



ELSEVIER

Contents lists available at ScienceDirect

European Economic Review

journal homepage: www.elsevier.com/locate/eer

Prize sharing in collective contests

Shmuel Nitzan^{a,*}, Kaoru Ueda^b^a Department of Economics, Bar Ilan University, Ramat Gan 52900, Israel^b Faculty of Economics, Nanzan University, Nagoya, Aichi 466-8673, Japan

ARTICLE INFO

Article history:

Received 23 September 2009

Accepted 21 September 2010

JEL classification:

D23

D72

D74

H41

Keywords:

Collective contest

Mixed public-good prize

Endogenous sharing rules

The group-size paradox

ABSTRACT

The characteristics of endogenously determined sharing rules and the group-size paradox are studied in a model of group contest with the following features: (i) The prize has mixed private–public good characteristics. (ii) Groups can differ in marginal cost of effort and their membership size. (iii) In each group the members decide how much effort to put without observing the sharing rules of the other groups. It is shown that endogenous determination of group sharing rules *completely* eliminates the group-size paradox, i.e. a larger group always attains a higher winning probability than a smaller group, unless the prize is purely private. In addition, an interesting pattern of equilibrium group sharing rules is revealed: The group attaining the *lower winning probability* is the one choosing the rule giving *higher incentives* to the members.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

Many kinds of public decisions determine allocations of economic rents among various groups in society. The groups of possible beneficiaries try to make the allocations favorable to them through lobbying. Some important cases of such “interest-group politics” can be described as contests for a prize. A typical example is a choice of location for public facilities with positive externalities to the chosen locality (e.g. special economic zones, agricultural experiment stations, a stadium for the Olympic games, etc.). The residents of the candidate regions expend time and money to appeal relevant politicians and bureaucrats for winning the prize associated with the selection of their region. Competitions for earmarked subsidies by local governments, conflicts in trade politics by industry groups, and ethnic conflicts within a country could also be modeled as such collective contests.¹ The main objective of this paper is to examine how endogenous “group sharing rules” in collective contests affect the validity of the “group size paradox.” Let us briefly explain the terms in the quotation marks.

An important feature of interest-group politics is that individuals attain or lose the rent as a group, i.e. the winning of the rent is a “collective good”. The benefit is shared in a group, which tempts individual members to free ride on the contribution by others. As Olson (1982) argues, how to manage such a collective-action or free-rider problem is an essential factor to understand the existence, nature and effectiveness of interest groups. He particularly emphasizes that the larger the number of individuals or firms sharing a collective good, the smaller the gains they can expect from contribution to the group interest. Hence, “*the incentive for group action diminishes as group size increases, so that large*

* Corresponding author. Tel.: +972 3 531 8345; fax: +972 3 535 3180.

E-mail address: nitzans@mail.biu.ac.il (S. Nitzan).

¹ On possible applications of contest models, see Konrad (2009). Epstein and Nitzan (2007) examine how the games of interest-group politics can be treated as contests. Esteban and Ray (forthcoming) apply a contest model for studying ethnic conflicts.

groups are less able to act in their common interest than small ones².” This famous conjecture is called the *group size paradox* by Esteban and Ray (2001).

As a device to manage the collective action problem, Olson emphasizes the “selective incentives,” which are the incentives applied selectively to the individuals depending on their actions. In a collective contest, how to share the prize plays a critical role in providing such incentives. Studies of collective contests on private-good prizes have considered alternative ways of prize division among the members of the winning group. One possibility is that, prior to the contest, members of a group commit to a *group sharing rule*.³ To study this type of collective contests, Nitzan (1991) focuses on a class of such rules that consists of linear combinations between the *egalitarian* and the *relative effort* sharing rules. In this case, part of the prize is divided equally among the group members (according to the egalitarian rule) and the rest is divided proportionally according to the members’ efforts (according to the relative effort rule), which works as a selective incentive.⁴ Lee (1995) and Ueda (2002) examine the endogenous determination of group sharing rules that belong to this class.

While much progress has been made in the study of prize sharing in collective contests for a private-good prize, the validity of the group size paradox has been questioned in collective contests for a public-good prize. Actually, as Chamberlin (1974) has pointed out, how much each member’s gain decreases as the group gets larger depends on the degree of the rivalry in the consumption of the collective good. Katz et al. (1990) and Riaz et al. (1995) examine the case where the prize is a pure public good for each group⁵ showing that, in this setting, a group with larger membership attains a winning probability larger than or equal to that of a smaller group. The group size paradox is not valid in such cases. In an important extension, Esteban and Ray (2001) study collective contests with a mixed private–public-good prize; part of the prize is a public good and part of the prize is a private good. Assuming increasing marginal effort costs of individuals, they have shown that a kind of economies of group size (so to speak, “multi-person” economies) works in such a general class of collective contests. With this finding, they have suggested that the group size paradox holds only in narrow cases.

The criticism by Esteban and Ray could have important implications for interest-group politics. We can interpret several rents sought by interest groups as a mixed prize. The object in regional, community or government division contests is often some budget, part of which can take the form of monetary transfers while the rest must be used to supply some local public goods.⁶ When a local government wins a contested subsidy earmarked for some public undertaking, part of it can be provided as an extra margin for the employed local people. Even an electoral competition can be conceived as a contest on a prize with mixed private–public good components, because a winning candidate is typically committed to the provision of both public and private benefits to his supporters.⁷ Furthermore, if members of the winning group jointly taste the delight of victory, any group contest for a private-good prize could be actually that for a mixed prize.

In the Esteban–Ray model, however, it is postulated that the private-good part of the prize is equally divided among the members of the winning group. As argued above, the private-good prize can be utilized as a source of selective incentives. If such incentives play an important role for real interest groups confronting the collective action problems, it is essential to examine how the introduction of endogenous group sharing rules to contests for impure public goods affects the advantage of group size. By using a generalized version of the Esteban–Ray model, we will show that endogenous determination of linear group sharing rules *completely eliminates* the group-size paradox, i.e. a larger group always attains a higher winning probability than a smaller group, unless the prize is purely private. The introduction of endogenous group sharing rules into collective contests for an impure public good prize dissolves the group size paradox, which has already been restricted by Esteban and Ray. Furthermore, we will provide a sufficient condition for a larger group to always get a higher per capita utility than that of a smaller group.

