Occupational Choice: Teacher Quality Versus Teacher Quantity

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Abstract

This article examines the relationship between skill-biased technological changes and the decline in both teacher quality and pupil–teacher ratio—called the "quality–quantity trade-off"—in the United States and other advanced economies during the past several decades. The study presents a theory of educational production that emphasizes teachers' occupational choices. A key assumption is that talented agents have a comparative advantage in learning. The model endogenously generates a teachers sector with intermediate abilities between two types of skilled workers with tertiary education: highly skilled workers and vocational workers. This unique feature helps specify *which* technological changes may lead to quality–quantity trade-offs. In particular, a crucial element is that the ratio of incomes and thus the income inequality rises *within* the skilled sector. In this case, the most talented teachers depart from the teachers sector to join the highly skilled sector, and as such, teacher quality declines. In other cases, both teacher quality and teacher quantity may increase. The results are consistent with the observed patterns of technology, educational attainment, educational expenditure, and wage inequality in advanced economies. Finally, another potential cause for the quality–quantity trade-off is a reduction in teacher certification requirement unless the reduction is implemented exclusively on high-ability workers.

Keywords: Human capital accumulation; Skill-biased technological change; Teacher quality; Pupil-teacher ratio; Education policy; Teacher certification

JEL classification: I21; O33

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I. Introduction

What are the implications of skill-biased technological changes (SBTCs) on the quality and quantity of teachers? Which types of SBTCs might result in quality–quantity trade-offs? How is the rising income inequality among skilled workers related to the declining quality of teachers over time? To address these questions, this research develops a theory of educational production with compulsory education and tertiary education that emphasizes teachers' occupational choices. The data in table 1 suggests that in the United States, real education expenditure per pupil has increased since 1955. However, the pupil–teacher ratio has consistently fallen despite the ups and downs in the enrollment dynamics, and teacher quality has also declined relative to the educated labor force.[1] This trade-off between the quality and quantity of teachers may have occurred in other OECD countries as well (Nickell and Quintini, 2002; OECD, 2005a).

Table 1: US Data on public elementary and secondary schools, 1955-2015

Year	Enrollment (in thousands)1	Teachers (in thousands)1	Pupil/teacher ratio1	Real expenditure per pupil4	Expenditure as a percentage of GDP2	Relative teacher salary3
1955	30,680	1141	26.9	3,090	3.3*	
1960	36,281	1408	25.8	3,441	3.6	43
1965	42,173	1710	24.7	4,398	3.9	
1970	45,894	2059	22.3	5,671	4.6	44
1975	44,819	2198	20.4	6,570	4.6	
1980	40,877	2184	18.7	6,796***	4.0	41
1985	39,422	2206	17.9	7,930***	3.8	
1990	41,217	2398	17.2	9,428	4.3	35
1995	44,840	2598	17.3	9,669	4.3	
2000	47,204	2941	16.0	11,254	4.5	36.5
2005	49,113	3143	15.6	12,230	4.5	
2010	49,386	3174	15.6	12,922**	4.6	
2015	50,824	3372	14.6			

Source:
1. Digest of Education Statistics, 2010, Table 68. Data for 2010 and 2015 is projected.
2. Digest of Education Statistics, 2010, Table 28.
3. Hanushek and Rivkin (2004). Percentage of college educated females, age 20-29, earning less than average female teacher, age 20-29.
4. Digest of Education Statistics, 2010, Table 190. Deflated by CPI and in 2008-2009 dollars.
*1959 data.**2007 data.***estimated. I basically reproduce table 1 in Gilpin and Kaganovich (2012) using updated data.

[1] The latter finding is robust to the sparse indicators used as a proxy for teacher quality, e.g., relative teacher salary (as table 1 displays), fraction of people entering teaching relative to other educated workers above the 80th percentile, fraction of prospective teachers being drawn from less selective institutions, and relative fraction of married female teachers with top-earning or highly-educated husbands assuming positive assortative mating. (e.g., Bacolod, 2006; Dolton and Marcenaro-Gutierrez, 2011; Hanushek and Rivkin, 1997; Lakdawalla, 2006; Stoddard, 2003; Hoxby and Leigh, 2004; Corcoran et al., 2004).

One goal of this article is to relate the quality–quantity trade-off to stylized facts in the United States and other advanced countries within a general equilibrium overlapping-generations framework. In particular, the results suggest the role of the increasing value of skill and thereby the rising income inequality among skilled workers in explaining the quality–quantity trade-off taking into account the non-pecuniary cost of higher education.

A common cause given in the literature for the quality–quantity trade-off is SBTC, which amplifies the demand for more college-educated workers, with a corresponding increase in their wages. As a result, decision makers tend to substitute quantity for quality in their resource allocation decisions. This study takes this explanation one step further and argues that only *certain* types of SBTCs lead to quality–quantity trade-offs. The few theoretical models that address this question (Gilpin and Kaganovich, 2012; Lakdawalla, 2006; Stoddard, 2003; Bacolod, 2007, 'existing TO models'), use a simplifying assumption that agents base their decisions only on income considerations. As a result, there is *excess* supply of low-ability teachers (whose earnings in the production sector are lower than teacher wages). Thus, the lower threshold of teachers is solely determined by the government. The current model is more comprehensive because it further emphasizes the occupational choice decisions of individuals taking into account the *leisure* implications of acquiring education and allowing for an optimal allocation of their time, consistent with Betts (1998) and Costrell (1994). The model further assumes that the innate ability reduces the effort needed to acquire higher education ('the comparative advantage assumption'). This is compatible with Spence (1973)'s assumption that the costs of signaling (in education, for example), are negatively correlated with productive capability. He mentions that signaling costs should be interpreted broadly to include psychic and other costs, as well as time. Under these assumptions, teachers endogenously have intermediate abilities typically between two types of skilled workers (with tertiary education): highly skilled and vocational. This unique division helps specify the crucial element in SBTCs that lead to quality–quantity trade-offs—that is, the rising income inequality *within* the skilled sector. Two strands of literature propose nuanced SBTCs as possible explanations for this element based on unobserved ability and the routinization hypothesis.[2] In the presence of nuanced SBTC, the model provides an explanation for the quality–quantity trade-off. Workers at the upper end of the ability distribution receive exponentially larger returns for their ability relative to their less talented peers. As a result, the highly skilled sector attracts the most talented teachers, which in turn generates a downward pressure on relative teacher quality. Moreover, as the pursuit of higher education becomes worthwhile for a broader population, workers with relatively low ability are added to the skilled sector[3]. In other types of SBTCs, when the ratio of incomes (and thus, the income inequality) does not change among skilled workers, both the supply of teachers and their quality increase. Additionally, the model relates SBTCs to observed patterns in the United States and other advanced countries since 1960: increasing educational expenditures, rising wage inequality between skilled and unskilled workers, rising college attendance, and equalized teacher incomes (Autor et al., 1998; Berman et al., 1998; Goldin and Katz, 1999; Katz and Murphy, 1992).

Another implication of ‘the comparative advantage assumption’ is that a reduction in the cost of becoming a teacher benefits more with low ability agents, causing an adverse selection to the teachers’ sector thereby worsening teacher quality and increasing their numbers. This result is consistent with the observations of UNESCO (2006) that several developing countries with limited budgets and serious teacher shortages (e.g., Burkina Faso, Bangladesh, India) have decided that the most viable option is to lower entry

[2] Much of the literature attributes the evidence of rising residual wage inequality (within education, experience, age, race, and gender groups) to SBTCs that increase the returns to unobserved learning *abilities*. Skill-biased revolutions trigger reallocations of capital from slow- to fast-learning workers, thus generating absolute gains for people with high cognitive ability (see Bartel and Sicherman, 1999; Caselli 1999; Galor and Moav, 2000; Juhn et al., 1993; Katz and Murphy, 1992; Murnane et al., 1995; Nelson and Phelps, 1966). The same phenomenon of rising polarization in the income distribution is addressed by the recent routinization literature. Theoretical contributions from Manning (2004) and empirical findings from Autor et al. (2003) and Spitz-Oener (2006) and Corsini (2010) suggest that jobs that require routine tasks (typically within the vocational sector, e.g., clerks, public servants, administrative employee, bookkeeping) are being substituted by new computer technologies. Thus, the technological change is beneficial for highly skilled workers, who hold a comparative advantage in nonroutine tasks, but detrimental for middle skilled jobs (see also Acemoglu and Autor, 2010).

[3] Gilpin and Kaganovich (2012) derive similar outcomes through a different mechanism – a dynamic process of human capital driven economic growth (as opposed to SBTC) in a different framework and a kinked human capital formation. Their competing view gives more weight to increasing dispersion in educational attainments (see their Theorem 1, Lemma 1 and Proposition 3 and their review about 'the rising talent premium'; see related discussions throughout my paper about the distinctions between the models). Note that the two perspectives may coincide. The literature on SBTCs points to the increase in the supply of skill due to growing availability of education as its underlying cause. For instance, Acemoglu (1998) argues that when the supply of skill rises, the market size of skill-complementary technologies grows, thus their invention is more profitable.

standards for the teaching profession. This may also be the case in California, where its class-size reduction program came at a cost of hiring teachers with lower qualifications (Jepsen and Rivkin, 2002). On the other hand, policies that combine teacher lower cost certification programs with access restrictions to high ability individuals, e.g., the well-known Teach For America, eliminate the adverse selection problem, restore teacher quality and increase the quality of education. This study further demonstrates the cost of requiring a relatively long time investment from teachers. In this case, in equilibrium even the top-quality teachers earn a higher income than their counterparts in the skilled sector, which compensates them for their greater effort in higher education.

A key insight in the analysis is that accounting for the heterogeneous learning effort in higher education is important for analyzing the quality–quantity trade-off. The evidence in Loewenstein and Thaler (1989) and Sizer (1984) suggests an extremely high discount rate of students on future incomes and a high emphasis of youth culture on current leisure and consumption. In Costrell (1994) and Betts (1998), time is optimally allocated between education and leisure, and they argue that student time and effort are the most important inputs to education, given the level of ability (see also Azariadis and Drazen, 1990; Glomm and Ravikumar, 1992; Tamura, 1991; Viaene and Zilcha, 2002). Huggett et al. (2006) show that differences in learning ability account for the bulk of the variation in earnings across agents. Accordingly, with heterogeneity in abilities, this study assumes that highly able agents have a comparative advantage in learning over low-ability workers. Therefore, low-ability workers are not interested in devoting the learning effort required for teacher certification. Instead, low-ability workers, who still desire tertiary education, enroll in shorter programs geared for entry into the labor market and designed to acquire practical/vocational/technical skills and know-how needed for employment in a particular occupation or trade[4]. As a result, they earn lower incomes than teachers (but higher incomes than unskilled workers). The model further assumes that teachers are equally paid because of collective bargaining agreements.[5]

[4] e.g. nurses, nannies, dental assistants, technicians, computer/network/internet/technical operators, QA (quality assurance), paramedics, investigators, bookkeepers, policemen, firemen, medical secretaries, practical engineers. Note that the classification and description of jobs within the model follows the International Standard Classification of Education (ISCED) developed by UNESCO (see UNESCO ,1999). The classification distinguishes between six levels of education ranging from preprimary to tertiary, with 3 levels of tertiary education. For further description of ISCED education programs and attainment levels and their mappings for each country, see Annex 3 (table 2) in OECD (2005b).

[5] This assumption is more suitable within a specific district or a small country and within the two periods of the model. Hoxby and Leigh (2004) highlight the substantial contribution of teachers' unions to wage

Under these assumptions, this study endogenously posits that teachers typically have intermediate abilities between vocational workers and highly skilled workers, who enroll in longer programs of higher education (typically academic and theoretically based/research preparatory). Thus, the main contribution of this article is the introduction of a more complete model of teacher self-selection that helps grasp the essential features of SBTCs that promote quality–quantity trade-offs.

The paper is organized as follows: Section II develops a general equilibrium model. Section III defines the equilibrium and provides conditions for its existence and uniqueness. Section IV characterizes the time investment in higher education and incomes across sectors. Section V details the numerical example. Section VI derives the comparative static results on teacher quality and quantity. Section VII analyzes the case of two pathways into teaching: formal and lower cost certification and section VIII concludes.

II. The Model

A. Timeline

Consider an overlapping-generations model with a continuum of consumers in each period and no population growth. Assume that agents live for two periods. In the first period, childhood (the education period), they are not productive: their parents support them, and they acquire compulsory public education at a uniform level. Then, they allocate their time to higher education. In the second period, adulthood (the working period), they work, pay taxes, give birth to one child and consume their after-tax income. Tax revenues are used by the government to support the children's public education.

