Incomplete Information Games with Ambiguity Averse Players

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What is Ambiguity

- In everyday language: Something open to more than one meaning, explanation or interpretation; vagueness; imprecision
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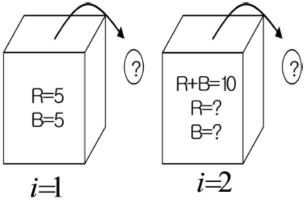
- In modern Economics and Decision Theory: (Subjective) uncertainty about probabilities
- Goes back at least to Ellsberg (1961), which contains some thought experiments illustrating how ambiguity might affect decision making in ways that challenged standard theories.
- Matters to the extent that changes behavior (actual and/or desired)
- Applications: finance (asset prices, form of contracts, portfolio choice); climate change policy; macroeconomics (monetary policy)

The (Two Color) Ellsberg Paradox



	Box 1			Box2	
	R	B		R	B
R_1	\$100	\$0	R_2	\$100	\$0
B_1	\$0	\$100	B_2	\$0	\$100

- Rank the four bets R_1, B_1, R_2 and B_2

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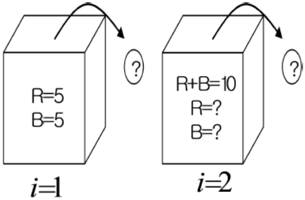

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- Rank the four bets R_1, B_1, R_2 and B_2
- $R_1 \sim B_1 \succ R_2 \sim B_2$
- SEU (Subjective Expected Utility) is not consistent with this behavior
  - What is the (subjective) probability of R in Box 2?
  - $R_1 \succ R_2 \implies p(R) < 1/2$; $B_1 \succ B_2 \implies p(R) > 1/2$

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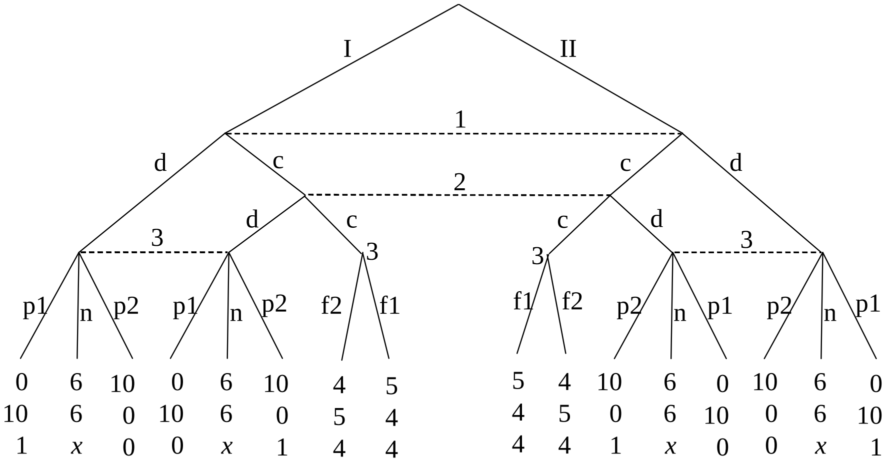
- Propose *sequential optimality* to capture perfection
  - Also propose refinement of sequential optimality:
    - Sequential Equilibrium with Ambiguity (SEA)

Multi-stage Games with Perfect Recall

- A **(finite) extensive-form multistage game with incomplete information, perfect recall and (weakly) ambiguity averse smooth ambiguity preferences** Γ consists of:
  - N a finite set of players
  - H a finite set of histories $h = (h_{-1}, (h_{0,i})_{i \in N}, \ldots, (h_{T,i})_{i \in N})$
    - $\Theta \equiv \{h_{-1} \mid h \in H\}$ the set of "parameters" or "types"
  - $\mathcal{I}_i \equiv \bigcup_{0 \le t \le T} \mathcal{I}_i^t$ are the information sets for player i
    - Perfect Recall: i's information sets reflect all her previously visited information sets
  - $u_i : H \to \mathbb{R}$ is the utility payoff of player i given the history
  - μ_i is a simple probability over $\Delta(\Theta)$, where $\Delta(\Theta)$ is the set of all probability measures over Θ and $\sum_{\pi \in \Delta(\Theta)} \mu_i(\pi)\pi(\theta) > 0$ for all $\theta \in \Theta$
  - $\phi_i : \mathrm{co}(u_i(H)) \to \mathbb{R}$ is a continuously differentiable, concave and strictly increasing function.