Our model also reveals an interesting pattern of equilibrium group sharing rules chosen by heterogeneous groups maximizing their per capita utility: If two competing groups have the same size, the group attaining the higher winning probability is the one that affords to use a *more egalitarian* group sharing rule, i.e. the rule divides a larger part of the prize according to the egalitarian rule. We can further identify the cases in which a larger group can choose a more egalitarian group sharing rule due to its group size advantage. These results seem strange on first glance, because the larger the part of the prize divided according to the relative effort rule, the stronger the selective incentives and, in turn, the higher the winning probability of the group. However, such unilateral comparative statics does not adequately describe the *equilibrium choices* of sharing rules by groups maximizing their welfare. As Sen (1966) has pointed out, the relative effort sharing rule can induce too much effort from the members of a group to attain a Pareto-efficient outcome (for the group).

² Olson (1982), p. 31.

³ Katz and Tokatlidu (1996), Wärneryd (1998) and Konrad (2004) consider another possibility; the division of the prize is determined by the within-group contest, subsequent to its award to the winning group. A third possibility is that the prize is utilized as a group-specific commons open to all members of the winning group. See Nitzan and Ueda (2009).

⁴ This class of group sharing rules has an alternative interesting interpretation. As argued by Baik (1994) and Baik and Lee (2001), it can be interpreted as a “winner-help-loser” agreement, or a self insurance device applied by the groups.

⁵ Ursprung (1990) provides an interesting application of this kind of collective contests to a two-candidate electoral competition. For another approach which applies all-pay auction to a contest for a public-good prize, see Baik, Kim and Na (2001).

⁶ See Nitzan (1994).

⁷ We could also conceive the prize in R&D contests as such a mixed prize, because the winning R&D team gets improved reputation (the status and recognition associated with the winning, which can be equally shared by all group members) and monetary benefits (the profit associated with winning the contest, that can be shared equally or not-equally by some or by all members).

If, in spite of it, a group assigns a higher weight to the relative effort rule than other groups, the reason is the need to compensate for its inferiority in ability to secure the prize.

Section 2 presents our extended model of collective contests. Section 3 contains the result on the disappearance of the group size paradox. Section 4 presents the basic relationship between the characteristics of a group and its selected sharing rule. Some concluding remarks appear in Section 5. All proofs are relegated to Section 6.

2. A model of group contests for an impure public good prize

2.1. Prize, benefit and cost

Let us consider a contest in which m groups compete for a prize. The number of members in group i is denoted by $N_i (i=1, \dots, m)$. We assume that the prize is a mixture of public and private goods. That is, a winning group gets some group-specific public goods and private goods that can be shared among its members.

For simplicity, we assume that every member of every group applies the same benefit function $B(q, G)$ to evaluate the prize, where q is the amount of the private good distributed to the individual and G is the amount of group-specific public good provided to the group to which the individual belongs. This function is twice differentiable, and $B(q, G) > 0$ unless $(q, G) = (0, 0)$. Furthermore, $B_q > 0$, $B_G \geq 0$, and $B_{qq} \leq 0$ hold for all $q > 0$, $G > 0$. The CES benefit function $B(q, G) = (b_1 q^\rho + b_2 G^\rho)^{1/\rho}$ with $0 < b_1 < 1$, $0 < b_2 < 1$ and $\rho \leq 1$, satisfies these conditions. We will refer to this useful special case later.

We normalize the total prize to unity, and denote the ratio of the private-good part by γ ($0 < \gamma \leq 1$). That is, the model covers all prize compositions but the pure public-good case. The ratio is given exogenously. We assume that, prior to the contest, the rule applied for sharing the private part of the prize is determined in each group. This rule is assumed to be chosen from the class of sharing rules that are linear combinations of the egalitarian and the relative-effort sharing rules. Denote the weight of the relative-effort rule in group i by δ_i . Then, if group i wins the contest, a member of the group having put effort $a \geq 0$ receives the benefit:

$$B\left(\gamma\left(\delta_i \frac{a}{A_i} + (1-\delta_i) \frac{1}{N_i}\right), 1-\gamma\right)$$

where A_i is the aggregate amount of effort put by the members of group i .

Every member of group i incurs the cost $v_i(a)$ when making an effort equal to a while trying to win the prize. The cost function is the same among the members of a group, but it can differ across the members in different competing groups. For every i , let $v_i(0)=0$, $v_i'(a) > 0$ and $v_i''(a) \geq 0$ for all $a > 0$. To guarantee that every individual chooses a positive effort in equilibrium, we also assume that $\lim_{a \rightarrow 0} v_i'(a) = 0$.⁸

2.2. The structure of the contest

Our model of group contest proceeds as follows. At the beginning, the decision on the value of δ_i , i.e. the group sharing rule of the private-good part of the prize, is made simultaneously in each group. We assume that this decision is made (and implemented after the winning) by a *group leader* who maximizes a group welfare function strictly increasing in the utility of every member of the group. Such maximization ensures the selection of a Pareto-efficient group outcome, constrained by the contest with the other groups. Our assumption on such a benevolent objective of a leader could be (at least partially) justified if the intrinsic objective of a leader is the position itself, and the assignment of this position requires the consensus of the group members. After observing the sharing rule chosen by his group leader, each member in the competing groups described in the previous subsection, chooses the effort level individually.

At this point, we introduce a departure from the standard models of collective contests with predetermined group sharing rules: Following Baik and Lee (2007), we choose to assume that the *individuals cannot observe the group sharing rules decided in the other groups*. Former studies presume that each group's sharing rule is observable from the outside. The decision on a group sharing rule is, however, made within a group, and it is plausible to assume that such a decision is changeable secretly (from the point of view of members in the other groups). Without restrictive assumptions that decisions made within a group are transparent and detection of changes is easy, a model of group contests with observable group sharing rules is questionable in its reality. Furthermore, it introduces a complicating factor in the decisions of the competing groups; *observable sharing rules work as strategic variables*. A group sharing rule determines how strong the selective incentives in a group are. If a group has an observable sharing rule, its change affects the effort of the individuals in the other groups by changing their expectations on the effort level of the group in question.⁹

⁸ This assumption rules out linear cost functions. The main reason for this assumption is to avoid cumbersome cases of "oligopolization," i.e. some groups put no effort in the contest. While many researches ignore such cases, they usually appear in models of group contests with linear costs. See Ueda (2002).

⁹ In the field of industrial organization, the strategic effects of internal contracts between competing vertical structures have been mentioned. Whether the contracts are observable or not is considered as an important factor there. See Katz (1991).

