

Collective Contests for Commons and Club Goods*

by

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Abstract

This paper focuses on collective contests for commons and club goods. Our main objective is to examine in this context the effect of group size on its performance. The main results specify conditions for the existence (non-existence) of the group-size paradox, namely, the situation where a larger group is less (more) effective in pursuing its interest because of (despite) the combined effect of the incentives that result in the free-riding problem and the tragedy of the commons. The paper also explains under what circumstances there exists a bias towards excessive or inadequate winning group size. Finally, it examines the effect of restricted excludability of the commons good within the winning group on the relationship between group-size and its winning probability.

Keywords: collective contest, commons and club good prize, the group-size paradox, excessive or inadequate winning group size.

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1. Introduction

How various characteristics of competing groups affect the outcome of collective contests has been of major concern in the relevant literature in economics and political science. In particular, the relationship between the size of a group and its performance is a notable topic in the collective action setting studies in the celebrated work of Olson (1965). It has been argued that, because of the free-rider problem, larger groups may be disadvantageous in pursuing their interest. In group contests, therefore, larger groups may be less successful than smaller ones. The main objective of our study is to examine the possibility of such “group-size paradox” in extended contests on a “commons” good, where consumption externalities in utilization of the acquired prize exist among the members of the winning group and the share of benefit is determined under the *limited user - open access* situation (Stevenson (1991)). That is, while the use of the prize is limited to the members of the winning group, no rule exists to regulate it. Such free access to the prize results in the “tragedy of the commons” (Hardin (1965)) within the winning group.

As examples of such a contest, we may consider conflicts between countries on territory that has some *common pool resources* (fisheries, oil, gas, water etc.) or provides access to valuable transportation routes. Such conflicts could be escalated to a militarized dispute, such as the 1932-1935 Chaco war between Bolivia and Paraguay (Rasler and Thompson (2006)). Disagreements on boundaries of coastal states, like the “cod wars” between Iceland and U.K., have also been caused by competition for marine resources. While the global framework of ocean governance with UNCLOS is forming recently, many distributional issues on marine resources are left unresolved (Hoel and Kvalvik (2006))³. Another possibility is a competition by groups on special funds for financing the provision of a common pool resource or a public facility. Consider, for example, the contest in Tuscany on special funds of the EU for financing restoration and promotion of arts or cultural goods, such as an archeological park or an Etruscan museum.⁴ Competition among groups of researchers on funding may also fit our extended contest, in those cases where there is

³ Even within a single country, disputes on the boundaries of coastal prefectures can arise due to the desire to secure good fisheries.

⁴ Certainly the importance of the archeological finds in the competing Tuscanian regions is partly determined by the lobbying efforts of their residents. The municipality of Cortona that won the competition allowed its residents free access to the two cultural common goods it provided (both the museum and the archeological park that were built using the EU funds).

free access to the research activity enabled by the awarded grant and provided that an increase in the research activity of the winning group has negative externalities (i.e. due to congestion in the usage of the research equipment purchased by the grant).⁵ In such examples, the members of the losing groups are excluded from using the prize on the basis of alternative legal, institutional or geographical criteria (typically, residence, affiliation or citizenship). The members of the winning group collectively get the prize as a disposable resource, encountering the problem of how to share the benefit. As we will subsequently argue, the limited user-open access situation is the most anarchical possible solution of the sharing problem.

Generally, how the benefit from the prize is shared by the members of the winning group depends on the governance structure of the group. And the form of the benefit sharing ultimately affects the effort levels of the participants in the contest. In their argumentative paper, Esteban and Ray (2001) study a one-stage model with an exogenous sharing rule. The prize in their contest is a mixed private-public-good and the private part of the prize is assumed to be shared equally among the members of the winning group⁶. They have been able to derive a rather weak sufficient condition ensuring that *larger groups have a higher winning probability*. In light of this result, they point out that the group-size paradox would have rather restrictive applicability.

The conclusions by Esteban and Ray are applicable to a wide range of group contests, containing as special cases contests for a pure private good prize and for a pure public good prize. They are derived, however, under the assumption that each competing group can commit to a distribution rule prior to the contest and the winning group can enforce the implied distribution of the benefit from the prize. So every group is assumed to have a rather sound governance power. One could examine the role of this assumption in restricting the possibility of the group-size paradox, by considering different cases where the intra-group division of the benefit from the prize is determined anarchically, i.e., through non-cooperative behavior of the members.