B. Human Capital Formation

Let $h_{i,t}$ be the human capital level in adulthood of an agent *i* born at date *t*–1. The term E_{t-1} denotes the public education level she acquired as a child. Public education is produced by two inputs, given from period $t-1$: teacher quality, $\bar{h}^{\eta}_{T,t-1}$, defined as the average level of human capital of the instructors, and teacher quantity , $P^{\gamma}_{T,t-1}$, defined

compression. It is well documented that unions tie teachers' incomes primarily to seniority, oppose linking incomes to performance, and insist on raising incomes across the board. Gilpin and Kaganovich (2012) note that the compression of teacher salaries is also attributed to the difficulty in measuring teacher productivity and determining criteria for performance-based pay.

as the proportion of teachers in the working population. I define the quality of public education in period t as a Cobb–Douglas function of teacher quantity and teacher quality in period t-1[6]

(1) $E_{t-1} = P_{T,t-1}^{\gamma}\overline{h}_{T,t-1}^{\eta}, \quad \gamma > 0, \eta > 0$

After graduation from high school, the agent chooses the fraction of time dedicated to higher education, $0 \leq e_{i,t-1} \leq 1$. This leads to the first assumption:

<u>(A1)</u> A minimal level of time investment in higher education is necessary to attain some tertiary education degree, $\hat{e}$. ∘

If this standard is not met, the human capital equals formal compulsory schooling. If the agent decides to acquire higher education above the minimal level, the human capital further depends on the time investment in higher education as well as the agent's innate ability, denoted by $\theta_{i,t-1}$. The term $\theta_{i,t-1}$ is i.i.d. and distributed as some random variable $\tilde{\theta}$ with values in the interval $[\underline{\theta},\overline{\theta}]$, where $\underline{\theta} < \overline{\theta} < \infty$. To simplify the exposition (but at no cost to the essence of the matter), let $\underline{\theta} = 1$. Note that 'abilities' may reflect any unobserved initial endowments related to home background or school background.[7] The production function of human capital is given by

(2) $h_{i,t} = \begin{cases} \rho E_{t-1} e_{i,t-1}^{\beta} \theta_{i,t-1}^{\lambda}, & if\ 1 \geq e_{i,t-1} \geq \hat{e} \\ E_{t-1}, & if\ 0 \leq e_{i,t-1} < \hat{e} \end{cases}$

for some $\beta < 1$, $\lambda < 1$, $\rho \hat{e}^{\beta} > 1$. Thus, acquiring higher education (above the minimal level) increases the human capital. Moreover, consistent with Ben-Porath (1967), Rosen (1976), Gilpin and Kaganovich (2012) and Laitner (2000), higher education and compulsory schooling are more productive for agents with higher initial endowments. In the following discussion, for simplicity of presentation, I omit the time index.

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[6] Theoretical models introduced by Viaene and Zilcha (2002), Eckstein and Zilcha (1994), Gilpin and Kaganovich (2008), Lakdawalla (2006), Stoddard (2003), Tamura (2001) and Hatsor (2008) refer to quality and quantity of teachers as the dominant inputs for public education. Numerous empirical studies estimate the contribution of teacher quality and teacher quantity to the success of the educational process in schools (e.g., Tamura, 2001; Rivkin et al., 2005; Hanushek, 2003; OECD, 2005a; Clotfelter et al., 2007, Hanushek and Woessmann, 2010, see a review in Hatsor (2008)). It is fair to assume that teacher human capital has an influence on the quality of their work, as it represents their basic raw knowledge.

[7] Cunha and Heckman (2007) argue that abilities are created, not solely inherited. The family plays a powerful role in shaping them through genetics, parental investments and choice of child environments.

C. Sectors of Workers: Teachers and Skilled and Unskilled workers

Individuals belong to a given sector on the base of their education. In particular, if the time investment in higher education is lower than $\hat{e}$, the individual is assigned to the unskilled sector. At the same time, if she decides to acquire tertiary education above $\hat{e}$, then she becomes a skilled worker *or* a teacher, based on the following classification:

> (A2) Teachers must invest at least $e_T \geq \hat{e}$ in higher education to attain a teacher certification. This level is exogenously given by governmental requirements. ∘

The time investment of teachers acts in the model as both a sorting mechanism to the teachers sector and a source of human capital (Betts, 1998; Weiss, 1983). Accordingly, agents who invest in higher education $e_i \in [\hat{e}, e_T)$ are not eligible for a teacher certification. Nonetheless, they are eligible for a tertiary education degree (recall (A1)-(A2)), thereby they are assigned to the skilled sector. Otherwise, if the requirement of e_T is met, i.e., $e_i \geq e_T$, they can choose whether to become skilled workers or teachers.[8]

Sectors differ in their income structure.[9] I assume that the income of unskilled workers is uniform, denoted by y_U , because abilities and higher education are secondary determinants of their incomes. According to Bishop (1988), employers of high school graduates rely almost exclusively on the diploma, rather than the more complete information contained in school transcripts or employment tests. Furthermore, because teachers' collective bargaining agreements tend to equalize their incomes, I assume that teachers are equally paid, and their income is denoted by y_T. In contrast, following Becker (1975), I assume that skilled workers are rewarded for their human capital, because skilled workers are employed in professions that require various levels of abilities and higher education. Their income equals $y_{s,i} = w_S h_{s,i}$, where w_S is the wage rate for an effective unit of human capital. Note that income and time investment in all sectors are determined in equilibrium, except for the time investment of teachers.

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[8] As will become apparent in the following section, at the optimum, teachers invest in their higher education exactly e_T, because their income is not based on their human capital.

[9] Note that the income variables in all professions, including teaching, represent lifetime incomes over the entire career. Accordingly, I do not model the wage dynamics over the career path as the worker accumulates experience (see a related discussion in section VII).

D. Agents' Decisions: Allocation of Time and the Labor Supply

In childhood, each agent is endowed with one unit of time, which she allocates between time investment in higher education, e_i , and leisure. In adulthood, each agent is endowed with an additional unit of time, which she inelastically devotes to labor. Lifetime utility of agent *i* depends on consumption, denoted by c_i, and effective leisure, l_i, for some $\delta, \mu < 1$:

(3) $U_i = u_{i,1} + \varphi u_{i,2}$, and

$$u_{i,1} = log\left(l_i^{\mu}\right), \quad for \quad l_i > 0,$$

$$u_{i,2} = log\left(c_i^{\delta}\right), \quad for \quad c_i > 0$$

where $u_{i,1}$ and $u_{i,2}$ are utility of agent *i* in period 1 and 2, respectively and φ is the discount factor. Rearranging equation (3) obtains

(4) $U_i = log\left(l_i\right)^{\mu} + \varphi \, log\left(c_i^{\delta}\right) \quad for \quad l_i > 0, \quad c_i > 0,$

The effective leisure of agent *i* is given by the following:

<u>(A3)</u> For some parameter $Z > 0$, $l_i = 1 - \frac{Ze_i}{\theta_i}$,

where $0 \leq l_i \leq 1$. That is, $0 \leq e_i \leq \frac{\theta_i}{Z}$.∘

The ratio $\frac{Ze_i}{\theta_i}$ represents the learning effort invested in higher education, where Z is the non-pecuniary cost of leisure. I assume that highly talented agents have a comparative advantage in learning. Therefore, the learning effort required to achieve a given level of higher education diminishes with the level of ability. Accordingly, less talented agents have lower incentives to invest in higher education at the expense of leisure. An additional assumption is necessary for the existence of the teachers sector:

<u>(A4)</u> The following condition holds:

(5) $e_T < \frac{\overline{\theta}}{Z}$.∘

If this condition is not satisfied, it is easy to verify from (A3) that no agent has a positive effective leisure as a teacher. I assume that the government avoids this scenario by ensuring that teachers' time investment is sufficiently low. Note that if the cost of leisure is sufficiently low, $Z < 1$, this condition holds for all e_T. I assume hereinafter that assumptions (A1)–(A4) hold. Given the income structure in the three sectors, $y_{s,i}$, y_T and y_U, each agent chooses whether to become a teacher, a skilled worker or an unskilled worker and how much time to invest in higher education by maximizing his or her utility, given in equation (4), such that his or her effective leisure, given in (A3), and consumption are positive:

$$
\begin{aligned}
&\underset{ci,ei}{Max}\; u_i = log\left(\left(1-\frac{Ze_i}{\theta_i}\right)^{\mu} c_i^{\varphi\delta}\right)\\
&s.t.\\
&c_i \geq 0 \quad and \quad 0 \leq e_i \leq \frac{\theta_i}{Z}\\
&One\ of\ the\ following\ options\ can\ be\ chosen:\\
&(a)\ Choose \quad e_i \geq e_T \quad and \quad c_i = (1-\tau)\,y_T \quad (teachers)\\
&(b)\ Choose \quad e_i \geq \hat{e} \quad and \quad c_i = (1-\tau)\,w_S h_i \quad (skilled)\\
&(c)\ Choose \quad e_i \quad and \quad c_i = y_U \quad (unskilled)
\end{aligned}
\tag{6}
$$

where h_i is defined in equation (2) and consumption equals the after-tax income. For simplicity, I assume the following progressive taxation: only the higher income sectors, teachers and skilled workers, pay taxes, and the tax rate, τ, is exogenously given. At the optimum, because teachers and unskilled workers are not rewarded for their human capital, teachers invest in their higher education exactly the time investment required to meet the standard, e_T, and unskilled workers exert zero effort, $e_U = 0$. I obtain the optimal time investment and effective leisure of skilled workers using (A3) and rearranging the first-order condition that equates their marginal utility from time investment in higher education to the marginal cost:

$$e_i^* = \frac{1}{Z}\left(\frac{\varphi\delta\beta}{\mu+\varphi\delta\beta}\right)\theta_i. \tag{7}$$

$$l_i = \frac{\mu}{\mu+\varphi\delta\beta}. \tag{8}$$

Note that innate ability has two distinct effects on the time investment in higher education: First, it amplifies the *returns* to education in the production of human capital (equation (2)), thereby the corresponding income level rises.[10] Second, it reduces the *cost* needed to acquire the education (in terms of effort) (recall (A3)), thereby the incentives to invest in higher education rise. Therefore, highly able workers prefer to spend more time on higher education than less talented ones (see equation (7)).

Substituting equation (7) in equation (2), I derive the human capital of skilled workers as a function of the quality of public education and ability:

$$(9) \qquad h_i = \rho\left(\frac{\varphi\delta\beta}{Z\left(\mu+\varphi\delta\beta\right)}\right)^{\beta}\left(\theta_i\right)^{\beta+\lambda} E \quad .$$

Accordingly, highly skilled workers accumulate larger levels of human capital directly (through λ) and indirectly (through β) by spending more time on higher education. Thus, they earn higher incomes than less talented skilled workers (recall equations (2), (A3) and (7)). If the weight of future consumption rises, skilled workers increase the time investment in higher education in order to increase their human capital (and thus their future incomes). Without loss of generality, I assume in the rest of the paper that $\varphi = 1$. Note that teachers are also compensated for having higher ability through the lower learning effort required to attain teacher certification (recall (A2)-(A3)). Accordingly,

Corollary 1: The utility from skilled professions and from teaching increases with ability, while the utility in the unskilled sector is independent on ability. ◦

Thus, the least talented workers join the unskilled sector. Because of their insufficient talent for schooling, they prefer not to acquire higher education at all and enjoy the extra leisure. That is, acquiring higher education would reduce their utility because their learning effort as skilled workers or as teachers is too high relative to their incomes. Only sufficiently talented workers may acquire the minimal level of higher education required from skilled workers or teachers:

Corollary 2: Agents with sufficiently high ability, such that $\theta_i \geq Z\hat{e}\left(\frac{\mu}{\delta\beta}+1\right)$ ($\theta_i > Ze_T$), are compatible with skilled professions (teaching), respectively. ◦

[10] The first effect cancels (its substitution and income effects offset each other, as a common feature of the Cobb-Douglas utility function). Therefore, the optimal effective leisure in equation (8) is identical for all skilled workers.