Example: Peace Negotiation (Greenberg 2000)



- μ: $\frac{1}{2}$-$\frac{1}{2}$ over distributions π_1 and π_2 with $\pi_1(\mathrm{I}) = 1$ and $\pi_2(\mathrm{I}) = 0$

Extensive Form Strategies

- **A (behavior) strategy** for player i is a function σ_i specifying the distribution over i's actions conditional on each possible information set.
- A strategy profile, $\sigma \equiv (\sigma_i)_{i \in N}$, is a strategy for each player.

Ex-ante Preferences:

$$V_i(\sigma'_i, \sigma_{-i}) \equiv \sum_{\pi \in \Delta(\Theta)} \phi_i \left(\sum_{h \in H} u_i(h) p_{(\sigma'_i, \sigma_{-i})}(h|h^0) \pi(h^0) \right) \mu_i(\pi),$$

where $p_\sigma(h|h^0)$ is the probability of reaching terminal history of play h from $h^0 \in \Theta$ according to strategies σ

Preferences and Ex-ante Equilibrium

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Preferences at an information set:

$$V_{i,I_i}(\sigma'_i, \sigma_{-i}) \equiv \sum_{\pi \in \Delta(I_i)} \phi_i \left(\sum_{h | h^t \in I_i} u_i(h) p_{(\sigma'_i, \sigma_{-i})}(h|h^t) \pi(h^t) \right) \nu_{i,I_i}(\pi)$$

Ex-ante Equilibrium

Definition

A strategy profile σ is an *ex-ante (Nash) equilibrium* of a game Γ if, for all players i and all σ'_i,

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- Azrieli and Teper (2011) applied to extensive form and smooth ambiguity preferences

Sequential Optimality

Definition

Fix a game Γ. A pair (σ, ν) consisting of a strategy profile and interim belief system is *sequentially optimal* if, for all players i, all information sets I_i and all $\sigma'_i \in \Sigma_i$,

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$$V_{i,I_i}(\sigma) \geq V_{i,I_i}(\sigma'_i, \sigma_{-i}).$$

- Under ambiguity neutrality, just ex-ante equilibrium plus Kreps and Wilson's sequential rationality.
- Implies subgame perfection.
- Satisfied by PBE and Sequential Equilibrium.

Negotiation Example: Equilibrium

Proposition

If players 1 and 2 are ambiguity neutral (i.e., ϕ_1 and ϕ_2 are affine), no ex-ante equilibrium results in a positive probability of peace.

Intuition: No threatened punishment by 3 is enough to simultaneously deter both 1 and 2 from scuttling the negotiations.

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Intuition: If 3's strategy is contingent on the realization of the payoff-irrelevant ambiguous parameter (I or II), this creates ambiguity in the minds of players 1 and 2 about who will be punished or favored. Given sufficient ambiguity aversion, this leads each to act as if the chance it will be punished is sufficiently high to support cooperation. Given cooperation, 3 is indifferent between all strategies.

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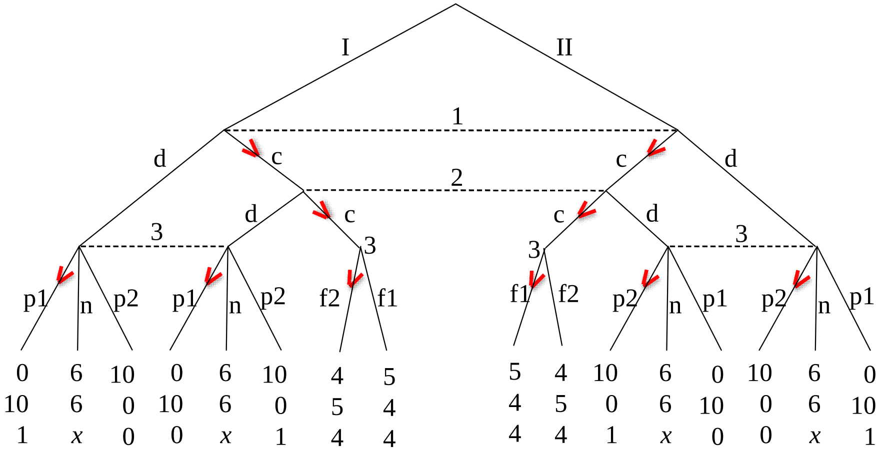

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If $x > 1$, then in all sequential optima players 1 and 2 play d with probability 1. If $x \leq 1$ and players 1 and 2 are sufficiently ambiguity averse, then there is a sequential optimum yielding peace with probability 1.