⁵ We are indebted to Silvia Fedeli, Gil Epstein and Nava Kahana for suggesting the last two applications of our extended contest.

⁶ In his model of a collective contest for a private good prize, Nitzan (1991) parameterizes the sharing rule applied by a group as a linear combination between the *equalitarian* and the *relative effort* sharing rules; part of the prize is divided equally, and the rest is divided proportionally to the members' efforts (also see Baik (1994) and Baik and Lee (2001)). We may therefore interpret Esteban and Ray's assumption as the winning group's application of the extreme equalitarian rule to divide the private part of the prize.

Katz and Tokatlidu (1996), Wärneryd (1998), and Konrad (2004) describe such a situation by adding the stage of intra-group contest; a similar kind of battle as in the inter-group contest is repeated in the winning group and each member secures his resulting own share in the private-good prize⁷. But we can argue that such a repeated-contest model still presupposes the existence of some well-established authority in the winning group. Otherwise, it would be difficult to allow a single battle determine the winner of the private property associated with the benefit stream from the prize. Such difficulty is even more severe if consumption externalities exist in utilization of the prize. To analyze the effect of the anarchical sharing of the prize with consumption externalities, we will therefore consider the situation in which anyone in the winning group can freely use the acquired prize at any desired rate.

We will consider a model of $m (\geq 2)$ competing groups with different membership. In the first stage of the contest, the group members determine their contributions to the collective effort to win the prize. In the second stage, the members of the winning group make, non-cooperatively, a decision on their levels of utilization of the prize. No authority or agreement exists to guide them to cooperative extraction of the benefit. By using this model, we will establish a simple relationship, in equilibrium, between the characteristics of a group and its winning probability. Our main results (Proposition 1 and Corollary 1) specify conditions for the existence (non-existence) of the group-size paradox, namely, the situation where a larger group attains a lower (higher) winning probability than a smaller group, where these probabilities are evaluated at a single equilibrium. We will see that the introduction of anarchical utilization of the prize strengthens the possibility of the group-size paradox.

We then make some observations on what is the socially desirable membership of the winning group and argue that the contest can have a bias towards selection of a socially undesirable group with high probability. We also examine what happens in the contest if the over-exploitation of the commons good can be mitigated within the winning group. By using an example, it is shown that such a change can dramatically enhance the advantage of larger groups in the contest.

The next section presents our two-stage model of group-specific contests for a commons good. In the first part of this section, we analyze endogenous prize

⁷ See Hausken (2005) and Konrad (2006) for a comprehensive survey of the studies that treat explicitly the related intra and inter- group conflicts.

utilization. In the second part, we study the equilibrium efforts of the contestants. Section 3 contains the main results on the effect of group size on its winning probability. In Section 4, we examine the possible biases towards excessive or inadequate winning group size. The effect of introducing regulation on the over-exploitation of the prize is studied in section 5. Some concluding remarks appear in Section 6. Proofs of Lemma 1 and Proposition 1 are relegated to an Appendix.

2. The extended two-stage group contest

In stage 1 of the collective contest on the commons good, the group members choose their contribution to the group effort. In stage 2, the members of the winning group make a decision on their extent of utilization of the prize won by the group. The contestants' efforts in stage 1 are determined strategically, in anticipation of the equilibrium levels of utilization in stage 2. The equilibrium analysis presented below starts with the determination of prize utilization and then proceeds with the determination of efforts.

2(a). *Endogenous prize utilization*

Consider a contest for the prize with the following characteristics. After winning the prize, it is used as a freely accessible resource among the members of the winning group. Every member of a competing group has the same form of the gross utility function given by

$$U(x \cdot b(z)),$$

where x denotes the extent of utilization of the prize by an individual, z is the aggregate group members' utilization of the prize and $b: (0, \infty) \rightarrow (0, \infty)$ denotes the average benefit of resource utilization. Clearly, there exist mutual externalities among members' utilization of the prize. The derivative $b'(z)$ can be interpreted as the externality for a group member caused by a change in z . When all the members choose zero utilization, each one is assumed to get zero benefit. We also assume that b is twice continuously differentiable and strictly concave. Further assumptions are:

$$U(0) = 0, U' > 0, U'' \leq 0, \lim_{z \rightarrow 0} b(z) > 0, \lim_{z \rightarrow \infty} b(z) < 0.$$

The last assumption together with the concavity of b imply that the mutual externality of utilization $b'(z)$ eventually becomes negative.

