First and Second Best Voting Rules in Committees*

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"Wasn't he sweet?" said Yossarian. "Maybe they should give him three votes."

Joseph Heller, *Catch-22*

Abstract. A committee of people with common preferences but different abilities in identifying the best alternative (e.g., a jury) votes in order to decide between two alternatives. The first best voting rule is a weighted voting rule that takes the different individual competences into account, and is therefore not anonymous, i.e., the voters' identities matter. Under this rule, it is rational for the committee members to vote according to their true opinions, or informatively. This is not necessarily true for an anonymous voting rule, under which members may have an incentive to vote non-informatively. Thus, strategic, sophisticated voters may vary their voting strategies according to the voting rule rather than naively voting informatively. This paper shows that the identity of the best anonymous and monotone (i.e., quota) voting rule does not depend on whether the committee members are strategic or naive or whether some are strategic and some are naive. One such rule, called the second best rule, affords the highest expected utility in all cases.

1 Introduction

A committee or team is given the task of deciding which of two possible states of the world actually obtains. For example, a jury has to decide whether a defendant is guilty or innocent, or a medical panel must determine whether a patient's condition warrants surgery. The committee may examine various pieces of information that can help reach this goal. However, if the information enables different interpretations and does not conclusively point to one state or the other, then even if the committee members are united in their desire to make the right decision, they may not agree on which decision is right. The members' opinions need not carry equal weights. Some may be more competent than others in identifying the actual state. Moreover, competences, or levels of expertise, may be state-dependent. For example, a juror who strongly believes in the goodness of human nature is more likely to identify the state correctly if the defendant is innocent than if he is guilty. A physician who places great weight on a test that tends to over-diagnose a particular medical condition may have a relatively high rate of success in identifying patients who actually have the condition but a relatively low rate of correctly diagnosing healthy patients. The committee aims to aggregate its members' opinions about the state in an efficient manner, taking into account the different individual competences, the prior probabilities of the two states, and the consequences of the two possible errors (e.g., convicting an innocent defendant and acquitting a guilty one). As shown in [6], it is always

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possible to reach an optimal decision by weighting the members' opinions so as to reflect their competences, and choosing one possible decision or the other according to whether or not the total weight of those favoring the former exceeds a certain threshold. Thus, if all the members vote naively, informatively, i.e., their votes always reflect their true opinions, then information is aggregated efficiently by an appropriately chosen weighted voting rule, which we refer to as the first best rule.[1]

A potential problem with a first best voting rule is that, for committees in which the members' competences differ, the rule is generally not anonymous: different members are assigned different weights. Such a rule may therefore be infeasible if anonymity is required, for example, because votes must be kept confidential. Even if all committee members have the same preferences, restriction to anonymous voting rules may lead to non-informative voting.[2] This is because, in a sense, an anonymous voting rule affords the same power to everyone, including the less competent members of the committee. Some of the latter may choose to suppress their own judgment and vote in a way that does not necessarily reflect their judgment if they think (correctly) that the other members can reach a better decision without them. Such strategic voting may increase the expected utility beyond the level of naive, informative voting. This raises the possibility that the identity of the best voting rule in a second best world, in which anonymity is required, may depend on whether the committee members are expected to vote strategically or naively. A third possibility is that only the more sophisticated members will vote strategically, and will choose their voting strategies to jointly maximize the expected utility, taking into account the voting rule used and assuming that the remaining, less sophisticated members will vote informatively. The main result in this paper shows that the identity of the best anonymous and monotone voting rule is in fact independent of these possibilities. A single such rule, referred to as the second best rule, affords the highest expected utility regardless of whether voting is strategic, naive or mixed.

The distinction between first and second best voting rules applies only to committees in which the members differ in their ability to correctly identify the state. Both rules are the same for a committee in which everyone is equally competent, since the first best rule is anonymous. Most related papers (e.g., [1], [10], [14], [17], [25], [35] and [36]) assume equal competences, and therefore do not consider issues arising when they are not. These issues are the main concern of the present paper.

Austen-Smith and Banks ([1]) demonstrated that the best voting rule for committees with equally competent members is characterized by the property that, under it, naive voting is (Nash) equilibrium behavior in that if all the members vote informatively, none has any incentive to unilaterally switch to a different voting strategy. This paper shows that with unequal competences, one direction in this characterization holds for first best rules and the other holds for second best rules. Since committee members are assumed to have no motivations other than to correctly identify the state, and all agree on the relative costs of the two possible errors, a sufficient (but not necessary) condition for naive voting to be equilibrium behavior is that a first best voting rule is used. In the class of anonymous and monotone voting rules, an essentially necessary (but not sufficient) condition is that the rule is second

[1] Optimal decision-making under the assumption of naive, informative voting has been studied extensively. Earlier studies of two-alternative models include [19], [27], [28] and [34]. Other related papers include [6], [7], [9], [20], [21], [31], [32] and [33]. Extensions of the two-alternative model have been suggested in [4], [5] and [8].

[2] Assuming commonality of preferences avoids confounding this kind of strategic voting with the kind that may result from the misalignment of different individuals' objectives. Strategic voting in the context of non-common preferences is studied, for example, in [12], [14], [15], [16], [17], [22], [23], [24], [26], [30] and [35]. For extensive strategic analysis of voting in committees, see [29].

best. If naive voting is not equilibrium behavior, at least one of the less competent committee members can increase the expected utility by not voting informatively when all the other members do so.

Naive voting is always equilibrium behavior if complete symmetry exists between the two states of the world, which are also treated in a symmetric manner by the voting rule (regardless of whether the rule is anonymous, but assuming it is monotone). Any setting in which naive voting is not equilibrium behavior necessarily involves some sort of asymmetry: differences between the states regarding prior probabilities, the members' abilities to identify the state, the cost of making the wrong collective decision, or treatment by the voting rule (e.g., an asymmetric tie-breaking rule). Asymmetry affects the rationality of naive, informative voting mainly by creating dependence between the state and the probability that each member is pivotal, i.e., his vote actually matters. State-dependent probabilities of being pivotal imply that the member's vote has a different effect in the two states, which may give an incentive to vote in a way that does not reflect the member's true opinion. Conversely, if the two probabilities are equal, it is perfectly rational for a member to act as if the decision is completely determined by his own vote.

This paper's main model is binary in that it does not allow different degrees of certainty about the better decision (a member either believes that one decision is better than the other, or vice versa) or abstentions (the member must vote for one decision or the other). However, both the members' confidence that their opinions are right and the ability to abstain may be important in the context of strategic voting. If different degrees of certainty are allowed, then the minimum level of confidence triggering a particular response (e.g., whether a juror votes "guilty" only if he is absolutely convinced of the defendant's guilt or also if he is only quite convinced) may be chosen strategically. As we show, our main result does not hold in this case: the identity of the best anonymous voting rule <u>may</u> depend on whether or not strategic voting occurs. If abstentions are allowed, they may be used strategically to improve the quality of the collective decision. We show that the improvement is likely to be most dramatic if the committee members' competences are such that, in some first best voting rule, some members have <u>half</u> the voting weight of the others. In this case, strategic abstention by the less competent members may lead to efficient information aggregation even under an anonymous, non-first best voting rule.

2 The Model

An n-person committee, e.g., a jury, must decide whether to accept (decision $+1$) or reject (-1) a particular proposal, e.g., to acquit a defendant. The <u>state of the world</u> may be that the proposal is "good" (state $+1$) or "bad" (-1), e.g., the defendant is innocent or guilty. The state is determined as a random variable z, which equals $+1$ with (prior) probability $0 < p < 1$ and -1 with probability $1 - p$. All committee members have the same utility from the collective decision, which depends on both the decision d and the state z. In both states, the utility is higher if $d = z$ than if $d \neq z$. The difference, which represents the state-dependant cost (for all members) of reaching the wrong collective decision, is denoted by c in state $+1$ and by c^{-} in state -1. Without loss of generality, the costs are normalized so that $c^{-} = 1$. Before the committee reaches its decision, each of its members i observes a random private signal s_i, which is either $+1$ or -1. The n signals, which together constitute the <u>signal vector</u> $\mathbf{s} = (s_1, s_2, \dots, s_n)$, are conditionally independent, given the state. They represent the members' opinions regarding the better decision. These opinions are based on their private information, life experience, and expertise.

We assume that the committee members' signals are not <u>negatively</u> correlated with the state. Equivalently, the <u>error probabilities</u> α_i and β_i for each member i, which are, respectively, the probability that i observes a -1 signal in state $+1$ (i.e., favoring rejection when the proposal is in fact good) and a $+1$ signal in state -1 (favoring acceptance of a bad proposal), satisfy

$$\alpha_i + \beta_i \leq 1 \, . \tag{1}$$

Most related papers (e.g., [1], [17] and [35]) make stronger assumptions about the signals, which imply that $0 < \alpha_i, \beta_i < 1/2$. Furthermore, nearly all of them also assume that the committee members are all equally competent, i.e., they have the same error probabilities. Our assumption is weaker, and does not even exclude the possibility that some members are more likely to be wrong than right.[3] Inequality (1), which can also be written as $1 - \alpha_i \geq \beta_i$ or as $\alpha_i \leq 1 - \beta_i$, only means that the likelihood of a $+1$ signal is at least as high in state $+1$ as in state -1, and that the converse is true for a -1 signal. If $0 < \beta_i < 1$, this can also be written as

$$\mathrm{LR}_i{+} \geq 1 \geq \mathrm{LR}_i{-} \, , \tag{2}$$

where $\mathrm{LR}_i{+} = (1 - \alpha_i)/\beta_i$ is the likelihood ratio of a $+1$ signal for member i and $\mathrm{LR}_i{-} = \alpha_i/(1 - \beta_i)$ is the likelihood ratio of a -1 signal (see [13]). As we show in the Appendix, our assumption (i.e., the assumption that the signals are not negatively correlated with the state) is a necessary and sufficient condition for the following to hold: For every fixed subset of committee members S, the posterior probability of each state weakly increases as the number of members in S observing the corresponding signal increases. Note that such monotonicity is not obvious. Since the identities of the members observing each signal matter, conditioning only on their numbers entails bundling of qualitatively different situations, e.g., a situation in which only the less competent members observe a +1 signal and a situation in which only the more competent members observe that signal.

After the signals are observed, the committee takes a vote. Each committee member i must vote either $+1$ or -1 (see the last section for an extension of the model in which abstention is allowed). A voting strategy for i is a rule that determines his vote x_i as a function σ_i of the private signal, i.e., $x_i = \sigma_i(s_i)$. (To keep our model tractable, we do not consider mixed strategies in this paper.) If $\sigma_i(+1) = +1$ and $\sigma_i(-1) = -1$, then i is said to vote informatively. If $\sigma_i(+1) = \sigma_i(-1)$ ($= +1$ or -1), then i votes non-informatively. These three voting strategies are monotone in that $\sigma_i(+1) \geq \sigma_i(-1)$. There is also one non-monotone voting strategy, which is given by $\sigma_i(+1) = -1$ and $\sigma_i(-1) = +1$. The n-tuple $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ is the committee's strategy profile. The collective decision of the committee is determined by a particular voting (or aggregation) rule, which prescribes either decision $+1$ or -1 for each voting vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. The rule is anonymous if the collective decision does not depend on the voters' identities but only on the number of voters who vote $+1$. This number will be denoted by $\mathbf{x}^+$. The rule is monotone if the following is true for every pair of voting vectors $\mathbf{x}$ and $\mathbf{x}'$: if $x_i \leq x'_i$ for all i and the collective decision $+1$ is prescribed to $\mathbf{x}$, then it is also prescribed for $\mathbf{x}'$. Anonymous and monotone voting rules are called quota (or cutoff) rules. Each such rule corresponds to an integer q, the quota, such that the collective decision is $+1$ if and only if $\mathbf{x}^+ \geq q$. If $q \leq 0$ or $q \geq n + 1$ then the rule is trivial in the sense that the decision is either always $+1$ or always -1, respectively, regardless of the votes. If the number of committee members is odd and $q = (n + 1)/2$, then the rule is the simple majority rule. Weighted voting rules are generalized quota rules. They are monotone but generally not anonymous. In such a rule, each member i is assigned a fixed voting weight $w_i \geq 0$ and the collective decision is $+1$ if and only if the sum of the weights of the members voting $+1$ equals or exceeds some fixed real number q.

