Demystifying the 'Metric Approach to Social Compromise with the Unanimity Criterion'

by

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Abstract

In a recent book and earlier studies, Donald Saari well clarifies the source of three classical impossibility theorems in social choice and proposes possible escape out of these negative results. The objective of this note is to illustrate the relevance of these explanations in justifying the metric approach to the social compromise with the unanimity criterion.

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1. Introduction

Saari (2008) well clarifies the source of three classical impossibility theorems in social choice and proposes possible escape out of these negative results. The objective of this note is to illustrate the relevance of these explanations in justifying the metric approach to the social compromise with the unanimity criterion.

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The three negative results discussed in Chapter 2 of his recent book and in Saari (1998), (2001), Saari and Petron (2006) and Li and Saari (2008) are Arrow's impossibility theorem (1951), Sen's Paretian-Liberal Paradox (1970) and Chichilinsky's topological dictatorship result (1982a). Saari shows that the common thread that relates these results is that they are based on conditions that force the associated aggregation rule to ignore crucial information about individual preferences. In Arrow's case, the Independence of Irrelevant Alternatives (IIA) condition forces the aggregation rule to concentrate solely on binary rankings, disregarding the consistency (transitivity) of individual preferences. In Sen's case, the Minimal Liberalism (ML) condition, again, prohibits the rule from using information about transitivity of individual preferences. In Chichilinsky's case, the Continuity (C) property emphasizes local behavior, ignoring the global structure of preferences.

The change of these results to positive ones requires modification of the (implausible) conditions that prohibit the aggregation rule from using valuable information, such that it does use the vital missing information.

Saari demonstrates the application of this approach in all three cases. He replaces IIA with condition IIIA, which ensures that the social preference relation is based on how individuals rank the alternatives as well as on the intensity of the ranking. This enables escape from Arrow's theorem and leads to the the Borda rule. The question 'how to evade Sen's result is not completely resolved', but Saari proposes ways and some guidelines to sidestep the difficulty. One suggestion is to allow an agent to be decisive only if his decision does not impose strong negative externalities on others. Finally, to escape from Chichilinsky's negative result, Saari proposes to secure global instead of local information on individual preferences. In his example, such information involves both the individuals' preferred points in the different regions of alternatives and the ranking of the regions.

It seems to me that the metric approach to social compromise with the unanimity criterion is based on recognition of Saari's plausible suggestion as to how to obtain positive results in the above three cases. In other words, the metric approach can be rationalized applying Saari's insightful diagnosis of the reason for the trouble in the above three well known impossibility results. Let me briefly present the metric approach (Farkas and Nitzan (1979), Lehrer and Nitzan (1985), Cambell and Nitzan (1986)) and then illustrate its effectiveness in obtaining positive conclusions.

2. The metric approach

The metric approach to compromise with the unanimity criterion, henceforth *the metric approach*, utilizes information on individual preferences taking into account the intensity of rankings. The unanimity criterion is defined as follows. Suppose that the individual preference relations are strict orderings on the set of alternatives X. Denote by $U(x,S)$ the set of profiles where alternative x is most preferred in the set S, which is a subset of X, from the point of view of every individual. A social choice function C(**P**,S) satisfies the unanimity criterion if

$$\mathbf{P} \in U(x,S) \Rightarrow C(\mathbf{P},S) = \{x\}$$

Since in most profiles unanimity regarding the most preferred alternative does not exist, the unanimity criterion does not lead to a unique social choice function. The multiplicity of social choice functions that satisfy the unanimity criterion raises the question whether there exists one that compromises with the criterion in the sense that it is "closer" (in terms of a metric δ between profiles) than all the other functions to implementing the criterion. A social choice rule C(**P**,S) compromises with the unanimity criterion according to the metric δ if, for every subset S and for every preference profile **P** defined on X,

$$C(\mathbf{P},S) = \{x \in S : \forall y \in S,\ d(\mathbf{P}|_S, U(x,S)) \leq d(\mathbf{P}|_S, U(y,S))\}$$

where $d(\mathbf{P},\mathbf{Q}) = \min_{Q \in V} \delta(\mathbf{P},\mathbf{Q})$ is the distance between a profile **P** and a set of profiles V and $\mathbf{P}|_S$ is the restriction of the profile **P** to the set S. Notice that the metric δ enables measurement of preference intensity (as well as intra-personal and inter-personal comparisons of preference intensities). The use of δ and of the profile sets

U(.,.) in the above definition of the compromising rule C (**P**,S) ensures that the global structure of individual preferences and their intensity are taken into account in determining the social choice.

3. Applications of the metric approach

3.1 The possibility of positional rules

A rule has a reasonable quasi-metric rationalization if it has a symmetric additively decomposable quasi-metric rationalization δ ($\delta = \sum_{i=1}^{n} \delta_i$, $\delta_i = \delta_j = \delta'$) and δ' is neutral and monotonic. Lehrer and Nitzan (1985) have proved that a rule has a reasonable quasi-metric rationalization, if and only if it is a positional rule. By replacing the IIA condition of Arrow's theorem with 'the reasonable metric approach', the admissible rules are the positional rules. In the special case where δ' is the inversion metric, Farkas and Nitzan (1979) have shown that the rule compromising with the unanimity criterion is the Borda positional rule. This positive result is closely related to Theorem 2.5 in Saari (2008).

3.2 The libertarian resolution of the Paretian-liberal paradox

Applying the metric approach to extend the individual preferences on alternatives to preferences on an individual's assigned rights (these second-order rankings are based on the intensity of the ordinal preferences), Harel and Nitzan (1987) demonstrate that the Paretian-liberal paradox can be resolved by allowing voluntary exchange of rights. As shown by Saari and Petron (2004), for any decision rule that satisfies Sen's (ML) condition, in each cycle, each and every agent suffers a strong negative externality that is caused by the choices made by some decisive agent. The libertarian approach of Harel and Nitzan is based, first, on the use of preference intensity to define preferences on rights and then on Coase's observation, the Coase theorem, that the externalities problem can be resolved by voluntary exchange; in our case, the exchange of rights.

3.3 Consistency between unanimity, anonymity and modified continuity

In a topological setting, the metric approach, which applies information of preference intensity, can again be used to establish a possibility result. Using a finite framework, Baigent (1986), following Chichilinsky's (1982b) topological approach, has considered a proximity preservation property for a social choice rule. This property requires that the "smaller" the change in individual preferences, the "smaller" the change in the social choice. It has been shown that this property is inconsistent with the plausible unanimity property and the anonymity property (a weaker form of dictatorship). The problem of this 'continuity' property is that it does not allow the rule to recognize the structure of the decision problem. A resolution of this problem by taking into account the missing information can be based on the use of an alternative property of proximity preservation, namely the metric respect for the unanimity criterion. A social choice rule has this property, if the "closer" a profile is to an ideal situation where some alternative is a unanimously preferred outcome, the "closer" the social choice to this alternative. This modified 'continuity' property succeeds in eliminating inconsistency with the unanimity and anonymity properties because it does not ignore the explicit global structure of preferences. This is illustrated in Nitzan (1989).

4. Conclusion

In his recent book, Saari (2008) accomplishes his objective of disposing dictators and demystifying voting paradoxes. I have tried to prove that he also demystifies the metric approach to social compromise with the unanimity criterion.

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