Performance and Prize Decomposition in Contests

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Abstract

This paper focuses on the effect of additive contest decomposition on performance: winning probabilities and efforts of the contestants. Our main result provides a sufficient condition for invariance of contest performance to the decomposition of a contest when the sum of the possibly differently valued prizes in the segmented contests is equal to the value of the prize in the original grand contest and the relative prizes in the sub-contests are equal for every contestant. It is shown that this condition is satisfied by the commonly used exponential logistic contest success functions. With these functions the contest designer does not have an incentive to split the prize and create additive, segmented sub-contests. We then prove that when the additive contest decomposition is asymmetric, contest decomposition may adversely affect the designer; that it, reduce the total efforts of the contestants.

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I. Introduction

The designers of contests are usually concerned about aggregate efforts (investment, influence activities, campaign contributions, rent seeking efforts, lobbying outlays) made by the contestants. The design of contests may focus on the number of contestants and their characteristics (ability, preferences, income), Amegashie (1999), Baye et. al. (1993), the nature of prizes (private or public goods,), Nitzan (1994b), the multi-stage sequential nature of the contest, Gradstein and Konrad (1999) or the contest success function, Epstein and Nitzan (2006a, 2007), Fang (2002). In this paper we focus on contest design that takes the form of additive contest decomposition; namely the decomposition of a contest into similar sub-contests, such that, for every contestant, the sum of the possibly differently valued prizes in the segmented contests is equal to the value of the prize in the original grand contest. Such decomposition may affect contest performance; that is, the winning probabilities of the contestants and their efforts.

For example, consider a privatization contest on an oil drilling concession in say, two oil fields of a country. One possibility is to assign the grand concession en-block to a single winner who is determined in the contest (an auction). Another possibility is to split the single concession conglomerate auction into some smaller sub-auctions and determine the winners of the different oil-field concessions (or any combination of these concessions) in the different sub-auctions. In our setting, there are constant returns to scale, so the value of the grand concession in the single auction is equal to the sum of the values of the separate concessions contested in the segmented auctions. In this paper we focus on the impact of such decomposition on the contestants' efforts and the governments' receipts, purposely disregarding other regulatory considerations, like enhancing competition by splitting a monopoly into several competing firms or enhancing the efficiency of a governmental firm through privatization.   When we divide a grand contest to a few contests, it is not clear that the stakes of the contestants are divided in the same way. Assume that one oil field is in the north and the other is in the south. There are technological location advantages to contestants and one of them might have a higher benefit from obtaining the oil field in the north while the other the oil field in the south (say, as a result of the different location of the already built supply facilities of the two competing firms). Therefore, if we divide the grand contest into two contests, one over the oil field in the north and the other for the one in the south, this division creates asymmetry in the prizes of the contestants in the segmented contests. Another possible illustration of such an effect arises in the context of inter-temporal decomposition. Namely, suppose that contest decomposition results in one contest held in the present while the other contest is held in the future. If one of the contestants prefers the present and the other prefers the future, then such contest decomposition would result in asymmetric stakes in the sub-contests held in the different periods. If, however, the contestants assign the same monetary valuation to the two oil fields, independent of their location and of the time of the awarded concession, then contest decomposition would result in symmetric stakes.

It is not clear whether the contest designer seeks to maximize or minimize contest efforts. Clearly, from the social welfare perspective, it may well be optimal to minimize the contest efforts, while from the direct viewpoint of the contest designer, it may be optimal to maximize these efforts. (see, for example, Epstein and Nitzan (2003, 2007)). In a paper related to ours, Fu and Lu (2007) consider a multiple-winner contest setting, examining how total efforts vary between a grand contest and a set of sub-contests. They show that the rent-dissipation rate increases when the number of contestants and of the prizes are "scaled up", showing that total efforts must increase when a set of identical sub-contests are merged into a grand contest. In their contest, total efforts decrease when a grand contest is evenly divided. Another important difference between the setting studied by Fu and Lu and our model is that they assume that contest splitting refers both to the prize and to the group of contestants. Namely, different contestants participate in the sub-contests. We assume a winner-takes-all contest and that only the prize is decomposed, allowing every contestant to take part in all sub-contests simultaneously dividing his effort between them. Our paper also differs from the paper by Fu and Lu (2007) because in our setting the contestants are not necessarily symmetric. Each contestant may have a different stake and/or have a different cost of investing effort in the contest. [4]

We provide conditions for the invariance of performance to contest decomposition and show that these conditions are satisfied by the exponential logistic

[4] Another paper related to ours is Moldovanu and Sela (2006). In this study the authors consider a perfectly discriminatory contest setting comparing two contest architectures with a grand contest that has *n* contestants who compete for one prize worth 1.

probability functions that are commonly used in the contest literature.[5] These contest success functions are the only logistic functions that are homogeneous of degree zero. With such contest success functions, the designer of the contest does not have an incentive to split the prize and create additive and symmetric segmented sub-contests.

In the second part of the paper, we consider the case of asymmetric (discriminative) distribution of relative prizes in the sub-contests. Our analysis clarifies how changes in stake asymmetry affect the efforts invested by each of the contestants and, in turn, the total effort invested in the contest. It is shown that additive but asymmetric decomposition can be inferior to a single grand contest from the point of view of a designer who is interested in maximizing the contestants' efforts.

II. The Segmented Contests

Consider two risk neutral agents who compete in two different independent contests. In contest k (k = 1,2) the prize of agent i (i=1,2) is n_i^k. The agents make efforts to increase their probability of winning. They allocate their efforts (time, expenditures, and resources) between the two different contests. If agent i expends x_i^k (i=1,2) in contest k (k =1,2) , the winning probability of agent i in contest k is $\Pr_i^k = \Pr_i^k\left(x_1^k, x_2^k\right)$. The function $\Pr_i^k\left(x_1^k, x_2^k\right)$ is usually referred to as the contest success function (CSF) in contest k. The expected net payoff of the risk neutral agent i in the *two* sub-contests is given by:

$$(1) \qquad E^S\left(w_i\right) = \Pr_i^1 n_i^1 + \Pr_i^2 n_i^2 - x_i^1 - x_i^2 , \quad i = 1,2$$

For contestant i, the sum of the prizes in the two contests is equal to $n_i = n_i^1 + n_i^2$. Contestant i's stake in the two contests can therefore be written in the following way: $n_i^k = \beta_i^k n_i$ where $\beta_i^k > 0$ *and* $\beta_i^1 + \beta_i^2 = 1$.

The parameters β_i^k, determined by the contest designer, reflect the splitting proportions of the grand contest into a bundle of segmented sub-contests. We consider

[5] A special case of these functions is the simple lottery function proposed in Tullock (1980)

two contingencies; egalitarian (symmetric) splitting and discriminating (asymmetric) splitting of a grand contest.