[3] For example, if $\alpha_i = 2/3$, $\beta_i = 1/4$ and $p = 3/4$, the (unconditional) probability that member i's signal is incorrect is $3/4 \cdot 2/3 + (1 - 3/4) \cdot 1/4 = 9/16$ ($> 1/2$).

3 Efficient Information Aggregation

The committee's decision-making process aggregates information efficiently (or completely) if, for every signal vector, the decision reached maximizes the conditional expected utility, given the signals. A first best voting rule is a rule under which information is aggregated efficiently if all the members vote informatively. Such a rule is not necessarily unique. However, multiple first best rules exist only if there are signal vectors that cannot possibly occur or for which both decisions give the same conditional expected utility. One first best rule has a particularly simple form.

Theorem 1. *There is always a first best rule that is a weighted voting rule*.

This result is proved in [6] under slightly more restrictive assumptions than in this paper. A proof suitable for the present setting is given in the Appendix, along with the proofs of all the other results in this paper. The members' weights in the first best voting rule reflect their competences. Specifically, we show in the Appendix that the weight w_i of each member i can usually be written as the logarithm of LR_{i+} / LR_{i-}, the quotient of the likelihood ratios of the $+1$ and -1 signals for member i. This quotient, which ultimately depends only on the error probabilities α_i and β_i, may be viewed as a measure of the quality, or informativeness, of i's signal. The higher the quotient, the greater the effect of i's signal on the (posterior) probabilities of the two states.

4 Anonymity

It follows from Theorem 1 that a first best voting rule is generally not anonymous. Committee members with different competences may have different voting weights, making their votes non-interchangeable. This may be a problem in situations (such as voting on an unpopular proposal) in which considerations of confidentiality or simplicity favor anonymity, thus raising the question of which anonymous voting rule is best.[4] A conceivable complication in identifying the best such rule is that anonymity may give rise to strategic voting, whereby one or more committee members vote non-informatively in order to increase the expected utility of the collective decision. This is demonstrated by the following simple example, in which an incentive to vote non-informatively exists under any non-trivial anonymous voting rule.

[4] One interpretation of the difference between unrestricted and anonymous voting rules, suggested to us by one of the referees, is the possibility or impossibility, respectively, of a communication phase prior to voting, in which committee members reveal their information so that its relative quality can be assessed. Since the committee members are assumed to have common preferences, the problem of misrepresentation, which is a central theme in the literature on strategic deliberation (e.g., [2], [3], [11] and [18]), does not arise. However, there may be non-strategic reasons for committee members not to speak their mind, such as a reluctance to express unpopular views or social dynamics, whereby less experienced members are overly influenced by the more experienced or more eloquent members. The effects of pre-voting communication on the members' voting may also depend on the nature of the signals. We interpret the signals as the members' opinions regarding the better decision, without precisely specifying the basis for these opinions. However, the consequences of information exchange may depend on whether the opinions are based mainly on factual knowledge, which is relatively easy to communicate, or intuition and gut feelings, which may be valuable reflections of the members' life experiences but are less easily passed on to others. The assessment of the signals' quality, which determines the members' weights in the first best, non-anonymous voting rule, can be based on content in the former case but only on reputation or credentials in the latter.

Example 1. A two-person committee has to identify the state of the world. Member 1, with $0 < \alpha_1, \beta_1 < 1$, does not always identify the state correctly, whereas member 2, with $\alpha_2 = \beta_2 = 0$, always does. A first best voting rule clearly assigns greater weight to member 2, whereas any anonymous voting rule by definition assigns equal weight to both members, and thus always prescribes the same decision when the votes differ. This implies that unless the anonymous rule is trivial, member 1 has an incentive to vote non-informatively. Specifically, if the decision in case of a disagreement is $+1$ or -1, and member 2 votes informatively, member 1 can increase the probability of a correct collective decision by disregarding his signal and always voting -1 or always voting $+1$, respectively. To see this, let the decisions when zero, one, or two members vote $+1$ be denoted by d_0, d_1, and d_2, respectively. Without loss of generality, $d_1 = +1$ (the analysis of the case $d_1 = -1$ is similar). If member 1 switches from informative voting to always voting -1, the collective decision is affected if and only if member 1's signal is $+1$ and either (i) the state of the world is -1 and $d_0 = -1$ or (ii) the state of the world is $+1$ and $d_2 = -1$. In both cases, the collective decision is changed from the wrong to the right one. If the voting rule is non-trivial, then (since $d_1 = +1$ by assumption) $d_0 = -1$ or $d_2 = -1$ (or both), and member 1's change of voting strategy thus has a positive probability of turning an incorrect collective decision into a correct one, and never has the opposite effect.

Example 1 illustrates an important aspect of strategic voting in the context of common preferences under an anonymous voting rule. Namely, rather than being a bad thing, strategic voting has the potential of increasing the expected utility to above the level of informative voting by all committee members. In Example 1, non-informative voting by one member only is required. However, as the next example shows, it may take several members to make a positive change.

Example 2. A three-person committee has to identify the state of the world. The prior probability and the cost of mis-identification are the same in both states (i.e., $p = 1/2$ and $c = 1$). The members' error probabilities are given by $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 1/3$ and $\alpha_3 = \beta_3 = 0$. With informative voting by all three members, the anonymous voting rule under which the probability of a correct identification of the state is greatest, and equals $1 - (1/3)^2 = 8/9$, is the simple majority rule. Under this rule, none of the members has an incentive to unilaterally switch to non-informative voting if the others vote informatively. If either member 1 or 2 deviates by always voting $+1$ or always voting -1, the probability of a correct identification decreases to $1 - 1/2 \cdot 1/3 = 5/6$, since one state is always correctly identified while the other is incorrectly identified with probability $1/3$. If only member 3 votes non-informatively, the probability is even lower, and equals $1/2 \cdot ((1 - (1/3)^2) + (1 - 1/3)^2) = 2/3$. However, if <u>both</u> members 1 and 2 deviate, and one of them always vote $+1$ and the other -1, the state is always correctly identified.

What strategy profile affords the highest expected utility under a given quota rule? The above examples suggest that some committee members should vote informatively and others non-informatively. Intuitively, the latter would be expected to be the less competent members, whose signals are less reliable indicators of the state of the world. However, since a member's error probabilities may be different in the two states, it is not generally possible to rank committee members according to them. Nevertheless, it is possible to <u>partially</u> order the members by saying that member *i* is <u>less competent</u> than *j* (and *j* is more competent than *i*) if $\alpha_i \geq \alpha_j$ and $\beta_i \geq \beta_j$ and at least one inequality is strict. (If both hold as equalities, the members are <u>equally competent</u>.) Member *i* has <u>minimal competence</u> if none of the others is less competent than *i*. The next proposition confirms this intuition by showing that, if *i* is less competent than *j*, then informative voting by *j* is always at least as good as by *i*. It also shows that it is not necessary to ever use the non-monotone voting strategy.

Proposition 1. *Suppose that a monotone voting rule (whether anonymous or not) is used. For every strategy profile, the following assertions hold for every member i:*

(i) *If i's voting strategy is non-monotone, it can be changed to a monotone strategy without decreasing the expected utility.*

(ii) *If i's voting strategy is monotone, and there is some committee member j more competent than i who votes non-informatively, then switching i and j's voting strategies does not decrease the expected utility.*

Proposition 1 does not completely specify the strategy profile yielding the highest expected utility. However, it may help narrow the search for it. In particular, if each member i is less competent than member $i+1$ or the two are equally competent, for $i = 1, 2, \dots, n-1$, there exists a utility-maximizing strategy profile of the following form: For some k_1 and k_2, with $0 \le k_1 \le k_2 \le n$, every member i with $i \le k_1$, $k_1 < i \le k_2$, or $k_2 < i$ votes +1 regardless of his signal, votes −1 regardless of his signal, or votes informatively, respectively. Note that the best strategy profile found in Example 2 has this form.

5 Second Best Rules

The examples in the previous section raise the possibility that the committee's behavior under an anonymous, non-first best rule may depend on whether its members are naive or strategic. Naive voting means that everyone simply votes informatively. Strategic (or sophisticated) voting means that the committee's strategy profile maximizes the expected utility under the voting rule used. Since our model does not allow for communication between committee members, which could be used for conditioning the votes on other members' signals, choosing such a strategy profile is the members' only possible mode of (tacit) cooperation. A more general possibility is that some members are strategic and others are naive. Strategic voting by the group of strategic members S (which in extreme cases may be the entire committee or an empty set) means that the strategies of the members in S (each of whom may vote informatively or non-informatively) jointly maximize the expected utility under the voting rule used and under the assumption that the members not in S will vote informatively. The expected utility thus achieved will be called the S-maximum under the voting rule. This is clearly determined by the set S as a monotone, non-decreasing function.

The possibility of strategic voting raises the question of whether, in a second best world in which only anonymous and monotone voting rules can be used, the voting rule should be chosen under the presumption that the committee will vote strategically, or vote naively, or that only a particular group of members (the more sophisticated ones) will vote strategically. Our main result in this paper shows that all of these possibilities essentially lead to the same voting rule. Although strategic voting may well affect the outcome (and, particularly, the efficiency) of the decision-making process, it does not affect the identity of the quota rule that affords the best outcome.

Theorem 2. *There is an integer q such that, for every group of committee members S, the S-maximum under the quota rule with quota q is greater than or equal to that under any other quota rule.*

A quota rule as in Theorem 2 will be referred to as a second best rule. The proof of the theorem, which is given in the Appendix, shows that the second best rule(s) can be identified by computing the expected utility for different values of the quota q (which specifies the minimum number of +1 votes required for the decision to be +1) in the special case in which S is empty, i.e., the committee votes naively. Lemma 2 in the Appendix and the proof of Theorem 2 show that the expected utility is determined by q as a unimodal function, whose peak is at the second best rule(s). The following example illustrates this, and the theorem itself.

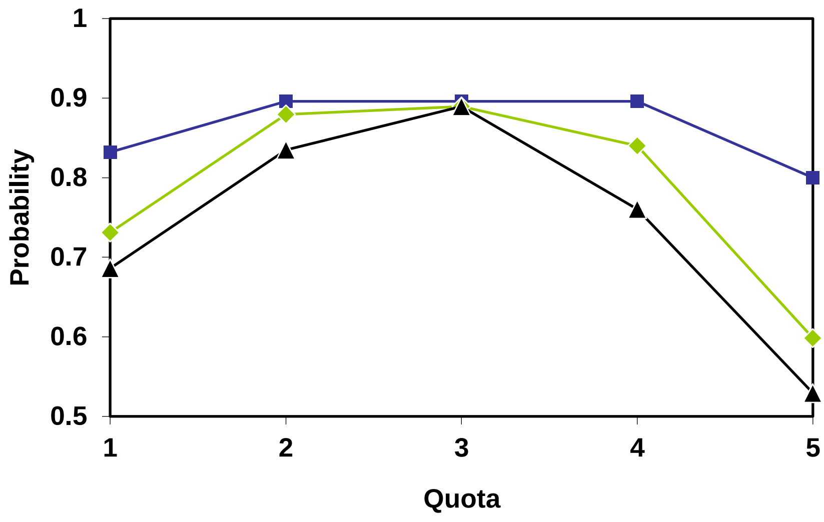

Figure 1 Quota rules in Example 3. The probability of a correct identification of the state under each value of the quota q is shown for: naive voting (triangles), strategic voting by member 1 only (diamonds), and strategic voting by the entire committee (squares). Naive voting means that everyone votes informatively. Strategic voting by a group of members means that their strategies jointly maximize the probability of a correct identification of the state under the quota q with the other members voting informatively. The probability of a correct identification is always highest under the second best rule ($q = 3$, simple majority).