Our assumptions satisfy the standard requirements for commons or club

goods⁸. Viewing x and z , respectively, as visitation of a club by an individual and by all members of the group, we can interpret the prize as a club good. Also, specifying

$$b(z) = \frac{F(z) - pz}{z},$$

where p is the unit cost of utilization and F is a strictly concave function with $F(0) = 0$, the utilization sub-game can be interpreted as the static model of open resource exploitation, as argued by Dasgupta and Heal (1979) and Cornes and Sandler (1983)⁹.

The group members choose their individual levels of utilization of the prize simultaneously and non-cooperatively. Let $N \geq 2$ be the size of the winning group. In the Nash equilibrium of this second stage of the contest, the ‘utilization sub-game,’ a member solves the problem:

$$\max_{x_i \geq 0} U\left(x_i \cdot b\left(\sum_{k=1}^N x_k\right)\right).$$

Since $\sum_{k=1}^N x_k = z$, the first-order condition for the solution of this problem is:

$$U'(x_i \cdot b(z)) \cdot (b(z) + x_i \cdot b'(z)) \leq 0,$$

where equality holds for $x_i > 0$. If $x_i = 0$, $b(z)$ must be non-positive and $x_k > 0$ holds for some other member k . But $b(z) \leq 0$ implies that $b'(z) < 0$, because of the assumption that $\lim_{z \rightarrow 0} b(z) > 0$ and the strict concavity of b . Hence, $x_k > 0$ cannot satisfy the first-order condition. On the other hand, if $x_i = 0$ holds under $z = 0$, the individual chooses zero utilization in attempting to maximize $U(x_i \cdot b(x_i))$, again contradicting $\lim_{z \rightarrow 0} b(z) > 0$. In an equilibrium of the utilization game, therefore, $x_i > 0$ and

$$U'(x_i \cdot b(z)) \cdot (b(z) + x_i \cdot b'(z)) = 0$$

must hold for all $i = 1, \dots, n$. These equations require that the equilibrium is symmetric.

The above first-order condition can therefore be modified to

$$b(Nx) + x \cdot b'(Nx) = 0, \tag{1}$$

where x denotes the symmetric equilibrium extent of utilization by every group member. Viewing the left-hand side of the equation as a function of x , it converges to $b(0) > 0$ as x goes to zero, and becomes negative when x increases. Also, it must be strictly decreasing at a value of x satisfying equation (1). There exists therefore unique

⁸ Unfortunately, the type of prizes treated by Esteban and Ray (2001) is beyond our scope. This is because of the strict concavity of $b(z)$. Also, the form of the gross utility function is more restrictive than usually assumed in the literature on club goods.

⁹ For a more advanced treatment of the problem, see Cornes and Hartley (2000).

symmetric pure-strategy equilibrium of the utilization sub-game. Denote the equilibrium level of utilization by a member of a group that includes N members by $x(N)$. That is, if a group of size N wins the contested prize, a member derives from it the benefit

$$G(N) = U(x(N) \cdot b(Nx(N))). \quad (2)$$

Notice that the equilibrium total utilization is excessive in the usual sense because (1) implies that $x \cdot b(Nx)$ is decreasing with respect to x .

Pretending that membership is a continuous variable which is denoted by n , we can define a continuous function $G(n)$ by using equation (2). The “benefit elasticity of membership,” $\chi(n)$, defined by

$$\chi(n) = -\frac{dG(n)}{dn} \cdot \frac{n}{G(n)},$$

plays an important role in the following analysis. It is an essential characteristic of the structure of the “utilization sub-game” indicating how the benefit from the prize for individual group members declines with membership.

Let $\delta(y)$ denote the “elasticity of the individual’s gross utility”,

$$\delta(y) = \frac{U'(y) \cdot y}{U(y)},$$

and let $\beta(n)$ denote the “elasticity of congestion,”

$$\beta(n) = \frac{b''(nx(n))}{b'(nx(n))} \cdot nx(n).$$

Because of equation (1), $\beta(n)$ is positive in equilibrium of the utilization sub-game. Then the following two equalities hold.

