

Increasing Sales by Introducing Non-Salable Items

by

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Abstract

Rationality implies that adding “irrelevant” and, in particular, inferior alternatives to the opportunity set cannot increase the choice probability of some other alternative. In this study we propose a novel approach that can rationalize an intended addition of such alternatives because it strictly increases the choice probability of some existing alternative. The driving force behind the existence and extent of such an increase is the random nature of individual preferences, that implies intransitivity, and the random nature of the applied choice procedures. We study the case of a firm interested in increasing the sales of some of its existing products by introducing a new and inferior (non-salable) product. Our main results focus on the feasibility and potential advantage of a successful such strategy. We first establish necessary and sufficient conditions for an increase in the sale probability and then derive the maximal possible absolute and relative increase in this probability, when the firm has extremely limited information on the characteristics of the consumers. We then derive analogous results, assuming that the existing line of products consists of just two items and that the firm has accurate information on the consumers' stochastic preferences over the existing products. These later results are illustrated using some experimental evidence. The applicability of the approach is finally briefly discussed in the context of branding policy.

Keywords: probabilistic scanning, stochastic preferences, asymmetrically dominated-alternatives effect (ADE), branding, sale probability.

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1. Introduction

Rationality in the standard economic sense implies that adding “irrelevant” and, in particular, inferior alternatives to the opportunity set of an individual cannot increase the choice probability of some other alternative¹. This means that a firm that wishes to increase the sale probability of some of its existing products cannot achieve this goal by introducing a new inferior product. Yet, inferior products that were never meant to be sold commonly appear in brochures, Internet sites, shop windows and firm product lines². The objective of this paper is to propose a model that rationalizes this versioning strategy and to evaluate its potential advantage. More specifically, we first wish to identify the characteristics of new inferior products the introduction of which strictly increases the sale probability of some other existing product and then compute the maximal possible absolute and relative increase in the sale probability of the targeted (dominant) product.

The proposed model has several basic features. First, in our setting, the identity of alternatives and their characteristics are not known in advance, like when choosing a book from among a pile of books or a vacation from a thick tourist brochure. In such situations the individual typically scans the opportunity set, De Palma, Myers and Papageoriou (1994), Osborne & Rubinstein (1998), Rubinstein (1998). The consumer is assumed to apply a particular choice procedure, namely an *Elimination Scan* where alternatives are scanned in the following manner: first, two alternatives are compared and the preferred one is compared with a new third alternative; the preferred alternative at this second stage is then compared with a new fourth alternative, and so

¹ This requirement is usually called independence of irrelevant alternatives, **IIA** - if an alternative x is considered the ‘best’ choice in some set X , and x belongs to some subset of X , then x would also be the ‘best’ choice in the subset. The stochastic counterpart of this requirement is the **Regularity principle** - if $x \in A \subset X$, and $P(x:A)$ is the probability that x is chosen from A , then $P(x:A) \geq P(x:X)$. That is, the probability that an alternative is chosen from a set cannot be greater than the probability that it is chosen from any of its subsets.

² Alternative explanations for the “Asymmetrically Dominated alternatives Effect” (ADE), where adding an alternative dominated by one alternative but not by others, strictly increases the selection probability of the dominant alternative, have been proposed in Simonson & Tversky (1992), Baharad & Nizan (2000), Bandyopadhyay, Dasgupta & Pattanaik (1999), Fischhoff (1991), Hoeffler & Ariely (1999), Sen (1993), (1997) Simon (1955), (1990), Slovic, Griffin & Tversky (1990) and Tversky and Simonson (1993).

on. The outcome of this scan is the preferred alternative in the last stage (comparison). The justification of the focus on this scan is based on three of its attributes:

(i) An elimination scan yields an acceptable choice; that is, when preferences are transitive, the outcome of the scan is the best alternative. When preferences are intransitive and there is no top cycle, the outcome of this scan is the best alternative and when preferences are intransitive and there is a top cycle, the outcome of this scan belongs to the top cycle.

(ii) An elimination scan is efficient in the sense that it results in an acceptable outcome applying the minimal number of comparisons.

(iii) An elimination scan requires minimal memory, in fact; only one memory "cell" is needed in order to reach the scan outcome.

Secondly, a company that produces n products does not know the consumers' preferences and the order in which they scan the opportunity set. It is assumed that both preferences and the order of scanning are stochastic³. In particular, each preference relation and every order is equally probable. For $n=2$, we also allow the company to have precise information on the actual distribution of the stochastic preferences of the consumers over the existing alternatives. When preferences are fixed and transitive, elimination scans always yield the same (best) alternative, regardless of the order the alternatives are scanned. When preferences and the scanning order are random, some preferences are intransitive and therefore different elimination scans can yield different outcomes. The assumptions regarding the random nature of the consumers' preferences and the random order of scanning are therefore the basic driving forces behind the possible existence of the company's incentive to add new inferior products.

Thirdly, even without detailed knowledge about the consumer preferences, a new asymmetrically dominated alternative (an alternative dominated by one alternative but not by the others) can be introduced. For example, consider a travel agent who offers similar vacations in various European capitals. Without detailed knowledge about the consumers' preferences, the travel agent can only assume that a

³ Various studies of stochastic preferences show (Starmer & Sugden (1989), Camerer (1989), Hey & Orme (1994), Ballinger & Wilcox (1997) and Loomes Moffatt & Sugden (2002)) that even in a controlled environment, where individuals are asked to choose from the same opportunity set, 20% to 30% of the individuals change their choice from time to time.