Example 3. A five-person committee has to identify the state of the world. The two states have different prior probabilities, given by $p = 0.6$ and $1 - p = 0.4$, but equal costs of mis-identification (i.e., $c = 1$). Members 1 and 2, with $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 0.35$, are less competent than members 3, 4 and 5, with $\alpha_3 = \beta_3 = \alpha_4 = \beta_4 = \alpha_5 = \beta_5 = 0.2$. Straightforward (if somewhat tedious) computation shows that state +1 has higher posterior probability than –1 if and only if at least two of the three more competent members observe the signal +1, or at least one of them observes +1 and so do both members 1 and 2. If follows that the weighted voting rule with $w_1 = w_2 = 1/2$, $w_3 = w_4 = w_5 = 1$, and $q = 2$ is first best (this can also be deduced from the explicit expression for the first best rule given in the proof of Theorem 1). Under this rule, the probability of a correct identification of the state with naive voting is 0.902. None of the anonymous voting rules is as good. The second best is the simple majority rule, under which the probability of correctly identifying the state with naive voting is 0.890. Strategic voting can improve on this. With non-informative voting by the two less competent members (only), where one always votes +1 and the other –1, the probability of a correct identification under the simple majority rule increases to 0.896, which is the highest probability that can be obtained by strategic voting under any quota rule. Thus, the efficient level of 0.902 cannot be obtained under a quota rule even with strategic voting. Figure 1 shows the probability of a correct identification of the state under all non-trivial quota rules for: (i) naive voting, (ii) strategic voting by member 1 only, and (iii) strategic voting by the entire committee. The highest expected utility is obtained under the second best rule in all three cases.

The second best voting rule in Example 3 is different from the first best rule, since the latter is not anonymous. The decision-making process thus does not aggregate information efficiently with naive voting under the second best rule. The same is true for the two previous examples. However, Example 3 differs in that the decision-making process under the second best rule (and hence also under any other quota rule) is inefficient also if a group of committee members (or the entire committee) votes strategically. According to Theorem 1, efficiency requires the use of a weighted voting rule, with different weights reflecting the members' diverse competences. In a quota rule, all the weights are unity. Strategic voting improves on this by effectively allowing zero weights: A member voting non-informatively in effect sets his own voting weight to zero, and may also change the quota. Therefore, it follows from Theorem 2 that efficiency can be achieved by strategic voting under the second best rule if and only if there is a first best weighted voting rule with all the weights 0 and 1. This is the case in Examples 1 and 2 (where only the expert should be assigned a unit weight), but not in 3.

6 Equilibrium

Strategic voting does not require that committee members communicate with each other. In principle, they could reach an identical conclusion regarding the best strategy profile by independently analyzing the strategic situation. However, reaching such silent agreement is arguably less likely if (as in Example 2) it involves non-informative voting by several committee members or if there is more than one strategy profile that maximizes the expected utility. In such cases, naive voting is perhaps not unlikely. On the other hand, naive voting is unlikely if it is not equilibrium behavior: there is a committee member who can increase the expected utility by unilaterally switching to another voting strategy when all the other members vote informatively. Thus, naive voting is said to be equilibrium behavior when informative voting by all committee members is a Nash equilibrium in the (strategic form) game in which the committee members choose their voting strategies and the common payoff is the expected utility of the collective decision. Whether naive voting is equilibrium behavior depends on the voting rule used. This is always true under a first best voting rule, since by definition the rule achieves the highest possible expected utility when everyone votes informatively. Thus, we observe the following. (Although the assertion is nearly obvious, for completeness we give a formal proof in the Appendix.)

Proposition 2. *Under a first best voting rule, naive voting is equilibrium behavior. Indeed, no strategy profile gives a higher expected utility than informative voting by all the members.*

Similar assertions do not hold for a second best voting rule. This is shown by Example 1, where naive voting is not equilibrium behavior under any non-trivial anonymous voting rule. In Example 3, naive voting is equilibrium behavior under the second best rule but not under the other non-trivial quota rules, all of which give member 1 an incentive to vote non-informatively. Clearly, such an incentive does not exist under a trivial voting rule, where the members' votes have no effect on the decision. Another kind of setting in which no single member can ever affect the collective decision is one in which a large number of members always observe the same state-independent signals and vote accordingly. These two settings share the feature that there is some decision (either $+1$ or -1) which is always reached, in both states, if at least $n - 1$ members vote informatively. If such a decision does not exist, and if naive voting is equilibrium behavior, then we will say that it is non-trivially so. The following proposition asserts that the only quota rules for which this may occur are the second best ones. Such rules therefore necessarily satisfy the condition in Theorem 2, i.e., they are optimal regardless of whether the committee votes naively or strategically.

Proposition 3. *Suppose that a quota rule is used. If naive voting is non-trivial equilibrium behavior, then the rule is second best.*

Checking whether naive voting is equilibrium behavior may be simplified by using Proposition 1, which implies that this is so if and only if none of the members with minimal competence can increase the expected utility by switching to a non-informative voting strategy. For example, the members with minimal competence in Example 3 are 1 and 2, and it thus suffices to check the consequences of non-informative voting by one of them.

Proposition 3 constitutes a partial converse to Proposition 2 for committees in which all the members are equally competent, so that the first and second best voting rules coincide. Thus, as already shown by Austen-Smith and Banks [1], an essentially necessary and sufficient condition for a non-trivial quota rule to be the (first and second) best one for such committees is that naive voting is equilibrium behavior under this rule.

7 Symmetry

Example 2, which shows that naive voting may be non-trivial equilibrium behavior also under a rule that is not first best, can be generalized. As the following proposition shows, naive voting is equilibrium behavior whenever there is complete symmetry between the two states of the world and the voting rule treats $+1$ and -1 votes symmetrically. This is mainly because, under these assumptions, the probability of each member being pivotal is the same in both states. Being pivotal means that the collective decision would be different if the member changed his vote. If the probability of being pivotal in one state is higher than in the other, it may be rational for the member to presume that the first state obtains, and to vote accordingly, regardless of the signal. This cannot happen in a symmetric setting and under a voting rule as above.

Proposition 4. *Suppose that the two states of the world are symmetric in that the prior probability, the cost of making the wrong collective decision and the members' error probabilities are the same in both states. Naive voting is then equilibrium behavior under any monotone voting rule (whether anonymous or not) that is neutral in that if all members reverse their votes the committee's collective decision is reversed.*

In the class of anonymous voting rules, the only monotone and neutral one (which only exists with an odd number of committee members) is the simple majority rule. By Proposition 4, if this rule is used and there is complete symmetry between the two states of the world, then naive voting is equilibrium behavior. In view of Proposition 3, this suggests that the simple majority rule is second best. The following proposition confirms this. Like Proposition 4, it only assumes symmetry between the two states. The n committee members do not have to be equally competent (cf. Example 2).

Proposition 5. *Suppose that the symmetry assumption in Proposition 4 holds. Then, a quota rule with quota $n/2$ or $n/2+1$ (n even) or $(n+1)/2$ (n odd) is a second best rule.*

8 A Richer Signal Space

A natural generalization of the above model is to allow for different degrees of confidence. Each member i observes a signal s_i that is a number between -1 and $+1$. The closer the signal is to either extreme, the surer is i that the corresponding decision (-1 or $+1$) is better. A zero signal indicates that the member does not incline to either side. A natural generalization of (1) (or (2)) is the assumption (essentially, a monotone likelihood ratio property; see [10], [14] and [25]) that, for each member i, the conditional probability that the state is $+1$, given i's signal s_i, can be expressed as a nondecreasing function of s_i. Such a richer signal space allows for more voting strategies. For example, a member's strategy may be to vote $+1$ if and only if he is almost sure that the state is $+1$ (e.g., $s_i \geq 0.95$). As shown below, such non-binary signals are qualitatively different from binary ones. In particular, Theorem 2 does not hold in this more general setting. Thus, the best anonymous and monotone voting rule may depend on whether voting is strategic or naive.

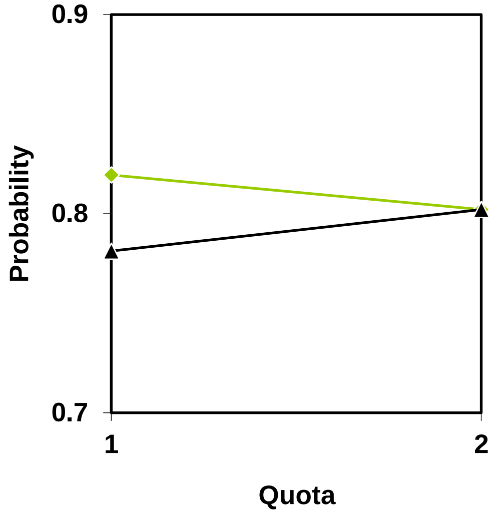

Figure 2 Quota rules in Example 4. The probability of a correct identification of the state under each value of the quota q is given for: naive voting (triangles), and strategic voting by member 2 only (diamonds). Unlike member 1, who may only observe a +1 or –1 signal, member 2 may also observe a third signal, which is positively correlated with the +1 state. Naive voting entails that 2 votes +1 in this case. With strategic voting, the vote may be either +1 or –1, depending on the quota.

The latter statement requires some elaboration. If there are more than two possible signals, but only two ways to vote, what does naive voting mean? One possible meaning is that a member observing a positive or negative signal (assuming, for simplicity, that zero signals do not occur) votes +1 or –1, respectively. Another possibility is to interpret naive voting as <u>sincere voting</u> ([1]): voting +1 or –1 according to whether, given the observed signal, the conditional expected utility of deciding +1 is greater or less than that for –1. However, sincere voting is not the same as naive voting even if there are only two possible signals. In addition, it is not really naive, since it depends on the data: the prior probability of the two states, the cost of making the wrong collective decision in each state and the member's error probabilities. However, as the following example demonstrates, this ambiguity is inconsequential. Theorem 2 fails under <u>any</u> reasonable interpretation of "naive voting".

Example 4. A two-person committee has to identify the state of the world. The prior probabilities and the costs of mis-identification are the same for both states (i.e., $p = 1/2$ and $c = 1$). In both states, member 1's signal can be either +1 or –1. The corresponding error probabilities are $\alpha_1 = 1/3$ and $\beta_1 = 1/4$. For member 2, the probabilities of not observing the signal corresponding to the true state are similar, $\alpha_2 = 1/3$ and $\beta_2 = 1/4$. However, when this happens, either in state +1 or –1, member 2 does not observe the signal corresponding to the other state but rather a third, distinct signal, "1/7". Simple computation shows that, when 2 observes this signal, state +1 has a higher conditional probability than state –1, and the conditional expected utility of deciding +1 is one-seventh of a unit higher than for –1. Therefore, naive voting, regardless of what it means in general, must entail that in this particular case member 2 votes +1 (in particular, this is so if 2 votes sincerely). Hence, with naive voting, member 2 always votes +1 in state +1, and does so with probability 1/4 in state –1. What about strategic voting? Suppose that only member 2 votes strategically (member 1 votes informatively) and a non-trivial quota rule is used, i.e., $q = 1$ or 2. If 2's signal is +1 or –1, then the signal represents the true state and 2 should vote accordingly. If the signal is 1/7, member 2's optimal vote depends on the quota: if $q = 1$ or 2, the expected utility is higher if he votes –1 or +1, respectively. Thus, strategic voting by member 2 alone is better than naive voting if $q = 1$ but gives the same result if $q = 2$. Figure 2 shows the probability of a correct identification of the state under each of the two non-trivial quota rules for: (i) naive voting, and (ii) strategic voting by member 2 only. As the figure shows, $q = 2$ is better than $q = 1$ in case (i), but worse than $q = 1$ in case (ii). Thus, neither rule is an unqualified "second best".

9 Abstaining from Voting

Another direction in which our model can be extended is allowing members to abstain. The definition of a weighted voting rule (and, as a special case, quota rule) can be generalized to accommodate abstention. For example, it may be defined as a voting rule prescribing the decision $+1$ when the difference between the total weights of the members voting $+1$ and those voting -1 exceeds some threshold.[5] Thus, the collective decision is a function of

$$\sum_i w_i x_i \,,$$

or equivalently a function of

$$\sum_i w_i \frac{x_i + 1}{2} \,,$$

where $x_i = 0$ if member i abstains. The latter expression equals the total weight of the members voting $+1$ plus one-half the total weight of those abstaining. Thus, in a sense, abstaining is equivalent to half supporting and half opposing the proposal.[6]

The possibility of abstaining may be used strategically. For example, a member may choose to abstain rather than vote against a proposal he considers bad. The potential advantage of doing this under an anonymous voting rule lies in the indication given by the abstention about the identity of the opposing member. As the following example shows, this may lead to a better collective decision.