Corollary 2 is derived from (A1)-(A3) and equation (7). Namely, the optimal time investment of skilled workers exceeds the minimal level necessary to attain some higher education degree, $\hat{e}$, and the effective leisure of teachers is positive. Now, I define the utility from skilled professions relative to teaching:

Definition 1: Using equations (6)–(9), the utility from skilled professions relative to teaching is the utility of agent *i* from skilled professions divided by his or her utility from becoming a teacher:

$$(10)\quad \left(\frac{y_{i,S}}{y_T}\right)^{\delta}\left(\frac{l_S}{l_{i,T}}\right)^{\mu},$$

where

$$\frac{y_{i,S}}{y_T}=\left(\frac{\delta\beta}{Z(\mu+\delta\beta)}\right)^{\beta}\frac{w_S\rho E}{y_T}\quad(\theta_i)^{\beta+\lambda}\qquad\text{and}\qquad\frac{l_S}{l_{i,T}}=\left(\frac{\mu}{\mu+\delta\beta}\right)\Big/\left(1-\frac{Ze_T}{\theta_i}\right).\ \circ$$

The utility from skilled professions relative to teaching in equation (10) can be rewritten by gathering ability-dependent factors in $\frac{u_S}{u_T}(\theta_i)$ and the other factors in π_1[11]:

$$(11)\quad \left(\frac{\pi_1 w_S}{y_T}\right)^{\delta}\left(\frac{u_S}{u_T}(\theta_i)\right),$$

where

$$\frac{u_S}{u_T}(\theta_i)=\frac{(\theta_i)^{\delta(\beta+\lambda)}}{\left(1-\frac{Ze_T}{\theta_i}\right)^{\mu}}\qquad\text{and}\qquad\pi_1=\left(\frac{(\delta\beta)^{\beta}\mu^{\frac{\mu}{\delta}}\rho}{Z^{\beta}(\mu+\delta\beta)^{\frac{\mu+\beta\delta}{\delta}}}\right)E.\ \circ$$

π_1 includes the skilled wage rate for an effective unit of human capital, teacher income, the quality of public education (given from the previous period) and parameters of the

[11] Recall that teacher income, y_T, and the optimal effective leisure of skilled workers, l_S, (see equation (8)) are uniform. However, when workers become more talented, their incomes as skilled workers, $y_{i,S}$, increase through $(\theta_i)^{\beta+\lambda}$ as a result of their greater time investment in higher education and their greater ability (recall equation (9)). Moreover, their effective leisure as teachers, $l_{i,T}=1-\frac{Ze_T}{\theta_i}$, increases because they have a comparative advantage in making the exogenous time investment required to become teachers, e_T (recall corollary 1).

preference structure and the human capital formation. The following proposition 1 and property 1 characterize important innovative features of the labor supply:

Proposition 1: The utility from skilled professions relative to teaching is *convex* in ability, θ_i, and it attains a minimum at

(12) $\hat{\theta} = \chi Z e_T$ = Argmin $\left(\frac{u_S}{u_T}(\theta_i) \right)$,

where $\chi = \frac{\mu}{\delta(\beta+\lambda)} + 1$. ∘

Proposition 1 is easily proved by deriving the utility from skilled professions relative to teaching by ability. The proofs of Proposition 1 and Property 1 are available on request.

Property 1[12]: The slope of $\frac{u_S}{u_T}(\theta_i)$ is steeper below $\hat{\theta}$ than above $\hat{\theta}$. ∘

The convexity in ability implies that both high-ability workers (above $\hat{\theta}$) and low-ability workers (below $\hat{\theta}$) prefer skilled professions rather than teaching and that the teachers sector consists of intermediate-ability workers. This feature is generated because ability contributes to utility through two channels: income and effective leisure. High-ability workers prefer to become skilled workers than teachers because skilled occupations compensate for their high talents with augmented incomes, whereas low-ability workers prefer the skilled sector because it is more costly for them in terms of effort to attain teacher certification (recall (A3)). Specifically, workers with sufficiently low ability ($\theta_i \to Ze_T^+$) have almost no effective leisure as teachers (recall (A3) and corollary 2), and thus their marginal utility from effective leisure is infinite. As a result, not only low-ability workers prefer skilled professions to teaching, but also the slope of $\frac{u_S}{u_T}(\theta_i)$ is steeper below $\hat{\theta}$ than above $\hat{\theta}$, as property 1 argues (A more detailed intuition for these results appears on the Appendix). Instead of attaining the uniform time investment required for teacher certification (recall (A2)), low-ability workers optimally alleviate their learning effort by enrolling in shorter programs of higher

[12] Property 1 is proved under the sufficient assumption (a): $\delta(\beta+\lambda)=\mu$, or (b) $\delta(\beta+\lambda)<\mu$ (i.e., effective leisure is sufficiently important relative to income, thereby for low-ability agents it is sufficiently more costly to study than for high-ability agents) and (c) $P_T > \frac{2Ze_T}{\bar{\theta}-1}\sqrt{\chi^2-4}$ (the teachers sector is sufficiently large). Note that according to the numerical example, these assumptions are not necessary. Moreover, assumption (c) is quite plausible as it reflects a pupil-teacher ratio lower than 17, which corresponds to pupil-teacher ratios in primary and secondary education in most OECD countries.

education with fewer requirements than teaching (recall (7) and see proposition 6 hereinafter), such as community colleges, vocational training or any practical courses beyond high school with occupational orientation. In contrast, highly skilled workers enroll in longer programs typically characterized by academic, theoretically based research preparation. This leads to definition 2:

Definition 2: The skilled sector is also referred to as the 'total skilled' sector. It contains two sub-sectors (not interested in teaching):

The high-skilled sector (The vocational sector) consists of all skilled agents with time investment in higher education greater (lesser) than that for teachers. ∘

The division of the labor force into unskilled workers, vocational workers, teachers and highly skilled workers is formalized in the following proposition 2 and illustration 1 using definitions 3–4 (I discuss an exception for this division in section IV; Note that the conditions that guarantee the existence of sectors are derived in section III):

Definition 3: The term θ_{jk} denotes the ability level of workers, who are indifferent between belonging to sector '*j*' and sector '*k*'. ∘

Definition 4: Assume that for some '*j*' and '*k*', θ_{jk} satisfies $\theta_{jk} < \bar{\theta}$. If all workers with ability below (above) θ_{jk} prefer sector '*j*' ('*k*') to all other sectors, sectors '*j*' and '*k*' exist and θ_{jk} is the threshold level between them. ∘

Proposition 2: Sectors of workers are organized as follows:

a. If both the high-skilled sector and vocational sector exist, then $\theta_{TH} > \hat{\theta} > \theta_{VT} > Ze_T$ and
   (1) The most talented agents, with abilities $[\theta_{TH}, \bar{\theta}]$, generate the high-skilled sector.
   (2) Agents with abilities $[\theta_{VT}, \theta_{TH}]$ are teachers.
   (3) The vocational sector comprises agents with abilities $[\theta_{UV}, \theta_{VT}]$ and $\theta_{UV} < \theta_{UT} < \theta_{VT}$.
   (4) The lowest ability agents, with abilities lower than θ_{UV}, generate the unskilled sector.
b. If the high-skilled (vocational) sector does not exist, then the upper (lower) threshold of the teachers sector is $\bar{\theta}$ (θ_{UT} and $\theta_{UV} > \theta_{UT} > \theta_{VT} > Ze_T$). ∘

Proof:

Because of the convexity of the utility from skilled professions relative to teaching (recall proposition 1), indifferent workers between these sectors are represented by a unique *pair* of abilities $\{\theta_{VT},\theta_{TH}\}$, for which equation (11) equals '1':

$$(13)\quad \left(\frac{y_T}{w_S\pi_1}\right)^{\delta}=\frac{u_S}{u_T}(\theta_{VT})=\frac{u_S}{u_T}(\theta_{TH}),\qquad \text{where}\quad \theta_{TH}>\hat{\theta}>\theta_{VT}>Ze_T\,.$$

The ability of indifferent workers between the unskilled sector and the vocational sector (the teachers sector), denoted by θ_{UV} (θ_{UT}), is obtained by equating the utility from unskilled professions and skilled professions (teaching), respectively:

$$(14)\quad \frac{y_U}{w_S}=(1-\tau)\pi_1(\theta_{UV})^{\beta+\lambda}$$

$$(15)\quad \frac{y_U}{y_T}=(1-\tau)\left(1-\frac{Ze_T}{\theta_{UT}}\right)^{\frac{\mu}{\delta}},\ \text{where}\ \theta_{UT}>Ze_T\,.$$

θ_{VT},θ_{UT} must be above Ze_T, because these workers are able to become teachers (recall corollary 2). The other inequalities are implied by consistency of preferences. This part of the proof is relegated to the Appendix. ◦

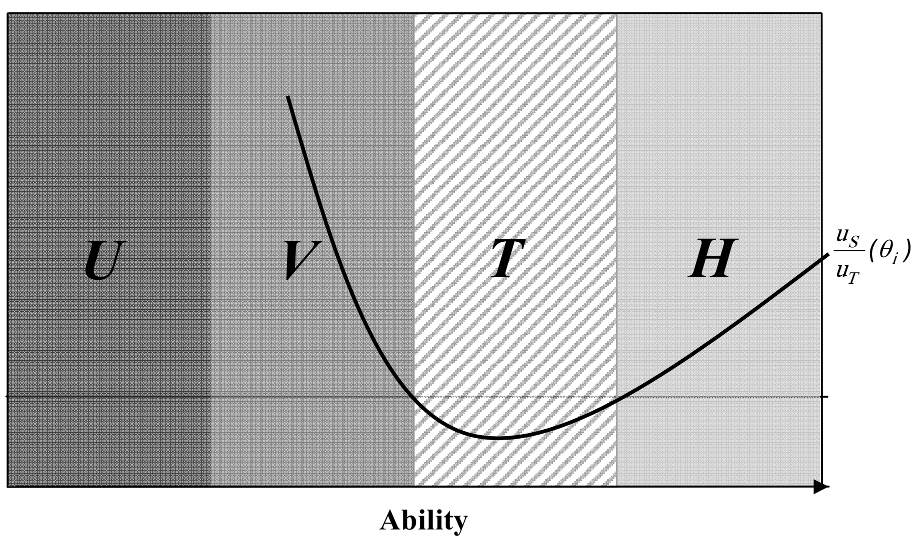

Illustration 1 – The composition of the labor force.

The numerical example is detailed in section V. Illustration 1 demonstrates the sectors defined in proposition 2: 'H', 'T', 'V' and 'U' denote the high-skilled sector, the teachers sector, the vocational sector and the unskilled sector, respectively. The X-axis denotes ability. The Y-axis denotes $\frac{u_S}{u_T}(\theta_i)$, which represents the utility from skilled professions relative to teaching (recall equation (11)). The intersection points define the threshold levels between skilled professions and teaching. Intermediate ability workers $(\theta_{VT}, \theta_{TH})$ prefer teaching.

My framework emphasizes the self-selection of workers. Because of the comparative advantage assumption, low-ability workers do not want to devote the exogenously given time investment required to gain teacher certification and thus do not want to become teachers. Instead, they join the vocational sector and alleviate their learning effort. Moreover, they respond to changes in their relative incomes. Therefore, ceteris paribus, to attract more teachers, teacher income must grow, which may break the quality-quantity trade-off.[13]

E. Labor demand

a. Firms

Competitive identical firms produce one consumption good, q, using total skilled labor (recall definition 2) and unskilled labor. I denote the proportions of total skilled labor and unskilled labor in the working population used by firm j by P_S^j, P_U^j. I assume that the per capita production function of firm j is the following[14]:

$$(16) \quad q^j(\overline{h_S}, P_S^j, P_U^j) = \left(\left(\overline{h_S}\right)^{\sigma} \left(P_S{}^j\right)^r + \left(\overline{h_S}\right)^{\phi} \left(P_U{}^j\right)^r \right)^{\frac{1}{r}}, \text{ where } r<1 \text{ and } 1>\sigma>\phi>0.$$

I also assume that the quality of skilled labor, $\overline{h_S}$, amplifies the productivity of skilled labor and unskilled labor with decreasing returns. This reflects the notion that skilled workers lead technological changes (e.g., Eicher 1996; Acemoglu, 1998; Galor and Moav, 2000; Nelson and Phelps, 1966). Though, the spillover is larger for skilled labor. This leads to the following notation:

---

[13] In contrast, in the existing TO models, the utility from skilled professions relative to teaching monotonically increases with ability because agents gain utility purely from income. Consequently, there is an excess supply of low-ability teachers (An exception is Bacolod (2007)). The government's choice of teacher income determines the top-quality teachers (with identical incomes as teachers and as skilled workers), and all college graduates with lower abilities (and thus lower incomes as skilled workers) are motivated to accept employment as teachers. Therefore, the results depend on the objective function of the government that determines the set of teachers. Accordingly, the government can decide to lower teacher income and still increase their numbers (i.e. substitute quantity for quality). This is a key element in generating the quality–quantity trade-off.

[14] Note that the results hold also under general functions $f_1\left(\overline{h_S}\right)$ and $f_2\left(\overline{h_S}\right)$ instead of $\left(\overline{h_S}\right)^{\sigma}$ and $\left(\overline{h_S}\right)^{\phi}$, respectively, assuming that they are strictly increasing, concave, and continuously differentiable.