‘vacation in London’ is equally probable to be preferred to a ‘vacation in Paris’. Now suppose that the travel agent introduces an ‘inferior vacation in London’, a vacation which is identical to the ‘vacation in London’ except that the hotel room has no bath. It is reasonable to assume that the probability that the ‘vacation in London’ is preferred to the ‘inferior vacation in London’ is higher than the probability that a ‘vacation in Paris’ is preferred to the ‘inferior vacation in London’.⁴ For simplicity, in the general multi-alternative case, we assume that the probability that any alternative (like a vacation in Paris) other than the dominant alternative (like a vacation in London) has the same probability to be preferred to the inferior added alternative.

Under the above assumptions, we first establish the necessary and sufficient condition in our setting for an increase in the sale probability of some good due to the introduction of some inferior (non-salable) item. This condition is first stated in terms of the relationship between the three parameters of the model, viz., the number of the existing alternatives, the probability that the dominant alternative is preferred to the new dominated one and the probability that the other alternatives are preferred to the dominated alternative. We then assess the potential advantage of the above versioning strategy by computing the maximal possible increase in the sale probability of the dominant product. This information is clearly critical for actual decision making, e.g., for determining the appropriate branding or marketing policy of the company. It is also essential for evaluating the validity of our proposed theory in light of the findings in the relevant experimental literature. Since this literature focused on the case where $n=2$, we also derive the necessary and sufficient condition for a successful branding strategy in this case. Here the condition is also stated in terms of the relationship between three parameters: the probability that the dominant existing product is preferred to the other product, the probability that the dominant product is preferred to the new dominated product and the probability that the non-dominant existing product is preferred to the dominated product.

The model is presented in section 2. The two first results are derived in section 3. In Section 4 we establish the two analogous results for the case where $n=2$ and then illustrate and evaluate the proposed approach in light of some existing experimental

⁴Whereas the clear-cut superiority in the former case naturally results in the choice of the superior London vacation, in the later case, such clear superiority does not exist and, in turn, the Paris vacation is not necessarily chosen when compared with the inferior London vacation.

evidence. Section 5 contains a brief discussion of some possible branding-policy implications of the model. We conclude in section 6 with a brief summary and concluding remarks.

2. The model

An individual chooses an alternative from a finite set of alternatives X with cardinality n , $n \geq 2$, given his strict complete and asymmetric preference relation \succ . Most economic models are based on the assumption that the alternatives the individual faces are known. However, if the alternatives are unknown, as when one chooses a book from among a pile of books or a vacation from a thick tourist brochure, scanning and some recognition processes are required. In this study the individual is assumed to apply a particular choice procedure. Specifically,

Assumption A1: The individual chooses according to an *Elimination Scan*.

An *Elimination Scan* has two elements:

- An ordered subset of $X \times X$ that consists of $n-1$ different elements; each element of this subset is called a comparison. The first element of this subset consists of any two alternatives. The second element consists of the preferred alternative in the first comparison and another alternative. The third element consists of the preferred alternative in the second comparison and a new alternative, and so on.
- An outcome which is the preferred alternative in the last $(n-1)$ th comparison.

Focusing on an *Elimination Scan* is plausible because:

(i) The outcome of this scan is always the best alternative according to \succ , if there exists such an alternative or an alternative that belongs to the top preference cycle, when a best alternative does not exist. More formally, an alternative x' is called an *acceptable choice* if $x' \in C(X, \succ)$, where

$$C(X, \succ) = \{ x' \in X \mid [(x' \succ x) \vee \exists y_1, \dots, y_k \in A: (x' \succ y_1 \succ y_2 \succ \dots \succ y_k \succ x)] \forall x \in X \setminus \{x'\} \}.$$

That is, an *acceptable choice* is an alternative that is preferred to all other alternatives, or, a preference chain can be constructed from this alternative to all other alternatives

in X . For example, suppose that $X = \{a, b, c, d\}$, $a \succ b$, $b \succ c$, $c \succ a$ (preferences are cyclic) and d is inferior to all other alternatives. By definition, $a \in C(X, \succ)$, since $a \succ d$, $a \succ b$ and $a \succ b \succ c$. Similarly, it can be verified that b and c belong to $C(X, \succ)$. Alternative d is not an acceptable choice; that is, $d \notin C(X, \succ)$. It is easy to verify that the outcome of an *Elimination Scan* always yields an *acceptable choice*.

(ii) An *elimination scan* yields an *acceptable choice* with a minimal number of comparisons, that is with $n-1$ comparisons⁵

The individual preferences in our setting are determined by a particular probability distribution over the set of possible preferences. Specifically,

Assumption A2: Every preference relation is equally probable.

Let $P(x_i \succ x_j)$ represent the probability that x_i is preferred to x_j . **A2** implies that for every $x_i, x_j \in A$, $i \neq j$, $P(x_i \succ x_j) = P(x_j \succ x_i) = 0.5$.