Symmetric splitting of a grand contest means that for every *k*, $\beta_1^k = \beta_2^k$. That is, the designer does not discriminate among the contestants by offering differential decomposition of the grand contest to the different participants. If the designer possesses a discrimination power, he may offer participant i a vector $\beta_i = \left(\beta_i^1, \beta_i^2\right)$, $\beta_i^1, + \beta_i^2 = 1$, reflecting a certain type of decomposition and at the same time offer another participant j, $j \neq i$, a different decomposition vector $\beta_j = \left(\beta_j^1, \beta_j^2\right)$, $\beta_j^1, + \beta_j^2 = 1$.

The expected net payoff of agent i $(i = 1,2)$ is given by:

(2) $$E^S(w_i) = \left(\Pr_i^1 \beta_i^1 + \Pr_i^2 \beta_i^2\right) n_i \; - x_i^1 - x_i^2 , \quad i = 1,2$$

It is assumed that $\frac{\partial \Pr_i^k(.)}{\partial x_i^k} > 0$, $\forall j \neq i$ $\frac{\partial \Pr_i^k(.)}{\partial x_j^k} < 0$ and that given x_j^k, there exists $\underline{x}_i^k$ such that, for $x_i^k \geq \underline{x}_i^k$, $\forall j \neq i$ $\frac{\partial^2 \Pr_i^k(.)}{\partial x_i^{k2}} < 0$ (the latter inequality ensures that the second order conditions are satisfied)[6].

We assume that all players participate in the contest $\left(x_i^k > 0 \quad \forall i, k\right)$. We therefore focus on the unique interior Nash equilibria of the contest.

The splitting of the contest may divide the stakes in an identical way for both contestants or it may split it differently. As clarified in the introduction, the splitting can yield symmetric or asymmetric stakes due, for example, to simple monetary division of the "grand prize" or due to a difference in the geographical location of the contested prize and of the contestants, or due to a difference in the time the contests are held and different time preferences of the contestants. To capture the effect of contest decomposition on the contestants' stakes, we introduce the instrument variable *I* that may affect one or two of the β_i's and examine how a change in *I* affects the efforts of the contestants. The index *I* may represent money, time or location as in the

[6] The functional forms of the CSF's commonly assumed in the literature, see for example Nitzan (1994a), satisfy these assumptions. We also assume that there is a unique pure strategy Nash equilibrium.

three different examples presented in the introduction. Let $\beta_i' = \frac{\partial \beta_i}{\partial I}$. Our subsequent analysis relates to the following five possible types of effects on the contestants' benefits from each of the contests:

Table 1: Types of changes in stakes

Type	β_i'	β_j'
(i)	>0	<0
(ii)	>0	=0
(iii)	=0	<0
(iv)	>0	>0
(v)	<0	<0

Types (ii) and (iii) mean that a change in *I* affects the stake of one contestant only. In the remaining types, a change in *I* affects the stakes of the two players not necessarily in the same direction. Type (i) means that a change in the instrument *I* has opposite effects on the two players. Under types (iv) and (v), a change in *I* has a similar positive or negative effect on the contestants.

III. Performance Invariance to Symmetric Decomposition

Let us start by examining the symmetric case. In the example presented in the introduction, this would capture the case of a straightforward "monetary decomposition", where the division results in identical stakes to both players. We now consider the relationship between the contestants' performance under the single grand (combined) contest where all agents compete for the sum of the prizes in the two contests, n_i, versus the case where they compete separately in the *two* sub-contests, as described above. Under the case of the overall contest, we obtain that the expected net payoff of contestant *i* is equal to,

(3) $$E^G(w_i) = \Pr_i n_i - x_i ,\ i = 1,2$$

where $\Pr_i$ is the probability of winning the contest and x_i is the total effort invested in this contest.

Consider the sub-class of the logistic contest success functions, Dixit (1987), $\Pr_i^k = \dfrac{\sigma_i g\left(x_i^k\right)}{\sigma_1 g\left(x_1^k\right) + \sigma_2 g\left(x_2^k\right)}$ $i = 1,2,\ k = 1,2$ where $\sigma_i > 0$, $g(0) \geq 0$, $g(t)$ increases in t[7] and $\dfrac{g(t_i)}{g(t_j)} = g\left(\dfrac{t_i}{t_j}\right)$. Clark and Riis (1998) show that $\dfrac{g(t_i)}{g(t_j)} = g\left(\dfrac{t_i}{t_j}\right)$ is equivalent to $g\left(t_i\right) = t_i^{\alpha}$.

Thus, the contest success function becomes:

(4) $$\Pr_i^k = \frac{\sigma_i \left(x_i^k\right)^{\alpha}}{\sigma_1 \left(x_1^k\right)^{\alpha} + \sigma_2 \left(x_2^k\right)^{\alpha}}, \quad \alpha \leq 2 \quad i = 1,2,\ k = 1,2$$

Solving the first order conditions for the maximization of (2), we obtain that in equilibrium the expenditure of players 1 and 2 in contest *k* is equal to (see Nti, 1999):

(5) $$x_1^{k*} = \frac{d\alpha\left(\beta_1^k n_1\right)^{\alpha+1}\left(\beta_2^k n_2\right)^{\alpha}}{\left(d\left(\beta_1^k n_1\right)^{\alpha} + \left(\beta_2^k n_2\right)^{\alpha}\right)^2} \quad and \quad x_2^{k*} = \frac{d\alpha\left(\beta_1^k n_1\right)^{\alpha}\left(\beta_2^k n_2\right)^{\alpha+1}}{\left(d\left(\beta_1^k n_1\right)^{\alpha} + \left(\beta_2^k n_2\right)^{\alpha}\right)^2}$$

for $d = \dfrac{\sigma_1}{\sigma_2}$

Thus the winning probabilities of agent 1 and 2 in contest k are equal to,

(6) $$\Pr_1^{k*} = \frac{d\left(\beta_1^k n_1\right)^{\alpha}}{d\left(\beta_1^k n_1\right)^{\alpha} + \left(\beta_2^k n_2\right)^{\alpha}} \quad and \quad \Pr_2^{k*} = \frac{\left(\beta_2^k n_1\right)^{\alpha}}{d\left(\beta_1^k n_1\right)^{\alpha} + \left(\beta_2^k n_2\right)^{\alpha}}$$

In the 'grand contest', the equilibrium expenditures and the winning probabilities are equal to:

$$x_1^* = \frac{d\alpha\left(n_1\right)^{\alpha+1}\left(n_2\right)^{\alpha}}{\left(d\left(n_1\right)^{\alpha} + \left(n_2\right)^{\alpha}\right)^2} \text{ and } x_2^* = \frac{d\alpha\left(n_1\right)^{\alpha}\left(n_2\right)^{\alpha+1}}{\left(d\left(n_1\right)^{\alpha} + \left(n_2\right)^{\alpha}\right)^2}$$

[7] In this special case, we retain the assumption that a contestant's marginal winning probability declines as his effort increases. This requires additional assumptions on the first and second derivatives of the function g(t).