Example 5. Consider again the setting in Example 3. As shown, even with strategic voting, information is not aggregated efficiently under the second best rule, the simple majority rule in this case, or any other quota rule. However, efficiency can be achieved with strategic abstention. Member 1 should vote $+1$ if his signal is $+1$ but should abstain if it is -1. Member 2's optimal voting strategy depends on the tie-breaking rule used (which is assumed to be deterministic). A "simple majority rule" may mean that the decision is $+1$ if and only if the number of $+1$ votes is greater than the number of -1 votes, or

[5] An alternative is to consider the ratio between the two total weights, or some other function. The space of applicable anonymous voting rules is considerably larger than without abstentions. This makes the issue of second best rules, which we do not pursue in this paper, more difficult to analyze than in the latter case.

[6] Abstention can also be defined this way. This definition can be extended by also allowing members to divide their votes in ways other than half–half. Chakraborty and Ghosh [10] showed that efficiency may be improved by allowing divisible votes. Their explanation for this is that allowing divisibility overcomes the problem of dimensionality: the rich signal space they consider may have more than two elements. The example and discussion below show that, for a committee in which the members have different competences, efficiency may be improved by abstention also when the problem is not dimensionality of the signal space (since the number of signals equals the number of possible votes) but anonymity of the voting rule.

also if they are equal. In the former case, member 2's optimal strategy is the same as 1's. In the latter, it is to abstain or vote -1 if the signal is $+1$ or -1, respectively. With these strategies and informative voting by the three more competent members 3, 4 and 5, information is aggregated efficiently: the probability of a correct identification of the state is 0.902. This is because, under both versions of the simple majority rule, the collective decision is +1 if and only if at least two of the three more competent members observe the signal $+1$, or at least one of them observes $+1$ and so do both members 1 and 2. Thus, the decision coincides with that under the first best rule with informative voting.

As indicated above, abstention may be viewed as halving the vote. Strategic abstention (which is a strategy, not an action) effectively halves the member's voting weight. This may help explain how strategic abstention can increase the expected utility beyond the level achievable with strategic voting. As explained above, informative and non-informative voting correspond to weights of 1 and 0, respectively. Strategic abstention corresponds to $1/2$. This applies to the strategy of abstaining upon observing -1 as well as to abstention prompted by a $+1$ signal (which are member 2's two possible strategies in Example 5). Indeed, changing from the former to the latter only leads to subtraction of a constant (unity) from the difference between the total number of $+1$ and -1 votes. It follows that the potential benefit of strategic abstention is likely to be greatest when, as in Example 3, the first best rule can be expressed as a weighted voting rule with only two weights, which are in a two-to-one ratio. In this case, strategic abstention by the members with the lower weight can lead to efficient information aggregation.

Appendix

The appendix presents the proofs of the two theorems and five propositions in this paper. In the proofs, the assumption that the signals are not negatively correlated with the state plays a central role. This assumption, (1), implies that if member i observes both signals with positive probability (i.e., $0 < \mathbf{P}(s_i = +1), \mathbf{P}(s_i = -1) < 1$), then

$$\mathbf{P}(z = +1 \mid s_i = +1) \geq \mathbf{P}(z = +1 \mid s_i = -1) . \tag{3}$$

In other words, the conditional probability that the state of the world z is $+1$, given that i's signal s_i is $+1$, is at least as high as that for the signal -1. This result can be generalized. It follows from the next lemma as a special case (namely, $l = m = \tilde{n} = 0$) that the posterior probability of each state weakly increases with the number of committee members observing the corresponding signal. Obviously, a similar result holds for any fixed subset of committee members S. Thus, the posterior probability of each state weakly increases with the number of members in S observing the corresponding signal. This result generalizes (3), which says the same when S is the singleton containing only member i. Therefore, the assumption that (1) holds for every i is a necessary and sufficient condition for a positive relation, in each subset of committee members, between the number of occurrences of each signal and the posterior probability of the corresponding state. The lemma's assertion is in fact stronger than that. For example, it allows conditioning on the event that the number of $+1$ signals outside S, i.e. in the complementary set $\tilde{S}$ (or alternatively in some fixed subset of S), is equal to some fixed number m. The stronger version is needed for the proof of Theorem 2.

Lemma 1. *For a given subset of committee members $\tilde{S}$, with $\tilde{n}$ members* ($0 \leq \tilde{n} \leq n$), *let* $\tilde{\mathbf{s}}$ *be the part of the signal vector* $\mathbf{s}$ *consisting only of the signals of the members in* $\tilde{S}$, *and* $\tilde{\mathbf{s}}^+$ *the number of* $+1$ *signals in that part*. *For any interval of integers of the form* $I = \{l, l + 1, \dots, m\}$, *with* $0 \leq l \leq m$, *and any pair of integers r and q with $r > q$, if* $\mathbf{P}(\mathbf{s}^+ = r, \tilde{\mathbf{s}}^+ \in I) > 0$ *and* $\mathbf{P}(\mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I) > 0$, *then*

$$\mathbf{P}(z = +1 \mid \mathbf{s}^+ = r, \tilde{\mathbf{s}}^+ \in I) \geq \mathbf{P}(z = +1 \mid \mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I)\,. \tag{4}$$

Since the proof of Lemma 1 is rather lengthy, we present it at the end of the appendix.

Proof of Theorem 1. First, consider the case in which the error probabilities satisfy $0 < \alpha_i, \beta_i < 1$ for all i.

Suppose that all committee members vote informatively. For every voting vector $\mathbf{x}$ occurring with positive probability (i.e., the probability $\mathbf{P}(\mathbf{s} = \mathbf{x})$ that the random signal vector $\mathbf{s}$ equals $\mathbf{x}$ is greater than zero), the conditional expected utility of deciding +1, given that the members' signal vector equals $\mathbf{x}$, is less than that for –1 if and only if

$$c\,\mathbf{P}(z = +1 \mid \mathbf{s} = \mathbf{x}) < \mathbf{P}(z = -1 \mid \mathbf{s} = \mathbf{x})\,.$$

The left-hand side of this inequality is the cost c of deciding –1 when the state is actually +1 multiplied by the conditional probability that the state is +1. The right-hand side is similar, with –1 and +1 interchanged. (The corresponding cost is assumed to be unity.) By Bayes' rule, the inequality is equivalent to

$$c\,p\,\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) < (1 - p)\,\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = -1)\,. \tag{5}$$

The conditional probability on the left-hand side of (5) equals

$$\prod_{\substack{i \\ x_i = +1}} (1 - \alpha_i) \prod_{\substack{i \\ x_i = -1}} \alpha_i\,. \tag{6}$$

(The first product involves all the members i voting +1, and the second those voting –1.) The logarithm of (6) equals

$$\sum_{\substack{i \\ x_i = +1}} \log\,(1 - \alpha_i) \;+\; \sum_{\substack{i \\ x_i = -1}} \log\,\alpha_i\,,$$

which can also be written as

$$\sum_{\substack{i \\ x_i = +1}} \log \frac{1 - \alpha_i}{\alpha_i} \;+\; \sum_{i} \log\,\alpha_i\,.$$

The logarithm of the conditional probability on the right-hand side of (5) is given by a similar expression, in which α_i is replaced (three times) by $1 - \beta_i$. Therefore, taking the logarithm of both sides of (5) and rearranging gives the following equivalent inequality:

$$\sum_{\substack{i \\ x_i = +1}} w_i < q\,,$$

where

$$w_i = \log \frac{(1 - \alpha_i)(1 - \beta_i)}{\alpha_i \beta_i} \quad \text{and} \quad q = \log \frac{1 - p}{c\,p} + \sum_i \log \frac{1 - \beta_i}{\alpha_i}\,.$$

These w_i's and q define a weighted voting rule that prescribes the decision +1 to a voting vector $\mathbf{x}$ if and only if this decision maximizes the conditional expected utility, given that the signal vector equals $\mathbf{x}$. This voting rule is, by definition, first best.

It follows from (1) that the voting weight w_i of each member i is nonnegative. The weight is zero if and only if $\alpha_i + \beta_i = 1$, which holds if and only if i's signal and the state are (statistically) independent. The voting weight can also be written as the logarithm of the quotient of the likelihood ratios of i's two possible signals:

$$w_i = \log \frac{(1-\alpha_i)\,/\,\beta_i}{\alpha_i\,/\,(1-\beta_i)} = \log \frac{\mathrm{LR}_{i}+}{\mathrm{LR}_{i}-}\,.$$

It remains to dispense with the initial assumption that α_i and β_i are not equal to 0 or 1. This can be preformed by approximating the error probabilities α_i and β_i of each member i by a pair $\hat{\alpha}_i$ and $\hat{\beta}_i$ that satisfies $0 < \hat{\alpha}_i, \hat{\beta}_i < 1$ and $\hat{\alpha}_i + \hat{\beta}_i = \alpha_i + \beta_i$ (≤ 1). These approximations can be arbitrarily close. Therefore, the continuity of (6) in the α_i's and of the analog expression in the β_i's implies that the $\hat{\alpha}_i$'s and $\hat{\beta}_i$'s can be chosen in such a way that the corresponding random signal vector $\hat{\mathbf{s}}$ satisfies the following condition: For every voting vector $\mathbf{x}$ that satisfies (5),

$$c\,p\,\mathbf{P}(\hat{\mathbf{s}} = \mathbf{x} \mid z = +1) < (1-p)\,\mathbf{P}(\hat{\mathbf{s}} = \mathbf{x} \mid z = -1)\,, \tag{7}$$

and similarly with the inequalities in (5) and (7) reversed. As shown above, for the approximate error probabilities ($\hat{\alpha}_i$'s and $\hat{\beta}_i$'s) there is some weighted voting rule that is first best. The <u>same</u> rule is also first best for the original probabilities (α_i's and β_i's). This is because, if $\mathbf{x}$ is such that $+1$ is a worse decision than -1 when the signal vector $\mathbf{s}$ equals $\mathbf{x}$ (i.e., (5) holds), then, by virtue of (7), the voting rule under consideration prescribes the decision -1 for $\mathbf{x}$. Similarly, this rule prescribes the decision $+1$ whenever it is a strictly better collective decision than -1. ■

Proof of Proposition 1. With fixed strategies for the other committee members, consider the effect of different voting strategies for i on the expected utility. The difference between the expected utility if i votes informatively or if he votes -1 regardless of his signal is given by

$$c\,p\,\mathbf{P}(s_i = +1, i \text{ is pivotal} \mid z = +1) - (1-p)\,\mathbf{P}(s_i = +1, i \text{ is pivotal} \mid z = -1)\,. \tag{8}$$

This is because i's vote matters only when he is pivotal, i.e., the decision is $+1$ or -1 if i's vote is $+1$ or -1, respectively. Similarly, the difference between the expected utility if i votes informatively or if he always votes $+1$ is

$$-c\,p\,\mathbf{P}(s_i = -1, i \text{ is pivotal} \mid z = +1) + (1-p)\,\mathbf{P}(s_i = -1, i \text{ is pivotal} \mid z = -1)\,. \tag{9}$$

Whether i is pivotal depends only on the votes, and hence on the signals, of the other members. Since the signals are conditionally independent, given the state of the world, (8) and (9), respectively, are equal to

$$c\,p\,(1-\alpha_i)\,\mathbf{P}(i \text{ is pivotal} \mid z = +1) - (1-p)\,\beta_i\,\mathbf{P}(i \text{ is pivotal} \mid z = -1) \tag{10}$$

and

$$-\,c\,p\,\alpha_i\,\mathbf{P}(i \text{ is pivotal} \mid z = +1) + (1-p)\,(1-\beta_i)\,\mathbf{P}(i \text{ is pivotal} \mid z = -1)\,. \tag{11}$$

The difference between the expected utility if i votes informatively or if he uses the non-monotone strategy of voting the opposite of his signal is given by the <u>sum</u> of (8) and (9), or equivalently the sum of (10) and (11). The latter two expressions cannot <u>both</u> be (strictly) negative, since this would imply that

$$\begin{aligned}
&\max\{c\,p\,\mathbf{P}(i \text{ is pivotal} \mid z = +1), (1-p)\,\mathbf{P}(i \text{ is pivotal} \mid z = -1)\} \\
&\qquad < c\,p\,\alpha_i\,\mathbf{P}(i \text{ is pivotal} \mid z = +1) + (1-p)\,\beta_i\,\mathbf{P}(i \text{ is pivotal} \mid z = -1) \\
&\qquad \le (\alpha_i + \beta_i)\max\{c\,p\,\mathbf{P}(i \text{ is pivotal} \mid z = +1), (1-p)\,\mathbf{P}(i \text{ is pivotal} \mid z = -1)\}\,,
\end{aligned}$$

which is not consistent with (1). Therefore, there are only three possibilities: both (10) and (11) are nonnegative, in which case i maximizes the expected utility by voting informatively; only (10) is negative, in which case the maximum expected utility is attained if i always votes -1; or only (11) is negative, in which case the maximum expected utility is attained if i always votes $+1$. This proves assertion (i).

