibrium size of the club is nk. That is; the existing n members approve the admission of $(k-1)n$ inferior low-quality candidates

Proof: See Appendix.

In a decentralized CRSC, when the size of the club is strategically determined by the existing members analogously to the voluntary provision of a public good, the lowest-quality member is actually decisive; the equilibrium number of club members maximizes his selection probability ($n_n k = nk$ is the most preferred club size of the lowest-quality member, the agent ranked n).[14] Note that, in contrast to the superiority of a decentralized club to the democratic self-governing club that applies a simple majority rule in the context of dichotomous (marginal) admittance decisions, in the current setting of non-marginal club size determination, the democratic club is preferable to the decentralized club. The reason for this reversal in the effect on quality under the two types of clubs is the following: Under marginal admittance decisions, the decisive club member is of lower quality in the democratic club than in the decentralized club. In the self-governing CRSC, the ranking of the decisive club member is $d = [(n+1)/k]+[(n+1)/2]$,[15] whereas in a decentralized CRSC his ranking is $[(n+1)/k]+1$. Consequently, admittance of a high-quality candidate (possibly with

[14] In fact, it can be shown that the same result is valid, even if the quality of potential entrants is not restricted to being inferior.

[15] By Proposition 1, the member ranked $(n+1)/k$ and the higher-quality members reject the potential new entrant. To be approved by a simple majority, the potential entrant must secure the support of at least $(n+1)/2$ of the highest quality members amongst the remaining members. This means that the ranking of the decisive club member; that is, the highest quality member that secures a simple majority, is equal to $[(n+1)/k]+[(n+1)/2]$.

above-average quality) of a ranking r^*, $d > r^* > [(n+1)/k]+1$, is prevented under the simple majority rule, but approved under decentralized decisions. Under non-marginal admittance decisions, the decisive club member is of higher quality under the simple majority rule than under decentralized decisions. In the former case, the ranking of the decisive club member is $(n+1)/2$, the decisive member is the median voter, whereas under the latter case, the ranking of the decisive member is n, the decisive member is the lowest-quality member. Consequently, more inferior low-quality candidates enter the club under decentralized decision making than under the simple majority rule, $\{kn - n\} > \{k(n+1)/2 - n\}$.

The last result implied by Proposition 2 relates to quality transparency in a decentralized CRSC.

Proposition 8: Suppose that the members of a fixed-size CRSC have equal ability to disseminate information on the quality of the club members and that this information positively affects the selector's fixed sample size k. Then in a decentralized CRSC, the (Nash) equilibrium of disseminated information on the club's members' quality is maximal.

Proof: See Appendix.

In a fixed-size decentralized CRSC whose members can disseminate information on the club members' quality, the highest-quality member, the member ranked 1, is actually decisive. This club member is interested in maximizing the selector's sample size k. In fact, his maximal selection probability of 1 is obtained when $k=n$. His maximal feasible selection probability is obtained when he disseminates as much information as he can on the quality of the club members. By Proposition 8, when the size of the club is fixed, the decentralized club is more transparent than the democratic self-governing club and, in turn, its quality is higher.

5. Applications

5(a). The quality of TV channels: The justification of external regulation

Suppose that the competitive club consists of TV channels that seek selection by potential watchers to improve their rating and that consumers of TV make their best channel choice after randomly sampling *k* out of the existing *n* channels.[16] To fit our CRSC setting, suppose that the government awards a certain privilege to the most popular channel. In this case our results imply that the expected quality of TV services can only deteriorate in a decentralized setting, where the existing club members, the operating channels, can establish new channels, that is, co-opt new club members while having in mind just their probability of winning the contest reward. The same conclusion is valid in a democratic setting where the operating channels are viewed as a self governing club. This assumption is realistic in a democratic environment where the decisions of the regulator of the TV industry directly reflect the interests of the existing channels and his decisions are made by applying a rule which is equivalent to the simple majority rule. In this case we also expect quality decay of the TV services. Our model can therefore provide a simple straightforward rationalization for quality regulation of the TV channels club.