Definition 5: The net productivity augmentation of skilled labor is given by $\sigma-\phi>0$ ∘

Given the quality of skilled labor and incomes, each firm *j* chooses its demand for skilled and unskilled labor by maximizing its profits:

$$(17)\quad \underset{P_S{}^j, P_U{}^j}{Max}\ \pi^j = \left(\left(\overline{h_S}\right)^{\sigma}\left(P_S{}^j\right)^r + \left(\overline{h_S}\right)^{\phi}\left(P_U{}^j\right)^r\right)^{\frac{1}{r}} - W_s\,\overline{h_S^j}\,P_S{}^j - y_U P_U{}^j\ .$$

By rearranging the first-order conditions, I obtain the demand of firm *j* for skilled labor relative to unskilled labor:

$$(18)\quad \frac{W_S\,\overline{h_S^j}}{y_U} = \left(\overline{h_S}\right)^{\sigma-\phi}\left(\frac{P_U^j}{P_S^j}\right)^{1-r}.$$

Because all firms are identical, by rearranging equation (18), I derive the total demand for skilled labor relative to unskilled labor as follows:

$$(19)\quad \frac{W_S}{y_U} = \left(\overline{h_S}\right)^{\sigma-\phi-1}\left(\frac{P_U}{P_S}\right)^{1-r},$$

where P_S, P_U are the aggregate proportions of total skilled labor and unskilled labor in the working population, respectively.

b. The government

Recall that in this model, the taxation is progressive in the sense that unskilled workers are not taxed to finance education, and the tax rate on the other sectors, τ, is exogenously given[15]. Tax revenues finance teachers' incomes at each date *t*, and the educational budget constraint is balanced:

$$(20)\quad y_T P_T = \tau\left(w_S\,\overline{h}_S P_S + y_T P_T\right).$$

By rearranging equation (20), teachers' incomes after tax are funded by the skilled sector:

$$(21)\quad (1-\tau)\,y_T P_T = \tau\left(w_S\,\overline{h}_S P_S\right).$$

That is, the teachers sector cannot exist without the funds from the skilled sector. On the other hand, if the skilled sector exists, then the tax revenues are positive. Because the educational budget is not disposed, the teachers sector must exist. Accordingly,

[15] While it is conceivable that the tax rate is set by a fiscal authority based on some decision process, I treat this as extraneous to the analysis. Nevertheless, I examine the effects of an exogenous increase of the tax rate in section VI(C). While this is not essential for the results, I allow for this possibility in order to reflect the fluctuations in public education expenditure as a percentage of GDP displayed in Table 1.

Corollary 3:

Given the budget constraint (21), the teachers sector exists if and only if the skilled sector exists (i.e. $P_T > 0 \Leftrightarrow P_S > 0$).

III. Equilibrium

A. Definition of Equilibrium

Let teachers' time investment in higher education, e_T, the tax rate, τ, the distribution of abilities, θ_i, and the quality of public education, E, be given in each period *t*. Then, $\{e_i, P_U, P_T, P_H, P_V, y_U, y_T, W_S\}$, for t=1,2…, constitutes an equilibrium, if it satisfies the following conditions:

a. Given $\{y_U, y_T, W_S\}$, for all workers, $\{e_i\}$ is the optimal time dedicated to higher education and no worker can improve his or her position by moving to another sector.
b. In production, $\{P_U, P_S\}$ are the optimal aggregate proportions of unskilled labor and total skilled labor, respectively, given $\{y_U, y_T, W_S\}$.
c. The educational budget constraint (21) holds.
d. The labor market clears. The demand for each sector equals supply. ◦

B. Existence of Equilibrium and Uniqueness

In this section, propositions 3-5 derive conditions for the existence and uniqueness of equilibrium, and for the existence of the two types of skilled workers in the equilibrium. All proofs in this section are relegated to the Appendix. Proposition 3 derives conditions for the existence of equilibrium.

Proposition 3

Under the aforementioned assumptions, equilibrium exists with at least three sectors: total skilled, teachers and unskilled (i.e. $P_S > 0$, $P_T > 0$, $P_U > 0$).◦

Thus, at least one of the two sub-sectors exists in equilibrium: the vocational sector or the high-skilled sector. In the rest of the paper, I assume the following –

(A5) The distribution of abilities is uniform.◦

I use the common uniformity assumption (A5), following e.g., Galor and Moav (2000) and Gilpin and Kaganovich (2012), in order to obtain tractable analytical results in the rest of the paper (besides Proposition 6), but it is not necessary for the overall intuition.[16] Now, Proposition 4 derives the conditions for the uniqueness of equilibrium.

Proposition 4: Under the aforementioned assumptions, and if the net productivity augmentation of skilled labor is sufficiently large, $\sigma - \phi > r$, equilibrium is unique with at least three sectors: total skilled, teachers and unskilled (i.e. $P_S > 0$, $P_T > 0$, $P_U > 0$). ◦

Additional technical assumptions (A6)–(A9), specified in the Appendix, are sufficient to ensure that the high-skilled sector and the vocational sector co-exist in equilibrium. [17]

Proposition 5:

Assume that assumptions (A5)–(A9) hold. Then, the number of highly skilled and vocational workers is positive in equilibrium. ◦

Note that in the following sections, I assume that the vocational sector and the high-skilled sector both exist. Nevertheless, I analyze the less probable case with no vocational sector in section VI(A).

IV. Time Investment in Higher Education and Income

This section characterizes time investment and incomes in each sector. Typically, time investment in higher education and related incomes are weakly increasing in ability (thereby sectors are organized according to proposition 2). The model generates this result in all sectors, though it may not hold at the higher threshold level between the teachers sector and the skilled sector, as proposition 6 and Illustration 2 depict.

Proposition 6:

High-ability workers are more educated and earn higher incomes than low-ability workers, with the following exception: some workers with higher abilities than teachers may be less educated and earn lower incomes than teachers, and thus they belong to the vocational sector. ◦

Illustration 2 depicts time investment in education and incomes as a function of ability:

[16] It has the advantage of simplicity and it is consistent with the empirical evidence regarding the positive relation of relative teacher quality to relative teachers' incomes (see discussion below Proposition 8). More general assumptions, that the high-skilled sector is sufficiently large near θ_{TH}, i.e., that teachers are *not* the top ability workers (which may not be correct only in Nordic countries) or that the effective leisure is sufficiently important (see Property 1) may produce the same qualitative results.

[17] See the intuition in the Appendix.

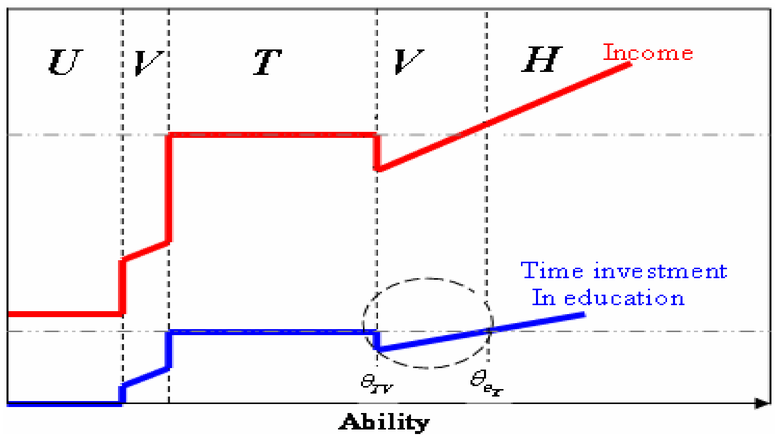

Illustration 2. – Time investment in education and income.

The numerical example is detailed in section V. 'H', 'T', 'V' and 'U' denote high-skilled sector, teachers sector, vocational sector and unskilled sector, respectively. Unskilled workers earn the lowest incomes and do not invest in higher education. As ability grows, workers become more educated and earn higher incomes but enjoy less effective leisure (substitute income with effective leisure), with the following exception: In illustration 2, the time investment of skilled workers is identical to the *exogenously* given time investment of teachers, e_T, when ability equals θ_{e_T}. The most talented workers, with ability above θ_{e_T}, given their comparative advantage in learning, naturally choose to be more educated and thus earn higher incomes than teachers (recall (A2) and equation (7)). However, because $\theta_{TV} < \theta_{e_T}$ in equilibrium, some workers, with higher abilities than teachers, i.e., $(\theta_{TV}, \theta_{e_T})$ (in the black circle), decide to become vocational workers. That is, they acquire less higher education and thus earn lower incomes than teachers. This phenomenon occurs when the intensity of ability, λ, is large. On the one hand, when the intensity of ability is low (see the Appendix for $\lambda = 0$), there exists a small high-skilled sector that includes the most talented workers, who choose to be more educated and earn higher incomes than teachers. On the other hand, when the intensity of ability is sufficiently large, the skilled sector expands and pushes the teachers' sector to the lower end of the ability distribution. That is, the top quality teachers join the skilled sector, and they may optimally choose to enroll in shorter higher education programs and enjoy more leisure but lower incomes than teachers.[18]

[18] Devoting the *exogenous* time investment of teachers in higher education, e_T, may be sub-optimal for them because their marginal cost is too high in terms of learning effort relative to the marginal utility from the income generated. See section VI for more details on an increase in the intensity of ability.

V. Numerical Example

The numerical example has several purposes. The first aim is to demonstrate the analytical results (specifically, illustrations 1-5, and figures 3-5 on the Appendix are calibrated based on the numerical example and demonstrate the corresponding Propositions). Note that illustration 2 provides an insightful example for the exception that proposition 6 depicts. The second goal is to verify that the restrictive assumptions in Property 1 are not necessary (by selecting the values displayed in table 2 of μ and δ that do not satisfy the assumptions). Moreover, I calibrated the values of assumption (c) of Property 1 to exhibit that it is quite plausible (see comment 12). The third goal, in the following section VII, is to obtain the effect of lower teacher certification cost program with access restrictions on relative teacher quality, which is too complicated to solve analytically.

The baseline is calibrated using conventional specifications. The population size is 300. The income distribution is uniform (recall assumption (A5)) and calibrates a Gini coefficient close to developed OECD countries (0.24). In addition, the average tax rate is between 0.11–0.25, the range of public expenditure per student for primary education as a percentage of GDP per capita in OECD countries. It lies in the range of the medium and high tax rates in Glomm and Ravikumar (2003), 0.05-0.6. I also set the standard parameters from the literature.

Table 2: parameter values

	Parameters' description	Parameters' value	Parameters' source
Firms' production	Substitution between skilled and unskilled workers	$r = 0.925$	Hamermesh's (1979) and Johnson's (1970) estimates of the substitution between high school graduates and college graduates. These are close to the result of Bowles (1970), 0.995, for the substitution between secondary education and higher education, and in the range of Psacharopoulos's (1972) estimates.
Education technology	Intensity of higher education	$\beta = 0.16$	In the range of Card's (1995) IV estimates.
	Intensity of ability	$\lambda = 0.5$	Orazem and Tesfatsion (1997) and Loury (1981).
	Intensity of human capital	$\rho = 6$	Glomm and Ravikumar (1992) and Su (2004)
Utility	Weights of effective leisure and consumption	$\mu = [0.2, 0.5], \delta = [0.3, 0.45]$	Greenwood et al. (1997) and Tamura (2001)

VI. Comparative Static to Explain the Quality–Quantity Trade-Off

In this section, I examine the possible causes for the trends in teacher quality and teacher quantity in advanced countries. First, in the following sub-sections A and B, I discuss two types of SBTCs:

(a) The returns to ability rise *linearly*: In this case, skilled incomes are multiplied by the same constant factor (without changing the ratio of incomes within the skilled sector). Thus, the income inequality within the skilled sector does not change (see definition 6 in the Appendix). This factor-augmenting SBTC is common in the literature. It is carried out through the following comparative static in line with the evidence reviewed in the introduction:

i. Augmented net productivity of skilled labor, $\sigma - \phi$ [19].
ii. Augmented human capital (and thus incomes) of skilled labor, through amplified intensity of human capital, ρ , or improved quality of public education, E (given at each period *t*) (recall equation (2)).

(b) The returns to ability rise *exponentially*: This SBTC is executed through an increase in the intensity of ability, λ. In this case, the marginal productivity of ability increases more than the marginal productivity of the other components of the human capital. As a result, workers at the upper end of the ability distribution receive exponentially larger returns for their ability. Consequently, the ratio of the incomes of highly skilled and vocational workers increases and the income inequality rises within the skilled sector. ∘

In addition to the two types of SBTC, I consider in sections C and D an increase in the tax rate and a reduction of the teacher certification requirement. Now, I define relative teacher quality in line with Gilpin and Kaganovich (2012).

<u>Definition 7</u>: Relative teacher quality refers to teacher mean quality relative to the mean quality of the working population. ∘

A. type (a) SBTC

This section demonstrates that when the returns to ability rise *linearly*, both teacher quality and teacher quantity increase, which in turn amplify the quality of education. Thus, the quality–quantity trade-off does *not* occur (see illustation 3 and proposition 7).

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[19] As a result, the relative demand for skilled workers increases in firms (recall the production function (16) and definition 5). Note that type (a)(i) SBTC allows for an increase in the productivity of *unskilled* labor, carried by an increase in ϕ. Nevertheless, the increase in the productivity of skilled workers must be larger than that for unskilled workers because σ rises more than ϕ.

As the incomes of skilled workers increase, low-ability agents decide to acquire higher education and join the vocational sector (i.e., θ_{UV} declines). Therefore, the total skilled sector expands and the unskilled sector shrinks. According to Rangazas (2002), schooling might not benefit from the increased labor productivity associated with technological changes. Much of the discussion considers schooling as suffering from the Baumol (1967) disease. Thus, in the absence of any increase in the total amount spent on teachers, the increase in the total skilled sector threatens both teacher quantity and teacher quality. However, because the funds for public education rise, teacher income grows to balance the educational budget (21). As a result, the teachers sector becomes more attractive for both highly skilled workers and vocational workers (i.e., θ_{TH} increases and θ_{VT} declines) thereby the teachers sector expands. Illustration 3 and proposition 7 summarize the changes in the sizes of sectors.