Lemma 1.

$$-\frac{nx'(n)}{x(n)} = 1 - \frac{1}{n+1+\beta(n)} \in \left(\frac{n}{n+1}, 1\right) \quad (3)$$

and

$$\chi(n) = \delta(x(n) \cdot b(nx(n))) \cdot \left(1 + \frac{n-1}{n+1+\beta(n)}\right). \quad (4)$$

The first equality implies that as the membership of a group grows, the aggregate extent of utilization rises, while the extent of utilization by each group member declines. The larger $\beta(n)$ is, the larger the decline. That is, larger congestion induces

the members of a larger group to refrain from over-utilization. Hence, a seemingly paradoxical implication is obtained from (4); an increase in $\beta(n)$ reduces $\chi(n)$.

2(b). *Equilibrium efforts in the contest*

Now consider the first stage. Let m be the number of the competing groups. The membership of group i is denoted by N_i ($i = 1, \dots, m$). The effort cost function for the contest is symmetric for every member in the competing groups and is given by $v(a)$, where $a \geq 0$ is the effort level. We assume that v is twice continuously differentiable, $v(0) = 0$, $v'(a) > 0$ and $v''(a) \geq 0$ for $a > 0$, and furthermore, that $\lim_{a \rightarrow 0} v'(a) = 0$. We denote by $\alpha(a)$ the elasticity of the marginal cost of effort,

$$\alpha(a) = \frac{v''(a)}{v'(a)} \cdot a.$$

The contest winning probability of group i is given by

$$\pi_i = \frac{A_i}{\sum_{h=1}^m A_h},$$

where A_h is the total amount of effort made by the members of group h ¹⁰. The total amount of effort put by all the competing groups is $A = \sum_{h=1}^m A_h$.

In a subgame-perfect equilibrium, each member of group i solves the problem:

$$\max_{a \geq 0} \frac{A_i}{A} G(N_i) - v(a),$$

where $G(N_i)$ is the member's equilibrium gross utility derived from the prize in the second stage. The first order condition for the solution is:

$$\frac{A - A_i}{A^2} G(N_i) - v'(a) \leq 0,$$

with equality if $a > 0$. Since $\lim_{a \rightarrow 0} v'(a) = 0$, the condition actually holds as an equation. In equilibrium, therefore, the members of group i make the same positive effort, and it must hold that

$$(1 - \pi_i) \cdot G(N_i) - v'\left(\frac{A}{N_i} \cdot \pi_i\right) \cdot A = 0, \quad (5)$$

¹⁰ We can assume that either π_i is equal to 0 or to $1/m$, when all groups exert zero effort. Note that the form of the lottery contest success function in our model is certainly specific. But the possible non-linearity of the effect of effort is taken into consideration by assuming increasing marginal costs of effort. In comparison to Tullock's general contest, it seems that our model is not less general (and, of course, not more general).

where $\pi_i = \frac{A_i}{A}$ is the winning probability of the group. Consistency among the winning probabilities, that is, $\sum_{i=1}^m \pi_i = 1$, is also necessary for equilibrium. It is easy to see that these conditions are also sufficient for equilibrium total effort and equilibrium winning probabilities.

Now, as a device to analyze equilibrium, let us apply the “pseudo” winning probability function $\pi(n, A)$ implicitly defined by the equation

$$(1 - \pi(n, A)) \cdot G(n) - v' \left(\frac{A}{n} \cdot \pi(n, A) \right) \cdot A = 0. \quad (5)'$$

Notice that this function is defined for all $n > 0$ and $A > 0$ and, in particular, for n and A that do not constitute an equilibrium of the extended contest. However, if A^* and the probabilities π_i^* ($i = 1, \dots, m$) constitute an equilibrium of the contest, $\pi_i^* = \pi(N_i, A^*)$ must hold for all i , with $\sum_{h=1}^m \pi(N_h, A^*) = 1$. Conversely, the total effort A^* and the probabilities given by $\pi_i^* = \pi(N_i, A^*)$ ($i = 1, \dots, m$) coincide with an equilibrium, if $\sum_{h=1}^m \pi(N_h, A^*) = 1$. These observations enable us to use the properties of $\pi(n, A)$ in the equilibrium analysis¹¹.

$\pi(n, A)$ is a continuously differentiable function and strictly decreasing with respect to A . Furthermore, $\lim_{A \rightarrow 0} \pi(n, A) = 1$ and $\lim_{A \rightarrow \infty} \pi(n, A) = 0$ hold for any $n > 0$. So $\sum_{h=1}^m \pi(N_h, A)$ is strictly decreasing with respect to A , $\lim_{A \rightarrow 0} \sum_{h=1}^m \pi(N_h, A) = m$ and $\lim_{A \rightarrow \infty} \sum_{h=1}^m \pi(N_h, A) = 0$. We can draw, therefore, the graph of $\sum_{h=1}^m \pi(N_h, A)$, as in Figure 1, and realize that there exists a unique equilibrium which is determined by the intersection of the graph of $\sum_{h=1}^m \pi(N_h, A)$ and the horizontal dotted line at 1 (point E).