(7) and

$$\Pr_1^* = \frac{d(n_1)^\alpha}{d(n_1)^\alpha + (n_2)^\alpha} \text{ and } \Pr_2^* = \frac{(n_1)^\alpha}{d(n_1)^\alpha + (n_2)^\alpha}$$

For the symmetric splitting of a grand contest (for every *k*, $\beta_1^k = \beta_2^k = \beta^k$), it is clear that:

(8) $$x_i^{k*} = \beta^k x_i^* \text{ and } \Pr_i^{k*} = \Pr_i^*$$

Using (2), (3) and (8) we obtain:

Proposition 1: Suppose that $\Pr_i^k = \frac{\sigma_i (x_i^k)^\alpha}{\sum_{i-1}^{2} \sigma_i (x_i^k)^\alpha}$, where $\alpha \leq 2 \, and \; \sigma_i > 0$. . If $\beta_1^k = \beta_2^k = \beta^k$, for all *k*=1,2, then

$$\text{(a) } \Pr_i^{k*} = \Pr_i^*, \quad \text{(b) } x_i^* = x_i^{1*} + x_i^{2*} \text{ and (c) } E^S(w_i^*) = E^S(w_i^*).$$

By part (a) of the Proposition, if the value of the prize in every sub-contest relative to the value of the grand prize is equal for all the contestants, $\beta_1^k = \beta_2^k = \beta^k$ for every *k*, then the weighted probability of winning the sub-contests is equal to the probability of winning the overall contest. Part (b) of the Proposition establishes that when $\beta_1^k = \beta_2^k = \beta^k$ for all *k*, the total effort incurred by the contestants is invariant with respect to additive contest decomposition. Part (c) of the Proposition implies that the expected payoff of the contestants is the same under the grand contest and its additive decomposition.

Note that $\frac{g(t_i)}{g(t_j)} = g\left(\frac{t_i}{t_j}\right)$ is equivalent to $g(t_i) = t_i^{\alpha}$ (see Clark and Riis, 1998), thus the CSF is the generalized Tullock function: $\Pr_i^k = \frac{\sigma_i \left(x_i^k\right)^{\alpha}}{\sum_{i-1}^{2} \sigma_i \left(x_i^k\right)^{\alpha}}$.[8] This means that under the commonly assumed exponential logistic contest success functions, a designer of contests cannot benefit (increase total efforts of the contestants) by contest decomposition. One should note that this result does not hold when (1) the CSF is an all pay auction (see, for example, Konrad ,2007, section 4.2, page 92) The contestants are non-identical (have different costs of efforts or different intrinsic valuations) . Note that there are several ways of decomposing contests, and we only consider one of them. Amegashie (1999) and Gradstein and Konrad (1999) show that, depending on the return parameter in Tullock function, a series of small sequential elimination contests leads to higher aggregate efforts than a grand contest. Konrad (2007) in his survey shows that splitting contests might dominate a grand contest. Moreover, Fu and Lu (2006) show that aggregate efforts are not invariant to contest decomposition. However, as noted above, they assume that the designer of the contest splits the group of contestants as well as the prize, while we assume that each contestant participates simultaneously in all sub-contests. All of this brings us to the next section where we consider the case of contest decomposition that yields asymmetry in the distribution of prizes in the sub-contests.

Note that in our analysis, the contestants are identical in their cost of investment in the contest. However, we assume that the contestants may have different stakes. Our assumption that the contestants may have different stakes captures the case of different costs of effort: Suppose that player 1's costs of exerting x_1 effort is λx_1 and player 2's cost of exerting the effort x_2 is x_2. In the grand contest the expected payoffs would be: $E^G(w_1) = \Pr_1 m_1 - \lambda x_1$ and $E^G(w_2) = \Pr_2 m_2 - x_2$. We can treat this contest as equivalent to a contest where player 1 has a valuation of $n_1 = \frac{m_1}{\lambda}$ for the prize and player 2 has a valuation of

[8] Clark and Riis (1998) show that the generalized Tullock probability functions are the only functions that satisfy both homogeneity of degree zero and independence of irrelevant alternatives. An alternative axiomatization of these contest success functions appears in Kooreman and Schoonbeek (1997).

$n_2 = m_2$. As we have shown above, in the symmetric decomposition the grand contest and the sub contests yield the same results.

Let us turn then to an extended setting with a general contest success function and asymmetric contest decomposition.

IV. Asymmetric (Discriminative) Distribution of Relative Prizes in the Sub-Contests

In this section we study how changes in the asymmetry in the stakes obtained by the players in each contest (β_i)[9] affect the contestants' expenditures. As stated above, the index *I* may change one or two of the β_i's and we examine how a change in *I* affects the efforts of the contestants. As noted in the three examples presented in the introduction, the index *I* may represent the symmetric effect of decomposition on the stakes under straightforward monetary decomposition or the asymmetric effect on the stakes, when the decomposition involves different timing of the two contests or different locations of the decomposed sub-prizes. A change in *I* may therefore split the stakes of the interest groups in the same way or in different ways. Moreover, it may affect the two players differently. It should be noted that several papers have shown that asymmetries in contests reduce aggregate efforts (for example, see Katz and Tokatlidu (1996), Che and Gale (1998), Baye, Kovenock and De Vries (1993) and Epstein and Nitzan (2006b, 2007)).

To consider the effect of such asymmetry in our framework, we consider below the different possibilities of the sign of $\beta_i' = \frac{\partial \beta_i}{\partial I}$, as presented in Table 1.

By our assumptions, both players participate in the contest $\left(x_i^k > 0 \quad \forall i,k\right)$. We therefore focus on the unique interior Nash equilibria of the contest. Solving the first order conditions $\left(\frac{\partial E\left(w_i^k\right)}{\partial x_i^k} = 0\right)$ we obtain that:

$$\Delta_i^1 = \beta_i \frac{\partial \Pr^1{}_i\left(x_i^1, x_j^1\right)}{\partial x_i^1} n_i - 1 = 0, \quad \forall i \neq j \quad and \quad i,j = 1,2$$

[9] Since we have two contests with two players we drop the superscript k from the β.