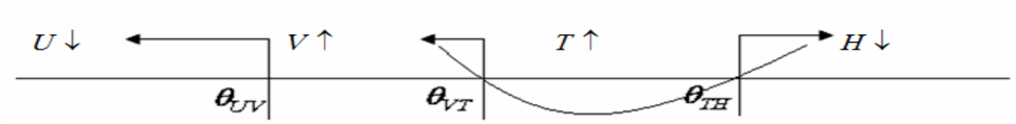

Illustration 3. – The effect of type (a) SBTC.

The corresponding simulation in figure 3 and the proof of Proposition 7 are relegated to the Appendix.

Proposition 7:

If type (a) SBTC occurs, then

a. The total skilled sector and the teachers sector expand.

b. The vocational sector expands, and the high-skilled sector shrinks. ◦

The following property 2 and proposition 8 analyze whether relative teacher quality increases or declines. The answer is not straightforward, because both highly skilled workers and vocational workers join the teachers sector.

Property 2:

The supply of highly skilled workers is more (less) elastic with respect to shocks in their relative incomes (leisure) than the supply of vocational workers. ◦

Property 2 derives immediately from Property 1 under (A5).

Proposition 8:

Assume that type (a) SBTC occurs. Then, relative teacher quality increases. ◦

<u>Proof</u>:

Proposition 8 derives from Property 2. Because highly skilled workers are more talented, it is less costly for them in terms of effort to obtain a teacher certification than for vocational workers (recall (A3))[20]. As a result, when teacher income increases, *more* highly skilled workers join the teachers sector than vocational workers (the rise in θ_{TH} is larger than the decline in θ_{VT}), thereby relative teacher quality increases. ∘

These results are consistent with the empirical evidence about a significant positive relation between teachers' incomes relative to other occupations and their quality[21]. As such, the quality–quantity trade-off does *not* occur: SBTC increases the supply of teachers without sacrificing their quality. Therefore, the quality of public education rises (in period t+1). In contrast, in existing TO models, there is *excess* supply of low-ability workers for the teachers sector. Thus, it is feasible to lower teacher income while increasing their numbers (i.e., substituting teacher quality with quantity). This policy may become optimal under SBTCs, because the cost of maintaining teacher quality rises (see Lakdawalla (2006)). In the current framework, this policy is not feasible because of the existence of vocational workers who do not want to become teachers: if teacher income declines, the supply of low-ability teachers shrinks (they join the vocational sector).

Note that under this framework, if there is *no* vocational sector (the less probable case), the quality–quantity trade-off occurs: as the incomes of skilled workers increase, the high-skilled sector expands and pushes the teachers sector to the lower levels of the ability distribution (i.e., θ_{TH} declines). Then, because the funds for public education increase, teacher income increases. As a result, the teachers sector expands towards the unskilled sector (i.e., θ_{UT} declines), and relative teacher quality declines, as illustration 4 and proposition 9 summarize (the proof is relegated to the Appendix):

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[20] Workers with sufficiently low abilities ($\theta_i \to Ze_T^+$) have almost no effective leisure as teachers (recall corollary 2), thereby their marginal utility from leisure is infinite. See the discussion after property 1.

[21] Bacolod (2007) finds that high-quality teachers are more sensitive to changes in relative teacher wages. Dolton and Marcenaro-Gutierrez (2011) using aggregate panel OECD data suggest that better pay for teachers will attract higher quality graduates into the profession and argue that teacher incomes can be used as a direct proxy for their (inherently unobservable) quality. Studying local labor markets, Figlio (1997) finds that a 1% increase in a school district's salary is associated with 0.75% higher probability of recruiting a teacher from a selective college (see also Gilpin, 2012; Player, 2009; review in Gilpin and Kaganovich, 2012 on the estimated effects of outside job market opportunities on teacher quality.

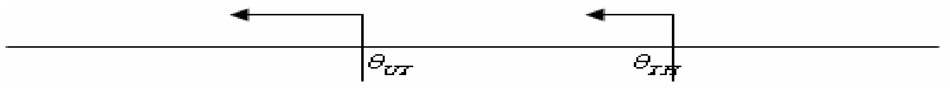

Illustration 4. – The effect of type (a) SBTC when the vocational sector does not exist.

<u>Proposition 9</u>:

Assume that (A5) holds and type (a) SBTC occurs. If the vocational sector does not exist, the teachers sector expands, and its relative quality declines.∘

In the following sub-sections, I analyze other potential causes for the quality–quantity trade-off, associated with observed trends in advanced countries: type (b) SBTCs, exogenous shifts in the tax rate and a reduction of the teacher certification requirement.

B. type (b) SBTC

When the intensity of ability, λ, increases, similarly to type (a) SBTC, more agents are attracted to the total skilled sector. Furthermore, because the funds for public education increase, teacher income increases, and thus the teachers sector expands. However, while type (a) SBTC multiplies the incomes of skilled workers by the same factor, under type (b) SBTC the growth in incomes is highly disproportionate within the skilled sector. Because they are more talented, highly skilled workers enjoy exponentially larger returns for their ability. That is, the ratio of the incomes of highly skilled and vocational workers increases, and thus income inequality rises within the skilled sector. Therefore, an additional substitution effect appears: The high-skilled sector becomes more attractive for teachers *than* the vocational sector. As a result, in contrast with type (a) SBTC, the high-skilled sector *expands* and pushes the teachers sector towards the lower levels of the ability distribution. Top-quality candidates depart the teachers sector and join the high-skilled sector, leaving the government with less talented teachers (i.e., both the upper and lower thresholds of the teachers sector, θ_{TH} and θ_{VT}, decline). Thus, the quality–quantity trade-off emanates from the occupational choice of high-ability workers, whereas the government is forced (by the market) to recruit teachers with lower qualifications. This result is consistent with the empirical finding that the decline in teacher quality was primarily driven by a decrease in the proportion of the most qualified teachers, who potentially faced higher returns to their ability (see Corcoran et al., 2004; and Bacolod, 2007). Accordingly, the fundamental disadvantage of type (b) SBTC relative to type (a) SBTC is that it entails a reduction in relative teacher quality. Illustration 5 and Proposition 10 summarize the effects of type (b) SBTC.

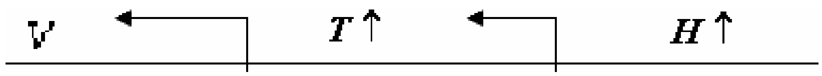

Illustration 5. – The effect of type (b) SBTC.

The corresponding simulation in figure 4 and the proof of Proposition 10 are relegated to the Appendix.

Proposition 10:

If type (b) SBTC occurs, then

a. The total skilled sector, the high-skilled sector and the teachers sector expand.

b. Relative teacher quality declines. ∘

C. Exogenous shifts in the tax rate[22]

In this section, I discuss an exogenous increase in the tax rate, τ. Within the model, all tax revenues go to education expenditures but, more in general, τ measures the share of tax revenues used in the public education sector[23]. An exogenous increase in the tax rate augments the funds for public education. To balance the educational budget (21), the government increases teacher income. Therefore, similarly to the type (a) SBTC, the quality–quantity trade-off does *not* occur and the quality of education rises. Proposition 11 depicts the effect on teacher quality and teacher quantity.

Proposition 11:

Assume that (A5) holds and the tax rate increases. Then, the teachers sector expands, and its relative quality rises. ∘

Proof:

When the tax rate increases, P_T rises to balance equation (22). The increase in relative teacher quality derives from Property 2 (similar to propositions 7(b) and 8). ∘

D. Reduction in the Teacher Certification Requirement

In this section, I discuss a reduction in the exogenous teacher certification requirement, e_T.[24] As a result, the learning effort required from teachers decreases (recall (A3)), and

[22] This section was added thanks to an anonymous referee and provides important insights.

[23] Rangazas (2002) attributes the growth in the United States since 1870 mainly to human capital accumulation, composed of sustainable balanced growth proportionately to that of GDP, and transitional growth because of the dramatic rise of education expenditures as a share of GDP. This section captures the effects of such transitional growth, which according to Rangazas (2002) accounts for 30 to 40 percent of the fivefold increase in worker productivity.

[24] Similar results are obtained for a reduction in the non-pecuniary cost of leisure, Z. Note that Costrell (1994), considering a shock in student preference for leisure in high school, argues that shifts in

the teachers sector becomes more attractive. Therefore, the supply of teachers grows. However, because they are less talented, with a higher marginal utility from effective leisure, more vocational workers are attracted to the teaching profession than highly skilled workers (recall Property 2). Then, to balance the budget constraint (21), the government reduces teacher income. Combining these effects, because highly skilled workers are more (less) sensitive to shocks in their incomes (leisure) than vocational workers (recall property 2), eventually the high-skilled sector expands, leaving the teachers sector with less talented candidates from the vocational sector. Accordingly, as both the upper and lower thresholds of the teachers sector (θ_{TH} and θ_{VT}) decline, relative teacher quality declines, and the quality–quantity trade-off occurs, as Proposition 12 summarizes. The corresponding simulation in figure 5 and the proof of proposition 12 are relegated to the Appendix.

Proposition 12:

If e_T declines, then

a. The teachers sector and the high-skilled sector expand.

b. Relative teacher quality declines. ∘

To summarize, a reduction in the cost of becoming a teacher reduces relative teacher quality because of two reasons: First, it benefits more with low ability agents, and thus plagued by adverse selection problem. Second, the lower time investment damages the human capital of teachers (see equation (2)). Does the prediction that relative teacher quality declines contradict the observation in the United States that the relaxation of licensing requirements is typically undertaken in order to *improve* the teacher quality? In the following section, I demonstrate that by fairly considering the *restricted access* to such programs, the model's predictions account for this observation.

VII. Two Pathways into Teaching: Formal and Lower Cost Certification[25]

The analysis so far has proceeded on the assumption that there is only one pathway into teaching, the 'formal' teacher certification. The purpose of this section is to discuss the implications of a well-known program in the United States for teacher training called

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preferences need not originate with the student. For example, changes in family structure might reduce the student's non-pecuniary cost of leisure.

[25] This generalization was added thanks to an anonymous referee and provides intriguing insights.

Teach For America (www.teachforamerica.org), the founder of a global program, Teach For All (www.teachforallnetwork.org), with more than 23 programs in many countries. Its prerequisites include a bachelor degree with undergraduate cumulative grade point average (GPA) of at least 2.50 on a 4.00 scale and a 3-step interview process. Then, the participants attend a short intense summer course and begin teaching by the next fall. To account for TFA, I introduce an additional pathway into teaching and demonstrate that it achieves the goal of increasing teacher quality.

Definition 8: An *Exclusive* lower cost teacher certification (ELCC) program is similar to the formal teacher certification program besides the following two elements: First, It requires a lower time investment in higher education. Second, *only* high-ability agents (who otherwise become highly skilled workers) are allowed to participate. ∘

The introduction of an ELCC program yields the following results: Indifferent agents between high skilled professions and formal teaching (or that slightly prefer high skilled professions) join the ELCC program. An additional secondary effect further amplifies teacher quality. As the program requires additional budget, teacher income must decline, which pushes low-ability agents away from the teachers' sector (A formal proof of the arguments so far is available on request). Now, I use the numerical example to determine whether relative teacher quality increases or declines.[26] The simulation results demonstrate that because of the exclusivity of the program for high-ability agents, relative teacher quality indeed rises. The high ability of the participants compensates for their lower time investment (recall equation (2)). Therefore, ELCC program, if combined with eligibility conditions restores teacher quality, and the quality–quantity trade-off does not occur.[27]

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[26] Specifically, the red graph in figure 5 denotes the preferences of low ability workers with a choice between formal teaching and the other sectors and the blue graph denotes the preferences of high ability workers with the additional alternative to participate in the ELCC program.

[27] This insight will not change if the model is further extended to include an option for a combined career path, that is, temporary employment as a teacher for some fraction of the working period. Actually, the TFA program requires a commitment to teach for two years, and then allows quitting. Assuming that teacher time devoted to educational activities does not pay off in the high skill labor market, it is easy to verify that in a combined career path, the *total* time investment, $e^{*}_{i,S+T} = \left(\frac{1}{\mu+\delta\beta}\right)\left(\frac{\delta\beta\theta_i}{Z} + \mu e_{i,T}\right)$, increases relative to a lifetime career in the skilled sector only (recall equation (7)), even for ELCC participants. Thus, in order to guerantee participation, the program must ensure that lifetime incomes of its eligible candidates relatively rise (unless they have other motivations to participate not captured in the model, such as heterogeneity in the preference for teaching or for leisure time). While participants in the TFA program receive the same salary as other beginning teachers in the district, the program provides financial incentives to attract high-ability candidates, e.g., education awards, loan forbearance, and

VIII. Conclusion

In the United States and other advances economies, the pupil–teacher ratio and teacher quality have declined over time. This study suggests that only certain types of SBTCs increase the income inequality *within* the skilled sector, thereby promote these quality–quantity trade-offs[28]. Therefore, a drawback of these SBTCs (as opposed to SBTCs that preserve the ratio of incomes among skilled workers) is the reduction in relative teacher quality, which may have a negative feedback effect on the education quality of the subsequent generation, and thus on human capital development as a factor of economic growth[29]. A reduction in the teacher certification requirement has similar effects, unless it is accompanied by access restrictions. While the model analyzes a one generation period, it predicts the quality of education and the aggregate human capital in the next period based on the current occupational choices of individuals (see equation (1)). Thus, an intriguing issue for future research is the long term dynamic impact of SBTCs.