To prove (ii), consider a member j who is more competent than i and votes non-informatively. If i also votes non-informatively, then interchanging their voting strategies clearly has no effect on the expected utility. If i votes informatively, then the interchange has the same effect as changing i's error probabilities to α_j and β_j. Therefore, it suffices to show that such a change of error probabilities (which makes i more competent) cannot decrease the expected utility. Consider (10), which gives the difference between the expected utility with informative voting by i and the expected utility if i votes -1 regardless of his signal. Clearly, only the former is affected by changing i's error probabilities. Changing α_i and β_i in (10) to α_j ($\le \alpha_i$) and β_j ($\le \beta_i$) either increases this expression or leaves it unchanged. Hence, it has the same effect on the expected utility. ■

The proof of Theorem 2 requires two additional lemmas, and some notation.

Notation. For an integer q, the quota rule with quota q is denoted by R_q. Under this rule, if all committee members vote informatively, the decision is $+1$ if and only if the number $\mathbf{s}^+$ of $+1$ signals is at least q. The expected utility under R_q with informative voting by all the members is denoted by $e(q)$. The smallest and largest integers q with $\mathbf{P}(\mathbf{s}^+ = q) > 0$ are denoted by $q_{\min}$ and $q_{\max}$, respectively.

The next lemma shows that the function $e(\cdot)$ is unimodal.

Lemma 2. *The following assertions hold for every integer* q:

(i) *If* $q \ge q_{\min}$ *and* $e(q) \ge e(q+1)$, *then* $e(q+1) \ge e(q+2)$.

(ii) *If* $q \le q_{\max} + 1$ *and* $e(q) \ge e(q-1)$, *then* $e(q-1) \ge e(q-2)$.

Proof. For every q, the collective decision prescribed by the quota rule R_q is different from that prescribed by R_{q+1} only when precisely q members vote $+1$. Therefore, $e(q) \ge e(q+1)$ if and only if

$$c\, p\, \mathbf{P}(\mathbf{s}^+ = q \mid z = +1) \;\ge\; (1-p)\, \mathbf{P}(\mathbf{s}^+ = q \mid z = -1)\,. \tag{12}$$

To prove (i), suppose that (12) holds and $q \ge q_{\min}$. It has to be shown that a similar inequality to (12), in which q is replaced by $q+1$, also holds. Suppose that this inequality does <u>not</u> hold. Then, $\mathbf{P}(\mathbf{s}^+ = q + 1 \mid z = -1) > 0$ and, by Bayes' rule,

$$c\, \mathbf{P}(z = +1 \mid \mathbf{s}^+ = q+1) \;<\; \mathbf{P}(z = -1 \mid \mathbf{s}^+ = q+1)\,. \tag{13}$$

If $\mathbf{P}(\mathbf{s}^+ = q) > 0$, then it follows from Lemma 1 (by choosing $l = m = \tilde{n} = 0$) that a similar inequality to (13) holds with $q+1$ replaced by q. However, this contradicts (12). If $\mathbf{P}(\mathbf{s}^+ = q) = 0$, then, since $\mathbf{P}(\mathbf{s}^+ = q + 1 \mid z = -1) > 0$, the number of members i with $\beta_i = 1$ (who in state -1 observe the signal $+1$ with probability 1) must be $q+1$. By (1), for each of these members i, $\alpha_i = 0$, which implies that i observes the signal $+1$ with probability 1 in <u>both</u> states. Therefore, $\mathbf{s}^+ \ge q + 1 > q_{\min}$ with probability 1, which contradicts the definition of $q_{\min}$. These contradictions prove (i). The proof of (ii) is similar, and can be obtained from the above proof essentially by interchanging the roles of $+1$ and -1. ■

Lemma 3. *For every profile of monotone voting strategies, there is an integer* q *such that the expected utility under the quota rule* R_q *is greater than or equal to that under any other anonymous voting rule*.

Proof. It suffices to consider the special case in which all the members vote informatively. This is because, for each member i, non-informative voting is equivalent to informative voting but with different error probabilities: either $\alpha_i = 0$ and $\beta_i = 1$, or $\alpha_i = 1$ and $\beta_i = 0$. Suppose, then, that everyone votes informatively, and let q^* be an integer such that $e(q^*) \geq e(q)$ for all q. To prove the assertion of the lemma, we assume that there is an anonymous voting rule R under which (with informative voting) the expected utility is greater than $e(q^*)$ (i.e., greater than under the quota rule R_{q^*}), and show that this assumption leads to a contradiction.

Our assumption implies that there is some integer q with $\mathbf{P}(\mathbf{s}^+ = q) > 0$ such that the conditional expected utility, given that $\mathbf{s}^+ = q$, is greater under R than under R_{q^*}. Suppose that the decisions prescribed by R and R_{q^*} when q members vote $+1$ are $+1$ and -1, respectively (similar analysis applies if the decisions are -1 and $+1$, respectively). The difference between the conditional expected utility under R and that under R_{q^*}, given that $\mathbf{s}^+ = q$, is

$$c\,\mathbf{P}(z = +1 \mid \mathbf{s}^+ = q) \;-\; \mathbf{P}(z = -1 \mid \mathbf{s}^+ = q)\,.$$

By assumption, this difference is (strictly) positive. This implies that (12) holds with strict inequality and, therefore, $e(q) > e(q+1)$. Since $\mathbf{P}(\mathbf{s}^+ = q) > 0$, and hence $q \geq q_{\min}$, repeated application of Lemma 2 gives that $e(q) > e(q')$ for all $q' > q$. The assumption that R_{q^*} prescribes the decision -1 when q members vote $+1$ implies that $q^* > q$. Therefore, by the previous conclusion, $e(q) > e(q^*)$, which contradicts the definition of q^*. This contradiction proves that an anonymous voting rule R as above does not exist. ■

Proof of Theorem 2. Let q^* be such that $e(q^*) \geq e(q)$ for all integers q. We claim that q^* has the property described in the theorem. In other words, the quota rule R_{q^*} is a second best rule. The proof of this is by contradiction. That is, we assume the following: For some group of members S and some quota $q \neq q^*$, the S-maximum under R_q is greater than under R_{q^*}. (Note that since $e(q^*) \geq e(q)$, S cannot be empty.) There may be more than one pair (S, q) with this property. Without loss of generality, we choose S and q in such a way that the following conditions hold for every other pair (S', q') with a similar property:

(a) S' is not a proper subset of S, and

(b) if $S' = S$, then $|q' - q^*| \geq |q - q^*|$.

We have to show that the assumption that such S and q exist leads to a contradiction.

Denote the S-maximum under R_q by M. It follows from the definition of S-maximum and Proposition 1 that M is the expected utility under R_q for some strategy profile $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ such that every member in S votes informatively, votes $+1$ regardless of his signal, or votes -1 regardless of the signal and every member not in S votes informatively. Let n^+ and n^- be the numbers of members in S voting $+1$ or -1, respectively, regardless of their signal. The following claim shows that one of these numbers is in fact the cardinality of the entire group S.

CLAIM. For the strategy profile $(\sigma_1, \sigma_2, \ldots, \sigma_n)$, the following assertions hold:

(i) None of the members in S votes informatively.

(ii) For every $i \in S$, $\alpha_i < 1$ and $\beta_i < 1$.

(iii) If $q > q^*$, then $n^+ = 0$, and if $q < q^*$, $n^- = 0$.

Assertion (i) follows immediately from assumption (a) above: There is no $i \in S$ who votes informatively, for otherwise the $(S \setminus \{i\})$-maximum under R_q would also be M. The following rather similar argument shows that there is no $i \in S$ with $\mathbf{P}(s_i = +1) = 1$. Suppose that such a member i exists. If i's strategy σ_i is to vote $+1$ regardless of the signal, switching to informative voting does not affect the way i actually votes. If σ_i is to always vote -1, switching to informative voting and raising the quota to $q + 1$ again does not change anything. In both cases, the $(S \setminus \{i\})$-maximum under some quota rule is equal to M, which contradicts assumption (a) above. This contradiction proves that for every $i \in S$, $\mathbf{P}(s_i = +1) \neq 1$, and hence $\alpha_i > 0$ or $\beta_i < 1$. In fact, $\beta_i < 1$ must hold, since by (1) the other inequality implies it. A very similar argument shows that there is no $i \in S$ with $\mathbf{P}(s_i = -1) = 1$, and therefore $\alpha_i < 1$ for every $i \in S$. This proves (ii). To prove (iii), take any integer $0 \le m \le n^+ + n^-$ and change the strategies of some (or all) of the members in S in such a way that m members (instead of n^+) vote $+1$ and $n^+ + n^- - m$ members (instead of n^-) vote -1 regardless of their signal. At the same time, in order not to change the expected utility, change the quota to $q + m - n^+$. Property (b) of S and q implies that $|(q - n^+ + m) - q^*| \ge |q - q^*|$ must hold. If $q > q^*$, this inequality holds for every $0 \le m \le n^+ + n^-$ only if $n^+ = 0$. If $q < q^*$, only if $n^- = 0$. This completes the proof of the Claim.

In the rest of the proof, we assume that $q > q^*$, and hence, by the Claim, $n^+ = 0$. (The alternative is that $q < q^*$ and $n^- = 0$. The proof in this case is very similar, and can be obtained from the following essentially by interchanging the roles of $+1$ and -1.) Let $\tilde{S}$ denote the complement of S, $\tilde{\mathbf{s}}$ the part of the signal vector $\mathbf{s}$ consisting only of the signals of the members in $\tilde{S}$, and $\tilde{\mathbf{s}}^+$ the number of $+1$ signals in that part. (If S is the entire committee, $\tilde{S}$ is the empty set and $\tilde{\mathbf{s}}^+ = 0$.) Everyone in $\tilde{S}$ votes informatively. Therefore, by (i) in the Claim and since $n^+ = 0$, the decision under the quota rule R_q is $+1$ if and only if $\tilde{\mathbf{s}}^+ \ge q$. If <u>all</u> the members voted informatively, the decision under R_q would be $+1$ if and only if $\mathbf{s}^+ \ge q$. Therefore, the two collective decisions differ if and only if $\mathbf{s}^+ \ge q > \tilde{\mathbf{s}}^+$, and the difference $M - e(q)$ between the S-maximum under R_q and the expected utility under informative voting by all committee members is

$$-c\, p\, \mathbf{P}(\mathbf{s}^+ \ge q > \tilde{\mathbf{s}}^+ \mid z = +1) + (1 - p)\, \mathbf{P}(\mathbf{s}^+ \ge q > \tilde{\mathbf{s}}^+ \mid z = -1)\ . \tag{14}$$

Since M is, by assumption, greater than the S-maximum under R_{q^*}, the latter is clearly equal to or greater than $e(q^*)$, and $e(q^*) = \max_{q'} e(q') \ge e(q)$, the difference $M - e(q)$ is (strictly) positive. Thus, (14) is positive, which implies that there is some $r \ge q$ with

$$c\, p\, \mathbf{P}(\mathbf{s}^+ = r, \tilde{\mathbf{s}}^+ < q \mid z = +1) < (1 - p)\, \mathbf{P}(\mathbf{s}^+ = r, \tilde{\mathbf{s}}^+ < q \mid z = -1)\ .$$