5(b). Disclosure of information on medical expertise

Information on medical expertise in different fields is almost never disseminated by the relevant professional organizations. In the United States, for instance, even the questionable individual ranking of heart surgeons according to success in bypass coronary has only appeared since 1991 in New York State and the only other state that reveals the ranking is Pennsylvania, Dyer (2003). This observation is rationalized by Proposition 5, assuming that medical professional organizations are democratic self-governing competitive clubs whose members wish to be approved or selected for offering their services to government agencies or medical insurance companies by government officials who randomly sample a fixed number of the club members. The doctors in this example have in mind the probability of winning the reward associated with selection to offer their professional services. The selectors in this context, the bureaucrats, are typically very much concerned about the quality of the chosen club member – the medical expert. Even when the bureaucrat's decision is affected by price, information on quality is crucial. When price is not an issue, for example when all of the payment for the medical service is borne by the medical insurance company, the quality of the expertise is the sole and main issue. The bureaucrats face a most difficult selection decision and the lack of transparency regarding the quality of the

[16] On the irrationality of consumers of TV channels, see Benesch, Frey and Stutzer (2006).

available medical experts is certainly not to their advantage (Proposition 6). The government can play a useful role in correcting the failure of the democratic competitive club of medical professionals. The objective difficulties of ranking medical experts, together with the fact that the politically powerful medical profession has no interest in disclosing this information, may explain the common disregard of the government in making available transparent medical expertise, especially when the medical services are offered directly to the public. Interestingly, in such cases, alternative non-profit organizations do help the patients. For example, in Israel, two ultra orthodox charity organizations disclose information on medical expertise. These organizations build and maintain a medical data base and grant accessibility to this data base to anybody free of charge. The outstanding popularity and efficiency of these organizations attests to their ability to identify and cope with the lack of quality transparency in the medical profession and its detrimental welfare implications. [17]

5©.. A new justification of branding

The last possible application that we propose is less directly related to our CRSC setting, but rather to the justification of branding, an issue that has been of much concern in the IO literature. However, this application makes a direct use of the results derived in our study. Suppose that a firm can produce different brands and, in particular, low-quality brands of its main product. In such a case the variety of brands can be conceived as a competitive club – the club of brands the firms wish to sell to consumers who make their best-brand purchase decision after randomly sampling k of the n existing brands. Notice that since the brands are ranked only according to quality, their price is disregarded, as in fixed price stores, or the brands quality takes into account all aspects, including their different prices. In this case, the sale probability of a leading brand may increase when new low-quality brands (or new firms that produce them) are introduced. This implies that a firm may have an incentive to introduce such low-quality brands, even if such brands have very small or zero sale probability. When the firm can produce low-quality brands at zero cost,[18] by

[17] The contribution of one of these organizations to the welfare of the Israeli society was recognized in 1997 when Rabbi Avraham Elimelech Firer, the chairman and founder in 1979 of "Ezra Le Marpeh" Association has received the Israel Award, the most prestigious civilian award granted to persons who contribute to social life in Israel.

[18] This is a plausible assumption when products are “virtual”, like some unattractive products in brochures that are never meant to be sold, or when the products are versions of an existing product, possibly a damaged product, that just carry new labeling.

Proposition 2, the optimal number of such brands; that is, the number that maximizes the sale probability of the leading brand, is equal to (kr-n). For example, if, there are already 13 brands in the market, n = 13, and, k = 5, a firm ranked 3rd, introduces two low-quality brands in order to maximize the sale probability of its leading brand. By Proposition 7, if every firm can produce any number of low-quality brands at zero cost, then under decentralized branding decisions, the (Nash) equilibrium industry size is nk. That is; the lowest-quality firm introduces $(k\text{-}1)n$ new low-quality brands. By Proposition 4, when the number of existing firms n is odd and every firm can produce any number of low-quality brands at zero cost, the equilibrium industry size in a self-regulating industry that applies a simple majority rule is $k(n+1)/2$. That is, the n existing firms introduce $\{n(k\text{-}2)+k)/2\}$ new low-quality brands.