(9) and

$$\Delta_i^2 = (1-\beta_i)\frac{\partial \Pr^2{}_i\left(x_i{}^2, x_j{}^2\right)}{\partial x_i^2} n_i - 1 = 0, \quad \forall i \neq j \quad and \quad i, j = 1, 2$$

By differentiating the first order conditions, (9), we get that the Nash equilibrium efforts satisfy the following conditions:

(10)
$$\frac{\partial x_i^{*k}}{\partial I} = \frac{\frac{\partial \Delta_i^k}{\partial x_j^k}\frac{\partial \Delta_j^k}{\partial I} - \frac{\partial \Delta_j^k}{\partial x_j^k}\frac{\partial \Delta_i^k}{\partial I}}{\frac{\partial \Delta_i^k}{\partial x_i^k}\frac{\partial \Delta_j^k}{\partial x_j^k} - \frac{\partial \Delta_j^k}{\partial x_i^k}\frac{\partial \Delta_i^k}{\partial x_j^k}}, \quad i \neq j, \quad k = 1, 2$$

We therefore get that:

(11)
$$\frac{\partial x_i^{*1}}{\partial I} = \frac{\beta_i \frac{\partial^2 \Pr_i^1}{\partial x_i^1 \partial x_j^1}\frac{\partial \Pr_j^1}{\partial x_j^1}\frac{\partial \beta_j}{\partial I} - \beta_j \frac{\partial^2 \Pr_j^1}{\partial x_j^{12}}\frac{\partial \Pr_i^1}{\partial x_i^1}\frac{\partial \beta_i}{\partial I}}{\beta_i \beta_j \left(\frac{\partial^2 \Pr_j^1}{\partial x_j^{12}}\frac{\partial^2 \Pr_i^1}{\partial x_i^{12}} - \frac{\partial^2 \Pr_i^1}{\partial x_i^1 \partial x_j^1}\frac{\partial^2 \Pr_j^1}{\partial x_i^1 \partial x_j^1}\right)}, \quad i \neq j$$

and

(12)
$$\frac{\partial x_i^{*2}}{\partial I} = \frac{(1-\beta_i)\frac{\partial^2 \Pr_i^2}{\partial x_i^2 \partial x_j^2}\frac{\partial \Pr_j^2}{\partial x_j^2}\left(\frac{\partial (1-\beta_j)}{\partial I}\right) - (1-\beta_j)\frac{\partial^2 \Pr_j^2}{\partial x_j^{22}}\frac{\partial \Pr_i^2}{\partial x_i^2}\left(\frac{\partial (1-\beta_i)}{\partial I}\right)}{(1-\beta_i)(1-\beta_j)\left(\frac{\partial^2 \Pr_j^2}{\partial x_j^{22}}\frac{\partial^2 \Pr_i^2}{\partial x_i^{22}} - \frac{\partial^2 \Pr_i^2}{\partial x_i^2 \partial x_i^2}\frac{\partial^2 \Pr_j^2}{\partial x_i^2 \partial x_j^2}\right)}, \quad i \neq j$$

Let us denote the elasticities of the contestants' stakes with respect to I by $\varepsilon_i^1 = \frac{I}{\beta_i}\frac{\partial \beta_i}{\partial I}$ and $\varepsilon_i^2 = \frac{I}{(1-\beta_i)}\frac{\partial (1-\beta_i)}{\partial I}$. Rewriting (11) and (12), taking advantage of (9), yields the fundamental equations that generate all subsequent comparative statics results: [10]

[10] Such comparative statics is similar to the analysis carried out in Epstein and Nitzan (2006b) that focuses on the effect of a change in public policy that, in turn, changes the stakes of the contestants on their effort and performance.

(13)
$$\frac{\partial x_i^{*1}}{\partial I}=\frac{1}{B^1}\left(\frac{\partial^2 \Pr_i^1}{\partial x_i^1\partial x_j^1}\frac{\varepsilon_j^1\beta_i}{n_j}-\frac{\partial^2 \Pr_j^1}{\partial x_j^{12}}\frac{\varepsilon_i^1\beta_j}{n_i}\right),\quad i\neq j$$

and

(14)
$$\frac{\partial x_i^{*2}}{\partial I}=\frac{1}{B^2}\left(\frac{\partial^2 \Pr_i^2}{\partial x_i^2\partial x_j^2}\frac{\varepsilon_j^2\left(1-\beta_i\right)}{n_j}-\frac{\partial^2 \Pr_j^2}{\partial x_j^{22}}\frac{\varepsilon_i^2\left(1-\beta_j\right)}{n_i}\right),\quad i\neq j$$

where

$$B^1=I\beta_i\beta_j\left(\frac{\partial^2 \Pr_j^1}{\partial x_j^{12}}\frac{\partial^2 \Pr_i^1}{\partial x_i^{12}}-\frac{\partial^2 \Pr_i^1}{\partial x_i^1\partial x_j^2}\frac{\partial^2 \Pr_j^1}{\partial x^1{}_i\partial x_j^1}\right)$$

$$B^2=I\left(1-\beta_i\right)\left(1-\beta_j\right)\left(\frac{\partial^2 \Pr_j^2}{\partial x_j^{22}}\frac{\partial^2 \Pr_i^2}{\partial x_i^{22}}-\frac{\partial^2 \Pr_i^2}{\partial x_i^2\partial x_j^2}\frac{\partial^2 \Pr_j^2}{\partial x_i^2\partial x_j^2}\right)$$

Suppose that all second-order partial derivatives are computed at the Nash equilibrium (x_1^{*k},x_2^k), k=1,2. The first term within the brackets of (13) and (14) represents the strategic rival's-stake effect. The second term represents the own-stake effect. The sign of this latter term is equal to the sign of ε_i^k since $\frac{\partial^2 \Pr_i^k\left(x_i^k,x_j^k\right)}{\partial x_i^{k2}}<0$ and since $\Pr_i^k\left(x_i^k,x_j^k\right)+\Pr_j^k\left(x_i^k,x_j^k\right)=1,\quad i\neq j$ $\frac{\partial^2 \Pr_i\left(x_i,x_j\right)}{\partial x_i\partial x_j}\cdot\frac{\partial^2 \Pr_j\left(x_j,x_i\right)}{\partial x_i\partial x_j}<0$, hence, $B^i>0$.

A. ***Results***

The above fundamental equations yield different types of results that can be classified into two categories; results that are obtained in cases where the stake of a single contestant changes and results that are obtained in cases where the stakes of the two contestants change (see the five types of changes in stakes that appear in Table 1).