Hence we assume that the value of δ_i is unobservable from outside, i.e. it is *private information* for the members of group i , and avoid the above unrealism and suspect strategic effects. In our model, therefore, the contest under any configuration of group sharing rules cannot be a proper subgame: Each player in the contest, as a member of one of the competing groups, cannot specify the payoff functions of players in the other groups, which depend on the unobservable group sharing rules. When making a decision on how much effort to make, each individual is required to infer the sharing rules in the other groups. We therefore apply the perfect Bayesian equilibrium notion, even though our model is not an incomplete information game.¹⁰

Given the effort levels put by all individuals, the contest winning probability of group i is determined by

$$\pi_i = \frac{A_i}{\sum_{k=1}^m A_k}, \tag{1}$$

where A_k is the total amount of effort made by the members of group k . Although we apply the common simple lottery contest success function, notice that our model allows heterogeneity in the contestants' effectiveness by allowing differences in the cost functions of the groups.

2.3. Equilibrium

To consider a perfect Bayesian equilibrium of our model, we assume for simplicity that each player can use only pure strategies. That is, we do not consider the possibility of randomization in each information set. For further characterization of the equilibrium, let us describe the beliefs in the information sets of the players. The decisions on the group sharing rules by the leaders are made simultaneously at the beginning of the game. For their information sets, therefore, the belief is trivial.

Consider the belief and the strategy of a group member making effort in the contest, say the k th member of group i . The individual's information set can be indexed by a value of δ_i corresponding to his group sharing rule announced by the group leader at the beginning of the contest. It is impossible to distinguish among the nodes at which different sharing rules are chosen in other groups. A strategy of the member is, therefore, described as a function of δ_i , which is denoted by $a_{ik}(\delta_i)$. This member's belief μ_{ik} with respect to the sharing rules of the other groups can be constructed, also depending only on the value of δ_i . For each δ_i , the value $\mu_{ik}(\delta_i)$ is a probability measure defined on $[0, 1]^{m-1}$, the space of possible configurations of the sharing rules in the other groups, $\delta_{-i} = (\delta_1, \dots, \delta_{i-1}, \delta_{i+1}, \dots, \delta_m)$.

Pick a profile of the sharing rules $\delta_1^*, \dots, \delta_m^*$ and individual decisions on effort, $(a_{jh}^*(\delta_j), \mu_{jh}^*(\delta_j))$, for all $j=1, \dots, m; h=1, \dots, N_j$, and $\delta_j \in [0, 1]$. Let us specify the necessary conditions for the profile to be a perfect Bayesian equilibrium. In a general representation, the expected utility of the k th member of group i at the information set indexed by δ_i' is calculated as

$$\int \frac{A_i^*(\delta_i')}{\sum_{j \neq i} A_j^*(\delta_j) + A_i^*(\delta_i')} B \left[\gamma \left(\delta_i' \frac{a_{ik}^*(\delta_i')}{A_i^*(\delta_i')} + \frac{1 - \delta_i'}{N_i} \right), 1 - \gamma \right] \mu_{ik}^*(\delta_i') (d\delta_{-i}) - v_i(a_{ik}^*(\delta_i')), \tag{2}$$

where $A_j^*(\delta_j) = \sum_{h=1}^{N_j} a_{jh}^*(\delta_j)$. At the information set indexed by δ_i^* , which lies on the equilibrium path, the requirement of consistency considerably simplifies the belief, which has to satisfy $\mu_{ik}^*(\delta_i^*) (\{\delta_{-i} = \delta_{-i}^*\}) = 1$. That is, the individuals must correctly infer the sharing rules in the other groups, given the strategies of the leaders.

Appealing to the “no-signaling-what-you-don't-know” condition of Fudenberg and Tirole (1991), we restrict the equilibrium beliefs at the un-reached information sets.¹¹ Intuitively, this condition requires that choices of sharing rules by a group leader tell nothing about what she does not know. At an un-reached information set, the associated individual is told a non-equilibrium group sharing rule by his leader. The individual knows that the group leader, just like him, cannot detect any change about the group sharing rules of the other groups, which are not observable from outside. Hence he believes that this deviation by the leader does not reflect any new information on the sharing rules of the other groups. The individual keeps the same belief as that held if he were told the equilibrium group sharing rule. Therefore, the equilibrium belief by the k th member of group i , μ_{ik}^* , must satisfy

$$\mu_{ik}^*(\delta_i) (\{\delta_{-i} = \delta_{-i}^*\}) = 1 \quad \text{for all } \delta_i \neq \delta_i^*.$$

Henceforth, we take such a *pure-strategy perfect Bayesian equilibrium* (with the “no-signaling-what-you-don't-know” condition) as the solution concept of our model.

Let us say that group i has an *interior group sharing rule*, if $0 < \delta_i^* < 1$ holds in equilibrium. Denoting the win probability of group i by π_i , we can state the following characterization of equilibrium:

Proposition 1. (i) *In a pure-strategy perfect Bayesian equilibrium, every individual belonging to the same group chooses a symmetric equilibrium effort (of course, those belonging to different groups can choose different effort levels), and attains a symmetric utility level.*

¹⁰ We thank Debraj Ray, who first called our attention to the necessity of this approach.

¹¹ The formal condition is given in Definition 6.1 of their paper.

(ii) An interior equilibrium sharing rule of the private-good component of the prize of group i is given by

$$\delta_i^* = \frac{1 - \pi_i}{\eta(N_i, \gamma)}, \tag{3}$$

where

$$\eta(N_i, \gamma) = \frac{\partial}{\partial q} B\left(\frac{\gamma}{N_i}, 1 - \gamma\right) \frac{\frac{\gamma}{N_i}}{B\left(\frac{\gamma}{N_i}, 1 - \gamma\right)} \tag{4}$$

is the elasticity of the benefit from the private part of the prize.

The proposition establishes that there exists a strong relationship between the winning probability and the endogenously determined share of the private part of the prize that is distributed according to the relative effort rule, as long as the sharing rule is interior. Notice that since the benefit function is concave with respect to the private part of the prize, we get that $\eta(N_i, \gamma) \leq 1$, with strict inequality unless the prize is purely private ($\gamma = 1$) and the benefit function is linear with respect to the private part of the prize. Proposition 1 helps us to confirm the following existence result.

Proposition 2. *In our model of group contest, there exists a unique pure strategy perfect Bayesian equilibrium.*

Henceforth we concentrate on equilibria in which every group chooses an interior sharing rule of the private component of the prize, i.e. $0 < \delta_i^* < 1$, for all $i = 1, \dots, m$.

3. Disappearance of the group size paradox

Let all members of the competing groups share the same cost function v . Following Esteban and Ray, we denote by $\alpha(a)$ the elasticity of the marginal cost,

$$\alpha(a) = \frac{av''(a)}{v'(a)}.$$

Also, let us pretend that the membership N_i is a continuous variable and view the winning probability π and the benefit elasticity η as its continuous functions. The membership size viewed as a continuous variable will be denoted by n . The following result presents the condition that determines the relation between group size and the winning probabilities of competing groups.