¹¹ See Esteban and Ray (2001) and Ueda (2002). In terms of Cornes and Hartley (2000, 2007), this is a special type of the “share function”.

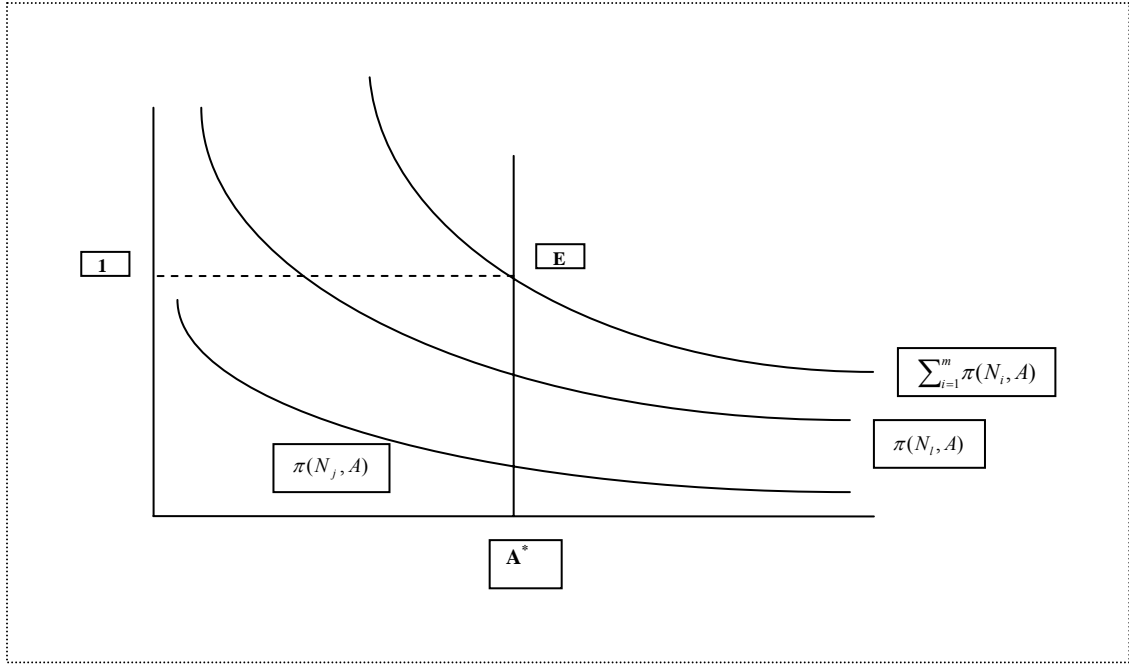


Figure 1

3. Group size and performance

Our fundamental result specifies the sufficient conditions for the validity and invalidity of the group-size paradox.

Proposition 1. *Consider groups l and j in the unique equilibrium of the extended group contest, with $N_l < N_j$. Then,*

(a) *if $\inf_{n \in [N_l, N_j]} \chi(n) > \sup_{a \in (0, \frac{A^*}{N_l})} \alpha(a)$, then the smaller group l attains a higher*

winning probability than that of the larger group j , and

(b) *if $\sup_{n \in [N_l, N_j]} \chi(n) < \inf_{a \in (0, \frac{A^*}{N_l})} \alpha(a)$, then the larger group j attains a higher*

winning probability than that of the smaller group l .¹²

The sufficient condition for the group-size paradox given in Part (a) of

¹² Notice that the phrase “a smaller (larger) group attains a higher winning probability” can be interpreted in two ways. In one interpretation the phrase is applied to the cross-section of groups in a single equilibrium, as we do in the above proposition. Another interpretation involves a comparison across equilibria; if the membership of a certain group declines (rises), the winning probability of that group in the new equilibrium is higher than that in the old equilibrium. Following the reasoning of Esteban and Ray (2001, Proposition 2), we can use the fact that the pseudo winning probability functions are strictly decreasing to show that the sufficient conditions in Proposition 1 are applicable to derive the assertion made under the second interpretation.