A.1 The effect of a change in the stake of a single contestant

When a change in *I* affects one of the contestants, as in cases (ii) and (iii), we obtain:

Lemma 1: In case (ii) with $i=1$ and case (iii) with $i=2$,

a. $\frac{\partial x_1^{*1}}{\partial \beta_1} > 0, \quad \frac{\partial x_1^{*2}}{\partial \beta_1} < 0$

b. $Sign\left(\frac{\partial x_2^{*1}}{\partial \beta_1}\right) = Sign\left(\frac{\partial^2 \Pr_2^1}{\partial x_1^1 \partial x_2^1}\right)$

c. $Sign\left(\frac{\partial x_2^{*2}}{\partial \beta_1}\right) = -Sign\left(\frac{\partial^2 \Pr_2^2}{\partial x_1^2 \partial x_2^2}\right)$

In cases (iii) with $i=1$ and case (ii) with $i=2$,

d. $\frac{\partial x_2^{*1}}{\partial \beta_2} > 0, \quad \frac{\partial x_2^{*2}}{\partial \beta_2} < 0$

e. $Sign\left(\frac{\partial x^{*1}_1}{\partial \beta_2}\right) = Sign\left(\frac{\partial^2 \Pr^1_1}{\partial x^1_1 \partial x^1_2}\right)$

f. $Sign\left(\frac{\partial x_1^{*2}}{\partial \beta_2}\right) = -Sign\left(\frac{\partial^2 \Pr_1^2}{\partial x_1^2 \partial x_2^2}\right)$

Proof: See Appendix.

The general comparative statics result stated in Lemma 1 focuses on the sensitivity of a contestant's effort to a change in his or his rival's stake. By this Lemma, under our general contest success function, the effort exerted by a contestant is positively related to his own stake. That is, the strategic own-stake ("income") effect is always positive (the effort of every player is a "normal good"). In contrast, the effort exerted by a player can be positively or negatively related to the stake of his rival. It can also be independent of the rival's stake. When the marginal winning probability of a contestant in equilibrium is positively (negatively) related to his rival's effort, his strategic substitution effect is positive (negative). Following Bulow, Geanakoplos and Klemprer (1985), in such a case we can say that a contestant's effort is a strategic complement (substitute) to his rival's effort. When the cross-partial derivative of the contest success function is equal to zero, the contestants' efforts are independent. Note that in our setting the strategic substitution effects are asymmetric; if a player's effort is a strategic complement to his opponent's effort, then his opponent's effort is a strategic substitute to his effort.

In the symmetric case, where $\forall x_i^k$, $k = 1,2$, $\Pr_1^k(x_1^k, x_2^k) = 1 - \Pr_1^k(x_2^k, x_1^k)$, assume, without loss of generality, that $x_1^{*k} > x_2^{*k}$. Then there exists a pure strategy Nash equilibrium where $\frac{\partial^2 \Pr_1^k}{\partial x_1^k \partial x_2^k} > 0$, which implies that $\frac{\partial^2 \Pr_2^k}{\partial x_2^k \partial x_1^k} < 0$. Hence,

Corollary 1.1: In cases (ii) and (iii), if $\forall x_1^k, x_2^k$, $\Pr_1^k(x_1^k, x_2^k) = 1 - \Pr_1^k(x_2^k, x_1^k)$ and $x_1^{*k} > x_2^{*k}$, then

a. For changes in β_1:

$$\frac{\partial x_1^{*1}}{\partial \beta_1} > 0, \quad \frac{\partial x_1^{*2}}{\partial \beta_1} < 0, \quad \frac{\partial x_2^{*1}}{\partial \beta_1} < 0, \quad \frac{\partial x_2^{*2}}{\partial \beta_1} > 0$$

b. For changes in β_2:

$$\frac{\partial x_1^{*1}}{\partial \beta_2} > 0, \quad \frac{\partial x_1^{*2}}{\partial \beta_2} < 0, \quad \frac{\partial \ \ x_2^{*1}}{\partial \beta_2} > 0, \quad \frac{\partial \ \ x_2^{*2}}{\partial \beta_2} < 0$$

This corollary clarifies that when the contest success function is symmetric, a change in the stake of contestant *i* results in unequivocal changes in the efforts that he and his opponent direct to the two decomposed sub-contests.

Consider, again, the particular asymmetric form of the logit contest success function of the preceding section, where $\Pr_i^k = \frac{\sigma g(x_i^k)}{\sigma g(x_1^k) + g(x_2^k)}$, $\sigma > 0$, $g(0) \geq 0$ and $g(t)$ increases with *t*. [11] Here the parameter σ represents the asymmetry between the abilities of the two contestants. Note that when σ<1, player 1 has an ability disadvantage relative to player 2. It can therefore be shown that under this particular contest success function, $Sign\left(\frac{\partial^2 \Pr_2^k}{\partial x_2^k \partial x_1^k}\right) = Sign\left(\Pr_2^k - \Pr_1^k\right)$ and, for some σ *<1,

[11] We no longer require that $\frac{g(t_1)}{g(t_2)} = g(\frac{t_1}{t_2})$.

$\Pr_1^k = \Pr_2^k = \frac{1}{2}$ *and* $\frac{\partial^2 \Pr_2^k}{\partial x_2^k \partial x_1^k} = \frac{\partial^2 \Pr_1^k}{\partial x_2^k \partial x_1^k} = 0$. By lemma 1, we get:

Corollary 1.2: In cases (ii) and (iii), if $\Pr_1^k = \frac{\sigma h\left(x_1^k\right)}{\sigma h\left(x_1^k\right) + h\left(x_2^k\right)}$, where $\sigma > 0$, $h(0) \geq 0$ and *h(t)* increases with *t*, then

a. $\frac{\partial x_1^{*1}}{\partial \beta_1} > 0$, $\frac{\partial x_1^{*2}}{\partial \beta_1} < 0$, $\frac{\partial x_2^{*1}}{\partial \beta_2} > 0$, $\frac{\partial x_2^{*2}}{\partial \beta_2} < 0$.

and

b. $\frac{\partial x_2^{*1}}{\partial \beta_1} > 0 \Leftrightarrow \frac{\partial x_1^{*1}}{\partial \beta_2} < 0 \Leftrightarrow \Pr_2^1 > \frac{1}{2}$ and $\frac{\partial x_2^{*2}}{\partial \beta_1} < 0 \Leftrightarrow \frac{\partial x_1^{*2}}{\partial \beta_2} > 0 \Leftrightarrow \Pr_2^1 > \frac{1}{2}$.