One interpretation of **A2** is that the individual has stochastic preferences. An alternative interpretation is that an outside observer \ firm does not know the preferences of a group of individuals and $P(x_i \succ x_j)$ represents the proportion of individuals preferring x_i over x_j .

An order $O = \langle x_1, x_2, \dots, x_n \rangle$, is an ordered n -tuple of alternatives in X . $O \in \mathbf{O}$, where \mathbf{O} is the set of possible orders. An order induces an *elimination scan* as follows. The first comparison is between x_1 and x_2 , the second between x_3 and the preferred alternative in the first comparison, and so on. The order that induces the scan applied by an individual is assumed to be determined by a particular probability distribution over the set of possible orders. Specifically,

⁵ *Elimination Scan* belongs to a family of scans called knockout scans, that is, scans in which an alternative that is compared and beaten never competes again. Although every knockout scan yields an *acceptable choice* with a minimal number of comparisons, it can be shown that an *Elimination Scan* requires minimal memory, in fact, only one memory "cell" is needed in order to reach the scan outcome.

Assumption A3: Every order is equally probable

One interpretation of **A3** is that the individual scans the alternatives randomly. An alternative interpretation is that an outside observer \ firm does not know the order in which each individual scans and assigns equal probability to each order.

Let X' denote the opportunity set that includes alternative y , in addition to all the alternatives in X ; that is, $X' = \{X\} \cup \{y\}$. Despite **A2**, even if preferences are unknown, in a controlled environment the added alternative y may be such that $P(x_i \succ y) \neq P(x_j \succ y)$ for some $x_i, x_j \in X'$. Consider, for example, the opportunity set mentioned above: {'vacation in London', 'vacation in Paris'}. Under assumption **A2**, $P(\text{'vacation in London'} \succ \text{'vacation in Paris'}) = 0.5$. Introducing an 'inferior vacation in London', a vacation which is identical to 'vacation in London' except that now the hotel room has no bath, it is possible that $P(\text{'vacation in London'} \succ \text{'inferior vacation in London'}) \neq P(\text{'vacation in Paris'} \succ \text{'inferior vacation in London'})$.

So let $P(\bar{x} \succ y) = \alpha$ and, for simplicity, assume that any alternative other than \bar{x} is preferred to the new alternative y in the same probability β . That is,

Assumption A4: $P(x_i \succ y) \equiv \beta$, for every $x_i \in X' \setminus \bar{x}$.

Notice that **A4** imposes weaker requirements, the smaller the number of the existing alternatives n . In particular, for $n=2$, **A4** can be dismissed because it is trivially satisfied.

3. Feasibility of Successful Versioning and its Potential Advantage

We say that an added alternative y is an *asymmetrically dominated alternative* relative to \bar{x} , if:

- $P(\bar{x} \succ y) = \alpha > 0.5$, that is, alternative y is dominated by \bar{x} .
- $P(\bar{x} \succ y) > P(x_i \succ y)$ for every $x_i \in X' \setminus \bar{x}$. That is, the probability that \bar{x}

dominates y is greater than the probability that y is dominated by any other alternative. It is easy to verify that if the above assumptions are satisfied, adding an

asymmetrically dominated alternative can increase the choice probability of the dominant alternative. For example, let $\bar{x} \in X, y \notin X$ and consider the extreme case where $P(\bar{x} \succ y) = 1$ and $P(x \succ y) = 0$ for all $x \in X \setminus \bar{x}$, that is $\alpha = 1$ and $\beta = 0$. In this case, if y is positioned after \bar{x} in the order inducing the applied scan, the choice probability of \bar{x} cannot decline. However, if y is positioned before \bar{x} in the order inducing the applied scan, the choice probability of \bar{x} increases, since y is the preferred alternative in any comparison until its comparison with \bar{x} . Since every order is equally probable, adding y strictly increases the choice probability of \bar{x} . We say that versioning is successful, if adding an alternative y strictly increases the selection probability of a targeted alternative \bar{x} . The following proposition provides the necessary and sufficient condition in our setting for such successful versioning.

Proposition 1: Let $X' = \{X\} \cup \{y\}$ and suppose that axioms **A1**, **A2**, **A3** and **A4** are satisfied, Then