The sale of a new product by a multi-product firm can be warranted because it may attract new consumers to the market, it may cause consumers who purchase products from competing firms to switch to the new product or it may "cannibalize" the firm's own line of products. According to the existing literature, the firm's incentives for introducing a new product are basically based on the existence of a tradeoff between the extra profit from the new product and the adverse effect of its introduction on the firm's profits from the existing line of products. In particular, new and inferior products (like the IBM Laser Printer that prints 5 pages per minute, that was created by adding a speed-limiting chip to IBM's Laser Printer of 10 pages p.m., Deneckere & Mcafee, (1996)), may be introduced for various reasons intended to exploit the heterogeneity in the consumers' preferences or income, to deter prospective entrants or to "fight" actual new entrants. Unlike any of the existing models, by Propositions 2 and 3, the sale probability of the leading brand in our setting may increase when new low-quality brands (or new firms that produce them) are introduced. This implies that, due to k being relatively small (the sufficiently bounded rationality of consumers) or n being relatively large (the fact that the existing size of the industry is sufficiently large relative to the number of firms sampled by the consumer), and not due to the heterogeneity of the consumers' income or preferences, a firm may have an incentive to introduce such low-quality brands, even if such brands have very small or zero sale probability. When there are no potential entrants and the size of the industry n cannot be changed by establishing new firms or by producing new brands, the existing firms may affect the sample size k, for example, by controlling consumers' information on the firms (allowing advertising, requiring that the firms' qualifications are known, etc.). In such a case, by Proposition 8, decentralized decisions by the firms will result in maximal disclosure of information regarding the quality ranking of the firms. In contrast, by Proposition 5, a collective democratic decision by the firms will result in no disclosure of information regarding the quality ranking of the firms.

6. Summary and concluding remarks

In this study we have focused on CRSC's whose members are of heterogeneous fixed quality and on a selector (bureaucrat or politician), who does not know the members' quality, randomly sampling a fixed number of club members and then choosing the best-sampled member. Selection of a member means that he receives the reward associated with a certain position, status, job or prize. Under the above assumptions, an existing club member may become better off due to enhanced competition; that is, the admission of a new member to the club. More specifically, by Proposition 1, the newly admitted member may harm the top-quality members, harm or not change the situation of the low-quality members and benefit the medium-quality club members. We then established in Proposition 2 that, given the fixed sample size k and the ranking of a club member r, that member maximizes his selection probability when $n_r = k*r$; that is, when the number of club members is equal to the fixed sample size multiplied by his ranking.

The analysis then focused on the implications of the model in the context of a self-governing CRSC; a club in which the existing members control entry, marginally or non-marginally, and in turn, the quality of the CRSC or control disclosure of information on the quality of the club member. By Proposition 3, under marginal admittance decisions, if the self-governing CRSC resorts to simple majority rule, then the outcome is either stability or continued growth in membership, which is accompanied by a decline in the quality of the club. A sufficient condition for such quality decay is that $k \leq 0.5n$; that is, the existing club size is sufficiently large relative to the number of club members sampled by the selector.[19]

We derived the non-marginal equilibrium club size when the democratic self-governing club resorts to simple majority rule. We have shown in Proposition 4 that under a simple majority rule, when $k \geq 3$, the club decides to increase its size. Under

[19] We showed that stability of a self-governing club is also secured under hierarchy, but not necessarily in a decentralized club.

the equilibrium club membership, the average quality is reduced. The reduction in quality is maximal when initially $k=n$. The reason is that in such a case of initial unbounded search, the club admits the largest number of new low-quality members. When the size of the club is fixed, it may affect the fixed number of club members sampled by the selector by disclosing information on the quality of the members. By Proposition 5, in such a case the democratic CRSC refrains from revealing such information.

We then focused on the welfare of the selector that in our case is equivalent to that of society. In our setting, the selector's preferences are represented by the expected quality of the chosen agent. By Proposition 6, if a new potential entrant has a sufficiently high quality that is beneficial to the selector, then he will not be admitted to the club. Furthermore, any information regarding the quality of the club members that can increase the expected quality of the selected club member will not be released. That is, the democratic CRSC has an incentive to prevent entry and quality transparency that are welfare enhancing.

Our model predicts some basic undesirable features of self-governing CRSC's. The focus is not on the standard aggregate rent-seeking efforts of the CRSC or on the efforts of its members to compete directly, by affecting their quality or by sending signals (to the selector) on their existing quality. Rather, given the limited search ability of the selector or his bounded rationality, we examine the effect of the decisions of the CRSC regarding its composition and regarding its transparency, respectively, on the competitive environment and on the assumed contest success function. The combination of selfish rent seekers who are interested in being chosen by the selector (civil servants, politicians), fixed random sampling of these rent seekers by the selector, and the fact that the self-governing CRSC resorts to simple majority rule, can only result in unchanged or deteriorating quality of the club and of the expected quality of the selected club member.[20] This is the inefficiency obtained in our quality contest which is due to the effective club members' incentives to welcome less able competitors and to object transparency regarding quality. The effectiveness of the incentives is rendered possible due the reliance of the self-

[20] Note that even in the standard rent-seeking literature, the source of inefficiency can be due to the reduced probability that the most efficient contestant is chosen rather than to increased (wasteful) rent-seeking expenditures. This was pointed out, for example, by Ellingsen (1991) and Fabella (1995) and, more recently, by Amegashie (2000).

governing CRSC on democratic decision making, viz., the application of the simple majority rule.