As in the previous corollary, Corollary 1.2 establishes that when the contest success function is of a more particular form, the particular asymmetric form of the logit contest success function of the preceding section, a change in the stake of contestant *i* results in definite changes in the efforts that he and his opponent direct to the two decomposed sub-contests. Let $X^{*k} = x_1^{*k} + x_2^{*k}$. Then, we directly obtain

Corollary 1.3: In case (ii) with $i = 1$ and case (iii) with $i = 2$,

$$\frac{\partial\left(X^{*1}\right)}{\partial \beta_i} = \frac{\partial\left(x_i^{*1} + x_j^{*1}\right)}{\partial \beta_i} = \frac{1}{B^1} \frac{\varepsilon_i^1 \beta_j}{n_i} \left[\frac{\partial^2 \Pr_j^1}{\partial x_1^1 \partial x_2^1} - \frac{\partial^2 \Pr_j^1}{\partial x_j^{1\,2}} \right] \text{and}$$

$$\frac{\partial\left(X^{*2}\right)}{\partial \beta_i} = \frac{\partial\left(x_i^{*2} + x_i^{*2}\right)}{\partial \beta_i} = \frac{1}{B^2} \frac{\varepsilon_i^2 \left(1 - \beta_j\right)}{n_i} \left[\frac{\partial^2 \Pr_j^2}{\partial x_1^2 \partial x_2^2} - \frac{\partial^2 \Pr_j^2}{\partial x_j^{2\,2}} \right]$$

As we can see, the effect of a change in the stakes on the contestants' total efforts depends on the relationship between $\frac{\partial^2 \Pr_j^1}{\partial x_1^1 \partial x_2^1}$ *and* $\frac{\partial^2 \Pr_j^1}{\partial x_j^{1\,2}}$ and on the relationship between $\frac{\partial^2 \Pr_j^2}{\partial x_1^2 \partial x_2^2}$ *and* $\frac{\partial^2 \Pr_j^2}{\partial x_j^{2\,2}}$. We will return to this issue in Proposition 2 when we consider the general case of a simultaneous change in the contestants' stakes.

Let $X^* = x_1^{*1} + x_1^{*2} + x_2^{*1} + x_2^{*2}$.

Corollary 1.4: In case (ii) with $i = 1$ and case (iii) with $i = 2$, if $\Pr_i = \frac{x_i}{x_i + x_j}$, that is, the contest success function is the special case of Tullock's lottery logit function, then for if $\beta_1 = \beta_2$, $\frac{\partial X^*}{\partial \beta_1} = 0$ and for $\beta_2 \leq 0.5$

a. if $\beta_1 > \beta_2$, then $\frac{\partial X^*}{\partial \beta_1} < 0$

b. if $\beta_1 < \beta_2$, then $\frac{\partial X^*}{\partial \beta_1} > 0$

This means that, with the lottery logit function, total efforts may be minimal when the contest designer creates an asymmetric contest between the contestants. That is, if the contest designer wishes to decreases effort invested in the contest, he/she should create asymmetric sub-contests. It follows directly that total effort is maximized when $\beta_1 = \beta_2$. That is, the designer of the contests does not have an incentive to create differential (asymmetric) sub-contests because the design of a discriminating system of sub-contests reduces the total efforts of the contestants.

A.2. The effect of a change in the stakes of the two contestants

When a change in *I* affects the stakes of both contestants, as in cases (i), (iv) and (v) of Table 1, the elasticities ε_i^k and ε_j^k are positive or negative. By the fundamental equations (13) and (14), when the contestants' efforts are independent, the sensitivity of every contestant's effort with respect to a change in stakes is always unequivocal. When the contestants' efforts are not independent, the sensitivity of one of the contestants' efforts with respect to such a change is also always unequivocal because the sign of his strategic rival's-stake ("substitution") effect is equal to the sign of his strategic own-stake ("income") effect. The sensitivity of his opponent's effort with respect to the change in stakes is ambiguous, depending on whether his strategic own-stake ("income") effect is larger than, equal to or smaller than his strategic rival's-stake ("substitution") effect. Using (13) and (14), see A1 and A2 in the Appendix, we thus get

(15) $$\frac{\partial x_i^{*1}}{\partial I}=\frac{1}{B^1}\left(\frac{\partial^2 \Pr_i^1}{\partial x_i^1 \partial x_j^1}\frac{\varepsilon_j^1\beta_i}{n_j}-\frac{\partial^2 \Pr_j^1}{\partial x_j^{12}}\frac{\varepsilon_i^1\beta_j}{n_i}\right),\quad i\neq j$$

and

(16) $$\frac{\partial x_i^{*2}}{\partial I}=\frac{1}{B^2}\left(\frac{\partial^2 \Pr_i^2}{\partial x_i^2 \partial x_j^2}\frac{\varepsilon_j^2(1-\beta_i)}{n_j}-\frac{\partial^2 \Pr_j^2}{\partial x_j^{22}}\frac{\varepsilon_i^2(1-\beta_j)}{n_i}\right),\quad i\neq j$$

By (15) and (16), we directly obtain:

Lemma 2:

In cases (i), (iv) and (v),

a. If $\frac{\partial^2 \Pr_i^k}{\partial x_i^k \partial x_j^k}=0$, then $\frac{\partial x_i^{*1}}{\partial \beta_i}>0$ *and* $\frac{\partial x_i^{*2}}{\partial \beta_i}<0$

b. If $\frac{\partial^2 \Pr_i^k}{\partial x_i^k \partial x_j^k}\neq 0$,

(17) $$\frac{\partial x_i^{*1}}{\partial \beta_i}>0 \Leftrightarrow \frac{\partial^2 \Pr_i^1}{\partial x_i^1 \partial x_j^1}\varepsilon_j^1>0$$ and

$$\frac{\partial x_i^{*2}}{\partial \beta_i}<0 \Leftrightarrow \frac{\partial^2 \Pr_i^2}{\partial x_i^2 \partial x_j^2}\varepsilon_j^2<0$$

(18) $$\frac{\partial x_i^{*1}}{\partial \beta_i}\,{}^{>}_{<}\,0 \Leftrightarrow \alpha_{i,j}^1\frac{\varepsilon_j^1}{\varepsilon_i^1}\,{}^{<}_{>}\,\frac{\beta_j n_j}{\beta_i n_i};\quad \frac{\partial x^{*1}{}_i}{\partial \beta_j}\,{}^{>}_{<}\,0 \Leftrightarrow \alpha_{i,j}^1\,{}^{<}_{>}\,\frac{\varepsilon_i^1}{\varepsilon_j^1}\frac{\beta_j n_j}{\beta_i n_i}$$

$$\frac{\partial x_i^{*2}}{\partial \beta_i}\,{}^{>}_{<}\,0 \Leftrightarrow \alpha_{i,j}^2\frac{\varepsilon_j^2}{\varepsilon_i^2}\,{}^{>}_{<}\,\frac{(1-\beta_j)n_j}{(1-\beta_i)n_u};\quad \frac{\partial x_i^{*2}}{\partial \beta_j}\,{}^{>}_{<}\,0 \Leftrightarrow \alpha_{i,j}^2\,{}^{>}_{<}\,\frac{\varepsilon_i^2}{\varepsilon_j^2}\frac{(1-\beta_j)n_j}{(1-\beta_i)n_i}$$

where $\alpha_{i,j}^k=\frac{\partial^2 \Pr_i^k}{\partial x^k{}_i \partial x_j^k}\bigg/\left(\frac{\partial^2 \Pr_j^k}{\partial x^k{}_j{}^2}\right)$ is a local measure of the asymmetry between the abilities of i and j.