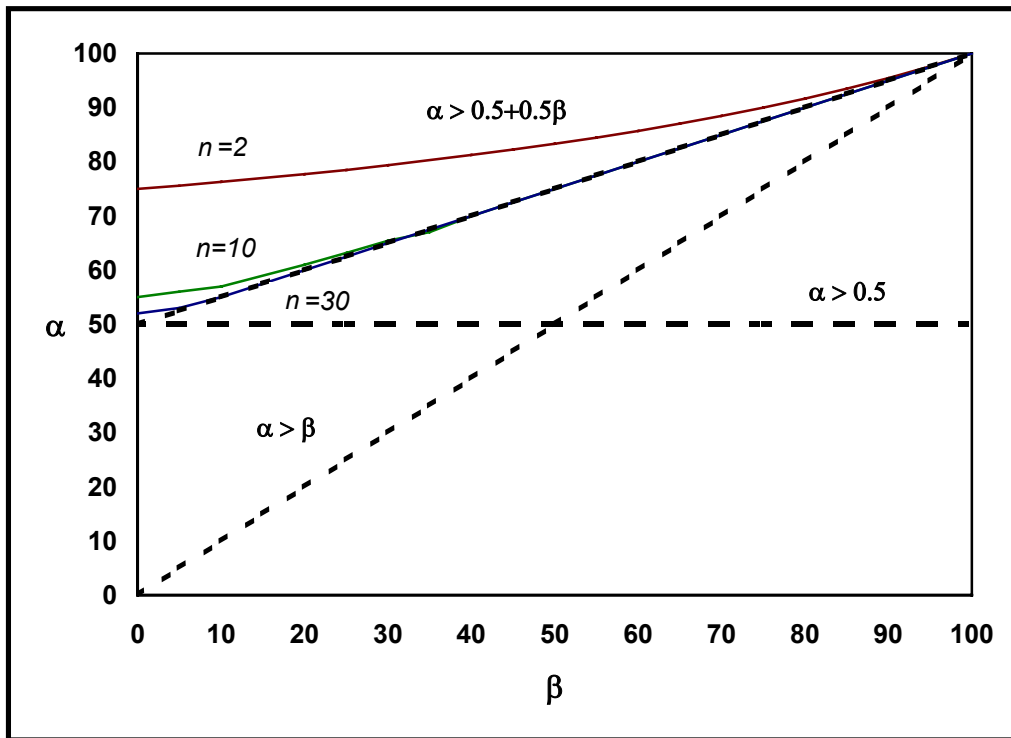
$$P(\bar{x} : X') > P(\bar{x} : X) \text{ iff } 1 \geq \alpha > \frac{1 + \beta - 0.5Z}{2 - Z}$$

$$\text{where } Z = (1 - \beta)0.5^{n-2} + (1 - 2\beta) \sum_{j=2}^n 0.5^{n-j} (1 - \beta)^{j-1}.$$

Proof: see Appendix. ■

The following figure illustrates the values of α that increase the choice probability of alternative \bar{x} given β and some particular values of n (the number of alternatives in X). Note that $\alpha > 0.5$; that is, alternative y is dominated by \bar{x} and $\alpha > \beta$; that is, the probability that \bar{x} dominates y is greater than the probability that y is dominated by any other alternative. This means that y must be asymmetrically dominated by \bar{x} . Note, however, that under axioms **A1-A4**, the necessary and sufficient condition stated in Proposition 1 requires a strong version of asymmetric domination because α has to be greater than $0.5 + 0.5\beta$ and not just greater than 0.5 and greater than β (see the graphs corresponding to $n=2$, $n=10$ and $n=30$). Also note that the extent of the required domination is inversely related to the number of

alternatives n . In other words, the larger the n , the weaker the required dominance of \bar{x} relative to the other x 's that ensures the existence of the ADE. When $n \rightarrow \infty$, $\alpha \rightarrow 0.5 + 0.5\beta$.



Our next concern is the maximal possible absolute and relative changes in the choice probability of some alternative due to the introduction of an inferior alternative. The following proposition specifies the maximal choice probability of a dominant alternative due to an introduction of an inferior one.

Proposition 2: Denote by r the maximal probability that \bar{x} is chosen, if alternative y is added to the opportunity set, and suppose that axioms **A1**, **A2**, **A3** and **A4** are satisfied, then:

$$r = \frac{2}{n+1}.$$

Proof: see Appendix. ■

Under assumptions **A1-A3**, the probability that some alternative \bar{x} is chosen from the

opportunity set X , without adding alternative y , is $1/n$. Hence, the maximal difference in the sale probability of the dominant alternative between the two cases is equal to:

$$\frac{2}{n+1} - \frac{1}{n} = \frac{n-1}{n(n+1)}$$

When $n = 2$, the maximal difference in the sale probability is equal to 16.67% (the sale probability is 66.67% with alternative y and 50% without alternative y). As n grows, the difference decreases, and converges to $1/n$ for large enough n .

The maximal increase in the sale probability with alternative y , in percentages terms, is:

$$\left[\left(\frac{2}{n+1} : \frac{1}{n} \right) - 1 \right] * 100 = 100 \frac{n-1}{n+1}$$

When $n = 2$, the maximal increase in the sale probability is equal to 33.33% (from 50% to 66.67%). As n grows, the maximal increase in the sale probability increases, and converges to 100% for large enough n , that is adding alternative y can double the sale probability of the dominant alternative.

4. Experimental Evidence: Illustration

The experimental evidence was obtained in cases where $n=2$ and the findings confirmed the existence of ADE. In the context of these controlled experiments assumption **A2** is not plausible. First, the control over the alternatives presented to the individuals participating in the experiment determines, at least to some extent, the probability λ that the dominant alternative is preferred to the other one. Furthermore, this probability can be actually estimated by the findings obtained in the experiment.