The important tasks of the government in the model are to impose and collect the taxes and manage the public education system, including hiring teachers and paying their wages, as to balance the educational budget. Given that the paper focuses on the occupational choice of individuals, I leave the process by which the tax rate and the teacher certification requirement are determined outside the scope of this article. I treat them as exogenously given (similar to the existing TO models) (though I analyze the implications of exogenous shocks in their values), whereas, in fact, they depend on endogenous governmental decisions[30]. On the one hand, an *overall* reduction in teacher

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interest payment coverage on their current and future student loans. Moreover, their skilled income as a fraction of their working period may also increase (because the marginal productivity of time investment in higher education is decreasing), though they lose the reward to experience in both sectors.

[28] Note that the model ignores the likely increase in the demand for schooling and enrollment as a response to SBTC. As I focus on OECD countries and especially the United States, with high enrollment rates, I assume that all children attain a similar level of compulsory public education, in line with the theoretical literature (see e.g., Loury, 1981; Glomm and Ravikumar, 1992; Viaene and Zilcha, 2002). Recall that the empirical evidence on the declining pupil-teacher ratio is valid despite the ups and downs in the enrollment dynamics (see table 1).

[29] In fact, the numerical example indicates that type (b) SBTC increases the *absolute* teacher quality even though the relative teacher quality declines. This may call for the introduction of *relative* teacher quality in theoretical production functions of public education similar to the empirical studies, instead of the common (and to my knowledge, the overall) use in *absolute* teacher quality; Note that the framework of analysis developed here is suitable to examine the spillover of income inequality and relative teacher quality across countries. For example, in the presence of imperfect technological diffusion, the income inequality among technological leaders is likely to be higher and relative teacher quality is likely to be lower than among followers.

[30] In a political dynamic equilibrium framework, Hatsor (2008) assumes, more realistically, that the budget authority determines the tax rate according to the widely used criterion of majority voting (e.g., Glomm and Ravikumar, 1992; Saint-Paul and Verdier 1993). Then, the public education agency allocates the educational budget. She compares an inefficient education agency which equates teacher quality to the population mean, similarly to Eckstein and Zilcha (1994), with the one that maximizes the quality of education subject to the budget constraint, similarly to Gilpin and Kaganovich (2012). The study highlights the implications of existing inefficiencies on growth, the income inequality and welfare and

time investment may be due to the optimal policy of the government to substitute teacher quality with teacher quantity in response to the rising cost of skilled workers (potentially caused by SBTC). This may have occurred in California and in several developing countries (see Jepsen and Rivkin, 2002; UNESCO, 2006). On the other hand, the analysis suggests that policy intervention can be designed to mitigate the trend of declining teacher quality through encouraging more linear (as opposed to exponential) increases in the returns to ability by upgrading the quality of public education, supporting the adaptation of low-ability workers to SBTCs, driving technological changes in low-ability sectors, or implementing *exclusive* lower certification cost programs for teacher training, similar to 'Teach for America'. Clearly, the role of the government includes the regulation, organization and implementation of such programs to the extent they comply with its objective function.

A key contribution of the model is the introduction of disutility from higher education (non-pecuniary cost) so that individuals directly 'pay' for higher education with a loss of leisure during youth. Moreover, I assume that the disutility diminishes with the ability level, because for highly talented agents it is less costly to study (Spence, 1973). As a result, a desirable feature of the model is that the teachers' sector is endogenously located between highly skilled workers and vocational workers, who avoid becoming teachers. The model generates a variety of sectors within a simplified one dimensional heterogeneity framework[31]. This unique division helps grasp the essential features in SBTCs that lead to quality–quantity trade-offs. Actually, another implication of the comparative advantage assumption is that a reduction in the cost of becoming a teacher adversely selects low ability agents to the teachers' sector unless access restrictions are imposed. As long as this common-knowledge assumption basically holds, other mechanisms can generate the results as well. For example, the disutility from higher education may arise in the form of forgone labor time (instead of leisure time) during college[32].

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provides a possible answer to why educational expenditures seem to be unrelated to educational achievements according to the empirical evidence (see discussion about budgetary and allocation decisions in section 2.6 there).

[31] Alternative frameworks may include a multi-dimensional skill Roy model that explicitly permits comparative advantage, or heterogeneity in preference for leisure, home production or teaching. See e.g., Chiappori et al. (2009)'s model for the joint determination of schooling and marriage patterns of men and women in which investment in schooling generates two kinds of returns and Willis and Rosen (1979). Bacolod (2007)'s Roy model highlights how occupational differences in the returns to skill determine teacher quality. These frameworks may enrich the model with more realism. However, they may be less tractable without additional insights on the matter.

[32] More specifically, assume that the time not spent on higher education is devoted to work (instead of leisure). Young agents with higher education either work for the remaining fraction of time in the skilled sector or, if qualified by the education authority, work as teachers. Accordingly, equation (7) denotes the effective labor, and assuming that $\mu = \varphi\delta$, agents gain utility from their effective income (income multiplied by effective labor). Alternatively, within the given framework of a tradeoff between higher

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education-leisure, an additional assumption can adjust the incomes of college educated workers for their shorter employment duration due to the time spent in college (similar to Gilpin and Kaganovich, 2012).

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Appendix

Proposition 1 and Property 1 – Intuition

Skilled workers are compensated for higher ability through larger incomes, while teachers are compensated through lower effective effort (recall corollary 1). When the ability of highly skilled workers increases, the marginal utility derived from enlarging their incomes as skilled workers more than offsets the increase in their effective leisure as teachers. This occurs because their effective leisure as teachers (recall (A3)), which is already high, is bounded by 1. Nonetheless, their skilled incomes are unbounded and thus increase more substantially ($l_{i,T} \xrightarrow[\theta_i \to \infty]{} 1$, $y_{i,S} \xrightarrow[\theta_i \to \infty]{} \infty$, $\frac{u_S}{u_T}(\theta_i) \xrightarrow[\theta_i \to \infty]{} (+\infty)$, recall equations (10)(11)). As a result, highly skilled workers prefer skilled professions rather than teaching. However, when the ability of low-talented workers decreases, the decline in their skilled incomes is negligible relative to the increase in their learning effort as teachers. Specifically, for sufficiently low ability workers $l_{i,T} \xrightarrow[\theta_i \to Ze_T^+]{} 0$, $\frac{u_S}{u_T}(\theta_i) \xrightarrow[\theta_i \to Ze_T^+]{} (-\infty)$ asymptotically.

Proof of proposition 2 – Consistency of preferences

If $\theta_{VT} \geq \theta_{UV}$, the vocational sector exists and agents θ_{VT}, θ_{UV} are the threshold levels between the vocational sector, the teachers sector and the unskilled sector. In this case, agents above θ_{VT} prefer teaching rather than vocational professions: $T \succ V$. They are also above θ_{UV} , and hence prefer vocational rather than unskilled professions $V \succ U$. Therefore, they prefer teaching rather than both vocational and unskilled professions: $T \succ V, V \succ U \Rightarrow T \succ U$. Similarly, agents below θ_{UV} prefer to become unskilled rather than vocational and unskilled workers, since $U \succ V, V \succ T \Rightarrow U \succ T$. Agents with abilities $[\theta_{UV}, \theta_{VT}]$ prefer to become vocational workers rather than being teachers or unskilled. However, if $\theta_{VT} < \theta_{UV}$, there is no vocational sector, since all agents prefer other sectors rather than the vocational sector: Agents above θ_{UV} prefer teaching rather than both vocational and unskilled professions since $T \succ V, V \succ U \Rightarrow T \succ U$. Agents below θ_{VT} prefer to become unskilled rather than vocational and skilled workers, since $U \succ V, V \succ T \Rightarrow U \succ T$. Additionally, agents between θ_{VT} , θ_{UV} do not desire to become vocational workers, since $U \succ V, T \succ V$. In this case, agent θ_{UT} is the threshold level between the unskilled sector and the teachers sector, such that $\theta_{VT} < \theta_{UT} < \theta_{UV}$.

Proof of Proposition 3

I prove by 3 steps that there is at least one feasible set of { $P_S > 0$, $P_T > 0$, $P_U > 0$ } that clears the labor market[33]: Step (a) combines the equations of labor supply, the educational budget constraint and the labor demand of firms; Step (b) characterizes the proportions of sectors according to the combined labor supply and educational budget; Step (c) intersects these results with the combined labor demand and educational budget and proves that the market must clear due to continuity considerations.

a. First, I intersect the labor supply equations with the educational budget constraint and the labor demand. In particular, I substitute the labor supply (13) in the educational budget constraint (21) to obtain:

$$(22) \quad \bar{h}_S P_S = \pi_1 \left(\frac{1-\tau}{\tau} \right) P_T \frac{u_S}{u_T} \left(\theta_{jk} \right), \; jk = TH, VT$$

When the vocational sector exists, substituting the labor supply (14) in the demand equation (19) obtains:

$$(23) \quad \left(\bar{h}_S \right)^{\sigma-\phi-1} \left(\frac{P_U}{P_S} \right)^{1-r} \left(\theta_{UV} \right)^{\beta+\lambda} = \frac{1}{\pi_1 (1-\tau)}$$

In case the vocational sector does not exist, multiplying the labor supply (13) in (15) and substituting in the demand equation (19) yields:

[33] Note that as r<1 (see equation (16)), the demand for skilled and unskilled workers must be positive (otherwise, their marginal productivity is infinite). Accordingly, because the skilled sector exists and the educational budget constraint holds, the teachers sector must exist either (recall corollary 3).

(24) $\left(\overline{h_S}\right)^{\sigma-\phi-1}\left(\frac{P_U}{P_S}\right)^{1-r}\left(\frac{\theta_{UT}-Ze_T}{\theta_{UT}}\right)^{\frac{\mu}{\delta}}\left(\frac{u_S}{u_T}(\theta_{TH})\right)=\frac{1}{\pi_1(1-\tau)}$

Then, substituting $P_S=1-P_U-P_T$ in equation (22) yields:

(25) $P_T=\pi_3(1-P_U)$

where $\pi_3=\dfrac{\overline{h}_S}{\pi_1\left(\frac{1-\tau}{\tau}\right)\left(\frac{u_S}{u_T}(\theta_{jk})\right)+\overline{h}_S}$ and $jk=TH,\quad VT$

If the vocational sector exists, $jk=VT$; if the high-skilled sector exists; $jk=TH$; if both sectors exist, $jk=TH,\quad VT$.

b. Second, I characterize the proportions of sectors, using equation (25) , $0<\pi_3<1$, $P_S=1-P_U-P_T$, and equation (22)[34]:

(26) $$\begin{Bmatrix} 1>P_T>0,\ 1>P_S>0 & ,if\ P_U\to 0 \\ 1>P_T>0,\ 1>P_S>0 & ,if\ 1>P_U>0 \\ P_T\to 0, P_S\to 0 & ,if\ P_U\to 1 \end{Bmatrix}$$

c. To clear the labor market, I intersect labor supply, educational budget and labor demand. Specifically, I substitute the proportions of sectors given in (26) in the LHS of equations (23)-(24) to obtain:

(27) $$\begin{Bmatrix} LHS(22)\to 0, & LHS(23)\to 0, & if\ P_U\to 0 \\ LHS(22)>0, & LHS(23)>0, & if\ 1>P_U>0 \\ LHS(22)\to\infty, & LHS(23)\to\infty, & if\ P_U\to 1 \end{Bmatrix}$$

To complete the proof, note that the RHS of equations (23)-(24) is always positive. Therefore, at least one intersection must occur with the LHS due to continuity. ∘

<u>Proof of proposition 4</u>:

Along the labor supply, when the relative wage of skilled workers, $\frac{w_S}{y_U}$, increases, their relative supply, $\frac{P_S}{P_U}$, grows. As θ_{UV} decreases (see equations (14) and (15)), $\overline{h}_S$ declines. Nevertheless, it is easy to verify using equation (9) that $P_S\overline{h}_S$ increases.