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Intuition: If $x > 1$ then punishment by 3 is not a credible threat. If $x \leq 1$ then some ambiguous punishment strategies become best responses to some beliefs of 3 following d. Enough of a deterrent given sufficient ambiguity aversion (though when $x \in (\frac{1}{2}, 1]$, may need more aversion than was needed to get peace as an ex-ante equilibrium).

Updating

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- We show that any sequentially optimal profile is sequentially optimal with respect to an interim belief system generated by one particular update rule.

Smooth Rule Updating (Hanany & Klibanoff 2009)

An interim belief system ν satisfies the *smooth rule* using σ if the following holds for each player i and information set I_i: except immediately following a deviation, for all $\pi \in \Delta(I_i)$,

$$\nu_{i,I_i}(\pi) \propto \sum_{\hat{\pi}\in\Delta(I_i^{-1})|\hat{\pi}_{I_i}=\pi} \frac{\phi_i'\left(\sum_{h|h^{t-1}\in I_i^{-1}} u_i(h)p_\sigma(h|h^{t-1})\hat{\pi}(h^{t-1})\right)}{\phi_i'\left(\sum_{h|h^t\in I_i} u_i(h)p_\sigma(h|h^t)\pi(h^t)\right)} \cdot \left(\sum_{h^t\in I_i} p_{-i,\sigma_{-i}}(h^t|h^{t-1})\hat{\pi}(h^{t-1})\right)\nu_{i,I_i^{-1}}(\hat{\pi}),$$

where

$$\hat{\pi}_{I_i}(h^t) = \frac{p_{-i,\sigma_{-i}}(h^t|h^{t-1})\hat{\pi}(h^{t-1})}{\sum_{\hat{h}^t\in I_i} p_{-i,\sigma_{-i}}(\hat{h}^t|\hat{h}^{t-1})\hat{\pi}(\hat{h}^{t-1})}$$

and similar requirements hold at initial information sets $I_i \subseteq \Theta$.

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Fix a game Γ. A strategy profile σ is sequentially optimal if and only if there exists an interim belief system $\hat{\nu}$ satisfying the smooth rule using σ such that $(\sigma, \hat{\nu})$ is sequentially optimal.

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Fix a game Γ. A strategy profile σ is sequentially optimal if and only if there exists an interim belief system $\hat{\nu}$ satisfying the smooth rule using σ such that $(\sigma, \hat{\nu})$ is sequentially optimal.

- Analysis of sequential optima of a game may be undertaken under the "as if" assumption that all players use smooth rule updating
- Under ambiguity neutrality, identifies the same strategy profiles as Kreps and Wilson (1982)'s sequential rationality plus the assumption of Bayesian updating given σ, which are, in turn, the same as weak PBE.

Sequential optimality and one-stage deviations

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Fix a game Γ *and a pair* (σ, ν) *such that* ν *satisfies the strong smooth rule using* σ. *Then* (σ, ν) *is sequentially optimal if and only if* (σ, ν) *has no profitable one-stage deviations.*

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Fix a game Γ and a pair (σ, ν) such that ν satisfies the strong smooth rule using σ. Then (σ, ν) is sequentially optimal if and only if (σ, ν) has no profitable one-stage deviations.

- Important implications shared with the standard ambiguity neutral case:
  - Allows the use of "folding back" to check whether some $\hat{\sigma}$ is sequentially optimal given beliefs.
  - Beliefs not determined by "folding back", but by updating given $\hat{\sigma}$

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- Extend Kreps and Wilson's *consistency* to accomodate ambiguity aversion by replacing Bayes' rule with the smooth rule:

Sequential Equilibrium with Ambiguity (SEA)

Definition

A pair (σ, ν) satisfies *smooth rule consistency* if there exists a sequence of completely mixed strategy profiles $\{\sigma^k\}_{k=1}^{\infty}$, with $\lim_{k\to\infty} \sigma^k = \sigma$, such that $\nu = \lim_{k\to\infty} \nu^k$, where each ν^k is the interim belief system determined by smooth rule updating using σ^k.

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A Sequential Equilibrium with Ambiguity (SEA) of a game Γ is a pair (σ, ν) that is sequentially optimal and satisfies smooth rule consistency.

- Show can replace sequential optimality by no profitable one-stage deviations in describing SEA.