For $n=2$, we therefore dismiss of assumption **A2** and for $X=\{\bar{x}, x\}$, let $P(\bar{x} \succ x) \equiv \lambda$ instead of $P(\bar{x} \succ x) \equiv 0.5$. For this case we obtain

Proposition 3: Let $X'=\{X\} \cup \{y\}$, $P(\bar{x} \succ y) \equiv \alpha$, $P(x \succ y) \equiv \beta$, $P(\bar{x} \succ x) \equiv \lambda$ and suppose

that axioms **A1** and **A3** are satisfied, then $P(\bar{x} : X') > P(\bar{x} : X)$ iff $1 \geq \alpha > \frac{\lambda(3-\beta)}{1+2\lambda-\beta}$.

Proof: see Appendix. ■

The next proposition specifies the maximal choice probability of a dominant alternative due to an introduction of an inferior one for the case $n=2$.

Proposition 4: Denote by r the maximal probability that \bar{x} is chosen, if alternative y is added to the opportunity set, and suppose that axioms **A1** and **A3** are satisfied.

Then $r = 1$.

Proof: see Appendix. ■

Consider the two relevant experiments that were conducted by Simonson & Tversky (1992). In the first experiment two groups of individuals were asked to choose between different alternatives: Group I chose between \$6 and an elegant Cross pen, and Group II chose between \$6, a Cross pen and a cheap, unattractive pen. The findings of the experiment are presented below:

| | Group I | Group II |
|-----------|---------|----------|
| \$6 | 64% | 52% |
| Cross pen | 36% | 46% |
| Other pen | ----- | 2% |

Although very few individuals chose the less attractive pen, adding this dominated alternative to the choice set strictly increased the choice probability of a dominant alternative - the Cross pen- from 36% to 46% (an increase of 10% in the choice probability of the dominant alternative, namely a relative increase of 28%).

In the second experiment, two groups of individuals were asked to choose between different alternatives: Group 1 chose between Panasonic microwave I on sale and Emerson microwave, and Group 2 chose between Panasonic microwave I on sale, a different inferior Panasonic microwave II and Emerson microwave. The findings of the experiment are presented below:

| Alternatives | Choice probabilities | |
|--------------|----------------------|---------|
| | Group 1 | Group 2 |
| Emerson | 57% | 27% |
| Panasonic I | 43% | 60% |
| Panasonic II | ----- | 13% |

Adding the dominated Panasonic II microwave increased the choice probability of the dominant Panasonic I microwave from 43% to 60%, a relative increase of almost 40%.

In the first experiment $\lambda=36\%$ and in the second $\lambda=43\%$. When y is added to X , the probability that \bar{x} is chosen is equal to (see proof of Proposition 3):

$$\frac{4\lambda\alpha + 2[\beta\lambda + (1 - \beta)\alpha]}{6}$$

and the maximal choice probability, $r(\lambda)$, which is obtained when $\alpha=1$ and $\beta=0$, is equal in the first experiment to $r(\lambda)=57\%$ (versus the actual 46%) and in the second experiment to $r(\lambda)=62\%$ (versus the actual 60%). Hence, the actual increase observed in this experiments is well within the range predicted by the model. This means that the evidence in these experiments can be rationalized by our model. In other words, rationalization of the findings does not require reliance on context-dependent preferences and, in particular, on psychological factors such as "local contrasts" (instead of overall value), "loss aversion" and "extremeness aversion", Simonson & Tversky (1992), Tversky and Simonson (1993), although such factors can play a role in determining the extent of the ADE. Note that our proposed model is insufficient to rationalize the experimental evidence in two cases. First, when the experiment results in $P(\bar{x} : X')$ that is larger than $r(\lambda)$, where λ is the actual probability $P(\bar{x} \succ x)$ observed in the experiment. Second, when $P(\bar{x} : X')$ is smaller than $r(\lambda)$, yet it is inconsistent with the observed λ , α and β (notice that the latter two magnitudes can also be estimated experimentally). Such inconsistency occurs when the observed $P(\bar{x} : X')$ sufficiently deviates from $P(\bar{x} : X')$ that corresponds to λ , α and β , which is equal to $\frac{4\lambda\alpha + 2[\beta\lambda + (1 - \beta)\alpha]}{6}$. In such cases, at least partial reliance on alternative psychological theories that, in contrast to our theory, allow dependence of preferences on context is warranted.

5. Branding-Policy Implications

Introducing new products as a means of increasing the firm's profitability is discussed in the literature on multi-product firms, versioning and product differentiation. This literature deals with discriminatory behavior of a monopoly (Stokey, 1979; Salant, 1989) as well as with competing multi-product firms. In the latter context, products

are differentiated by horizontal (location) characteristics (Shaked & Sutton, 1990; Klemperer, 1992), vertical (quality) characteristics (Champsaur & Rochet, 1989; Johnson & Myatt, 2003; Preyas *et. al.* 2001), or both (Katz, 1984; Canoy & Peitz, 1997). Sale of a new product by a multi-product firm can be warranted because it may attract new consumers to the market, it may cause consumers who purchase products from competing firms to switch to the new product or it may “cannibalize” the firm’s own line of products. In particular, new and inferior products (like the IBM Laser Printer that prints 5 pages per minute, that was created by adding a speed-limiting chip to IBM’s Laser Printer of 10 pages p.m., Deneckere & McAfee, 1996), may be introduced for various reasons intended to exploit the heterogeneity in the consumers’ preferences, budget or income, to deter prospective entrants or to “fight” actual new entrants. The firm’s decision concerning the introduction of the new product hinges on the tradeoff between the extra net profit from the new product and the adverse effect of its introduction on the firm’s profits from its existing line of products.