Proposition 1 makes sense. As Lemma 1 shows, a larger membership implies a smaller per capita benefit from the prize. Confronting the smaller benefit, each member puts less effort. The extent of this incentive can be measured by $\chi(n)$. On the other hand, the larger membership also implies a lower marginal cost for an individual at a given level of group effort, which induces more effort from each member. The extent of this second incentive can be measured by $\alpha(a)$. If the first incentive is larger than the second in the relevant interval of membership size, the effort made by the larger group (the N_j -member group) is smaller and, consequently, its winning probability is lower. An analogous intuition can be provided for the sufficient condition for the invalidity of the group-size paradox given in Part (b).

To see the implications of Proposition 1, it is convenient to concentrate on the special “constant elasticity case,” where the elasticity of the individual’s gross utility $\delta_0 > 0$, the elasticity of congestion $\beta_0 > 0$, and the elasticity of marginal effort cost $\alpha_0 > 0$ are all constants¹³. The following Corollary is directly obtained from Lemma 1 and Proposition 1 (The Corollary can be easily extended to general cases by substituting the constants to sup or inf in an adequate way).

Corollary 1. *Consider groups l and j with $N_l < N_j$. In the constant elasticity case,*

$\delta_0 \cdot \left(1 + \frac{N_l - 1}{N_l + 1 + \beta_0}\right) > \alpha_0$ implies that the smaller group l attains a higher winning

probability than that of the larger group j . Also, $\delta_0 \cdot \left(1 + \frac{N_j - 1}{N_j + 1 + \beta_0}\right) < \alpha_0$ implies that

the larger group attains a higher winning probability than that of the smaller group.

Hence, the larger the membership of the competing groups, and the smaller the elasticity of congestion β_0 , the broader the incidence of the group-size paradox¹⁴. The effect of β_0 can be explained by the argument discussed at the end of subsection 2(a). Corollary 1 also implies coarser but simpler conditions for the existence (non-existence) of the group-size paradox: in the constant elasticity case, if $\delta_0 > \alpha_0$, the

¹³ Such a case is obtained when $U(y) = U_0 \cdot y^{\delta_0}$, $b(z) = b_0 - z^{1+\beta_0}$, and $v(a) = v_0 \cdot \frac{a^{1+\alpha_0}}{1+\alpha_0}$,

($0 < U_0, b_0, v_0, \delta_0, \beta_0, \alpha_0$).

¹⁴ Contrast this result with the corresponding arguments of Esteban and Ray. In their model, the larger the membership of the competing groups, the more likely it is that the group-size paradox is invalid.

smaller group attains a higher winning probability than that of the larger group. If $2\delta_0 < \alpha_0$, the larger group attains a higher winning probability.

Our two-stage model rather symmetrically gives rise to the possibilities of existence and non-existence of the group-size paradox. It should be emphasized that this is in marked contrast to the situation in the contest studied by Esteban and Ray (2001), in which very restrictive room is left for the group-size paradox. Recall that in their model the exogenous rule is set beforehand to make the benefit of the acquired prize be shared equally. In contrast, in our model anarchical encroachment of the prize determines the intra-group sharing of the benefit as its consequence. This suggests that the extent of control a group has on the intra-group sharing of the prize plays an important role on the possible emergence of the group-size paradox.

4. Group size and selection bias

Viewing the contest as a system for selecting the group that exclusively utilizes the commons good, the equilibrium total effort A^* can be conceived as the selection cost. Given this cost, the selected group to receive the commons and share its benefit should be the one attaining the highest possible value of $nG(n)$, provided that we apply the utilitarian criterion..

Suppose that the largest group competing for the commons good has the membership N_{\max} and the smallest one has the membership N_{\min} . If $\chi(n)$ is larger (smaller) than unity on $[N_{\min}, N_{\max}]$, $nG(n)$ is strictly decreasing (increasing) on this interval and it is more desirable that the smallest (largest) group wins the prize. Proposition 1 implies, however, that the winning probability is determined by the relationship between $\chi(n)$ and $\alpha(a)$. When $\inf_{a \in (0, \frac{A^*}{N_{\min}})} \alpha(a) > \chi(n) > 1$ holds on the interval $[N_{\min}, N_{\max}]$, a larger group wins the prize with a higher probability than a smaller group, although it is socially undesirable. We can say therefore that in this case the contest has a bias to induce excessive winning group size. In fact, in this case the expected winning group size is larger than the socially desirable size.