This inequality implies that

$$\mathbf{P}(\mathbf{s}^+ = r, \tilde{\mathbf{s}}^+ < q \mid z = -1) > 0\ , \tag{15}$$

and by Bayes' rule,

$$c\, \mathbf{P}(z = +1 \mid \mathbf{s}^+ = r, \tilde{\mathbf{s}}^+ < q) < \mathbf{P}(z = -1 \mid \mathbf{s}^+ = r, \tilde{\mathbf{s}}^+ < q)\ . \tag{16}$$

By the Claim, every member i in S has $\beta_i < 1$, which means that i has positive probability of <u>not</u> observing the signal $+1$ in state -1. Together with (15) and the inequality $r \ge q$, this implies that $\mathbf{P}(\mathbf{s}^+ = q - 1 \mid z = -1) > 0$. Therefore, it follows from Lemma 1 (by setting $I = \{0, \dots, q - 1\}$ in (4)) that a similar inequality to (16) holds with r replaced by the smaller integer $q - 1$. Therefore,

$$c\, \mathbf{P}(z = +1 \mid \mathbf{s}^+ = q - 1) < \mathbf{P}(z = -1 \mid \mathbf{s}^+ = q - 1)\ .$$

This inequality implies that when $q - 1$ members observe the signal $+1$, the conditional expected utility of deciding $+1$ is less than that of deciding -1. However, since it was assumed that $q > q^*$, the decision prescribed by the quota rule R_{q^*} is $+1$. Changing the decision to -1 would give a new anonymous voting rule such that, with informative voting, the expected utility under this rule is greater than under R_{q^*}, and therefore greater than under <u>any</u> quota rule. However, this contradicts Lemma 3. This proves that the initial assumption cannot be true. Thus, S and q as above do not exist.

■

Proof of Proposition 2. Let a first best voting rule and a strategy profile (σ_1, σ_2, … , σ_n) be given. It has to be shown that under the given rule, the expected utility with the given strategies is less than or equal to that with informative voting by all the members. The special case in which all the members but one vote informatively shows that naive voting is equilibrium behavior.

Consider the voting rule which, for each voting vector (x_1, x_2, … , x_n), prescribes the same collective decision prescribed by the given first best rule for ($\sigma_1(x_1)$, $\sigma_2(x_2)$, … , $\sigma_n(x_n)$). Clearly, the following are equal: (i) the expected utility under this rule with informative voting by all the members, and (ii) the expected utility under the first best rule with the given strategies σ_1, σ_2, … , σ_n. Since (i) is by definition less than or equal to the expected utility under the first best rule with informative voting by all the members, the same is true for (ii). ■

Proof of Proposition 3. Suppose that a quota rule R_q is used, and that all committee members, except perhaps i, vote informatively. The difference between the expected utility if i also votes informatively or if his strategy is to vote −1 regardless of his signal is given by (8). The conditional probabilities in this expression can be computed by summing over all voting vectors in which i's vote is +1 and he is pivotal, which means that the total number of members voting +1 equals the quota q. Thus, (8) is equal to

$$\sum_{\substack{\mathbf{x} \\ x_i = +1 \\ \mathbf{x}^+ = q}} \left[c\,p\,\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) - (1-p)\,\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = -1)\right] .$$

If naive voting is equilibrium behavior under R_q, this expression is nonnegative for all $1 \le i \le n$, and summation over i gives

$$\sum_{\substack{\mathbf{x} \\ \mathbf{x}^+ = q}} q\left[c\,p\,\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) - (1-p)\,\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = -1)\right] \ge 0\,, \tag{17}$$

where the factor q comes from the fact that each voting vector $\mathbf{x}$ appearing in the summation does so exactly q times (since this is the number of members i with $x_i = +1$). If naive voting is <u>non-trivial</u> equilibrium behavior, then in addition $q > 0$ and $q_{\min} \le q \le q_{\max} + 1$. The first inequality, $q > 0$, implies that (17) is equivalent to (12), and hence to $e(q) \ge e(q+1)$. A very similar argument, based on (9) rather than (8), shows that if naive voting is non-trivial equilibrium behavior, then also $e(q) \ge e(q-1)$, and therefore repeated application of Lemma 2 yields $e(q) \ge e(q')$ for all integers q'. As shown in the proof of Theorem 2, this property of q implies that R_q is a second best rule. ■

Proof of Proposition 4. By assumption, $p = 1/2$, $c = 1$, and $\alpha_i = \beta_i$ for all members i. Suppose that a monotone and neutral voting rule is used. The key to proving that naive voting in this case is equilibrium behavior is to show that, if all committee members vote informatively, the probability that each is pivotal is the same in both states of the world. As shown below, this implies that each member i should vote as if the collective decision is determined by his vote alone.

By definition, member i is pivotal in a voting vector $\mathbf{x}$ if and only if replacing $\mathbf{x}$ with the voting vector $\mathbf{x}^{(i)}$ defined by $x_i^{(i)} = -x_i$ and $x_j^{(i)} = x_j$ for all $j \ne i$ changes the collective decision. Since the voting rule is assumed to be neutral, it prescribes different decisions for $\mathbf{x}$ and $-\mathbf{x}$ (the components of which have opposite signs), as well as for $\mathbf{x}^{(i)}$ and $-\mathbf{x}^{(i)}$. It follows, since $-\mathbf{x}^{(i)} = (-\mathbf{x})^{(i)}$, that i is pivotal in $\mathbf{x}$ if and only if he is pivotal in $-\mathbf{x}$. This proves the left equality in the following, and the right equality follows from the assumption that $\alpha_j = \beta_j$ for all j:

$$\sum_{\substack{\mathbf{x} \\ i \text{ is pivoal in } \mathbf{x}}} \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) \quad = \quad \sum_{\substack{\mathbf{x} \\ i \text{ is pivoal in } \mathbf{x}}} \mathbf{P}(\mathbf{s} = -\mathbf{x} \mid z = +1) \quad = \quad \sum_{\substack{\mathbf{x} \\ i \text{ is pivoal in } \mathbf{x}}} \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = -1)\,. \tag{18}$$

With informative voting by all the members, the leftmost sum in (18) equals $\mathbf{P}(i \text{ is pivotal} \mid z = +1)$, the probability that member i is pivotal in state $+1$. Similarly, the rightmost sum equals $\mathbf{P}(i \text{ is pivotal} \mid z = -1)$. The equality between these two conditional probabilities, together with (1) and the assumption that $p = 1/2$ and $c = 1$, imply that (10) is nonnegative. Therefore, with informative voting by all the other members, the expected utility if i also votes informatively is greater than or equal to the expected utility if he votes -1 regardless of his signal. A similar argument shows that with informative voting by all the other members, the expected utility if i also votes informatively is greater than or equal to that if he always votes $+1$. In view of Proposition 1, this proves that naive voting is equilibrium behavior. ■

Proof of Proposition 5. The main idea of the proof is as follows. By Lemma 2, the function $e(\cdot)$, which gives the expected utility with informative voting under each quota rule, is unimodal. The proof of Theorem 2 shows that this function peaks at the second best rule(s). The symmetry assumption implies that the peak can only lie at the midpoint between $q = 1$ and $q = n$. A more detailed argument follows.

By the symmetry assumption and (1), $\alpha_i = \beta_i \le 1/2$ for all i. Therefore, $q_{\min} = 0$ and $q_{\max} = n$. In addition, for any q, the probability that in state $+1$ q members observe the signal $+1$ and $n - q$ observe -1 is equal to the probability that in state -1 q members observe -1 and $n - q$ observe $+1$. In other words, $\mathbf{P}(\mathbf{s}^+ = q \mid z = +1) = \mathbf{P}(\mathbf{s}^+ = n - q \mid z = -1)$. Since by the symmetry assumption $p = 1/2$ and $c = 1$, this implies that if all the members vote informatively, the expected utility under the anonymous voting rule that prescribes the decision $+1$ if and only if the number of $+1$ signals is at least q is equal to the expected utility under the rule prescribing -1 if and only if the number of -1 signals is at least q. In other words, for every q, $e(q) = e(n - q + 1)$, and hence also $e(q + 1) = e(n - q)$, which implies that

$$e(q) - e(q + 1) = e(n - q + 1) - e(n - q)\,. \tag{19}$$

CLAIM. If $0 \le q \le n/2$ or $n/2 \le q \le n$, then $e(q) \le e(q + 1)$ or $e(q) \ge e(q + 1)$, respectively.

To prove this, suppose that $0 \le q \le n/2$. If $e(q) > e(q + 1)$, repeated application of Lemma 2 yields $e(n - q) \ge e(n - q + 1)$, but (19) implies that $e(n - q + 1) > e(n - q)$. This contradiction proves that $e(q) \le e(q + 1)$. A similar argument shows that, if $n/2 \le q \le n$, then $e(q) \ge e(q + 1)$.

It follows from the Claim that, for an integer q^* with $n/2 \le q^* \le n/2 + 1$, $e(q^*) \ge e(q)$ for all q. As shown in the proof of Theorem 2, this property of q^* implies that the quota rule R_{q^*} is second best. ■

Proof of Lemma 1. By Bayes' theorem, for any interval I and integer q with $\mathbf{P}(\mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I) > 0$,

$$\mathbf{P}(z = +1 \mid \mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I) = \frac{p\,\mathbf{P}(\mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I \mid z = +1)}{p\,\mathbf{P}(\mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I \mid z = +1) + (1 - p)\,\mathbf{P}(\mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I \mid z = -1)}\,. \tag{20}$$

The right-hand side of (20) can be expressed as a function of the members' error probabilities, the α_i's and β_i's. This function is continuous (indeed, rational). Therefore, it suffices to prove the assertion of the lemma under the additional assumption that $0 < \alpha_i, \beta_i < 1$ for all i. Under this assumption, for any interval of integers of the form $I = \{l, l+1, \dots, m\}$ with $0 \le l \le m$ and integers r and q with $r > q$, $\mathbf{P}(\mathbf{s}^+ = r, \tilde{\mathbf{s}}^+ \in I) > 0$ and $\mathbf{P}(\mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I) > 0$ if and only if r and q lie in the interval $\{l, l+1, \dots, m+n-\tilde{n}\}$ and $\tilde{n} \ge l$. In this case, $q+1$ also lies in that interval, so that $\mathbf{P}(\mathbf{s}^+ = q+1, \tilde{\mathbf{s}}^+ \in I) > 0$. Therefore, it suffices to consider in the proof of the lemma only the special case $r = q+1$. Since $0 < p < 1$, it follows from (20) that (4) holds for $r = q+1$ if and only if

$$\mathbf{P}(\mathbf{s}^+ = q+1, \tilde{\mathbf{s}}^+ \in I \mid z = +1)\, \mathbf{P}(\mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I \mid z = -1)$$
$$\ge \mathbf{P}(\mathbf{s}^+ = q, \tilde{\mathbf{s}}^+ \in I \mid z = +1)\, \mathbf{P}(\mathbf{s}^+ = q+1, \tilde{\mathbf{s}}^+ \in I \mid z = -1)\,. \tag{21}$$

Hence, we only have to show that (21) holds, for an interval $I = \{l, l+1, \dots, m\}$ and integer q which are kept fixed throughout the rest of the proof.

For each voting vector $\mathbf{x}$ and member i, let $\mathbf{x}^{(i)}$ be the voting vector defined by $x_i^{(i)} = -x_i$ and $x_j^{(i)} = x_j$ for all $j \ne i$.

