Problem Set 4 – Limits and Continuity

Calculate the following limits

A. $\lim_{x \to 2} \quad x^2 - 4$

B. $\lim_{x \to \infty} \quad x^3 - x^2 + x - 5$

C. $\lim_{x \to -\infty} \quad x^3 + x^2 + x - 5$

D. $\lim_{x \to -\infty} \quad 3x^4 + 2x^3 + x - 1$

E. $\lim_{x \to 3} \quad \frac{x^2 - 4}{x^2 - 5x + 6}$

F. $\lim_{x \to \infty} \quad \frac{2x + 3}{4x - 5}$

G. $\lim_{n \to \infty} \quad \frac{\sum_{i=1}^{n} i}{n^2}$

H. $\lim_{x \to \infty} \quad \frac{x^3 - x^2 + x - 5}{-(x^2 - 1)^2}$

I. $\lim_{x \to -\infty} \quad \frac{(x^2 - 1)^2}{-x^3 - x^2 + x - 5}$

J. $\lim_{x \to 2} \quad \frac{x^2 - 4}{x^2 - 5x + 6}$

K. $\lim_{x \to a} \quad \frac{x^3 - a^3}{x^2 - a^2}$

L. $\lim_{h \to 0} \quad \frac{(x + h)^3 - x^3}{h}$

M. $\lim_{x \to 1} \quad \frac{1}{1 - x} - \frac{1}{1 - x^2}$

N. $\lim_{n \to \infty} a_n = \begin{cases} \dfrac{3n}{n+3} & \text{if } n \text{ even} \\ 2 & \text{otherwise} \end{cases}$

O. $\lim_{x \to \infty} \dfrac{\sqrt{x+2} - \sqrt{x+1}}{\sqrt{x}}$

P. $\lim_{x \to 4} \dfrac{\sqrt{2x+1} - 3}{\sqrt{x-2} - \sqrt{2}}$

Q. $\lim_{x \to \infty} x(\sqrt{x^2+6} - x)$

R. $\lim_{x \to \infty} \sqrt{x}(\sqrt{x+1} - \sqrt{x})$

2. Find the points of discontinuity in each of the following functions, and categorise which type of discontinuity you have found at each such point:

$$f(x) = |x-2|$$

$$f(x) = \frac{x^3-8}{x-2}$$

$$f(x) = \begin{cases} x^2 & x \geq 1 \\ x & 0 < x < 1 \\ 5 & x \leq 0 \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{x}\{(1+x)^2 - 1\} & x \neq 0 \\ 3 & x = 0 \end{cases}$$

$$f(x) = \frac{1}{x-2} - \frac{6}{x^2+2x-8}$$

$$f(x) = \begin{cases} \frac{1}{x+3} & x < -1 \\ \frac{x^2+|x|}{|x|} & x > -1 \end{cases}$$

$$f(x) = \left\lfloor x + \frac{1}{2} \right\rfloor$$

$$f(x) = \begin{cases} \frac{1}{x+4} & x < 1 \\ -1 & x = 1 \\ \frac{x^2-9x+14}{x^2+3x-10} & x > 1 \end{cases}$$

$$f(x) = \frac{|x+2|}{x^2-4}$$

$$f(x) = \begin{cases} x^2-2 & x \leq 0 \\ \frac{\sqrt{x}+1}{3\sqrt{x}-2} & x > 0 \end{cases}$$