Lemma 2a states that if the contestants are symmetric in equilibrium in terms of their abilities, then the strategic rival's-stake ("substitution") effects vanish (efforts are independent) and the positive strategic own-stake ("income") effect solely determines the direct effect of a change in the stake decomposition (splitting) on a contestant's effort. In the perfectly symmetric case, where $\forall x_i^k, \mathrm{x}_\mathrm{j}^\mathrm{k}, \mathrm{k}=1,2,$ $\Pr_i^k(x_i^k, x_j^k) = 1 - \Pr_i^k(x_j^k, x_i^k)$ and $n_1 = n_2 = n$, there exists a symmetric pure strategy Nash equilibrium, $x_i^{*k} = x_j^{*k}$, and $\frac{\partial^2 \Pr_i^{*k}}{\partial x_j^k \partial x_l^k} = \frac{\partial^2 \Pr_j^{*k}}{\partial x_j^k \partial x_l^k} = 0$ [12]. Hence, by Lemma 2a**,** **w**hen both contestants' stakes change; namely, in cases (i), (iv) and (v), if $\forall x_i^k, x_j^k$, $n_j = n_i$ and $\beta_j = \beta_i = \beta$, then

$$\frac{\partial x_i^{*1}}{\partial \beta} = \frac{\partial x_j^{*1}}{\partial \beta} > 0 \text{ and } \frac{\partial x_i^{*2}}{\partial \beta} = \frac{\partial x_j^{*2}}{\partial \beta} < 0$$

Note that Lemma 3b determines the sensitivity of the contestants' efforts in all possible situations corresponding to the three types of changes affecting both of their stakes when $\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j} \neq 0$.

Finally, let us consider how changes in stakes affect the total amount of efforts directed to the different sub-contests by the two contestants. Let $X^{*k} = x_1^{*k} + x_2^{*k}$.

Proposition 2:

$$\frac{\partial X^{*1}}{\partial I} \underset{<}{\overset{>}{}} 0 \Leftrightarrow \frac{\partial^2 \Pr_1^1}{\partial x_2^1 \partial x_1^1}\left(\varepsilon_2^1 \beta_1 n_1 - \varepsilon_1^1 \beta_2 n_2\right) \underset{<}{\overset{>}{}} \frac{\partial^2 \Pr_1^1}{\partial x_1^1} \varepsilon_2^1 \beta_1 n_1 + \frac{\partial^2 \Pr_2^1}{\partial x_2^1} \varepsilon_1^1 \beta_2 n_2$$

and

$$\frac{\partial X^{*2}}{\partial I} \underset{<}{\overset{>}{}} 0 \Leftrightarrow \frac{\partial^2 \Pr_1^2}{\partial x_2^2 \partial x_1^2}\left(\varepsilon_2^2 \left(1-\beta_1\right) n_1 - \varepsilon_1^2 \left(1-\beta_2\right) n_2\right) \underset{<}{\overset{>}{}} \frac{\partial^2 \Pr_1^2}{\partial x_1^2} \varepsilon_2^2 \left(1-\beta_1\right) n_1 + \frac{\partial^2 \Pr_2^2}{\partial x_2^2} \varepsilon_1^2 \left(1-\beta_2\right) n_2$$

Proof: See appendix.

12 See Dixit (1987).

As we can see, the total expenditure in the contest is a function of many parameters. It is a function of the local measure of the asymmetry between the abilities of i and j and of the elasticities of the contestants' stakes with respect to *I.* To better understand the significance of this proposition, let us consider some of its specific implications in the different corollaries presented below:

Corollary 2.1

An increase in β_1 and β_2 results in an increase in X^{*1} if

$$\frac{\partial^2 \Pr_1^1}{\partial x_2^1 \partial x_1^1} > 0 \quad and \quad \frac{\varepsilon_2^2}{\varepsilon_1^1} > \frac{\beta_2 n_2}{\beta_1 n_1}.$$

Notice, however, that an increase in both β_1 and β_2 may result in a decrease in X^{*1} and yet, possibly in an increase or a decrease in X^{*2}.

Corollary 2.2

For $\varepsilon_2^1 = 0$, $\frac{\partial X^{*1}}{\partial \beta_1} \gtrless 0 \Leftrightarrow \alpha_{1,2}^1 \gtrless -1$ and, therefore,

$$\frac{\partial X^{*1}}{\partial \beta_1} > 0 \Leftrightarrow \frac{\partial^2 \Pr_1^1}{\partial x_2^1 \partial x_1^1} < 0 \text{ and } \frac{\partial X^{*2}}{\partial \beta_1} \gtrless 0 \Leftrightarrow \alpha_{1,2}^1 \lessgtr -1.$$

We conclude by noting that an increase only in β_1 may decrease total expenditure in the contest. A sufficient condition for this possibility is that $\alpha_{1,2}^1 < -1$ *and* $\alpha_{1,2}^2 > -1$. The results thus hinge on $\alpha_{i,j}^k$; the local measure of asymmetry between the abilities of contestants i and j.

Corollary 2.3

If $n_1 = n_2$, $\beta_1 = \beta_2 = \beta$ and $\varepsilon_1 = \varepsilon_2$ (the stakes of the contestants change in the same direction), then $\frac{\partial X^{*1}}{\partial \beta} > 0$ and $\frac{\partial X^{*2}}{\partial \beta} < 0$

By this Corollary, even though in the general case the effect of an increase in stake on the total effort of the contestants is ambiguous; here, when the players are fully symmetric, the total efforts directed to one of the contests is increased. If the CSF satisfies the requirement of Proposition 1, then, since the total effort X is equal to the effort in the case where there is just one contest, *X* is a constant, which implies that $\frac{\partial X^{*1}}{\partial \beta} = -\frac{\partial X^{*2}}{\partial \beta}$.