Proposition 3. *Suppose that all individuals share the same cost function v . Let N and N' be two group sizes with $N < N'$. Then the winning probability of an N' -member group is larger than that of an N -member group, if*

$$1 + \inf_{a \geq 0} \alpha(a) > \max_{n \in [N, N']} \eta(n, \gamma). \tag{5}$$

A larger membership implies a smaller per capita private-good component of the prize. Confronting the smaller benefit, each member puts less effort. This is the effect indicated by Olson to cause the group size paradox, the extent of which can be measured by $\eta(N_i, \gamma)$. With increasing marginal effort costs, however, the larger membership also implies lower individual's marginal costs at a given level of group effort, which induces more effort from each member. This is the multi-person economies effect emphasized by Esteban and Ray, and its extent can be measured by $\alpha(A_i/N_i)$. Their sufficient condition (that the larger N' -member group has advantage to the smaller N -member group) can be written as

$$\inf_{a \geq 0} \alpha(a) > \max_{n \in [N, N']} \eta(n, \gamma),$$

for our generalized model.¹² Hence the term “1” in the left-hand side of inequality (5) reflects the new effect added by endogenous group sharing rules.

Actually, this added term closes the lid on the group size paradox. As we have already pointed out, $\eta(n, \gamma)$ is always less than 1, unless the prize is purely private. It is therefore straightforward to verify that.

Corollary 1. *In a contest for a mixed private–public good where $\gamma \neq 1$, a larger group always attains a higher winning probability.*

That is, a larger size always enhances the winning probability of the group, provided that the prize is not a pure private good.¹³

To see what is happening, notice that (interior) equilibrium group sharing rules allow the competing groups to induce (group) optimal contribution from the members. Therefore, each member's marginal effort cost should be equal to the sum

¹² The model of Esteban and Ray can be interpreted as the special case in which it is common knowledge that every group leader always chooses the pure egalitarian rule. Set $\delta_i = 0$ for all i , derive the first-order condition for the members to put effort under this condition, and follow the similar procedure in their proof (Esteban and Ray, 2001, Proposition 1). To surely eliminates the group size paradox, it is needed to require that $\inf_a \alpha(a) > 1$, as they actually state in their Proposition 1—a.

¹³ Even in such a case, the group size paradox is not valid when the benefit function is strictly concave.

of the marginal benefits within the group, because winning of the prize is a collective good for a group. In equilibrium, therefore, a member of group i contributes as if the value of the prize is $N_i B(\gamma/N_i, 1-\gamma)$, the sum of the actual values of the prize within the group.¹⁴ This is the reason why the term “1” appears in (5); externalities of members’ efforts within a group can be internalized by the group leader with an adequate group sharing rule. The higher the sum, the more contribution in equilibrium is made by each member. As long as the public-good part of the prize is positive, an expansion of group size raises the sum even though the per-capita private-good declines.

We can also provide a sufficient condition for a (large) group to attain a higher per capita utility than the smaller groups even if the prize is not a pure public good for the winning group.

Proposition 4. *Suppose that all individuals share the same cost function v , and $\inf_{a \geq 0} \alpha(a) \geq 1$. Let N and N' be two group sizes with $N < N'$. Then the per capita (expected) utility in an N' -member group is larger than that in an N -member group, if the winning probability of the former group is less than one half and $\max_{n \in [N, N']} \eta(n, \gamma) < 4/5$.*

When the number of competing groups is larger than two, the winning probability of a group is less than one half, at least unless it is the most advantageous group attaining the highest probability. For such groups, a larger size implies a higher per capita utility, provided that the elasticity of the marginal cost is sufficiently high and the elasticity of the benefit from the private part of the prize is sufficiently low.¹⁵

4. Group characteristics and prize sharing

4.1. Determinants of differences in winning probabilities

Eq. (3) in Proposition 1 reveals a simple and interesting relationship between the preferred egalitarianism of a group and its winning probability: A group attaining a high winning probability π_i prefers a highly egalitarian group sharing rule (i.e. a low value of δ_i), provided that the value of $\eta(N_i, \gamma)$ is constant. The equation does not imply that either π_i or δ_i is a cause of the other, because both of them are simultaneously determined in equilibrium. However, if we identify some group characteristics enhancing the group winning probability, using Eq. (3) we can see that they also enhance the degree of egalitarianism in sharing the prize. Such identification is accomplished in the next proposition.

Proposition 5. *If two groups k and l have the same group size and the same effort cost function for the members, i.e. $N_k = N_l$ and $v_k = v_l$, then $\delta_k^* = \delta_l^*$ holds and these groups share the same winning probability and per-capita utility. Furthermore, if all competing groups are symmetric, then their common weight of the relative effort rule δ^* is given by*

$$\delta^* = \frac{1-1/m}{\eta(N, \gamma)}. \quad (6)$$

Hence, if two groups choose different values of the weight of the relative effort sharing rule, they must be different either in their costs or in their membership size. The assignment of a larger weight to the relative effort rule is actually due to inefficiency or size disadvantage of the group. Complete reliance on the relative effort rule induces the members to make excessive efforts that prohibit the attainment of Pareto optimum, while reliance just on the egalitarian rule also results in an inefficient outcome (Sen, 1966). The egalitarian sharing rule causes free-riding in a group because each member’s effort has a positive externality for the other members under this rule. On the other hand, if the members are rewarded for their relative effort, each member’s effort has a negative externality for the others, and the result is an excessive group effort.¹⁶ An advantageous group that can secure a higher winning probability has room to loosen up this negative externality (which is due to selective incentives), still providing enough utility gain to compensate for the reduction in the winning probability.

Proposition 5 also discloses (see Eq. (6)) the effect of the number of competing groups on the group sharing rules. When the number of the groups is large, each group uses more of the private prize to provide selective incentives to its members (δ^* is increasing with respect to m).

4.2. Different efficiency

We have identified possible causes of the difference in the winning probabilities of competing groups; differences in costs or group sizes. On the effect of cost differences among the competing groups, we can derive the following result.¹⁷

¹⁴ It is easy to confirm that Eq. (9) in the proof of Proposition 1, which characterizes the equilibrium contribution of an individual, is equivalent of the (interior) first-order condition for the maximization problem $\max_a \frac{A_i}{\sum_{j=1}^m A_j} N_i B(\frac{a}{N_i}, 1-\gamma) - v_i(a)$.

