THE SEVEN GOOD YEARS: ON THE ECONOMICS OF HOARDING

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Abstract. An economic agent has private information about a future drop in the supply of a certain product. What quantities of the product should the agent buy and store if the goal is: (1) to maximize profit from selling the product later at a higher price, or (2) to maximize social welfare?

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> Then Pharaoh said to Joseph, "In my dream I was standing on the bank of the Nile, when out of the river there came up seven cows, fat and sleek, and they grazed among the reeds. After them, seven other cows came up—scrawny and very ugly and lean. I had never seen such ugly cows in all the land of Egypt. The lean, ugly cows ate up the seven fat cows that came up first. But even after they ate them, no one could tell that they had done so; they looked just as ugly as before. Then I woke up."
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> GENESIS 41:17–21

Introduction

In the biblical story of Joseph and Pharaoh, Joseph interprets the king's dreams as auguring seven years of abundance, to be followed by seven years of famine. He then adds this piece of practical advice: "Let Pharaoh appoint commissioners over the land to take a *fifth* of the harvest of Egypt during the seven years of abundance. … This food should be held in reserve for the country, to be used during the seven years of famine that will come upon Egypt, so that the country may not be ruined by the famine" (Genesis 41:34, 36).

The supply of agricultural or other products may vary from year to year. In extreme cases, such as a drought year or locust infestation, it may drop to near zero. Under such variable conditions, storing quantities of a product during years of abundance, with the intention of selling it in lean years, may be a good idea: both for a benevolent government, which is concerned with social welfare, and for an individual trader, who only considers the profits that can be made from buying when the supply is high and selling when it is low. The question is: How much to store? Realistically, the answer to this question should involve strategic

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considerations. For example, since the market value of the stored good depends on the quantities stored by others, economic agents with private information about future shortage may not want to take full advantage of that information for fear of revealing it to others. Thus the scale of hoarding may depend on whether it can be done in a discreet manner. For simplicity, however, all strategic considerations are ignored in this paper. Specifically, only one decision maker is assumed capable of acting in a non-myopic way, buying the product in one period and selling it in the next. All the other agents, producers and consumers alike, are assumed to act myopically and non-strategically, either because they have no knowledge about the future or because they lack adequate storage facilities. All the producers in a given period are represented by a single supply curve, and all the consumers, by a single demand curve. Equilibrium in a given period is defined as equality between supply and demand, the latter comprising both the quantity consumed in the period and that stored for later consumption.

The results of this paper are given in terms of the optimal ratio between the total quantity produced in a good period immediately preceding a bad one (in which there is no production) and the quantity stored in the same period for consumption in the next, bad period. Two kinds of optimal ratios are considered: optimal for a benevolent government, which seeks to maximize social welfare; and optimal for a profit-seeking trader. These results do not depend on any quantitative data related to the supply or demand curves. The first result is that, regardless of the exact characteristics of these curves, a benevolent government should purchase and store exactly *half* the total quantity produced in the good period. In this case, the consumption in the bad period that follows equals that in the good period and the government's budget is balanced in the long run. The second result is that, if the demand curve is linear, then a trader who only considers the profits to be made from buying cheap and selling expensively optimally purchases between a *third* and a *fourth* of the quantity produced in the good period. (The exact optimal ratio depends on the relative elasticities of supply and demand.) These optimal ratios may be viewed as benchmarks for the comparison of the results of more sophisticated models.

A benevolent government's optimization problem

The market for some product—call it wheat—in period 1 is described by a downward sloping (but not necessarily linear) demand curve, and by a supply curve that is flat (i.e., perfectly elastic supply), vertical (i.e., perfectly inelastic supply), or upward sloping (but not necessarily linear). In period 2, the demand curve is the same as in period 1 but the supply is zero. Under

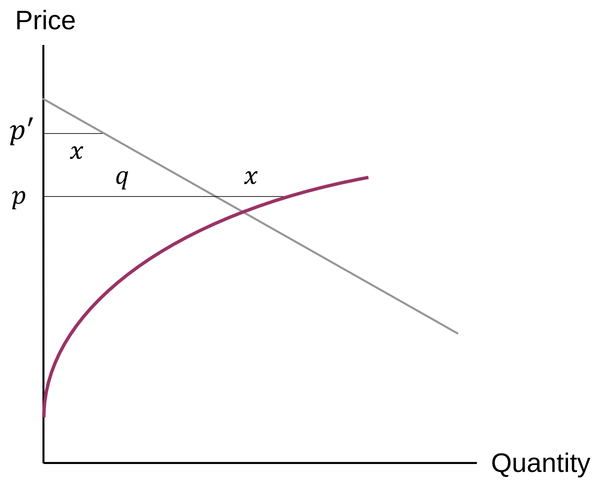

FIGURE 1. THE SUPPLY CURVE IN PERIOD 1 (HEAVY LINE) AND THE DEMAND CURVE IN PERIODS 1 AND 2 (LIGHTER LINE). THE SUPPLY IN PERIOD 2 IS ZERO.

these conditions, transfer of wheat from period 1 to period 2 by a benevolent government may have welfare-increasing effects. The question is: To maximize these effects, how much wheat should the government buy in period 1 and sell in period 2?

Assume, more specifically, that the government's goal is to maximize total surplus. That is, it seeks to maximize the sum of these four terms: consumer surplus in period 1, consumer surplus in period 2, producer surplus, and the government's own budget surplus or deficit. Let x be the quantity of wheat the government buys (and stores) and q the quantity the public buys (and consumes) in period 1. These two quantities, as well as the price of wheat p in that period, are interconnected: the point $(q + x, p)$ lies on the supply curve and (q, p) lies on the demand curve (see Figure 1). In period 2, the government sells the wheat it bought in period 1 at price p', such that (x, p') lies on the demand curve. (It is not difficult to see that, since the government's goal in period 2 is to maximize the sum of its revenue and the consumer surplus, it should sell in that period *all* its stored wheat.)