Unlike any of the existing models, in our setting, by Proposition 1, the sale probability of a firm's leading brand may increase when a new low-quality brand is introduced. Note that usually the firm has to bear some costs when introducing a new inferior or ‘decoy’ product, even if these are virtual ones (like products presented as part of a promotion campaign, yet were never meant to be sold). The firm has an incentive to introduce such a product, if the expected extra gain from the increased sale probability of the leading (dominant) product exceeds the costs associated with the introduction of the new inferior product, including the adverse effect on the sale probability of its other products and the corresponding decline in its expected revenue.⁶

⁶ The applicability of our model is not confined to the context of marketing or branding policy. An example of an alternative contest application of our results is the following one. Suppose that a candidate takes part in a contest among n candidates, the members of the set X . The outcome of the contest is determined according to an elimination scan. By Proposition 1, a candidate may benefit (his choice probability may increase) if an asymmetrically dominated candidate is added to the opportunity set X . For example, suppose that a paper is presented at an academic conference, all the presented papers are reviewed and a merit prize is given to one of them. In such a contest, an author interested in increasing the winning probability of his paper would prefer that an additional paper be presented, provided that it is very similar to his paper, yet distinctly inferior to it.

Our model can also be applied to shed light on the relationship between optimal product variety and brand value. In a recent study, Hui (2004) examined the relationship between brand value and similarity in product variety from the consumers' point of view. Using our setting, it is possible to study the relationship between brand value and similarity in product variety from the firm's point of view. Assuming that high-value brands generate relatively high marginal profits, a firm producing such a brand has a stronger incentive to introduce a new and similar inferior product in order to enhance the sales of its leading product. A firm producing a low-value brand also has an incentive to add new similar inferior products, but this incentive is weaker. In other words, in this later case the expected profit of versioning is less likely to exceed the expected marginal costs of adding the new brand and, therefore, the firms producing the low-value brands may find it more profitable to introduce new products in different market segments. To sum up, when marginal profit increases with brand value, our model predicts positive correlation between brand value and product similarity.

6. Conclusion

Given the assumption regarding the particular choice procedure (the elimination scan) consumers apply (**A1**), the assumption regarding the random nature of the consumers' preferences (**A2**), which implies intransitivity, and the assumption regarding the random scanning order (**A3**) are the basic driving forces behind the possible existence of the company's incentive to add new inferior products (a simple definition of asymmetric dominance using just two parameters, α and β , is enabled by **A4**). Given **A1-A4**, our first result (Proposition 1) provides the necessary and sufficient condition for the ADE, which in our setting implies that the addition of an inferior product increases the company's sale probability of some other product. This condition that requires a sufficiently strong asymmetric domination is stated in terms of the three parameters of the model: the number of existing products, n , the probability that the targeted product is preferred to the inferior new product, α and the probability that any other existing product is preferred to the inferior product, β . This result implies that the extent of the required (probabilistic) domination is inversely related to the number of alternatives n . In other words, to ensure the existence of the ADE, the larger the n , the weaker the required dominance of the targeted product with respect

to the new product relative to the dominance of the other existing products. The second result (Proposition 2) specifies the maximal possible sale probability of the targeted product in terms of n . This result implies that the maximal absolute increase in the sale probability of a product is 16.67%. This increase is obtained when $n=2$. It also implies that the maximal percentage increase in the sale probability of a product converges to 100% when n is large enough.

The results have been illustrated in light of the findings of two relevant experiments conducted by Simonson and Tversky (1992). The findings in these experiments suggest that in the particular context of these experiments where there are just two existing alternatives, $n=2$, assumption **A2** is not satisfied. We have shown that an appropriate modification of this assumption enables rationalization of the ADE observed in these experiments by exclusively using our proposed theory. The psychological theories proposed by Simonson & Tversky (1992) and Tversky and Simonson (1993) suggest that stochastic preferences are context dependent. In particular, adding a dominated alternative changes the probability that the individual prefers the dominating alternative relative to some other alternatives. In these studies it has been argued that experimental findings, like those mentioned in section 4, validate the claim that preferences are context dependent. Our results imply that a change in the choice probability of some alternative does not necessarily result from the context effect on the probability that the individual prefers one alternative to the other, but rather it can be due to the fixed stochastic nature of preferences and the random order of scanning, as in our model. In general, for $n \geq 3$, the rationalization of the ADE can be based on some version of our model (allowing modifications in **A1-A4**) as well as on some alternative theories that may take into account context effects on preferences and, in particular, psychological factors affecting the individual's choice, such as "local contrasts" (instead of overall value), "loss aversion" and "extremeness aversion", Simonson & Tversky (1992), Tversky and Simonson (1993). We have also delineated two possible situations where the experimental findings cannot be rationalized by applying exclusively the proposed model. In these cases the ADE is apparently partly due to the dependence of preferences on the context and not just to the intransitivity implied by the stochastic nature of preferences and to the random nature of the order of scanning. Finally, some implications of the model in the context of branding policy have been briefly discussed. In particular, we have argued

that our results lend support to the prediction that, from the supply side, product similarity tends to vary directly with brand value. These and other potential applications of the model certainly deserve more attention, and are left for future research.