(28) $$P_S\overline{h_S}=P_S\left(\frac{P_H}{P_S}\overline{h_H}+\frac{P_V}{P_S}\overline{h_V}\right)=P_H\overline{h_H}+P_V\overline{h_V}=\pi\left(\left(\frac{\overline{\theta}-\theta_{TH}}{\overline{\theta}-1}\right)\left(\frac{1}{\overline{\theta}-\theta_{TH}}\int_{\theta_{TH}}^{\overline{\theta}}\theta^{\beta+\lambda}\right)+\left(\frac{\theta_{VT}-\theta_{UV}}{\overline{\theta}-1}\right)\left(\frac{1}{\theta_{VT}-\theta_{UV}}\int_{\theta_{UV}}^{\theta_{VT}}\theta^{\beta+\lambda}\right)\right)$$

Thus, $P_S\overline{h_S}=\left(\frac{\pi}{\overline{\theta}-1}\right)\left(\int_{\theta_{TH}}^{\overline{\theta}}\theta^{\beta+\lambda}+\int_{\theta_{UV}}^{\theta_{VT}}\theta^{\beta+\lambda}\right)$

Rewriting the labor demand (19) yields:

(29) $$\frac{W_S}{y_U}=\left(\overline{h_S}\right)^{\sigma-\phi-r}\left(\frac{P_U}{P_S\overline{h_S}}\right)^{1-r}$$

Along the labor supply, the RHS of equation (29) is monotonically decreasing in $\frac{w_S}{y_U}$. Therefore, there is one intersection between the labor supply and demand, and $\frac{w_S}{y_U}$ that clears the market is unique.

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[34] It is easy to verify that $\infty>\frac{u_S}{u_T}(\theta_{jk})>0$, $\infty>\overline{h}_S>0$ and thus $0<\pi_3<1$ (because $\overline{\theta}<\infty$, $h_i(\underline{\theta}=1)>0$ (recall equation (9)), and if the high-skilled (vocational) sector exists, then $\overline{\theta}>\theta_{TH}>\hat{\theta}$ ($\hat{\theta}>\theta_{VT}>Ze_T$)(recall equations (12)-(13)).

<u>Proposition 5 – assumptions</u> (Note that these assumptions do not contradict the previous ones)

<u>(A6)</u> $\theta^{**} \geq \overline{\theta} \geq \theta^{*}$, where $\theta^{*} = \widehat{\theta}\left(1+\left(\frac{\tau}{1-\tau}\right)\left(1+\frac{\delta\beta}{\mu}\right)^{\frac{\mu}{\delta}}\right)$ and $\theta^{**} = \widehat{\theta} Ze_T$

<u>(A7)</u> Public education is sufficiently large , i.e., $E > \left(\frac{v}{\rho}\right)\left(\frac{I}{F}\right)^{\frac{1}{\sigma-\phi}}\left(\frac{1}{e_T}\right)^{\chi}\left(\frac{1}{Z}\right)^{\chi-\beta}$

where $\chi = \frac{(\beta+\lambda)(2(\sigma-\phi)-1)-(1-r)}{\sigma-\phi}$, $I = \left(\frac{\mu}{\delta(\beta+\lambda)}+1\right)^{(\beta+\lambda)(1+\phi-\sigma)+1-r}$

$F = \left(\frac{\mu}{\mu+\delta\beta}\right)^{\frac{\mu}{\delta}}(1-\tau)$, $v = \left(\frac{\mu+\delta\beta}{\delta\beta}\right)^{\beta}$ 。

<u>(A8)</u> The net productivity augmentation of skilled labor, $\sigma-\phi$, the returns to ability, λ, and the returns to time investment in higher education, β, are sufficiently large, such that $\sigma-\phi-1 > \frac{1-r-\lambda}{\beta+2\lambda}$. In this case, $\chi-\beta>0$, and hence (A7) does not contradict (A8).

<u>(A9)</u> Effective effort is costly, i.e., $Z > \frac{1}{e_T}\left(1+\left(\frac{\tau}{1-\tau}\right)\left(1+\frac{\delta\beta}{\mu}\right)^{\frac{\mu}{\delta}}\right)$.

<u>The intuition for the co-existence of the two types of skilled workers</u>

Assumption (A6) posits that $\overline{\theta}$ has intermediate levels. The intuition is that highly talented workers (with high θs) prefer the high-skilled sector to teaching because of the returns to their ability whereas low-ability workers (with low θs) prefer vocational professions to teaching to alleviate their learning effort. Assumptions (A7)–(A8) guarantee that the total skilled sector is sufficiently attractive: If the quality of public education; the net productivity augmentation of skilled labor; the intensity of ability; and the intensity of the time investment in higher education are sufficiently large, then incomes in the skilled sector are relatively amplified. Moreover, assumption (A9) guarantees that the vocational sector exists, as it posits that the effective effort is sufficiently costly. In this case, the marginal utility from effective leisure increases. As a result, for low-ability workers, the teachers sector (with the exogenously given time investment in higher education) becomes less attractive relative to the vocational sector (in which they can optimally alleviate their learning effort; recall equation (7)). Moreover, as Z rises, skilled workers become cheaper to firms relative to unskilled workers (because they reduce their time investment in higher education (recall equation (7)). Thus, the relative demand for vocational workers increases at the expense of unskilled workers. A weaker secondary effect is that the supply of vocational workers declines relative to the supply of unskilled workers (see equation (14)).

<u>Proof of Proposition 5</u>

a. Let us assume by contradiction that the high-skilled sector does not exist in equilibrium, i.e., $P_H = 0$. Since the skilled sector exists in equilibrium (see proposition 4), it is composed of vocational workers only, i.e., $P_S = P_V$. Additionally, $P_H = 0$ implies that agent $\overline{\theta}$ prefers teaching rather than high-skilled professions. Therefore, using a monotonic transformation of equation (6)(4), I obtain:

$$u^H\left(\overline{\theta}\right) = \left((1-\tau)w_S\left(\overline{\theta}\right)^{\beta+\lambda}\pi\right)^{\delta}\left(\frac{\mu}{\mu+\delta\beta}\right)^{\mu} < u^T\left(\overline{\theta}\right) \to \left((1-\tau)y_T\right)^{\delta}\left(1-\frac{Ze_T}{\overline{\theta}}\right)^{\mu}$$

$$\Leftrightarrow \frac{y_T}{w_S} > \left(\frac{\mu}{\mu+\delta\beta}\right)^{\frac{\mu}{\delta}}\left(\overline{\theta}\right)^{\beta+\lambda}\left(\frac{\overline{\theta}}{\overline{\theta}-Ze_T}\right)^{\frac{\mu}{\delta}}\pi \quad \text{where } \pi = \left(\frac{\delta\beta}{Z(\mu+\delta\beta)}\right)^{\beta}\rho E$$

Substituting the educational budget constraint (21) in the LHS obtains

$$(30) \quad \frac{\tau \overline{h}_S P_S}{(1-\tau)P_T} > \left(\frac{\mu}{\mu+\delta\beta}\right)^{\frac{\mu}{\delta}}\left(\overline{\theta}\right)^{\beta+\lambda}\left(\frac{\overline{\theta}}{\overline{\theta}-Ze_T}\right)^{\frac{\mu}{\delta}}\pi$$

Using equation (9), the skilled quality equals

$$(31) \quad \overline{h_S} = \pi\overline{\left(\theta_S\right)^{\beta+\lambda}}$$

where $\overline{(\theta_S)^{\beta+\lambda}}$ is the mean of $(\theta_i)^{\beta+\lambda}$ for all skilled workers.
Substituting in (30) and rearranging obtains

$$(32)\quad \left(\frac{\overline{\theta}}{\overline{\theta}-Ze_T}\right)^{\frac{\mu}{\delta}}\left(\frac{P_T}{P_S}\right)\frac{\overline{\theta}^{\beta+\lambda}}{\overline{(\theta_S)^{\beta+\lambda}}}<\left(\frac{\tau}{1-\tau}\right)\left(1+\frac{\delta\beta}{\mu}\right)^{\frac{\mu}{\delta}}$$

Now, I prove that inequality (32) does not hold. Since $Ze_T>0$, then $\frac{\overline{\theta}}{\overline{\theta}-Ze_T}>1$. Moreover, using assumption (A5), $P_S=P_V$ and $\hat{\theta}>\theta_{VT}$ (see equation (13)), then

$$(33)\quad \frac{P_T}{P_S}=\frac{P_T}{P_V}=\frac{\overline{\theta}-\theta_{VT}}{\theta_{VT}-\theta_{UV}}\geq\frac{\overline{\theta}-\hat{\theta}}{\hat{\theta}}$$

Furthermore, since $P_S=P_V$, all skilled workers are below θ_{VT}, and as $\hat{\theta}>\theta_{VT}$ (see equation (13)) they are below $\hat{\theta}$. As a result,

$$(34)\quad \overline{(\theta_S)^{\beta+\lambda}}<\left(\hat{\theta}\right)^{\beta+\lambda}$$

Using inequalities (33)-(34), the LHS of inequality (32) is bounded by

$$\left(\frac{\overline{\theta}}{\overline{\theta}-Ze_T}\right)^{\frac{\mu}{\delta}}\left(\frac{P_T}{P_S}\right)\frac{\overline{\theta}^{\beta+\lambda}}{\overline{(\theta_S)^{\beta+\lambda}}}>\left(\frac{\overline{\theta}}{\hat{\theta}}-1\right)\frac{\overline{\theta}^{\beta+\lambda}}{\left(\hat{\theta}\right)^{\beta+\lambda}}$$

Inserting the lower bound of $\overline{\theta}$, given in assumption (A6)

$$\left(\frac{\overline{\theta}}{\hat{\theta}}-1\right)\left(\frac{\overline{\theta}}{\hat{\theta}}\right)^{\beta+\lambda}\geq\left(\frac{\tau}{1-\tau}\right)\left(1+\frac{\delta\beta}{\mu}\right)^{\frac{\mu}{\delta}}\left(1+\left(\frac{\tau}{1-\tau}\right)\left(1+\frac{\delta\beta}{\mu}\right)^{\frac{\mu}{\delta}}\right)^{\beta+\lambda}>\left(\frac{\tau}{1-\tau}\right)\left(1+\frac{\delta\beta}{\mu}\right)^{\frac{\mu}{\delta}}$$

This contradicts inequality (32). Thus, the most talented worker, $\overline{\theta}$, prefers high-skilled professions rather than teaching. Hence, the high-skilled sector exists. ∘

b. According to corollary 2, all agents $[1,\breve{\theta}]$, where $\breve{\theta}=Ze_T$, are incompatible for teaching. Thus, they are vocational or unskilled workers. For $Ze_T>1$ (under assumption (A9)), this set is not empty (recall that $\underline{\theta}=1$). Assume by contradiction that the vocational sector does not exist in equilibrium, i.e., $P_V=0$. Since the skilled sector exists in equilibrium (see proposition 4), it is composed of highly skilled workers only, i.e., $P_S=P_H$. Another implication of $P_V=0$ is that the utility of agent $\breve{\theta}$ is larger as an unskilled worker than as a vocational worker. Substituting equation (9) in equation (6),

$$u^V\left(\breve{\theta}\right)=\left((1-\tau)w_S\left(\breve{\theta}\right)^{\beta+\lambda}\pi\right)^{\delta}\left(\frac{\mu}{\mu+\delta\beta}\right)^{\mu}<u^U\left(\breve{\theta}\right)=\left(y_U\right)^{\delta}$$

$$\Leftrightarrow\ \frac{y_U}{w_S}>\left(\frac{\mu}{\mu+\delta\beta}\right)^{\frac{\mu}{\delta}}\pi(1-\tau)\left(\breve{\theta}\right)^{\beta+\lambda}$$

Substituting the total demand for skilled relative to unskilled workers, (19), and $\breve{\theta}=Ze_T$ obtains

$$(35)\quad \frac{1}{\left(\overline{h}_S\right)^{\sigma-\phi-1}}\left(\frac{P_S}{P_U}\right)^{1-r}>\left(\frac{\rho FE}{\nu}\right)\left(e_T\right)^{\beta+\lambda}Z^{\lambda}$$

Inserting equation (31) in inequality (35), derives:

$$(36)\quad \left(\frac{P_S}{P_U}\right)^{1-r}\left(\overline{(\theta_S)^{\beta+\lambda}}\right)^{1+\phi-\sigma}>Z^{\beta(\phi-\sigma)}(F)\left(\frac{\rho E}{\nu}\right)^{\sigma-\phi}\left(Ze_T\right)^{\beta+\lambda}$$

Now, I prove that inequality (36) does not hold. Since all workers are below $\overline{\theta}$ and the returns to skilled quality is decreasing in production, $1+\phi-\sigma>0$ (recall equation (16)),

$$(37)\quad \left(\overline{(\theta_S)^{\beta+\lambda}}\right)^{1+\phi-\sigma}<\overline{\theta}^{(\beta+\lambda)(1+\phi-\sigma)}$$

Using assumption (A5), $P_S=P_H$, $\theta_{TH}>\hat{\theta}$ (see equation (13)) and $\theta_{UT}>Ze_T$ (see corollary 2), derives