CLAIM 1. For every pair of voting vectors $\mathbf{x}$ and $\mathbf{y}$ and every member i with $x_i = +1$ and $y_i = -1$,

$$\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1)\, \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1) \ge \mathbf{P}(\mathbf{s} = \mathbf{x}^{(i)} \mid z = +1)\, \mathbf{P}(\mathbf{s} = \mathbf{y}^{(i)} \mid z = -1)\,. \tag{22}$$

This can be proved as follows. Since the only difference between $\mathbf{x}$ and $\mathbf{x}^{(i)}$ is that i's vote in the former is $+1$ and in the latter -1, $\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1) = (1 - \alpha_i)\, A$ and $\mathbf{P}(\mathbf{s} = \mathbf{x}^{(i)} \mid z = +1) = \alpha_i\, A$, where A is a nonnegative expression that depends only on the error probabilities of the members other than i (see (6)). Similarly, $\mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1) = (1 - \beta_i)\, B$ and $\mathbf{P}(\mathbf{s} = \mathbf{y}^{(i)} \mid z = -1) = \beta_i\, B$, for some nonnegative expression B. Thus, if both sides of (22) are explicitly written as products of probabilities, the only difference between them is that the product $(1 - \alpha_i)(1 - \beta_i)$ on the left-hand side is replaced on the right-hand side by $\alpha_i\beta_i$. Since $(1 - \alpha_i)(1 - \beta_i) = (1 - \alpha_i - \beta_i) + \alpha_i\beta_i \ge \alpha_i\beta_i$ by (1), the inequality (22) holds.

To use Claim 1 to prove (21), we need to consider the following set of pairs of voting vectors:

$$V = \left\{(\mathbf{x}, \mathbf{y}) \;\middle|\; \mathbf{x}^+ = q+1,\, \mathbf{y}^+ = q\,,\, \tilde{\mathbf{x}}^+ \in I \text{ and } \tilde{\mathbf{y}}^+ \in I\right\}\,,$$

where a tilde ($\tilde{\ }$) over a voting vector indicates that only the votes of the members in $\tilde{S}$ are considered. Two pairs $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$ in V will be said to be <u>joined</u> by committee member i if $x_i = +1$, $y_i = -1$, $\mathbf{x}^{(i)} = \mathbf{y}'$ and $\mathbf{y}^{(i)} = \mathbf{x}'$. These conditions are equivalent to the following, symmetric ones: $x'_i = +1$, $y'_i = -1$, $\mathbf{x}'^{(i)} = \mathbf{y}$ and $\mathbf{y}'^{(i)} = \mathbf{x}$. Two pairs in V can be joined by at most one member i (since $\mathbf{x}^{(j)} = \mathbf{y}' = \mathbf{x}^{(i)}$ cannot hold if $j \ne i$). This relation between elements of V can be described by an undirected graph Γ, with the vertex set V, in which two vertices are joined by an edge if and only if they are joined by some committee member i. More precisely, Γ is a multigraph, or pseudograph. It does not have multiple edges, but may have loops, which represent elements of V that are joined with <u>themselves</u> by some (unique) committee member i.

CLAIM 2. Positive weights can be assigned to the edges in Γ such that, for each vertex $(\mathbf{x}, \mathbf{y})$, the weights of the edges incident with $(\mathbf{x}, \mathbf{y})$ (including loops) sum up to 1.

Before presenting the proof of Claim 2, which is rather involved, we show how the existence of such weights implies (21). For every two pairs $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$ in V (possibly, $(\mathbf{x}, \mathbf{y}) = (\mathbf{x}', \mathbf{y}')$), let $W(\mathbf{x}, \mathbf{y}; \mathbf{x}', \mathbf{y}')$ be the weight of the edge joining them, if such an edge exists, and 0 otherwise. For every $(\mathbf{x}, \mathbf{y}) \in V$,

$$\sum_{(\mathbf{x}', \mathbf{y}') \in V} W(\mathbf{x}, \mathbf{y}; \mathbf{x}', \mathbf{y}') = \sum_{(\mathbf{x}', \mathbf{y}') \in V} W(\mathbf{x}', \mathbf{y}'; \mathbf{x}, \mathbf{y}) = 1 ,$$

and therefore, by Claim 1,

$$\mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1)\, \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1)$$

$$\geq \sum_{(\mathbf{x}', \mathbf{y}') \in V} W(\mathbf{x}, \mathbf{y}; \mathbf{x}', \mathbf{y}')\, \mathbf{P}(\mathbf{s} = \mathbf{y}' \mid z = +1)\, \mathbf{P}(\mathbf{s} = \mathbf{x}' \mid z = -1) .$$

Summing over all pairs in V gives

$$\sum_{(\mathbf{x}, \mathbf{y}) \in V} \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = +1)\, \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = -1)$$

$$\geq \sum_{(\mathbf{x}, \mathbf{y}) \in V} \sum_{(\mathbf{x}', \mathbf{y}') \in V} W(\mathbf{x}, \mathbf{y}; \mathbf{x}', \mathbf{y}')\, \mathbf{P}(\mathbf{s} = \mathbf{y}' \mid z = +1)\, \mathbf{P}(\mathbf{s} = \mathbf{x}' \mid z = -1) \tag{23}$$

$$= \sum_{(\mathbf{x}, \mathbf{y}) \in V} \sum_{(\mathbf{x}', \mathbf{y}') \in V} W(\mathbf{x}', \mathbf{y}'; \mathbf{x}, \mathbf{y})\, \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = +1)\, \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = -1)$$

$$= \sum_{(\mathbf{x}, \mathbf{y}) \in V} \mathbf{P}(\mathbf{s} = \mathbf{y} \mid z = +1)\, \mathbf{P}(\mathbf{s} = \mathbf{x} \mid z = -1) .$$

(The first equality holds since its two sides differ only in the order of summation.) The first and last sums in (23) are equal to the expressions on the left- and right-hand sides of (21), respectively. Therefore, (23) shows that (21) holds.

To complete the proof of the lemma, it remains to prove Claim 2. The proof requires two additional claims, and the following notation. For $(\mathbf{x}, \mathbf{y}) \in V$, denote:

$$n^{+-} = |\{j \mid x_j = +1 \text{ and } y_j = -1\}| , \quad n^{-+} = |\{j \mid x_j = -1 \text{ and } y_j = +1\}| ,$$

$$\tilde{n}^{+-} = |\{j \in \tilde{S} \mid x_j = +1 \text{ and } y_j = -1\}| , \tilde{n}^{-+} = |\{j \in \tilde{S} \mid x_j = -1 \text{ and } y_j = +1\}| ,$$

$$\tilde{n}^{++} = |\{j \in \tilde{S} \mid x_j = y_j = +1\}| .$$

($|T|$ is the cardinality of set T.) It is not difficult to see that the following identities hold:

$$\tilde{n}^{++} + \tilde{n}^{+-} = \tilde{\mathbf{x}}^{+} , \quad \tilde{n}^{++} + \tilde{n}^{-+} = \tilde{\mathbf{y}}^{+}, \tag{24}$$

$$n^{+-} - n^{-+} = \mathbf{x}^{+} - \mathbf{y}^{+} = (q + 1) - q = 1 .$$

CLAIM 3. In every (connected) component of the graph Γ, n^{+-}, n^{-+}, $\tilde{n}^{+-} + \tilde{n}^{-+}$ and $\tilde{n}^{++}$ are constants, i.e., they have the same values at all vertices.

To prove the claim, is suffices to consider two elements in V, $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$, that are joined by some i, so that $x_i = +1$, $y_i = -1$ and $(\mathbf{x}', \mathbf{y}') = (\mathbf{y}^{(i)}, \mathbf{x}^{(i)})$. Clearly, $\tilde{n}^{++}$ is the same at $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{y}^{(i)}, \mathbf{x}^{(i)})$, and so is also $\tilde{n}^{+-} + \tilde{n}^{-+}$, which equals $|\{j \in \tilde{S} \mid x_j \neq y_j\}|$. Since by (24) $n^{+-} = n^{-+} + 1$, it remains only to show that the value of n^{+-} at $(\mathbf{x}, \mathbf{y})$ is equal to the value of $n^{-+} + 1$ at $(\mathbf{y}^{(i)}, \mathbf{x}^{(i)})$. The former is equal to the cardinality of the set $\{j \mid x_j = +1 \text{ and } y_j = -1\}$, and the latter to that of $\{j \mid y_j^{(i)} = -1 \text{ and } x_j^{(i)} = +1\}$ plus one. Since the latter set is obtained from the former by deleting from it the single element i, the two values are indeed equal. This completes the proof of Claim 3.

CLAIM 4. Each component of the graph Γ satisfies at least one of the following two conditions:

(i) All the vertices in the component have the same number $\nu \geq 1$ of neighbors (possibly, including themselves).

(ii) At each of the vertices, $1 \leq \tilde{n}^{+-} \leq n^{-+}$.

To prove this, note first that the number of neighbors of a vertex $(\mathbf{x}, \mathbf{y})$ is equal to the number of committee members i such that $x_i = +1$, $y_i = -1$ and $(\mathbf{y}^{(i)}, \mathbf{x}^{(i)}) \in V$. If $x_i = +1$ and $y_i = -1$, the last condition holds if and only if $l \leq \widetilde{\mathbf{x}^{(i)+}} \leq m$ and $l \leq \widetilde{\mathbf{y}^{(i)+}} \leq m$. This is automatically so if $i \notin \tilde{S}$, but if $i \in \tilde{S}$, the condition holds if and only if $\tilde{\mathbf{x}}^+ > l$ and $\tilde{\mathbf{y}}^+ < m$. This shows that the number of neighbors of a vertex $(\mathbf{x}, \mathbf{y})$ is n^{+-} if $\tilde{\mathbf{x}}^+ > l$ and $\tilde{\mathbf{y}}^+ < m$, and $n^{+-} - \tilde{n}^{+-}$ if at least one inequality does not hold. In the first case, $(\mathbf{x}, \mathbf{y})$ is joined with elements of V by each of the members i with $x_i = +1$ and $y_i = -1$, and in the second case, only by such i who do not belong to $\tilde{S}$. Since by Claim 3 the value of n^{+-} is the same at all vertices in a component, and by (24) $n^{+-} \geq 1$, this shows that a component of Γ does not satisfy condition (i) only if it includes at least one vertex $(\mathbf{x}, \mathbf{y})$ with $\tilde{n}^{+-} \geq 1$ such that $\tilde{\mathbf{x}}^+ = l$ or $\tilde{\mathbf{y}}^+ = m$. We have to show that the existence of such a vertex implies that the component satisfies condition (ii).

Let $(\mathbf{x}, \mathbf{y})$ be as above. By (24), $\tilde{\mathbf{x}}^+ + \tilde{\mathbf{y}}^+$ is equal to $2\tilde{n}^{++} + (\tilde{n}^{+-} + \tilde{n}^{-+})$. Since by Claim 3 the latter is constant in the component, every other vertex $(\mathbf{x}', \mathbf{y}')$ in it satisfies $\tilde{\mathbf{x}}'^+ + \tilde{\mathbf{y}}'^+ = \tilde{\mathbf{x}}^+ + \tilde{\mathbf{y}}^+$. By definition of V, all four terms in this equality are between l and m. Since by assumption $\tilde{\mathbf{x}}^+ = l$ or $\tilde{\mathbf{y}}^+ = m$, this implies that

$$l \leq \tilde{\mathbf{x}}^+ \leq \tilde{\mathbf{x}}'^+ \leq m \ \underline{\text{and}}\ l \leq \tilde{\mathbf{y}}'^+ \leq \tilde{\mathbf{y}}^+ \leq m\,. \tag{25}$$

Since $\tilde{n}^{++}$ is constant in the component, it follows from the first part of (25) and the first equality in (24) that the value of $\tilde{n}^{+-}$ at $(\mathbf{x}', \mathbf{y}')$ is greater than or equal to that at $(\mathbf{x}, \mathbf{y})$. This shows that the minimum value of $\tilde{n}^{+-}$ in the component is attained at the vertex $(\mathbf{x}, \mathbf{y})$. Since it is assumed that at that vertex $\tilde{n}^{+-} \geq 1$, the same is true at all vertices in the component. It remains to show that the other inequality in (ii), $\tilde{n}^{+-} \leq n^{-+}$, also holds at all vertices. This inequality is equivalent to each of the following two: (a) $\tilde{n}^{++} + \tilde{n}^{-+} \geq \tilde{n}^{++} + (\tilde{n}^{+-} + \tilde{n}^{-+}) - n^{-+}$, and (b) $\tilde{n}^{+-} + \tilde{n}^{++} \leq \tilde{n}^{++} + n^{-+}$. By Claim 3, the value of the expression on the right-hand side of inequality (a) at any vertex in the component is equal to the value at $(\mathbf{x}, \mathbf{y})$, and the same is true for the expression on the right-hand side of (b). Since, necessarily, $n^{-+} \geq \tilde{n}^{-+}$, it follows from (24) that the right-hand side of (a) is less than or equal to $\tilde{\mathbf{x}}^+$ and the right-hand side of (b) is greater than or equal to $\tilde{\mathbf{y}}^+$. By assumption, $\tilde{\mathbf{x}}^+ = l$ or $\tilde{\mathbf{y}}^+ = m$. Therefore, it suffices to show that the following inequalities hold at all vertices in the component: (a') $\tilde{n}^{++} + \tilde{n}^{-+} \geq l$, and (b') $\tilde{n}^{+-} + \tilde{n}^{++} \leq m$. At each vertex $(\mathbf{x}', \mathbf{y}')$, these inequalities can be written as $\tilde{\mathbf{y}}'^+ \geq l$ and $\tilde{\mathbf{x}}'^+ \leq m$, which hold by (25). Therefore, condition (ii) holds. This completes the proof of Claim 4.