Symmetrically, if $\sup_{a \in (0, \frac{A^*}{N_{\min}})} \alpha(a) < \chi(n) < 1$ holds on $[N_{\min}, N_{\max}]$, the socially undesirable small group size is realized with a higher probability.

We can conclude therefore that a group contest for a commons good might be biased towards selection of a group with excessive or inadequate winning

group size, depending on the relationship between three values; the elasticity of the marginal cost of effort, the benefit elasticity of congestion and unity¹⁵.

5. The effect of reducing over-exploitation

It has been pointed out by authors like Ostrom (1990) and Stevenson (1991) that resources commonly owned by a well-delineated group are not necessarily over-exploited. The possessing group may devise some rules or institutions to manage the common resource, instead of leaving it under free access. Different devices can be employed, depending on the characteristics of the resource and on the governance power of the group, which result in different levels of the resource exploitation. The outcome of group contests for a commons-good prize would be affected by the existence of such devices. We conclude with some preliminary observations on the effect of such devices that are based on the comparison between the case of efficient utilization vs. that of free access.

Let us suppose then that every competing group has some effective device to achieve efficient utilization of the prize, once it is won. Our assumptions on the benefit function b imply that $x \cdot b(nx)$ is a strictly quasi-concave function of x . There exists therefore a unique level of utilization x_E that maximizes $x \cdot b(nx)$, the per-capita benefit from the prize, which satisfies

$$b(nx_E) + nx_E \cdot b'(nx_E) = 0.$$

The resulting symmetric individual level of utilization x satisfies

$$U'(x \cdot b(nx)) \cdot (b(nx) + x \cdot b'(nx)) + U'(x_E \cdot b(nx_E)) \cdot (n-1) \cdot x_E \cdot b(nx_E) = 0.$$

Let $G^E(n) = U(x_E \cdot b(nx_E))$ be the corresponding benefit of each member. By assumption, each group can accomplish the efficient utilization, once it wins the prize.

Now the benefit elasticity of congestion has the form

$$\chi^E(n) = -\frac{dG^E(n)}{dn} \cdot \frac{n}{G^E(n)} = \delta(x_E \cdot b(nx_E)) \cdot \left\{ n - (n-1) \left(-\frac{nx'_E(n)}{x_E(n)} \right) \right\} = \delta(x_E \cdot b(nx_E)), \quad (6)$$

¹⁵ If $\inf_{a \in (0, \frac{A^*}{N_{\min}})} \alpha(a) > 1 > \chi(n)$ or $\chi(n) < \inf_{a \in (0, \frac{A^*}{N_{\min}})} \alpha(a) < 1$ holds on $[N_{\min}, N_{\max}]$, a larger group ought to hold the prize and the contest also tends to select a larger group as the prize winner. Similarly, if $\chi(n) > 1 > \sup_{a \in (0, \frac{A^*}{N_{\min}})} \alpha(a)$ or $1 < \sup_{a \in (0, \frac{A^*}{N_{\min}})} \alpha(a) < \chi(n)$ holds on $[N_{\min}, N_{\max}]$, a smaller group ought to hold the prize and the contest is inclined to award the prize to such a smaller group. In these cases, the contest outcome is consistent with social desirability.

because $-\frac{nx'_E(n)}{x_E(n)} = 1$. This should be contrasted with (4).

Hence the improvement in utilization of the prize reduces the benefit elasticity of membership in every competing group. By Proposition 1, this change is favorable for larger groups. For example, consider the constant elasticity case used in Section 3. Under free access to the prize, we get that $\chi(n) = \delta_0 \cdot \left(1 + \frac{n-1}{n+1+\beta_0}\right)$ and $\min_{n \in [N_{\min}, N_{\max}]} \chi(n) = \chi(N_{\min})$ is larger than δ_0 . Assume that $\delta_0 < \alpha_0 < \chi(N_{\min})$. When the prize is utilized under the limited user-open access situation, a smaller group attains a higher winning probability. But when efficient utilization prevails, $\chi^E(N_i) = \delta_0$ for all i , and a larger group attains a higher winning probability¹⁶. We conjecture that group schemes reducing over-exploitation of common resources bring advantage to larger groups in contests for such resources.