Corollary 2.4

If $n_1 = n_2,\ \beta_1 = \beta_2\ and\ 0 < \varepsilon_1 = -\varepsilon_2$ (the percentage change in the stakes of the contestants is equal; however, the changes are in opposite directions), then

$$\frac{\partial X^{*1}}{\partial \beta_1} \underset{<}{\overset{>}{}} 0 \Leftrightarrow 2\frac{\partial^2 \Pr_1^1}{\partial x_2^1 \partial x_1^1} \underset{>}{\overset{<}{}} \left(\frac{\partial^2 \Pr_1^1}{\partial x_1^1} - \frac{\partial^2 \Pr_2^1}{\partial x_2^1} \right)$$

and

$$\frac{\partial X^{*2}}{\partial \beta_1} \underset{<}{\overset{>}{}} 0 \Leftrightarrow 2\frac{\partial^2 \Pr_1^2}{\partial x_2^2 \partial x_1^2} \underset{<}{\overset{>}{}} \left(\frac{\partial^2 \Pr_1^2}{\partial x_1^2} - \frac{\partial^2 \Pr_2^2}{\partial x_2^2} \right)$$

Using this Corollary, we may conclude that in the symmetric case where $\frac{\partial^2 \Pr_1^1}{\partial x_2^1 \partial x_1^1} = 0$, the sign of the affect of a change in β_1 on the total outlays hinges on the relationship between $\frac{\partial^2 \Pr_2^1}{\partial {x_2}^1}\ and\ \frac{\partial^2 \Pr_1^1}{\partial {x_1}^1}$ and on the relationship between $\frac{\partial^2 \Pr^2{}_2}{\partial {x_2}^2}\ and\ \left(\frac{\partial^2 \Pr^2{}_1}{\partial {x_1}^2} \right)$. Specifically,

Proposition 3

If $\frac{\partial^2 \Pr_1^1}{\partial x_2^1 \partial x_1^1} = 0$, $\frac{\partial^2 \Pr_2^1}{\partial {x_2}^1} \underset{>}{\overset{<}{}} \frac{\partial^2 \Pr_1^1}{\partial {x_1}^1}$ and $\frac{\partial^2 \Pr^2{}_2}{\partial {x_2}^2} \underset{<}{\overset{>}{}} \left(\frac{\partial^2 \Pr^2{}_1}{\partial {x_1}^2} \right)$ then $\frac{\partial \left(X^{*1} + X^{*2} \right)}{\partial \beta_1} \underset{<}{\overset{>}{}} 0$

By this last proposition, the effect of a change in the stake of one of the players on the total amount of resources invested in the contests may increase or decrease. In light of the ambiguity of contest decomposition on the total outlays invested by the contestants, it may well be optimal to split a grand contest both when the objective is to minimize or to maximize the contest efforts.

V. Conclusion

We have studied contest performance under symmetric and asymmetric contest decomposition that preserves the contest success function and the group of contestants, allowing different additive net prizes in the sub-contests. Such contests are common in open, nationwide lobbying games, political contests or R&D races, where the designer often splits a grand prize (budget), however, the potential contestants voluntarily decide how to allocate their efforts between the sub-contests.

Our first result states sufficient conditions for the invariance of contest performance to such contest decomposition and showed that these conditions are satisfied by the commonly used exponential logistic contest success functions; the only logistic functions that are homogeneous of degree zero. With these functions a contest designer, who is interested in maximizing the contestants' total efforts, does not have an incentive to split the prize and create symmetric and additive segmented sub-contests.

In the second part of the paper, we consider the case of asymmetric (discriminative) distribution of relative prizes in the sub-contests. The focus in this part is on the effect of changes in stake asymmetry on the efforts invested by each of the contestants and on the total effort invested in the sub-contests. It has been shown that contest decomposition may adversely affect the designer; that it, reduce the total efforts of the contestants, when the additive segmented sub-contests are asymmetric; that is, when the distribution of the relative values of the prizes vary across the contestants.

Finally note that further research is needed to take into account additional considerations that were disregarded in our analysis. In particular, one could explore the effect of contest decomposition on performance when one allows increasing returns to scale, risk-averse contestants or splitting the prize as well as the group of contestants in the grand contest.

VI. Appendix

Proof of Lemma 1: When a change in *I* only affects one of the contestants, as in cases (ii) and (iii), ε_i or ε_j is equal to zero and (13) and (14) reduce to

(A1) $$\frac{\partial x^{*1}{}_i}{\partial I} = \frac{1}{B^1}\frac{\partial^2 \Pr_i^1}{\partial x_i^1 \partial x_j^1}\frac{\varepsilon_j^1 \beta_i}{n_j}, \quad or \quad \frac{\partial x^{*1}{}_i}{\partial I} = -\frac{1}{B^1}\frac{\partial^2 \Pr_j^1}{\partial x_j^{12}}\frac{\varepsilon_i^1 \beta_j}{n_i}, \quad i \neq j$$

and

(A2) $$\frac{\partial x^{*2}{}_i}{\partial I} = \frac{1}{B^2}\frac{\partial^2 \Pr_i^2}{\partial x_i^2 \partial x_j^2}\frac{\varepsilon_j^2 \left(1-\beta_i\right)}{n_j} \quad or \quad \frac{\partial x^{*2}{}_i}{\partial I} = -\frac{1}{B^2}\frac{\partial^2 \Pr_j^2}{\partial x_j^{22}}\frac{\varepsilon_j^2 \left(1- \beta_j\right)}{n_i}, \quad i \neq j$$

In these cases the change in player *i*'s effort, corresponding to the change in β_j, is equal to the strategic rival's ("substitution") effect, when $i \neq j$, or to the strategic own ("income") effect, when $i = j$. The former effect is ambiguous, depending on the sign of the cross-partial derivative of the contest success function. The latter effect is clear-cut, due to our assumption that the marginal winning probability of a contestant is declines in his effort.

By (A1) and (A2), we can conclude that in case (ii) with *i=1* and case (iii) with $i = 2$, $Sign\frac{\partial x_1^{*k}}{\partial I} = Sign\left(\varepsilon_1^k\right)$ and

$Sign\left(\frac{\partial x_2^{*k}}{\partial I}\right) = Sign\left(\frac{\partial^2 \Pr_2^k}{\partial x_1^k \partial x_2^k}\varepsilon_1^k\right)$ and in case (iii) with $i = 1$ and case (ii) with $i = 2$, $Sign\left(\frac{\partial x_2^{*k}}{\partial I}\right) = Sign\left(\varepsilon_2^k\right)$ and

$$Sign\left(\frac{\partial x_1^{*k}}{\partial I}\right) = Sign\left(\frac{\partial^2 \Pr_1^k}{\partial x_2^k \partial x_1^k}\varepsilon_2^k\right).$$ ***QED***

Proof of Proposition 2: Follows from (A1) and (A2).

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