When the size and transparency of the club are determined in a decentralized way, the most active members play a dominant role. Specifically, the least qualified and the most qualified members play a decisive role in determining, respectively, the size and the transparency of the club. By Proposition 7, the size of the decentralized CRSC is larger than the size of the democratic self-governing CRSC and its quality is lower. The superior performance of the democratic club is due to the decisiveness of the less active median-quality member, relative to the more active harmful role played by the lowest-quality member in a decentralized CRSC. By Proposition 8, when the size of the club is fixed, the decentralized CRSC is more transparent than the self-governing CRSC and its quality is higher. The advantageous functioning of the decentralized CRSC is due to the decisiveness of the more active highest-quality club member, who disseminates as much information as he can, relative to the less active detrimental role played by the decisive median-quality member in a democratic self-governing club.

From the welfare point of view, the outcome of a self governing CRSC is inefficient because it results in more inferior members and less information on the quality of the club members (actually no information). Transforming such a club into a decentralized CRSC changes the outcome; the number of inferior members is increased, however, more information is disclosed. This means that the efficiency effect of such an institutional transformation is ambiguous; the outcome does not necessarily become more efficient.

Our analysis of CRSC's has focused on democratic self-governing clubs and decentralized clubs that have two main simple features. First, the benefit of the club members from being chosen by the selector and from consuming other excludable club goods are independent of the size and the quality of the club. Second, the club faces a particular form of limited search ability or bounded rationality of the selector who samples a fixed number of club members before making his choice. Alternative assumptions regarding the benefit of the club members and the behavior of the selector can be made. For example, the benefit from selection can be positively related to the quality of the club. The benefit from consuming other excludable club goods, such as reputation, can be inversely related to the size of the club and positively related to its size. The sample size of the selector need not be fixed; it can be inversely related to the quality of the club. And the selector need not sample club members randomly. Examining the implications of these alternative, perhaps more realistic and less stylized, assumptions on the size, transparency and quality of CRS's is certainly worth pursuing. However, this task is beyond the scope of the current paper and is left for future research.

The reward of the selected club member can be associated with the selling of some good or (professional) service that he and the other members attempt to sell to the government or to the public. In such a case, our findings can shed some light on the issue of whether industry regulation is warranted. In particular, they can provide rationalization for the regulation of industries that satisfy the basic features of our CRSC, as illustrated in the example of the TV channels. We can also apply our model to shed light on the observed lack of transparency in certain professions, such as the medical profession example we have briefly discussed and on the reason that charity organizations have to disclose information, e.g., on medical expertise. In the latter case, the reason seems to be at least partly a response of the charity organizations to their realization that the democratic competitive club of medical professionals and the government have no interest in disclosing this information. Finally, we have presented a possible application of our model to branding, an issue that is of much concern in the IO literature. In this application we have conceived the variety of brands produced by a firm as a decentralized or a self-governing competitive "club of products or brands". In this case, our model can provide interesting rationalization of versioning, and the endogenous determination of club membership could relate to the choice of the extent of versioning or branding by a multi-product firm.

Appendix

Proposition 1: Let r be the ranking of member x. A new entrant, ranked r*, increases the selection probability of member x iff:

$$\min\left[(n-k+3), r^*\right] > r > \frac{n+1}{k}.$$

Proof:

The probability that a selector chooses club member x, when k members are sampled, is:

$$(1) \qquad \frac{(n-r)!(n-k)!k}{(n-r-k+1)!n!}$$

The probability that a selector chooses member *x*, when *k* members are sampled, and a new inferior member joins the club, is:

(2) $$\frac{(n - r + 1)!(n - k + 1)!k}{(n - r - k + 2)!(n + 1)!}.$$

Thus, the entrance of a new inferior agent increases the selection probability of member *x* if:

(3) $$\frac{(n - r + 1)!(n - k + 1)!k}{(n - r - k + 2)!(n + 1)!} > \frac{(n - r)!(n - k)!k}{(n - r - k + 1)!n!}$$

or, by rearranging (3), if:

(4) $$r > \frac{n + 1}{k}.$$

Furthermore, members ranked in the last *k*-1 positions (the last *k*-2 positions when an inferior member is added) have zero selection probability. Since ranking in one of the last *k* positions is equal to ranking in position (*n*-*k*+1) or onward (for example, ranking in one of the last 10 positions when there are 100 agents is equal to ranking in the 91st position or onward), members ranked in position (*n*-*k*+2) or onward (positions (*n*-*k*+3) or onward when an inferior member is admitted) have a zero selection probability. Hence, a necessary condition for the selection probability of a member to be positive when an inferior member joins the club is:

(5) $$(n - k + 3) > r$$

By (4) and (5), if *k* agents are sampled, the admission of a new inferior member to the club increases the selection probability of member *x*, ranked in position *r*, iff:

(6) $$(n - k + 3) > r > \frac{n + 1}{k}.$$

The selection probability of member *x* when *k* club members are sampled, and a new *superior* member is admitted to the club, is:

(7) $$\frac{(n - r)!(n - k + 1)!k}{(n - r - k + 1)!(n + 1)!}$$

There is no positive *k* such that the expression in (7) is larger than the expression in (1), hence a member of quality *r** decreases the selection probability of all the members that are ranked lower than *r**. Combining this result with (6), we get that entry of a new agent ranked *r** increases the selection probability of member *x* iff:

$\min[(n - k + 3), r^*] > r > \frac{n+1}{k}$. ■

Proposition 2: Suppose that k club members are sampled. The maximal selection probability of a member ranked r is obtained when the number of club members is equal to $n_r = kr$.

Proof:

When $n = rk$ or $n = (rk\text{-}1)$, by (1), the selection probability of a member of quality r is the same and equal to $\frac{(n - r)!(n - k)!k}{(n - r - k + 1)!n!} = \frac{(rk - r)!(rk - k)!k}{(rk - r - k + 1)!(rk)!}$.

The selection probability of a member with quality r when there are (rk+1) members is smaller than the selection probability when there are (rk) members. By induction, it can be verified that the selection probability of a member with quality r when there are (rk+a+1) members is smaller than the selection probability when there are (rk+a) members. The selection probability of a member with quality r when there are (rk-2) members is smaller than the selection probability when there are (rk-1) members. By induction, it can be verified that the selection probability of a member with quality r when there are (rk-a-2) members is smaller than the selection probability when there are (rk-a-1) members. Hence, the maximal selection probability of a club member ranked r is obtained when there are (rk) or (rk-1) members, which completes the proof. ■

Proposition 3: A self-governing CRSC that applies a simple majority rule to admit members is either stable or decaying. Assuming that the selector samples $k \geq 3$ agents, a sufficient condition for decay is that $k \leq 0.5n$.

Proof:

The club faces a dichotomous choice; whether or not to approve the admission of a prospective entrant. An entrant with above-average quality is rejected by:

1. All the members of lower quality, in particular, all the members of equal or less than average quality.
2. ((n+1)/k) top quality members.

Hence, the majority of members never approves such an entrant. Only entrants with less than average quality can be approved. To prove that the club is decaying, it suffices to show that the self-governing club is unstable, approving admittance of inferior candidates. So suppose that $k \geq 3$ and that the club considers the candidacy of an inferior agent and let us prove that $k \leq 0.5n$ is a sufficient condition for the majority approval of this candidate. Denote by INT[i] the integer part of any real number [i]. By Proposition 1, when a new inferior agent enters the club, the number of top- quality members whose selection probability does not increase is:

$$(8) \qquad \text{INT}[(n+1)/k]$$

and the number of low-quality members whose selection probability does not increase is:

$$(9) \qquad k\text{-}2$$

Hence, by (8) and (9), the proportion of members whose selection probability does not increase, when a new inferior candidate is admitted to the club, is:

$$(10) \qquad \frac{\text{INT}[(n+1)/k]+k-2}{n}$$

If $n = 2k$, then (10) takes the form:

$$(11) \qquad \frac{\text{INT}[(2k+1)/k]+k-2}{2k} = \frac{2+\text{INT}[1/k]+k-2}{2k} = \frac{1}{2}$$

and the new member is admitted since only half of the members vote against him.