6. Conclusion

This paper has studied a collective contest for a prize which is allowed to be of a nature that so far has been disregarded; the prize has consumption externalities and is utilized under the limited user - open access situation. Our collective contest is a two-stage game; in stage 1, the group members choose their contribution to the group effort and, in stage 2, the members of the winning group make a decision on their utilization levels of the acquired prize. The contestants' efforts to seek the prize are determined strategically, in anticipation of the open access prize exploitation in stage 2. It has been shown that the introduction of such anarchical utilization of a prize definitely increases the incidence of the group-size paradox.

The two main results (Proposition 1 and Corollary 1) provide sufficient conditions for the validity (invalidity) of the group-size paradox; namely, the situation where a larger group attains a lower (higher) winning probability at a single equilibrium. One secondary issue that we examine is under what conditions the contest is biased in favor of excessive (inadequate) winning group size. The effect of group schemes that secure efficient exploitation of the prize on the group-size paradox

¹⁶ We are comparing equilibrium in which *every* competing group introduces a scheme of efficient management with equilibrium in which free access prevails in the utilization sub-game. When only some of the competing groups move to the regime of efficient utilization, it is clear that such groups can increase their winning probabilities, independent of their size.

is also examined. The results stress the role of two elasticities; the elasticity of the marginal cost of effort and the benefit elasticity of membership.

Appendix

Proof of Lemma 1.

Notice that the equilibrium individual utilization level $x(n)$ in the utilization sub-game satisfy equation (1), and then, $b'(nx(n)) < 0$. This property allows us to apply the implicit function theorem to derive

$$x'(n) \cdot \{(n+1) \cdot b'(nx(n)) + b''(nx(n)) \cdot nx(n)\} + b'(nx(n)) \cdot x(n) + b''(nx(n)) \cdot (x(n))^2 = 0.$$

We therefore get equation (3). Since $b'(nx(n)) < 0$ implies $\beta(n) > 0$,

$$\frac{n}{n+1} < 1 - \frac{1}{n+1+\beta(n)} < 1.$$

On the other hand,

$$\chi(n) = \frac{U'(x(n) \cdot b(nx(n))) \cdot x(n) \cdot b(nx(n))}{U(x(n) \cdot b(nx(n)))} \cdot \left\{ n - (n-1) \cdot \left(-\frac{nx'(n)}{x(n)} \right) \right\}.$$

Hence equation (4) can be easily derived from (3). **Q.E.D.**

Proof of Proposition 1.

Since $\pi_l^* = \pi(N_l, A^*)$ and $\pi_j^* = \pi(N_j, A^*)$ hold at the unique equilibrium of our contest, comparison of the winning probabilities of group l and j can be executed by using $\pi(N_l, A^*)$ and $\pi(N_j, A^*)$. Totally differentiating equation (5)' at $A = A^*$, we get:

$$-\left(G(n) + v''\left(\frac{A^*}{n}\pi\right) \cdot \frac{(A^*)^2}{n} \right) \cdot \frac{\partial \pi}{\partial n} + (1-\pi) \cdot \frac{dG}{dn}(n) + v''\left(\frac{A^*}{n}\pi\right) \cdot \frac{(A^*)^2}{n^2} \pi = 0.$$

Dividing both sides of the above equation by $v'(\frac{A^*}{n}\pi) \cdot A^*$ and using again equation

(5)', we can derive the following fundamental equation:

$$\frac{\partial \pi}{\partial n}(n, A^*) = \frac{\pi}{n} \cdot \frac{(1-\pi) \cdot \left(-\chi(n) + \alpha\left(\frac{A^*}{n}\pi\right) \right)}{\pi + (1-\pi) \cdot \alpha\left(\frac{A^*}{n}\pi\right)}.$$

Hence $\frac{\partial \pi}{\partial n}(n, A^*) < 0$ holds for all $n \in [N_l, N_j]$, if $\inf_{n \in [N_l, N_j]} \chi(n) > \sup_{a \in (0, \frac{A^*}{N_l})} \alpha(a)$.

Geometrically, this means that the graph of $\pi(n, A)$, considering n as a parameter, shifts down with a rise of n , see Figure 1. Along the perpendicular line at the

equilibrium total effort A^* , the graph of $\pi(N_l, A)$ is therefore located above that of $\pi(N_j, A)$. Thus $\pi(N_l, A^*) > \pi(N_j, A^*)$. In short,

$$\pi(N_j, A^*) - \pi(N_l, A^*) = \int_{N_l}^{N_j} \frac{\partial \pi}{\partial n}(n, A^*) dn < 0.$$

Part (b) of the theorem can be proved similarly.

Q.E.D.

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