¹⁵ With the notation of Esteban and Ray (2001) and their linear specification $B(q, G) = Mq + PG$, the condition $\max_{n \in [N, N']} \eta(n, \gamma) < 4/5$ can be expressed as $\theta(\lambda, N_{\min}) > 1/5$, or $\frac{N_{\min}}{4P + \frac{M}{N_{\min}}} < \lambda \leq 1$, where $\lambda = 1-\gamma$ (the ratio of the public-good part of the prize), $N_{\min} = N$, and $\theta(\lambda, N_{\min}) = 1 - \eta(N_{\min}, 1-\lambda)$.

¹⁶ We owe this intuition to one of the anonymous reviewers.

¹⁷ Baik and Lee (2007) have shown, in their model of two-group contest with equal group membership and a pure private-good prize, that a more efficient group chooses a more egalitarian rule. Their result can be considered as a special case of this proposition.

Proposition 6. Pick two competing groups k and l with the same number of members, say N . Let the members of group k have lower marginal costs than those of members of group l . That is, $v_k'(a) < v_l'(a)$ for all $a > 0$. Then, in equilibrium, $\pi_k > \pi_l$, which implies (see Eq. (3)) that $\delta_k^* < \delta_l^*$. Also, the per capita utility is larger in group k than in group l .

Proposition 6 can be applied to shed light on the role of differences in the valuation of the prize¹⁸ and in lobbying capability. To study the effect of variability in the evaluation of the prize, we modify the model by letting members of group i have the benefit function $w_i B(q, G)$, where $w_i > 0$ is the augmenting factor. The decision by an individual with the benefit function $w_i B$ and the cost function v_i is equivalent to that by someone with the benefit function B and the cost function v_i/w_i . If $w_k > w_l$, therefore, Proposition 6 is applicable, pretending that the members of group k have a lower marginal cost than those of group l . To study the effect of variability in the political influence or lobbying power of the individuals, we interpret A_i as aggregate effort measured by an efficiency unit, and assume that each individual of group i needs to make $e_i a$ ($e_i > 0$) units of effort to produce the efficiency units a . Then the cost function has the form $v_i(e_i a)$ and differences in political capabilities can be transformed to differences in the marginal cost of effort across groups.

4.3. Different group sizes

We have already discussed how group size enhances the winning probability of a group. But we need to introduce a reservation regarding the relationship between a large group size and a highly egalitarian group sharing rule, because η also varies with group size in Eq. (3). Nevertheless, if $\eta(n, \gamma)$ is non-decreasing with respect to the number of group members n (and if $\gamma \neq 1$ holds), then the sharing rule applied by a larger group is more egalitarian.

The CES family of benefit functions, $B(q, G) = (b_1 q^\rho + b_2 G^\rho)^{1/\rho}$, with $0 < b_1 < 1$, $0 < b_2 < 1$ and $\rho \leq 1$, is a convenient specification to illustrate cases that fit the above reservation. With this form, we get that

$$\eta(n, \gamma) = \frac{b_1 (\gamma/n)^\rho}{b_1 (\gamma/n)^\rho + b_2 (1-\gamma)^\rho}.$$

Let $\gamma \neq 1$. If $\rho < 0$, i.e. the public and the private parts of the prize are not good substitutes, we can see that $\eta(n, \gamma)$ becomes non-decreasing with respect to n , and indeed the larger the group, the higher its egalitarianism in sharing the prize.

5. Concluding remarks

We have examined a model of a group contest for a mixed private–public-good prize, in which each group can choose a sharing rule to distribute the private-good part of the prize among its members. Our main findings are the following: When each competing group can choose its sharing rule:

- (i) the larger a group, the higher its winning probability (at least unless the contested prize is a pure private good): the selective incentives provided by endogenous group sharing rules are sufficient to eliminate the group size paradox and
- (ii) strong incentives appearing in the sharing rule of a group are not a sign of its advantage, but rather of its disadvantage relative to the other groups.

The first finding suggests that the group size advantage is a normal outcome in a contest by well-organized interest groups. The free-rider problems may prevent the individuals from organizing a large interest group. However, once somehow it is established and equipped with adequate selective incentives, group size becomes advantageous, which prompts the group's further expansion of its membership. The second finding suggests that adjustment of the sharing rule applied by a group in order to provide stronger incentives to its members might be a prelude to the decline of the group. The revision of the sharing rule could reflect some disorders in the group. Empirical investigation of these predictions is an interesting worthy task for future research. The extent of egalitarianism in our setting is not determined by moral values, religious commitments or social ideology. It is the outcome of rational strategic incentives that arise in the contest environment. In this competitive environment, groups with higher valuation of the prize or larger lobbying capabilities tend to be more egalitarian. Under the sufficient conditions we have stated, larger groups also tend to be more egalitarian. Testing empirically these predictions is another worth pursuing direction.

Finally, we wish to make some remarks on the robustness of the two main results. We have restricted the class of possible group sharing rules to linear combinations between the egalitarian and the relative effort rules. In real interest group politics, a group leader could choose a rule not belonging to this class. But our point is to enable a group to introduce selective incentives and adjust their strength. Even our restrictive feasible set of group sharing rules is sufficient to eliminate the group size paradox. Extension of the range of possible sharing rules would only reinforce the result. Also, with any incentive schemes, it seems plausible to conjecture that a disadvantaged group will provide its members with strong incentives to catch up with the other groups.

¹⁸ The early arguments on the role of different valuations of the prize in the contests can be found in Hillman and Riley (1989).

Some readers might question the validity of the assumption that leaders are benevolent. Group leaders may maximize the winning probability rather than the per capita utility of their groups to enhance their own benefit. If a leader is a *political entrepreneur* (Laver, 1997), i.e. an outsider hired by the group members to determine and enforce a group sharing rule, the reward contract might be written conditionally not on the welfare, but on the winning of the group. The former is much more difficult to verify than the latter. In other cases, the competing groups are firms and the leaders are naturally managers. They are responsible for the shareholders, who only care about the success of the firms. In response to such possible criticism, we would like to argue that a group leader cannot do well without the support of the members, and therefore, at least partially he takes care of their welfare. A possible plausible generalization of our model can therefore be based on the assumption that group leaders are concerned both about the winning probability and the welfare of their group. Let the group welfare functions be the averaged utility of the group members, and let θ and $(1-\theta)$ be the common weights assigned, respectively, to the winning probability and the group welfare in such mixed-motive objective function of the leaders. Then, the following statements can be proved.¹⁹

Statement 1. An interior equilibrium sharing rule of the private-good component of the prize of group i is given by

$$\delta_i^* = \frac{N_i - 1 + \theta}{(1-\theta)(N_i - 1)} \frac{1 - \pi_i}{\eta(N_i, \gamma)} \tag{3'}$$

Statement 2. Proposition 3 holds in this generalized model without any change.