For $n > 2k$ then, let $n = 2k+a$, where a is a positive integer. (10) now takes the form:

$$(12) \qquad \frac{\text{INT}[(2k+a+1)/k]+k-2}{2k+a} = \frac{2+\text{INT}[(a+1)/k]+k-2}{2k+a}$$

The new candidate is approved if half or fewer of the members vote against him; that is, if:

$$(13) \qquad \frac{1}{2} \geq \frac{k+\text{INT}[(a+1)/k]}{2k+a}$$

Let $b = \text{INT}[(a+1)/k]$. Then (13) can be rewritten as:

$$(14) \qquad \frac{1}{2} \geq \frac{k+b}{2k+a}$$

and (14) holds iff:

$$(15) \qquad a \geq 2b$$

or:

$$(16) \qquad a \geq 2\text{INT}[(a+1)/k]$$

and, in particular, (16) holds if:

(17) $$a \geq 2(a+1)/k$$

or

(18) $$k \geq 2(a+1)/a$$

If $k = 3$, it can be verified that (16) is satisfied for $a = 1$, and that (18) is satisfied for every positive integer a, $a > 1$. If $k \geq 4$, (18) is satisfied for every positive integer a. Hence, for $n \geq 2k$, $k \geq 3$, the majority of the existing club members always approves the admission of a new inferior candidate. We have thus proved that the club is decaying. ■

Proposition 4: Suppose that the number of existing club members n is odd. Then the equilibrium size of a self-governing CRSC that applies a simple majority rule is $k(n+1)/2$. That is, when $k \geq 3$ the n existing club members approve the admission of $\{k(n+1)/2 - n\}$ new inferior low-quality members.

Proof:

By Proposition 2, $n_r = kr$ is the most preferred club size of a member ranked r. The distribution of these most preferred sizes satisfies the single-peakedness property (see proof of Proposition 3). Hence, by the median voter theorem, the most preferred club size of the median voter $k(n+1)/2$ is the equilibrium club size under simple majority rule (the proposal $k(n+1)/2$ is a Condorcet winner: a proposal that defeats any other alternative club size by a simple majority). ■

Proposition 5: Suppose that the dissemination of information on the quality of the club members positively affects the selector's fixed sample size k. Then a self-governing CRSC that applies a simple majority rule will not disseminate information on its quality.

Proof:

By Proposition 2, $k_r = \frac{n}{r}$ is the most preferred sample size of a member ranked r. The distribution of these most preferred sizes satisfies the single-peakedness property. Hence, by the median voter theorem, the most preferred fixed sample size of the median voter $\frac{2n}{n+1}$ is the equilibrium fixed sample size under simple majority rule.

This means that $k^*=1$ *or* $k^*=2$ is the voting equilibrium, because it yields the same selection probability to the median voter. Since, by assumption, dissemination of information on the quality of the club members positively affects the actual fixed sample size *k*, the median voter prefers that no information on the club members' quality is disclosed and this will be the voting equilibrium outcome. ■

Proposition 6: Suppose that a competitive self-governing CRSC applies a simple majority rule when making marginal or non-marginal admittance decisions or when making a decision on disclosure of information about the quality of its members. Then the CRSC never makes a decision that is advantageous to the selector.

Proof:

(i) The probability that a selector chooses member *x* when *k* members are sampled is given in (1). The expected ranking of the selected member is therefore equal to:

$$(19) \qquad \sum_{r=1}^{n-k+1} r(1) = \sum_{r=1}^{n-k+1} \frac{r(n-r)!(n-k)!k}{(n-r-k+1)!n!} = \frac{n+1}{k+1}$$

The selector can benefit only if the entrant's ranking is lower than $\frac{n+1}{k+1}$. By Proposition 1, if a prospective entrant is of such quality, then all the members of higher quality; members whose ranking is lower than or equal to $\frac{n+1}{k}$, vote against entry and, in particular, all the members whose ranking is smaller than $\frac{n+1}{k+1}$, which is smaller than $\frac{n+1}{k}$, vote against entry. By Proposition 1, all the members of quality lower than that of the entrant vote against him. Since all the existing members vote against entry, there is no democratic voting rule that can benefit the selector when entry decisions are marginal.