The proof of Claim 2 can now be completed. Clearly, it suffices to show that for any given component of the graph Γ there is an assignment of weights to the edges in the component such that the total weight of the edges incident with each vertex is unity. If condition (i) in Claim 4 holds for the component, all the edges in it may simply be assigned the weight $1/\nu$. In the rest of the proof, we assume that condition (ii) in Claim 4 holds for the component.

Consider two neighboring vertices in the component, $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$ (which can be distinct or the same vertex), that are joined by some (unique) member i. The weight that needs to be assigned to the edge joining the vertices depends on whether or not i is in $\tilde{S}$. If $i \notin \tilde{S}$, the weight is

$$\frac{1}{n^{+-}}\left[1 + \frac{C}{\binom{n^{-+}}{\tilde{n}^{-+}}\binom{n^{-+}}{\tilde{n}^{+-}}}\right], \tag{26}$$

where C is a positive integer, given explicitly below, which is the same for all the edges in the component. If $i \in \tilde{S}$, the weight is

$$\frac{1}{n^{+-}}\left[1 - \frac{C}{\binom{n^{-+}}{\tilde{n}^{-+}}\binom{n^{-+}}{\tilde{n}^{+-}-1}}\right]. \tag{27}$$

(As shown below, the explicit expression of C is such that this weight is positive.) Since by assumption $n^{-+} \geq \tilde{n}^{+-} \geq 1$ everywhere in the component, the combinatorial coefficients in (26) and (27) are well defined. Each coefficient may have different values at $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$. However, if $i \notin \tilde{S}$, the product

$$\binom{n^{-+}}{\tilde{n}^{-+}}\binom{n^{-+}}{\tilde{n}^{+-}}$$

has the same value at both vertices, since the values of $\tilde{n}^{-+}$ and $\tilde{n}^{+-}$ at $(\mathbf{x}, \mathbf{y})$ are equal, respectively, to those of $\tilde{n}^{+-}$ and $\tilde{n}^{-+}$ at $(\mathbf{x}', \mathbf{y}')$ and, by Claim 3, n^{-+} is the same at both vertices. Similarly, if $i \in \tilde{S}$, then

$$\binom{n^{-+}}{\tilde{n}^{-+}}\binom{n^{-+}}{\tilde{n}^{+-}-1}$$

has the same value at both vertices, since the values of $\tilde{n}^{-+}$ and $\tilde{n}^{+-}-1$ at $(\mathbf{x}, \mathbf{y})$ are equal, respectively, to those of $\tilde{n}^{+-}-1$ and $\tilde{n}^{-+}$ at $(\mathbf{x}', \mathbf{y}')$. Since, by Claim 3, $1/n^{+-}$ is constant in the component, this shows that the weight ((26) or (27)) of the joining edge can be computed at either vertex.

We have to show that the weights of the edges incident with each vertex $(\mathbf{x}, \mathbf{y})$ in the component sum up to 1. There are two cases to consider. In the first case, $\tilde{\mathbf{x}}^{+} > l$ and $\tilde{\mathbf{y}}^{+} < m$, and in the second, $\tilde{\mathbf{x}}^{+} = l$ or $\tilde{\mathbf{y}}^{+} = m$. As shown in the proof of Claim 4, in the first case $(\mathbf{x}, \mathbf{y})$ is joined with elements of V by $n^{+-} - \tilde{n}^{+-}$ ($= n^{-+} + 1 - \tilde{n}^{+-}$, which is a positive number since it is assumed that $n^{-+} \geq \tilde{n}^{+-}$) members not in $\tilde{S}$ and $\tilde{n}^{+-}$ (≥ 1 by assumption) members in $\tilde{S}$. Hence, the total weight of the edges incident with $(\mathbf{x}, \mathbf{y})$ is

$$\frac{n^{+-}-\tilde{n}^{+-}}{n^{+-}}\left[1 + \frac{C}{\binom{n^{-+}}{\tilde{n}^{-+}}\binom{n^{-+}}{\tilde{n}^{+-}}}\right] + \frac{\tilde{n}^{+-}}{n^{+-}}\left[1 - \frac{C}{\binom{n^{-+}}{\tilde{n}^{-+}}\binom{n^{-+}}{\tilde{n}^{+-}-1}}\right] \tag{28}$$

$$= 1 + \frac{C}{n^{+-}\binom{n^{-+}}{\tilde{n}^{-+}}}\left[\frac{(n^{-+}+1-\tilde{n}^{+-})\,(n^{-+}-\tilde{n}^{+-})!\,\tilde{n}^{+-}!}{n^{-+}!} - \frac{\tilde{n}^{+-}\,(\tilde{n}^{+-}-1)!\,(n^{-+}-(\tilde{n}^{+-}-1))!}{n^{-+}!}\right]$$

$$= 1\,.$$

In the second case, in which $\tilde{\mathbf{x}}^{+} = l$ or $\tilde{\mathbf{y}}^{+} = m$, $(\mathbf{x}, \mathbf{y})$ is joined with elements of V only by $n^{+-} - \tilde{n}^{+-}$ members not in $\tilde{S}$. In this case, the total weight is

$$\frac{n^{+-}-\tilde{n}^{+-}}{n^{+-}}\left[1 + \frac{C}{\binom{n^{-+}}{\tilde{n}^{-+}}\binom{n^{-+}}{\tilde{n}^{+-}}}\right].$$

The requirement that this equals 1 determines C uniquely. Specifically, C has to be such that the second term on the left-hand side of (28) is zero, which is the case if

$$C = \binom{n^{-+}}{\tilde{n}^{-+}}\binom{n^{-+}}{\tilde{n}^{+-}-1}. \tag{29}$$

It has to be shown that this expression for C is independent of $(\mathbf{x}, \mathbf{y})$, i.e., it has the same value at every other vertex $(\mathbf{x}', \mathbf{y}')$ in the component with $\tilde{\mathbf{x}}'^{+} = l$ or $\tilde{\mathbf{y}}'^{+} = m$. For such a vertex, (25) implies that $\tilde{\mathbf{x}}'^{+} = \tilde{\mathbf{x}}^{+} = l$ or $\tilde{\mathbf{y}}'^{+} = \tilde{\mathbf{y}}^{+} = m$. Since, by Claim 3, n^{-+}, $\tilde{n}^{+-} + \tilde{n}^{-+}$ and $\tilde{n}^{++}$ have the same values at $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$, this implies that

$$\binom{n^{-+}}{\tilde{n}^{+-} + \tilde{n}^{-+} + \tilde{n}^{++} - \tilde{\mathbf{x}}^{+}} \binom{n^{-+}}{\tilde{\mathbf{x}}^{+} - 1 - \tilde{n}^{++}} = \binom{n^{-+}}{\tilde{n}^{+-} + \tilde{n}^{-+} + \tilde{n}^{++} - \tilde{\mathbf{x}}'^{+}} \binom{n^{-+}}{\tilde{\mathbf{x}}'^{+} - 1 - \tilde{n}^{++}} \tag{30}$$

or

$$\binom{n^{-+}}{\tilde{\mathbf{y}}^{+} - \tilde{n}^{++}} \binom{n^{-+}}{\tilde{n}^{++} + \tilde{n}^{+-} + \tilde{n}^{-+} - \tilde{\mathbf{y}}^{+} - 1} = \binom{n^{-+}}{\tilde{\mathbf{y}}'^{+} - \tilde{n}^{++}} \binom{n^{-+}}{\tilde{n}^{++} + \tilde{n}^{+-} + \tilde{n}^{-+} - \tilde{\mathbf{y}}'^{+} - 1}. \tag{31}$$

For the vertex $(\mathbf{x}, \mathbf{y})$, the expressions on the left-hand sides of (30) and (31) are both equal to the expression in (29). For $(\mathbf{x}', \mathbf{y}')$, the expressions on the right-hand sides are equal to the expression in (29). The value of C is therefore the same regardless of whether $(\mathbf{x}, \mathbf{y})$ or $(\mathbf{x}', \mathbf{y}')$ is used to define it, and in addition,

$$C = \binom{n^{-+}}{\tilde{n}^{+-} + \tilde{n}^{-+} + \tilde{n}^{++} - l} \binom{n^{-+}}{l - 1 - \tilde{n}^{++}} \quad \text{or} \quad C = \binom{n^{-+}}{m - \tilde{n}^{++}} \binom{n^{-+}}{\tilde{n}^{++} + \tilde{n}^{+-} + \tilde{n}^{-+} - m - 1}. \tag{32}$$

It remains to show that the weight (27) is positive for every vertex $(\mathbf{x}', \mathbf{y}')$ in the component under consideration with $\tilde{\mathbf{x}}'^{+} > l$ and $\tilde{\mathbf{y}}'^{+} < m$. (For a vertex with $\tilde{\mathbf{x}}'^{+} = l$ or $\tilde{\mathbf{y}}'^{+} = m$, (27) is zero, since (29) holds. However, such a vertex is not incident with any edge whose weight is given by (27).) Clearly, it suffices to show that if the first equality in (32) holds, then at $(\mathbf{x}', \mathbf{y}')$

$$\binom{n^{-+}}{\tilde{n}^{+-} + \tilde{n}^{-+} + \tilde{n}^{++} - l} \binom{n^{-+}}{l - 1 - \tilde{n}^{++}} < \binom{n^{-+}}{\tilde{n}^{-+}} \binom{n^{-+}}{\tilde{n}^{+-} - 1},$$

and if the second equality holds, then

$$\binom{n^{-+}}{m - \tilde{n}^{++}} \binom{n^{-+}}{\tilde{n}^{++} + \tilde{n}^{+-} + \tilde{n}^{-+} - m - 1} < \binom{n^{-+}}{\tilde{n}^{-+}} \binom{n^{-+}}{\tilde{n}^{+-} - 1}.$$

The inequality that has to be proven has a similar form in both cases. With $\theta = n^{-+}$ and $\phi = \tilde{n}^{-+}$, it has the form

$$\binom{\theta}{\rho} \binom{\theta}{\tau} < \binom{\theta}{\phi} \binom{\theta}{(\rho + \tau) - \phi}, \tag{33}$$

where, in the first case, $\rho = \tilde{n}^{+-} + \tilde{n}^{-+} + \tilde{n}^{++} - l$ and $\tau = l - 1 - \tilde{n}^{++}$, and in the second case, $\rho = m - \tilde{n}^{++}$ and $\tau = \tilde{n}^{++} + \tilde{n}^{+-} + \tilde{n}^{-+} - m - 1$. Since, by the assumption concerning $(\mathbf{x}', \mathbf{y}')$ and the definition of V, $l < \tilde{\mathbf{x}}'^{+} \le m$ and $l \le \tilde{\mathbf{y}}'^{+} < m$, it follows from equations similar to (24) that in both cases $\rho > \phi > \tau$. Therefore, regardless of which equality in (32) holds, the corresponding inequality holds as a special case of the following general claim.

CLAIM 5. Inequality (33) holds for any four integers satisfying $\theta \ge \rho > \phi > \tau \ge 0$.

The proof of this claim, which involves standard manipulations of combinatorial coefficients, is omitted. ■

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