Thus, result (i) is immune to this generalization of the leaders' objectives. From Eq. (3'), we can observe that the group leaders will choose the pure relative effort rule when θ is sufficiently close to 1, i.e. when the leaders' incentives are sufficiently solely dependent on the winning probability of their group. As long as the leaders choose interior sharing rules, however, the negative correlation between π_i and δ_i^* is preserved. This implies that result (ii) is also robust.

6. Proofs

6.1. Proof of Proposition 1

We start with a lemma characterizing the equilibria of our model.

Lemma 1. A profile of strategies $\delta_1^*, \dots, \delta_m^*$, and $a_{jh}^*(\delta_j)$, for all $j=1, \dots, m$, $h=1, \dots, N_j$, and $\delta_j \in [0, 1]$ constitute a pure strategy perfect Bayesian equilibrium, if and only if the following two conditions hold:

Condition 1: For all $i=1, \dots, m$; $k=1, \dots, N_i$, and $\delta_i \in [0, 1]$, $a_{ik}^*(\delta_i) = A_i^*(\delta_i)/N_i$ with

$$\left(1 - \frac{A_i^*(\delta_i)}{A}\right) B\left(\frac{\gamma}{N_i}, 1-\gamma\right) + \gamma \delta_i \frac{\partial B(\gamma/N_i, 1-\gamma)}{\partial q} \left(1 - \frac{1}{N_i}\right) - v_i' \left(\frac{A_i^*(\delta_i)}{N_i}\right) A = 0, \tag{7}$$

where $A = \sum_{j \neq i} A_j^*(\delta_j^*) + A_i^*(\delta_i)$ is the total effort made by the contestants.

Condition 2: $\delta_i^* \in \operatorname{argmax}_{0 \leq \delta_i \leq 1} A_i^*(\delta_i)/AB(\gamma/N_i, 1-\gamma) - v_i(A_i^*(\delta_i)/N_i)$, for all $i=1, \dots, m$.

Proof of Lemma 1. Only-if-part: Under a belief profile satisfying the “no-signaling-what-you-don’t-know” condition, the k th member of group i must choose effort for an arbitrary group sharing rule δ_i that satisfies

$$a_{ik}^*(\delta_i) \in \operatorname{argmax}_{a \geq 0} \frac{\sum_{h \neq k} a_{ih}^*(\delta_i) + a}{\sum_{j \neq i} A_j^*(\delta_j^*) + \sum_{h \neq k} a_{ih}^*(\delta_i) + a} B\left[\gamma \left(\frac{a \cdot \delta_i}{\sum_{h \neq k} a_{ih}^*(\delta_i) + a} + \frac{1 - \delta_i}{N_i}\right), 1 - \gamma\right] - v_i(a).$$

Since $\lim_{a \rightarrow 0} v_i'(a) = 0$, the maximization problem requires $a_{ik}^*(\delta_i) > 0$ and the fulfillment of the first order condition:

$$\frac{A - A_i^*(\delta_i)}{A^2} B\left(\gamma \left(\frac{a_{ik}^*(\delta_i)}{A_i^*(\delta_i)} + (1 - \delta_i) \frac{1}{N_i}\right), 1 - \gamma\right) + \frac{1}{A} \gamma \delta_i \frac{\partial B(\gamma \delta_i a_{ik}^*(\delta_i)/A_i^*(\delta_i) + (1 - \delta_i)1/N_i, 1 - \gamma)}{\partial q} A_i^*(\delta_i) - \frac{a_{ik}^*(\delta_i)}{A_i^*(\delta_i)} v_i'(a_{ik}^*(\delta_i)) = 0.$$

As B is concave with respect to the private part of the prize, the left-hand side of the equation is strictly decreasing with respect to a_{ik} . Hence we can confirm that every member of group i chooses a symmetric effort level. Condition 1 is established.

Condition 1 implies that $A_i^*(\delta_i)$ is strictly increasing with respect to δ_i . Because of the “no-signaling-what-you-don’t-know” condition of the belief profile, the leader of group i can change the aggregate effort of the group by a unilateral deviation from the equilibrium group sharing rule δ_i^* , keeping constant the belief of the members on the other groups' efforts. Condition 1 also implies that every member attains a symmetric per capita utility at each value of δ_i . The criterion

¹⁹ Notice that a change in the objective functions of group leaders only changes their payoffs in the extensive form of our model. Thus it does not affect Condition 1 of Lemma 1 in Section 6. Also notice that maximization of the winning probability π_i is equivalent to maximizing $\pi_i B(\gamma/N_i, 1-\gamma)$. In turn, instead of Condition 2 of Lemma 1, as the equilibrium condition for the leaders with mixed motives, we would obtain the following expression: $\delta_i^* \in \operatorname{argmax}_{0 \leq \delta_i \leq 1} \theta(A_i^*(\delta_i)/A)B(\gamma/N_i, 1-\gamma) + (1-\theta)(A_i^*(\delta_i)/AB(\gamma/N_i, 1-\gamma) - v_i(A_i^*(\delta_i)/N_i))$, for all $i=1, \dots, m$. With this slightly modified condition, we can repeat the proofs of Propositions 1 and 3 to derive the above two statements.

for choosing a group sharing rule is reduced to the maximization of per capita utility, as long as the leader wishes to ensure the selection of a Pareto efficient outcome. Hence Condition 2 is necessary to prevent deviations by the group leaders.

If-part: Let Conditions 1 and 2 hold. By Condition 1, each member of each group maximizes the expected utility over all of his information sets under the belief profile satisfying the “no-signaling-what-you-don’t-know” condition. By symmetry and Condition 2, each leader chooses a group sharing rule that maximizes the welfare function of his group, given the sharing rules in the other groups and the succeeding actions of the individual contestants. □.

Part (i) of Proposition 1 is implied by the part of Condition 1 in Lemma 1. We only need to prove part (ii). By the Eq. (7), $A_i^*(\delta_i)$ is differentiable with respect to δ_i and $\partial A_i^*/\partial \delta_i > 0$. As a necessary and sufficient condition of Condition 2 for group i , we therefore have

$$\frac{A - A_i^*(\delta_i^*)}{A} B\left(\frac{\gamma}{N_i}, 1 - \gamma\right) - v_i\left(\frac{A_i^*(\delta_i^*)}{N_i}\right) \frac{A}{N_i} \geq (\leq) 0 \quad \text{if } \delta_i^* > 0 (< 1). \tag{8}$$

By Eq. (7), we can see that $\delta_i^* = 0$ is impossible. Notice that the left-hand side of (8) is strictly decreasing with respect to the value of A_i^* . Therefore, if (8) holds as an equality for some $\delta_i < 1$, it must be the unique solution and equal to δ_i^* . Otherwise, the per capita utility is strictly increasing with respect to δ_i on $[0, 1]$, and $\delta_i^* = 1$ must hold.