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Suppose there is an ex-ante equilibrium yielding peace with probability 1*. Then there exists an SEA yielding peace with probability* 1 *if and only if* $x \leq 0.5$*.*

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Intuition: Smooth rule consistency implies that 3 must have the same interim beliefs after observing a defection d no matter the value of the ambiguous payoff-irrelevant type (I or II). If $x > 0.5$, then following d no mixture placing positive weight on both $p1$ and $p2$ can be a best response to any belief. Thus, 3 cannot credibly punish both players 1 and 2 and one of them will play d. If $x \leq 0.5$ then all mixtures over $p1$ and $p2$ are best responses for 3 to the belief putting all weight on the measure assigning probability 0.5 to each player being the defector, and these interim beliefs for 3 satisfy smooth rule consistency.

Sequential Equilibrium with Ambiguity (SEA)

Theorem

An SEA exists for any game Γ *(no matter how ambiguity averse players are or what are their initial beliefs).*

Comparative Statics in Ambiguity Aversion

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  - Taking the union over all equilibria generated by *any* beliefs, the set of ex-ante or sequentially optimal or SEA strategies is the same for any ambiguity aversions.
  - Relation of this last result to Battigalli et al. (2015) and Battigalli et al. (2019) on self-confirming equilibria with ambiguity aversion.

Ambiguity Aversion and Robustness

Two robustness notions:

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- Belief robust (in words, formal definition in paper):

Definition

Ambiguity aversion makes an equilibrium σ *belief robust* if sufficient increases in players' ambiguity aversion, holding the π's in the supports of players' beliefs $(\mu_i)_{i \in N}$ fixed, make all beliefs placing sufficient weight on each such π support σ as an equilibrium.

Ambiguity Aversion and Robustness

Assumption (1)

For each player i, $\sum_{h \in H} u_i(h) p_\sigma(h|h^0) \pi(h^0)$ can be strictly ordered across the π in the support of μ_i

Robustness to increased ambiguity aversion implies belief robustness:

Theorem

If an equilibrium σ is robust to increased ambiguity aversion and Assumption (1) holds, then ambiguity aversion makes σ belief robust.

- Holds for both ex-ante equilibrium and sequential optimum notions.
- Holds with an additional condition for SEA.

Ambiguity Aversion and Robustness

Intuition for the result:

- Robustness to increased ambiguity aversion implies that σ_i must be a best response given the minimizing π
- If one were infinitely ambiguity averse (i.e., all effective weight placed on the minimizing π), then the beliefs over the π cease to matter and all beliefs with the same support make σ_i a best response.
- Along the way, as ambiguity aversion is increased, the proof uses concave transformations tailored to generate specific shifts in effective beliefs that maintain optimality for all beliefs placing sufficient weight on each π in the support.
- Assumption 1 ensures enough flexibility in the manner in which more ambiguity aversion can shift the effective weight placed on expected payoffs for the various π.

Ambiguity Aversion and Robustness

- Consider a population having heterogeneous beliefs. Equilibria that, under ambiguity neutrality, are not supported by many beliefs might not be expected to occur often. Our robustness result offers ambiguity aversion as a possible explanation for unexpected prevalence of such equilibria. Specifically, if such an equilibrium is robust to increased ambiguity aversion, ambiguity aversion can make it an equilibrium for more of the population (i.e., for more beliefs).

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- Apply to show conditions under which entrant ambiguity aversion makes Limit Pricing equilibria more robust

Related Literature

- Extensive form games with ambiguity and incomplete information
  - Battigalli et al. (2019) – SCE, smooth ambiguity prefs, Bayesian updating, not sequential opt even on path, focus on comparative static in ambiguity aversion
  - Pahlke (2018) – sequential equil., Recursive MEU, ex-ante preference not a primitive – analyze different game for each σ
- Specific extensive form applications with ambiguity and incomplete information – all work by violating sequential optimality
  - Eichberger and Kelsey (1999, 2004), Dominiak and Lee (2017) – signaling games
  - Bose and Daripa (2009), Bose and Renou (2014) – dynamic mechanism design
  - Kellner and Le Quement (2017, 2018) – sender-receiver cheap talk games
  - Beauchêne, Li and Li (2019) – persuasion
  - Auster and Kellner (2018) – descending price auctions