(ii) When the club-membership decision is not marginal, by Proposition 4 inferior members are admitted to the club and this necessarily reduces the expected quality of the selected member.

(iii) Proposition 5 directly implies that a decision of the club regarding disclosure of information regarding the quality of its members is not advantageous to the selector.■

Proposition 7: In a decentralized CRSC, the (Nash) equilibrium size of the club is *nk*. That is; the existing *n* members approve the admission of *(k-1)n* inferior low-quality candidates.

Proof:

By Proposition 2, $n_r = kr$ is the most preferred club size of a member ranked *r*. It is straightforward to verify that in such a case, the game associated with decentralized club-size decisions possesses the unique Nash equilibrium $(e_1^*,...,e_n^*) =$ (0,0,…,(k-1)n). Hence, $e_1^* + e_2^* + ... + e_n^* = (k-1)n$. That is, the (Nash) equilibrium size of the club is *nk* ■

Proposition 8: Suppose that the members of a fixed-size CRSC have equal ability to disseminate information on the quality of the club members and that this information positively affects the selector's fixed sample size *k*. Then in a decentralized CRSC, the (Nash) equilibrium of disseminated information on the club's members' quality is maximal.

Proof:

By Proposition 2, $k_r = \frac{n}{r}$ is the most preferred sample size of a member ranked *r*. It is straightforward to verify that in such a case, the game associated with decentralized decisions on disclosure of information regarding the quality of the club members possesses the unique Nash equilibrium where the highest-quality club member disseminates the maximal possible amount of information and the other members do not disseminate information. ■

References

Amegashie, A.J., 2000. Some results on rent-seeking contests with shortlisting, *Public Choice*, 105, 245-253.

Baik, K.H., 2008, Contests with group-specific public-good prizes, *Social Choice and Welfare*, 30(1), 103-117.

Baik, K.H., 1999, Rent-seeking firms, consumer groups, and the social costs of monopoly. *Economic Inquiry*, 37(3), 542-554.

Baik, K.H., Kim, I-G. and Na, S-H., 2001, Bidding for a group-specific public-good prize, *Journal of Public Economics*, 82, 415-429.

Benesch, C, Frey, B. and Stutzer, A., 2006, TV channels, self control and happiness, Working Paper 301, Institute for Empirical Research in Economics, University of Zurich.

Congleton, R.D., Hillman, A.L. and Konrad, K.A. (Eds), 2008, *Forty Years of Research on Rent Seeking*, Springer- Verlag.

Cornes, R. and Sandler, T., 1996. *The Theory of Externalities, Public Goods, and Club Goods*, 2nd edition. Cambridge: Cambridge University Press.

Deneckere, R. J. and McAfee, R. P., 1996, Damaged goods, *Journal of Economics & Management Strategy*, 5-(2), 149-174.

Dyer, O., 2003. Heart surgeons are to be rated according to bypass surgery success, *BMJ;*326: 1053-a. (17 May.).

Ellingsen, T., 1991. Strategic buyers and the social cost of monopoly, *American Economic Review*, 81, 648-657.

Fabella, R.V., 1995. The social cost of rent seeking under countervailing opposition to distortionary transfers, *Journal of Public Economics*, 57, 235-247.

Gans, J., Smart, M., 1996. Majority voting with single crossing preferences, *Journal of Public Economics*, 59, 219-237.

Gradstein, M. (1995). 'Intensity of competition, entry and entry deterrence in rent-seeking contests', *Economics and Politics*, vol. 7, pp. 79-91.

Hausken, K., 2005, Production and conflict models versus rent seeking models. *Public Choice*, 123 (1-2), 59-93.

Konrad, K.A., 2008, *Strategy and Dynamics in Contests*, Oxford University Press, forthcoming.

Nitzan, S., 1994, Modelling rent-seeking contests, *European Journal of Political Economy* 10, 41–60.

Nti, K.O., 1997, Comparative statics of contests and rent seeking games. *International Economic Review*, 38, 43-59.

Rubinstein, A., 1998. *Models of Bounded Rationality*, MIT Press.

Rubinstein, A., Salant, Y., 2006. A model of choice from lists, *Theoretical Economics*, 1, 3-17.

Sah, R.K., Stiglitz, J.E., 1986. The architecture of economic systems, hierarchies and polyarchies, *American Economic Review*, 76, 716-727.