Suppose that group i has an interior group sharing rule $0 < \delta_i^* < 1$ in equilibrium. Denoting the winning probability of group i by $\pi_i = A_i^*(\delta_i^*)/A$, Eq. (8) has the form

$$(1 - \pi_i) B\left(\frac{\gamma}{N_i}, 1 - \gamma\right) - v_i\left(\frac{A}{N_i} \pi_i\right) \frac{A}{N_i} = 0. \tag{9}$$

Substitution of (9) into (7) yields Eq. (3). □.

Proof of Proposition 2. Consider, hypothetically,²⁰ Eq. (9) as the condition implicitly defining π_i as a function of A , γ , and the membership N_i . Then, π_i is continuous and strictly decreasing in A . Also, $\lim_{A \rightarrow 0} \pi_i = 1$ and $\lim_{A \rightarrow \infty} \pi_i = 0$. As A increases, the value of δ_i derived from Eq. (3), which is required from Conditions 1 and 2 of Lemma 1 as long as $0 < \delta_i^* < 1$, approaches 1. If $\eta(N_i, \gamma)$ is less than 1, δ_i satisfying (3) can exceed 1 for the value of A larger than some level, say A_R . If the total effort A actually attains such a value in an equilibrium, group i must have $\delta_i^* = 1$, and then, π_i satisfies the following equation:

$$(1 - \pi_i) B\left(\frac{\gamma}{N_i}, 1 - \gamma\right) + \gamma \frac{\partial B(\gamma/N_i, 1 - \gamma)}{\partial q} \left(1 - \frac{1}{N_i}\right) - v_i\left(\frac{A}{N_i} \pi_i\right) A = 0, \tag{10}$$

which is derived from (7), setting $\delta_i = 1$. Confirm that π_i is again continuous and strictly decreasing in A and $\lim_{A \rightarrow \infty} \pi_i = 0$, when Eq. (10) is hypothetically seen as the condition defining π_i as an implicit function.

Now, consider the share function of group i that depends on A , $\pi_i^P(A): (0, \infty) \rightarrow \mathbb{R}$, which is defined as follows: For any A in $(0, A_R]$, this function assigns the value of π_i given by Eq. (9), and for any A larger than A_R , it assigns the value of π_i determined by Eq. (10). The derived function is continuous and strictly decreasing, with $\lim_{A \rightarrow 0} \pi_i^P(A) = 1$ and $\lim_{A \rightarrow \infty} \pi_i^P(A) = 0$.

Let us consider the value A^* with $\sum_{i=1}^m \pi_i^P(A^*) = 1$. Such a value certainly exists and is unique. It can be viewed as a candidate of the total equilibrium effort put by all the competing groups. Then, $\pi_i^P(A^*) A^*$ must be the aggregate effort put by group i in equilibrium. By using the definition of the share function and Eq. (3), we can uniquely specify the group sharing rule δ_i^* . We can confirm that this rule δ_i^* and the aggregate group effort $A_i^*(\delta_i^*) = \pi_i^P(A^*) A^*$ satisfy the conditions of Lemma 1. The existence of equilibrium has thus been confirmed.

On the other hand, if we have an equilibrium with the total equilibrium effort A^* , Lemma 1 requires that the aggregate effort by group i in equilibrium satisfies $A_i^*(\delta_i^*) = \pi_i^P(A^*) A^*$. In equilibrium, however, the sum of the winning probabilities of the m groups must be equal to 1, and $\sum_{i=1}^m \pi_i^P(A^*) = 1$ has to be satisfied. This implies the uniqueness of the equilibrium. □.

Proof of Proposition 3. Keep the total effort unchanged at its equilibrium value A^* . Pretending that N_i is a continuous variable in the Eq. (9), we can derive from the equation that

$$\left. \frac{\partial \pi_i}{\partial n} \right|_{n=N} = \frac{\pi_i (1 - \eta(N, \gamma) + \alpha(A^* \pi_i / N))}{N \pi_i / (1 - \pi_i) + \alpha(A^* \pi_i / N)}. \tag{11}$$

If inequality (5) holds, this derivative is positive at all values of n in the closed interval $[N, N']$. This establishes the validity of Proposition 3. □.

Proof of Proposition 4. Again, keeping the total effort unchanged at its equilibrium value A^* and viewing N_i as a continuous variable, we can examine the behavior of the per capita utility

$$u_i = \pi(A^*, N_i) B\left(\frac{\gamma}{N_i}, 1 - \gamma\right) - v_i\left(\frac{A^*}{N_i} \pi(A^*, N_i)\right),$$

²⁰ We will use the same technique as in Esteban and Ray (2001) and Ueda (2002). This technique is called the *share function approach* by Cornes and Hartley (2003, 2005).

where $\pi(A^*, N_i) = \pi_i$ is the value of the winning probability given by Eq. (9). By using Eqs. (9) and (11), we get that

$$\frac{\partial u}{\partial n} \Big|_{n=N} = \frac{\pi}{N} \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) \left\{ 1-\eta + \frac{(1-\pi)\pi}{\pi+(1-\pi)\alpha} \left(1-\eta - \frac{\pi}{1-\pi}\right) \right\},$$

where $\pi = \pi(A^*, N)$, $\alpha = \alpha(A^*/N)\pi$ and $\eta = \eta(N, \gamma)$. If $\inf_{a \geq 0} \alpha(a) \geq 1$ and $0 < \pi < 1/2$, we can see that $0 < (1-\pi)\pi / \{\pi + (1-\pi)\alpha\} \leq \frac{1}{4}$ holds. Then, $\max_{n \in [N, N']} \eta(n, \gamma) < 4/5$ implies that $\partial u / \partial n \Big|_{n=N} > 0$ holds on the interval $[N, N']$, because

$$1-\eta + \frac{(1-\pi)\pi}{\pi+(1-\pi)\alpha} \left(1-\eta - \frac{\pi}{1-\pi}\right) \geq 1-\eta - \frac{(1-\pi)\pi}{\pi+(1-\pi)\alpha} \eta \geq 1 - \frac{5}{4} \eta \quad \square.$$

Proof of Proposition 5. If two groups are symmetric, they have the same schedule of the share function. At the unique equilibrium total effort A^* , therefore, the groups attain the same values of aggregate group effort. This implies that they have the same group sharing rule, the winning probability, and the per-capita utility. When all groups are symmetric, every group attains the winning probability $1/m$, and the Eq. (6) holds trivially. \square .