Proposition 1. *Total surplus is maximal if and only if the public buys the same amount of wheat in both periods (in period 1, from the producers, and in period 2, from the government). In this case, exactly half the wheat produced and sold in period 1 is bought by the government, whose expenditure in that period equals its revenue from selling the wheat in period 2.*

Proof. Suppose the government increases the quantity of wheat it buys in period 1 by a small amount dx, which changes the market price of wheat in periods 1 and 2 by dp and dp', respectively. The consumer surplus then changes by $-q\,dp$ in period 1 and by $-x\,dp'$ in period 2, the producer surplus changes by $(x + q)\,dp$ and the budget surplus or deficit changes by $(p' - p)\,dx + x(dp' - dp)$. Summing up, the change in the total surplus is $(p' - p)\,dx$. The government should therefore increase the quantity of wheat x it buys if $p' > p$ and decrease it if $p' < p$. Since increasing (decreasing) x decreases (respectively, increases) the difference $p' - p$, the total surplus is maximal if and only if $p' = p$. At this point, the government's budget is balanced and $q = x$. Thus, exactly half the wheat produced and sold in period 1 is bought by the government, which then sells it to the public in period 2 in its original price. ■

A trader's optimization problem

A trader wishes to make profit by buying wheat in period 1 at the market-clearing price and selling it in period 2, when there is no production of wheat. For given supply and demand curves, and neglecting transaction and other costs (such as the cost of storage), what quantity of wheat would maximize the trader's profit?

Denote by x the quantity of wheat the trader buys in period 1. This quantity uniquely determines the market price of wheat p in that period and the quantity of wheat q the public buys by the condition that the point $(q + x, p)$ lies on the supply curve and (q, p) lies on the demand curve. In period 2, the quantity of wheat sold by the trader is x' ($\leq x$) and its price is p', with (x', p') lying on the demand curve.

Proposition 2. *Suppose that the demand curve is linear. If the trader's profit is maximal, then he necessarily buys between a third and a fourth of the total quantity of wheat produced and sold in period 1, and sell it all in period 2. The upper and lower limits (*$1/3$ *and* $1/4$*) are approached if supply is much more elastic or inelastic, respectively, than demand, and they are exactly reached if it is perfectly elastic or inelastic.*

Proof. Although, in principle, the trader may buy more wheat in period 1 than he intends to sell in period 2 (i.e., $x' < x$), it is not difficult to see that it would not be optimal for him to do so. Indeed, suppose the trader increases the quantity of wheat he buys in period 1 by a positive amount Δx. Denote the corresponding changes to the market price and the quantity of wheat bought by the public in period 1 by Δp and Δq, respectively. Since the demand curve is downward sloping, either Δp and Δq have opposite signs or they are both zero. Hence, if $\Delta p < 0$, then $\Delta q + \Delta x > 0$. However, by the assumption concerning the supply curve, the last inequality would imply $\Delta p \geq 0$. This contradiction proves that $\Delta p \geq 0$, so that either $\Delta p > 0 > \Delta q$ and $\Delta q + \Delta x \geq 0$, or $\Delta p = \Delta q = 0$. This establishes the unsurprising finding that the price of wheat in period 1 is not reduced if the trader buys more of it, and it follows that, for the profit to be maximal, x must equal x'. More importantly, the above inequalities prove the following ones (which also hold if Δx is negative):

$$-1 \leq \frac{\Delta q}{\Delta x} \leq 0. \tag{1}$$

If supply is perfectly elastic, the right inequality in (1) holds as equality, since changing the quantity of wheat the trader buys does not affect the price of wheat and hence does not affect the public's demand for it. If supply is perfectly inelastic, the left inequality holds as equality, since any additional quantity of wheat the trader buys is wholly subtracted from the quantity bought by the public. It is easy to show, by manipulating the definitions, that the *limit* ratio between Δq and Δx is largely determined by (the absolute value of) the ratio between the elasticity of supply ε_s and the elasticity of demand ε_d at price p. More specifically,

$$\lim_{\Delta x \to 0} \left|\frac{\Delta q}{\Delta x}\right| = \frac{1}{1 + \left(1 + \frac{x}{q}\right)\left|\frac{\varepsilon_s}{\varepsilon_d}\right|}. \tag{2}$$

A positive profit for the trader implies $0 < x < q$, and therefore the coefficient $1 + x/q$ is between 1 and 2. (Tighter bounds are obtained below.)

Suppose the trader's choice of x maximizes his profit $(p' - p)x$. Because of the assumed linearity of the demand curve and its negative slope, maximizing this expression is equivalent to maximizing $(q - x)x$. At a maximum point, $0 \geq \Delta[(q - x)x] = (q - x)\Delta x + (\Delta q - \Delta x)(x + \Delta x)$ for all Δx, negative or positive. By (1), this condition implies $(x - q)/(x + \Delta x) + 1 \leq \Delta q/\Delta x \leq 0$ if $0 > \Delta x > -x$ and $(x - q)/(x + \Delta x) + 1 \geq \Delta q/\Delta x \geq -1$ if $\Delta x > 0$. Letting Δx tend to zero gives $2 \leq q/x \leq 3$. Therefore, the optimal ratio between the quantity of wheat x bought by the trader in period 1 and the total amount produced $q + x$ is between $1/3$ and $1/4$. It follows from (2) that the optimal ratio is close to the first or second number if the absolute value of the ratio between the supply and demand elasticities is close to infinity or zero, respectively. ■

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