Sequential optimality, consistent planning and one-stage deviations

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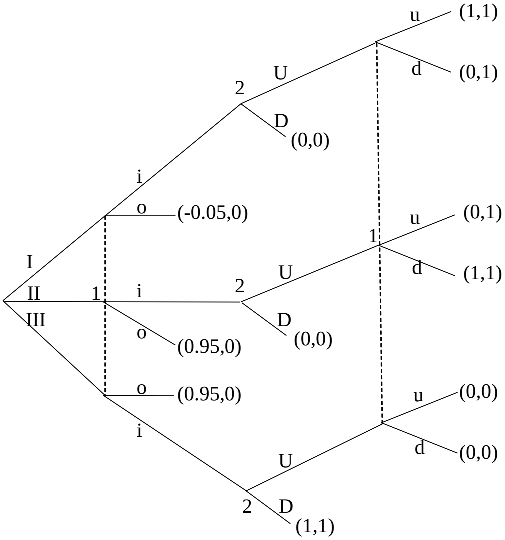
- For a fixed interim belief system, sequential optimality $\Rightarrow$ consistent planning optimality $\Rightarrow$ no profitable one-stage deviations
- Our results show that under smooth rule updating, all three are equivalent. (Generalizes result under ambiguity neutrality that Bayesian updating makes the three equivalent).
- When sequential optimality is strictly stronger what kinds of behavior does it rule out?

Sequential optimality vs consistent planning



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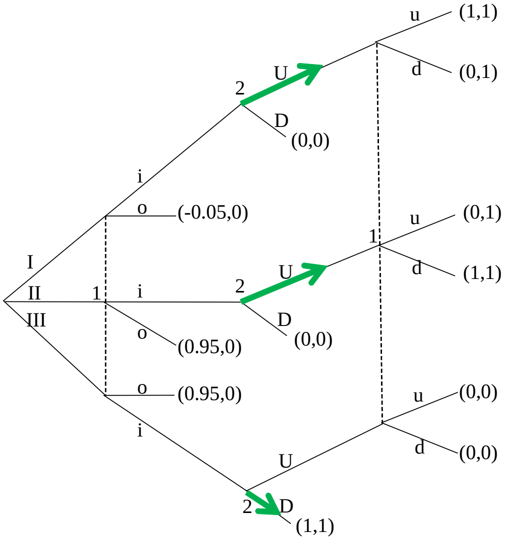

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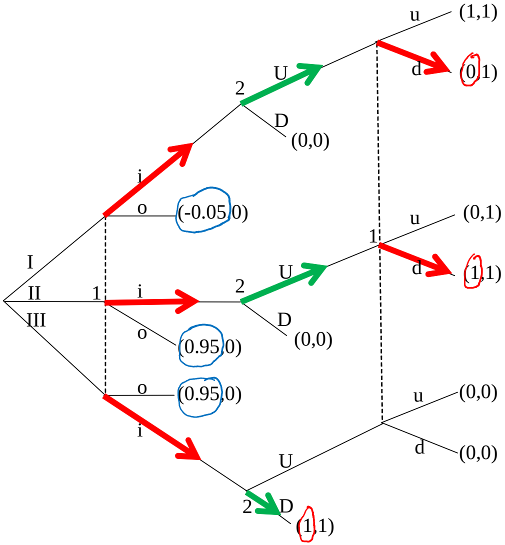

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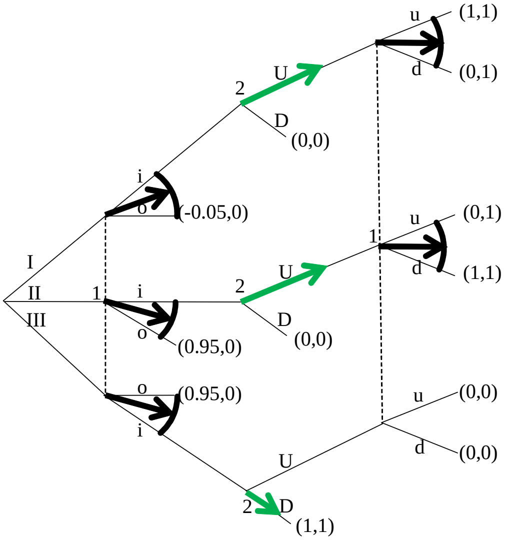

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- Framework allows modeling strategic use of ambiguity and profitable ambiguous cheap talk
- Tractable enough to handle some classical game theoretic economic models such as Milgrom Roberts (1982) -style limit pricing.