Proof of Proposition 6. Eq. (9) implies that

$$\pi_k B\left(\frac{\gamma}{N}, 1-\gamma\right) \frac{N}{A^*} + v'_k \left(\frac{A^*}{N} \pi_k\right) = \pi_l B\left(\frac{\gamma}{N}, 1-\gamma\right) \frac{N}{A^*} + v'_l \left(\frac{A^*}{N} \pi_l\right).$$

Suppose that $\pi_k \leq \pi_l$. We therefore obtain the strict inequality: $\tau'_k(A^* \pi_k / N) < v'_l(A^* \pi_l / N)$, which makes the above equation impossible. This means that $\pi_k > \pi_l$. Since $\eta(N_k, \gamma) = \eta(N_l, \gamma) = \eta(N, \gamma)$, we can derive $\delta_k < \delta_l$. Finally, notice that $A_k^*(\delta_k^*)$ which maximizes the per capita utility of group k , is larger than $A_l^*(\delta_l^*)$. Denoting the per capita utility of group i by u_i , and noticing that $v_k(a) < v_l(a)$ for all $a > 0$, we get that

$$\begin{aligned} u_k &= \frac{A_k^*(\delta_k^*)}{A^*} B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_k^*(\delta_k^*)}{N}\right) \geq \frac{A_l^*(\delta_l^*)}{\sum_{j \neq k} A_j^*(\delta_j^*) + A_l^*(\delta_l^*)} B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_l^*(\delta_l^*)}{N}\right) \\ &> \frac{A_l^*(\delta_l^*)}{A^*} B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_l^*(\delta_l^*)}{N}\right) > \frac{A_l^*(\delta_l^*)}{A^*} B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_l\left(\frac{A_l^*(\delta_l^*)}{N}\right) = u_l \quad \square. \end{aligned}$$

Acknowledgements

We are very much indebted to two anonymous referees and an associate editor for their most useful suggestions. Financial support from the Adar Foundation of the Economics Department at Bar-Ilan University is gratefully acknowledged by Shmuel Nitzan. Kaoru Ueda is grateful for the funding by the Nitto Foundation, Aichi.

References

- Baik, Kyung H., 1994. Winner-help-loser group formation in rent-seeking contests. *Economics and Politics* 6 (July), 147–162.
- Baik, Kyung H., Lee, Sanghack, 2001. Strategic groups and rent dissipation. *Economic Inquiry* 39 (October), 672–684.
- Baik, Kyung H., Lee, Sanghack, 2007. Collective rent seeking when sharing rules are private information. *European Journal of Political Economy* 23 (September), 768–776.
- Baik, Kyung H., In-Gyu, Kim., Sunghyun, Na., 2001. Bidding for a group-specific public-good prize. *Journal of Public Economics* 82 (December), 415–429.
- Chamberlin, John R., 1974. Provision of Collective goods as a function of group size. *American Political Science Review* 68 (June), 707–716.
- Cornes, Richard, Hartley, Roger, 2003. Risk aversion, heterogeneity and contests. *Public Choice* 117 (1), 1–25.
- Cornes, Richard, Hartley, Roger, 2005. Asymmetric contests with general technologies. *Economic Theory* 26 (4), 923–946.
- Epstein, Gil S., Nitzan, Shmuel, 2007. *Endogenous Public Policy and Contests*. Springer-Verlag, Berlin, Heidelberg.
- Esteban, Joan-Maria, Ray, Debraj, 2001. Collective action and the group size paradox. *American Political Science Review* 95 (September), 663–672.
- Esteban, Joan-Maria, Debraj Ray, forthcoming. A model of ethnic conflict. *Journal of the European Economic Association*.
- Fudenberg, Drew, Tirole, Jean, 1991. Perfect bayesian equilibrium and sequential equilibrium. *Journal of Economic Theory* 53 (2), 236–260.
- Hillman, Arye L., Riley, John G., 1989. Politically contestable rents and transfers. *Economics and Politics* 1 (March), 17–39.
- Katz, Eliakim, Nitzan, Shmuel, Rosenberg, Jacob, 1990. Rent-seeking for pure public goods. *Public Choice* 65 (April), 49–60.
- Katz, Eliakim, Tokatlidu, Julia, 1996. Group competition for rents. *European Journal of Political Economy* 12 (December), 599–607.
- Katz, Michael L., 1991. Game-playing agents: unobservable contracts as precommitments. *Rand Journal of Economics* 22 (Autumn), 307–328.
- Konrad, Kai A., 2004. Bidding in hierarchies. *European Economic Review* 48 (December), 1301–1308.
- Konrad, Kai A., 2009. *Strategy and Dynamics in Contests*. Oxford University Press, New York.
- Laver, Michael, 1997. *Private Desires, Political Action*. SAGE Publications Ltd., London.
- Lee, Sanghack, 1995. Endogenous sharing rules in collective-group rent-seeking. *Public Choice* 85 (October), 31–44.
- Nitzan, Shmuel, 1991. Collective rent dissipation. *Economic Journal* 101, 1522–1534.
- Nitzan, Shmuel, 1994. Transfers or public goods provision? A political allocation perspective. *Economics Letters* 45 (August), 451–457.
- Nitzan, Shmuel, Ueda, Kaoru, 2009. Collective contests for commons and club goods. *Journal of Public Economics* 93 (1), 48–55.
- Olson, Mancur, 1982. *The Rise and Decline of Nations*. Yale University Press, New Haven, London.
- Riaz, Khalid, Shogren, Jason F., Johnson, Stanley R., 1995. A general model of rent-seeking for public goods. *Public Choice* 82 (March), 243–259.
- Sen, Amartya K., 1966. Labour allocation in a cooperative enterprise. *Review of Economic Studies* 33 (October), 361–371.
- Ueda, Kaoru, 2002. Oligopolization in collective rent-seeking. *Social Choice and Welfare* 19 (July), 613–626.
- Ursprung, Heinrich W., 1990. Public goods, rent dissipation, and candidate competition. *Economics and Politics* 2 (July), 115–132.
- Wärneryd, Karl, 1998. Distributional conflict and jurisdictional organization. *Journal of Public Economics* 63 (September), 435–450.