(38) $$\frac{P_S}{P_U}=\frac{P_H}{P_U}=\frac{\overline{\theta}-\theta_{TH}}{\theta_{UT}-1}<\frac{\overline{\theta}-\widehat{\theta}}{Ze_T-1}$$

Using inequalities (37) and (38), the LHS of inequality (36) is bounded by

$$\left(\frac{P_S}{P_U}\right)^{1-r}\left(\overline{(\theta_S)^{\beta+\lambda}}\right)^{1+\phi-\sigma}<\left(\frac{\overline{\theta}-\widehat{\theta}}{Ze_T-1}\right)^{1-r}\overline{\theta}^{(\beta+\lambda)(1+\phi-\sigma)}$$

Inserting the upper bound of $\overline{\theta}$, given in assumption (A6) and substituting $\widehat{\theta}$, given in equation (12) yields

$$\left(\frac{\overline{\theta}-\widehat{\theta}}{Ze_T-1}\right)^{1-r}\overline{\theta}^{(\beta+\lambda)(1+\phi-\sigma)}<I\left(Ze_T\right)^{2(\beta+\lambda)(1+\phi-\sigma)+1-r},$$

Under assumption (A7), we derive

$$I\left(Ze_T\right)^{2(\beta+\lambda)(1+\phi-\sigma)+1-r}<Z^{\beta(\phi-\sigma)}F\left(D\right)^{\sigma-\phi}\left(Ze_T\right)^{\beta+\lambda}$$

This contradicts inequality (36). Thus, agent $\breve{\theta}=Ze_T$ prefers vocational jobs rather than unskilled jobs and teaching. Hence, the vocational sector exists. Note that assumption (A8) guarantees that as Z or e_T increase, the RHS in assumption (A7) decreases. Thus, assumptions (A7) and (A9) co-exist. ∘

c. Under assumption (A6), the high-skilled sector and the vocational sector co-exist for $\frac{\overline{\theta}}{\widehat{\theta}}$ in the range:

(39) $$\frac{\overline{\theta}}{\widehat{\theta}}\in\left[1+\left(\frac{\tau}{1-\tau}\right)\left(1+\frac{\delta\beta}{\mu}\right)^{\frac{\mu}{\delta}},\quad Ze_T\right].$$

This set of $\frac{\overline{\theta}}{\widehat{\theta}}$ is not empty for sufficiently large Z, defined in assumption (A9). Note that assumption (A8) guarantees that as Z increases, the RHS of assumption (A7) decreases. Thus, assumption (A7) is compatible with assumption (A9). ∘

Proof of proposition 6:

Since unskilled workers choose not to invest at all in their higher education, and thus enjoy the maximum level of leisure, the other sectors must be compensated by larger incomes than in the unskilled sector. If the vocational sector exists, combining equations (13) and (14) yields:

(40) $$\frac{y_U}{y_T}=(1-\tau)\left(\frac{\theta_{UV}}{\theta_{VT}}\right)^{\beta+\lambda}\left(\frac{\theta_{VT}-Ze_T}{\theta_{VT}}\right)^{\frac{\mu}{\delta}}$$, where $\theta_{VT}>Ze_T$ and $\theta_{VT}>\theta_{UV}$

It is easy to verify that the RHS is lower than 1. Additionally, if the vocational sector does not exist, it is easy to see that the RHS of equation (15) is lower than 1. For $\lambda=0$, the Argmin ability equals $\widehat{\theta}_0=\left(\frac{\mu+\delta\beta}{\delta\beta}\right)Ze_T$ (recall equation (12)). Note that workers with the Argmin ability are teachers. Workers with the Argmin ability have the highest utility from teaching relative to skilled professions. Thus, if workers with the Argmin ability are reluctant to become teachers, the teachers sector is empty. Though, if they were obligated to become skilled workers, their optimal time investment in higher education would be identical to the time investment of teachers, i.e., $e_i^*\left(\widehat{\theta}_0\right)=e_T$ (recall equation (7)). Accordingly, highly skilled workers, who are more talented than $\widehat{\theta}_0$, spend more time on higher education than teachers, whereas vocational workers with lower ability than $\widehat{\theta}_0$, spend less time on higher education than teachers**.** Now, the skilled incomes relative to teachers are derived easily. Since highly skilled workers (teachers) invest more time than teachers (vocational workers) in higher education, they must be compensated by relatively higher incomes. In the presence of $\lambda>0$, it is easy to verify from equation (12) that $\widehat{\theta}_1=$ the Argmin of $\left(\frac{u_S}{u_T}(\theta_i)\right)$ is lower than $\widehat{\theta}_0$. Similarly, workers with the Argmin ability $\widehat{\theta}_1$ are teachers. Also note that $e_i^*\left(\widehat{\theta}_0\right)=e_T$ for all λ, since the optimal time allocation in the skilled sector does not depend on λ (recall (7)). As a result, vocational workers who are less talented than $\widehat{\theta}_1$, have lower ability than $\widehat{\theta}_0$. Therefore, similar to the case of $\lambda=0$, they spend less time on higher education and have lower incomes than teachers. However, workers with higher ability than $\hat{\theta}_1$ but lower ability than $\hat{\theta}_0$, i.e., with $e_i^* < e_T$, may become highly skilled workers, as figure 2 illustrates.

Proof of Proposition 7

a. It is easy to verify from the equilibrium equations (22)-(23) that both the skilled sector and the teachers' sector expand: Under type (a)(i) SBTC, both π_1 and $\overline{h_S}$ increase by the same factor (recall equations (2) and (11)), thereby equation (22) does *not* change (because the educational budget and skilled income increase by the same factor). However, $\left(\overline{h_S}\right)^{\sigma-\phi}$ rises in equation (23), which occurs also under type (a)(ii) SBTC. Thus, under each one of these shocks, to balance equation (23), θ_{UV} declines thereby P_S rises. The decline in θ_{UV} further implies an increase in $P_S\overline{h_S}$ (recall equation (28)(28)). Now, in order to balance equation (22), P_T increases.

b. This part emanates from the increase in P_T and P_S. P_T increases if and only if θ_{TH} rises and θ_{VT} declines, i.e., the teachers' sector pushes both the high-skilled and the vocational sector (recall equations (11) and (13), and note that the graph $\frac{u_S}{u_T}(\theta_i)$ does *not* shift under type (a) SBTC). Because θ_{TH} rises, the high-skilled sector shrinks. Thus, the increase in the total skilled sector occurs through an increase in the vocational sector towards the unskilled sector.

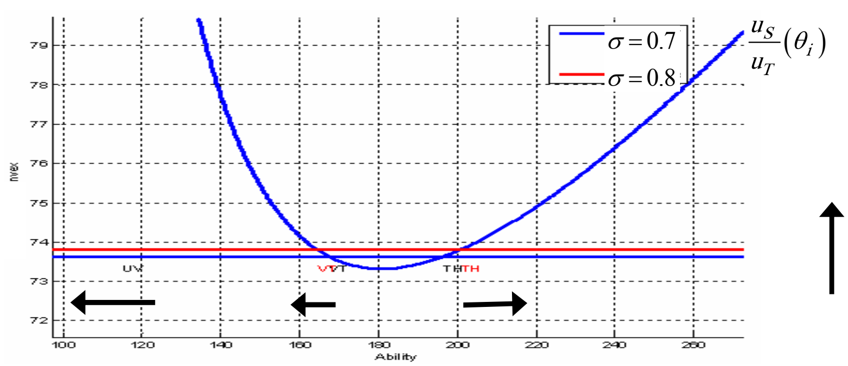

Fig. 3. – The effect of type (a) SBTC – an inc[illegible] net productivity augmentation of skilled labor (recall equation (16)).

Proof of Proposition 9

It is easy to verify from the equilibrium equations (22) and (24) that both the skilled sector and the teachers' sector expand: Most of the proof is similar to Proposition 7(a) (referring to equation (24) instead of equation (23)). Because $\left(\overline{h_S}\right)^{\sigma-\phi}$ rises, to balance equation (24), θ_{TH} *or* θ_{UT} decline. It is easy to verify that both P_S and P_T increase, and thus both θ_{TH} *and* θ_{UT} decline (otherwise equation (22) does not hold)[35]. Thus, the relative teacher quality declines.

Definition 6: Consider two income distributions represented by the random variables X and W. X is more equal than W if the Lorenz curve corresponding to X is everywhere above that of W. Thus, if X is more equal that W, it has a lower Gini coefficient. According to Atkinson (1970), a larger Lorenz curve is equivalent to second-degree stochastic dominance.

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[35] On the one hand, if θ_{TH} declines, then P_S rises. The decline in θ_{TH} further implies an increase in, $P_S\overline{h_S} = P_H\overline{h_H} = \left(\frac{\pi}{\overline{\theta}-1}\right)\left(\int_{\theta_{TH}}^{\overline{\theta}} \theta^{\beta+\lambda}\right)$. Now, in order to balance equation (22), P_T must increase thereby θ_{UT} declines. On the other hand, if θ_{UT} declines, then P_T rises. In order to balance equation (22), $P_S\overline{h_S}$ must increase, which occurs through a decline in θ_{TH} thereby P_S rises.

Proof of Proposition 10

a. When λ increases, $\overline{h_S}$ increases (recall equations (2)). Thus, to balance equation (23), θ_{UV} declines thereby P_S rises. In equation (22), when λ increases, then $\frac{u_S}{u_T}(\theta_{TH})$, $\frac{u_S}{u_T}(\theta_{VT})$ and $\overline{h_S}$ increase. As $\frac{u_S}{u_T}(\theta_{TH})$ sharply increases, then to balance equation (22) for $jk = TH$, θ_{TH} declines thereby P_H rises; As $\frac{u_S}{u_T}(\theta_{VT})$ moderately increases, then to balance equation (22) for $jk = VT$, θ_{VT} declines. Combining the effects, the educational budget rises thereby P_T rises.

b. Relative teacher quality declines because both θ_{TH} and θ_{VT} decline.

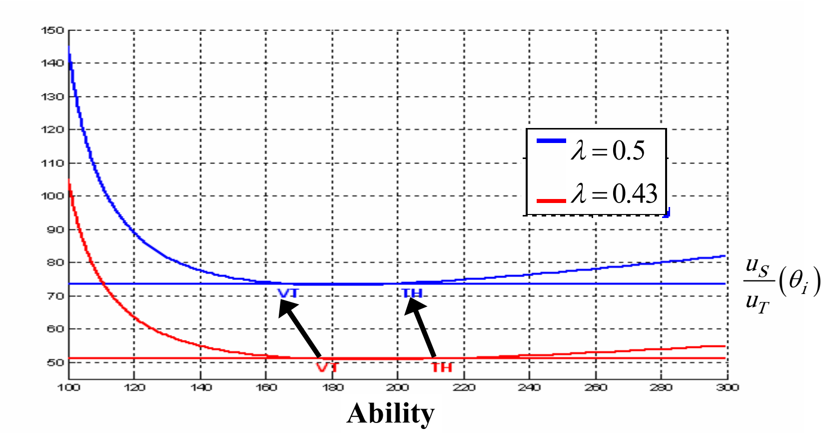

Fig. 4. – The effect of type (b) SBTC – an increase in the intensity of ability

Proof of Proposition 12

a. When e_T declines, $\frac{u_S}{u_T}(\theta_i)$ (see equation (11)) declines, thereby P_T increases. To balance the budget constraint, teacher income declines. The increase in the high-skilled sector (i.e., θ_{TH} declines) derives from its higher (lower) sensitivity to income (leisure) changes relative to the vocational sector, according to Property 2 ($\frac{u_S}{u_T}(\theta_i)$ declines and shifts to the left). Thus, the teachers' sector increases towards the vocational sector, i.e., θ_{VT} declines.

b. Relative teacher quality declines as teachers shift to the lower end of the distribution (θ_{TH} and θ_{VT} decline).

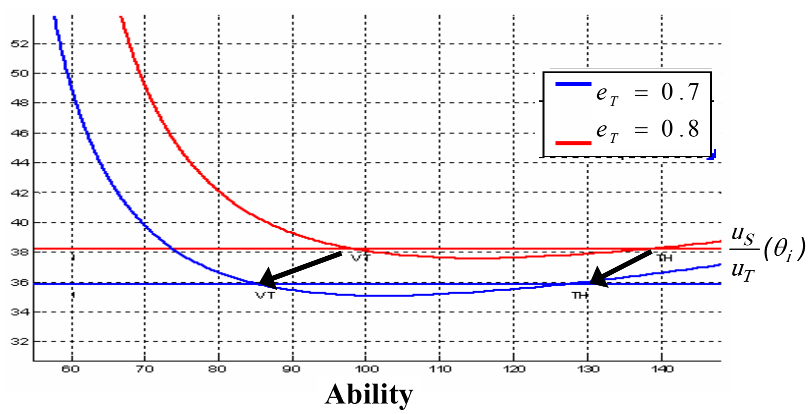

Fig. 5. – The effect of reduction in the time investment of teachers in higher education, e_T.