Appendix

Proof of proposition 1:

If assumptions **A1-A3** are satisfied, the probability that some alternative \bar{x} is chosen from the opportunity set X is $1/n$.

If assumptions **A1-A4** are satisfied, the probability that some alternative \bar{x} is chosen, with an alternative y added to the opportunity set, is computed as follows:

Let $j, n \geq j \geq 1$, be the position of \bar{x} in the order, which induces a scan when y is added. The probability that y is positioned before \bar{x} is $j/(n+1)$.

The probability that y is preferred in all the comparisons up to, but not including, the comparison with \bar{x} is:

$$(1) \quad Q = \frac{(1-\beta) + (2\beta-1)(1-\beta)^{j-1}}{j\beta}.$$

The probability that \bar{x} is the preferred alternative in the $(j-1)^{\text{th}}$ comparison (the first comparison that includes it) is $\alpha Q + 0.5(1-Q)$, while the probability that \bar{x} is chosen is equal to:

$$(2) \quad [\alpha Q + 0.5(1-Q)]0.5^{n-j}.$$

If $j=1$, the probability that \bar{x} is chosen is equal to:

$$(3) \quad (j/(n+1)) \alpha 0.5^{n-j} + [1-(j/(n+1))] \alpha 0.5^{n-j} = \alpha 0.5^{n-1}.$$

If $j=2 \dots n$, the probability that \bar{x} is chosen is equal to:

$$(4) \quad (j/(n+1))[\alpha Q + 0.5(1-Q)]0.5^{n-j} + [1-(j/(n+1))] \alpha 0.5^{n-j+1}.$$

The probability that \bar{x} is in any place in the order is fixed and equal to $1/n$.

Hence, the probability that \bar{x} is chosen, if alternative y is added to the opportunity set, is:

$$(5) \quad 1/n \left\{ \alpha 0.5^{n-1} + \sum_{j=2}^n [(j/(n+1))[\alpha Q + 0.5(1-Q)]0.5^{n-j} + [1-(j/(n+1))] \alpha 0.5^{n-j+1}] \right\}.$$

The probability that \bar{x} is chosen, if alternative y is added to the opportunity set, increases if:

$$(6) \quad 1/n \{ \alpha 0.5^{n-1} + \sum_{j=2}^n [(j/(n+1))[\alpha Q + 0.5(1-Q)]0.5^{n-j} + [1-(j/(n+1))]\alpha 0.5^{n-j+1}] \} > 1/n.$$

or, by rearranging terms in (6),

$$(7) \quad \alpha > \frac{1 - \sum_{j=2}^n \left[\frac{j}{n+1} (1-Q) 0.5^{n-j+1} \right]}{0.5^{n-1} + \sum_{j=2}^n \left[\left(\frac{j}{n+1} Q + \left(1 - \frac{j}{n+1} \right) 0.5 \right) 0.5^{n-j} \right]}.$$

Let

$$(8) \quad \sum_{j=2}^n \left[\frac{j}{n+1} (1-Q) 0.5^{n-j+1} \right] = \frac{1}{2(n+1)} \sum_{j=2}^n j(1-Q) 0.5^{n-j} = \frac{1}{2(n+1)} [a],$$

and

$$(9) \quad \sum_{j=2}^n \left[\left(\frac{j}{n+1} Q + \left(1 - \frac{j}{n+1} \right) 0.5 \right) 0.5^{n-j} \right] = \frac{1}{2(n+1)} \sum_{j=2}^n (2jQ + n + 1 - j) 0.5^{n-j} \\ = \frac{1}{2(n+1)} [b].$$

Applying (8) and (9), (7) can be rewritten as:

$$(10) \quad \alpha > \frac{1 - \frac{1}{2(n+1)} [a]}{0.5^{n-1} + \frac{1}{2(n+1)} [b]} = \frac{2(n+1) - \sum_{j=2}^n j(1-Q) 0.5^{n-j}}{2(n+1) 0.5^{n-1} + \sum_{j=2}^n (2jQ + n + 1 - j) 0.5^{n-j}} \\ = \frac{2(n+1) + \sum_{j=2}^n jQ 0.5^{n-j} - \sum_{j=2}^n j 0.5^{n-j}}{2(n+1) 0.5^{n-1} + 2 \sum_{j=2}^n jQ 0.5^{n-j} - \sum_{j=2}^n j 0.5^{n-j} + (n+1) \sum_{j=2}^n 0.5^{n-j}} \\ = \frac{2(n+1) + \sum_{j=2}^n jQ 0.5^{n-j} - \sum_{j=2}^n j 0.5^{n-j}}{2(n+1) + 2 \sum_{j=2}^n jQ 0.5^{n-j} - (2n-2)} \\ = \frac{2 + 0.5 \sum_{j=2}^n jQ 0.5^{n-j}}{2 + \sum_{j=2}^n jQ 0.5^{n-j}}.$$

Since,

$$(11) \quad \sum_{j=2}^n jQ0.5^{n-j} = \sum_{j=2}^n \frac{0.5^{n-j} [(1-\beta) + (2\beta-1)(1-\beta)^{j-1}]}{\beta}$$

$$= \frac{1}{\beta} \sum_{j=2}^n \frac{0.5^{n-j} [(1-\beta) + (2\beta-1)(1-\beta)^{j-1}]}{\beta} = \frac{1}{\beta}[c],$$

(10) can be rewritten as:

$$(12) \quad \alpha > \frac{2 + \frac{0.5}{\beta}[c]}{2 + \frac{1}{\beta}[c]} = \frac{2\beta + 0.5[c]}{2\beta + [c]}$$

$$= \frac{2\beta + 0.5[(1-\beta)\sum_{j=2}^n 0.5^{n-j} + (2\beta-1)\sum_{j=2}^n 0.5^{n-j}(1-\beta)^{j-1}]}{2\beta + (1-\beta)\sum_{j=2}^n 0.5^{n-j} + (2\beta-1)\sum_{j=2}^n 0.5^{n-j}(1-\beta)^{j-1}}.$$

Let $\sum_{j=2}^n 0.5^{n-j}(1-\beta)^{j-1} = [d]$. Since,

(13)

$$(1-\beta)\sum_{j=2}^n 0.5^{n-j} = (1-\beta)(2 - 0.5^{n-2}) = 2 - 0.5^{n-2} - 2\beta + \beta 0.5^{n-2} = 2 - 2\beta + 0.5^{n-2}(\beta - 1),$$

(12) requires that:

$$\alpha > \frac{2\beta + 0.5[2 - 2\beta + 0.5^{n-2}(\beta - 1) + (2\beta - 1)[d]]}{2\beta + 2 - 2\beta + 0.5^{n-2}(\beta - 1) + (2\beta - 1)[d]}.$$

or,

$$(14) \quad \alpha > \frac{1 + \beta - 0.5Z}{2 - Z}$$

where $Z = (1-\beta)0.5^{n-2} + (1-2\beta)\sum_{j=2}^n 0.5^{n-j}(1-\beta)^{j-1}$. ■

Proof of proposition 2:

If assumptions **A1-A4** are satisfied, the maximal increase in the sale probability of alternative \bar{x} occurs when $\alpha = 1$ and $\beta = 0$. In such a case, the probability that alternative \bar{x} is chosen, with alternative y added to the opportunity set, is computed as follows:

Let $j, n \geq j \geq 1$, be the position of \bar{x} in the order, which induces a scan when y is added. The probability that y is positioned before \bar{x} is $j/(n+1)$. The probability that y is preferred in all the comparisons up to, but not including, the comparison with \bar{x} , when $\beta=0$ is 1. The probability that \bar{x} is the preferred alternative in the $(j-1)^{\text{th}}$ comparison (the first comparison that includes it) when $\alpha=1$ is 1, while the probability that \bar{x} is chosen is equal to 0.5^{n-j} .

If $j=1$, the probability that \bar{x} is chosen is equal to:

$$(15) \quad (j/(n+1)) 0.5^{n-j} + [1-(j/(n+1))] 0.5^{n-j} = 0.5^{n-1}.$$

If $j=2 \dots n$, the probability that \bar{x} is chosen is equal to:

$$(16) \quad (j/(n+1)) 0.5^{n-j} + [1-(j/(n+1))] 0.5^{n-j+1} = 0.5^{n-j+1} (j+n+1)/(n+1)$$

Denote by r the maximal probability that \bar{x} is chosen, if alternative y is added to the opportunity set. Since the probability that \bar{x} is in any place in the order is fixed and is equal to $1/n$,

$$(17) \quad r = 1/n \left\{ 0.5^{n-1} + \sum_{j=2}^n 0.5^{n-j+1} (j+n+1)/(n+1) \right\}$$

or, by rearranging terms in (3),

$$r = \frac{2}{n+1}. \blacksquare$$

Proof of Proposition 3:

The probability that \bar{x} is chosen if y is not added is λ .

If y is added, there are six possible orders of x, \bar{x} and y . In four of these orders, namely $(\bar{x}, x, y), (x, \bar{x}, y), (\bar{x}, y, x)$ and (y, \bar{x}, x) , the probability that \bar{x} is chosen is $\alpha\lambda$.

In the other two orders, namely (x, y, \bar{x}) and (y, x, \bar{x}) , the probability that \bar{x} is chosen is $[\beta\lambda + (1-\beta)\alpha]$. By **A3**, each of the possible orders is selected in equal probability of $1/6$. Hence, adding y increases the choice probability of \bar{x} iff:

$$(18) \quad \frac{4\lambda\alpha + 2[\beta\lambda + (1-\beta)\alpha]}{6} > \lambda$$

or by rearranging (18), iff:

$$(19) \quad \alpha > \frac{\lambda(3-\beta)}{1+2\lambda-\beta}. \blacksquare$$

Proof of Proposition 4:

If y is added, and axioms **A1** and **A3** are satisfied, then the probability that \bar{x} is chosen is:

$$(20) \quad \frac{4\lambda\alpha + 2[\beta\lambda + (1 - \beta)\alpha]}{6}$$

and the maximal choice probability is achieved when $\alpha=1$, $\lambda=1$ and $\beta=0$. In such case, $r=1$. ■

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