Discrimination in Contests - A Survey

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Abstract

The objective of this paper is to provide a comprehensive answer to some fundamental questions related to discrimination within the context of contests. For example, what forms of discrimination are possible? Can discrimination be justified? What mode of discrimination is expected? Does discrimination necessarily result in the elimination of polarization? How effective are the different modes of discrimination in inducing efforts (revenue)? How do the most widely studied contests based on an all-pay-auction and on a lottery compare under different modes of discrimination?

Applying a contest-design approach, we examine four alternative types of discrimination that can be selected by a contest designer who maximizes the contestants' efforts (his revenue). Our survey focuses on the leading principles of the separate and joint effective application of the alternative modes of direct, overt covert and head starts-discrimination that are assumed to be exercised under the widely studied family of (logit) contest success functions (CSFs). Whereas direct discrimination refers to differential taxation of the contested prize subject to a balanced-budget constraint, overt, covert and head starts-discrimination relate to structural discrimination that involves the parameters of the CSF. While the direct mode of discrimination is legally feasible, the structural modes of discrimination are more subtle and more difficult to implement and, sometimes, may even involve legal barriers.

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1. Introduction

The numerous applications of contest theory include promotional competitions, litigation, internal labor market tournaments, rent-seeking, R&D races, political and public policy competitions and sports, Konrad (2009), Congleton et al. (2008). Contest design may involve the endogenous determination of relevant institutional characteristics by contest designers; economic and political entrepreneurs who wish to maximize the total efforts made by the contestants. These characteristics include various forms of discrimination between the contestants. In fact, evidence on discrimination is abounding in all the above mentioned applications and it can take various forms. More specifically, the various types of discrimination can be observed in the admission policy of universities that is often based on affirmative action, Fu (2006), Franke (2012), in procurement auctions, Feess et al. (2008), in political rent-seeking contests, Epstein et al. (2013), in litigation games, Bernardo et al. (2000), in promotion competitions and bonus tournaments designed by employees and in the design of sport competitions, Szymanski (2003). In the latter context, handicapping is very common.[1] The application of our model is not limited to the social sciences. In particular, it also seems to fit disciplines in the life sciences such as ethology and biology, because the assumptions of contest resolution based on some lottery and effort maximization seem plausible.[2]

The contest designer can discriminate on the basis of moral, social or welfare considerations. Typically, such considerations are not related to his interest of increasing the contestants' efforts, but to his attempt to correct a moral, social or economic distortion. Indeed discrimination is commonly defined as differential treatment of individuals with respect to an exogenous marker that is not related to the designer's task and, in particular, to his objective. But in the present study we broaden the standard concept of discrimination such that it serves as a means of attaining the maximal expected effort by the contestants. In our study discrimination therefore

[1] Handicapping in sports and games is the practice of assigning advantage through scoring compensation or other advantage given to different contestants to equalize the chances of winning. The handicap system is used in many games and sports, including go, chess, croquet, golf, bowling, polo, basketball and track and field events.

[2] Consider, for example, a wooing game where several males compete for a single female. Efforts are the energy (time) spent by the males to woo the female. Efforts maximization is reasonable because it enables the female to evaluate the competing males' 'true potential' in terms of genetic promise for off-springs and, in turn, choose the fittest one. In this case, the designer is either the female (which is also the contested prize) or the evolutionary process.

includes differential treatment of contestants based on their identity (overt or head starts-discrimination), on the extent of their incurred efforts (covert discrimination) and on their different valuations of the contested prize (direct discrimination). These different means of affecting the contestants' efforts are presented in more detail in section 3. Discrimination motivated by the designer's objective to maximize the contestants' efforts in other contexts has been adopted in different contest frameworks. This is notable in bribery games, Clark and Riis (2000), Lien (1990), Hillman and Katz (1987), Hillman (2013) where a bribe to the contest winner is conceived as a form of non-standard discrimination as long as it induce increased efforts by the contestants. In fact, sometimes the bribes even entail creation and extraction of rents that are ultimately contestable, see Hillman and Katz (1987) and Hillman (2013). In other contests, attention has been devoted to the effect of using tie-breaking rules, caps and minimum bids, Che and Gale (1998), Gavious et al. (2003), Sahuguet (2006), Cohen and Sela (2007), Szech (2015). A broader view of discrimination can also cover the control by the designer of the group of contestants and, in particular, their number, the optimal order of the contestants in a simultaneous or in a multi-stage sequential contest and the control of the rules of the game such that the designer openly or latently prefers one of the contestants assigning him a head starts, see Moldovanu and Sela (2006), Li and Yu (2012), Franke et al. (2014b), Segev and Sela (2014a, 2014b). Finally note that two common discriminatory methods are employed to achieve distributional goals in the context of auctions, Athey et al. (2011). One approach is to set aside a fraction of contracts for targeted contestants (firms). An alternative method is to provide bid subsidies for favored contestants. These modes of discriminatory policies are disregarded in our survey because they have been studied in the auction setting but not in the context of our all-pay auction and lottery contest setting.

In light of the numerous applications of contest theory and the significance of discrimination in contest design, the objective of the current essay is to shed light on the following questions:

(i) What are the means of reducing asymmetry in contests?

(ii) Under what circumstances leveling the playing field by applying discrimination increases the contestants' efforts?

(iii) Is there a reason for the contest designer to preserve some asymmetry between the contestants?

(iv) What are the maximal possible efforts discrimination can induce and how can they be reached? In particular, what combination of CSF and discrimination policy induces these efforts?

(v) Are two modes of discrimination better than one?

(vi) In light of the answers to the above questions, what conclusions can be drawn regarding the effectiveness of an all-pay-auction (APA) relative to that of simple or general lotteries?

Our survey attempts to present a comprehensive synthesizing view of the existing results on discrimination in contests and their interrelationships assuming a simultaneous game of investment by the contestants under complete information.[3] It is intended to have a clear added value to researchers of contest theory as well as to those who are not particularly interested in contest theory, but are aware of the significance of the topic to economics and, in particular, to political economy. To introduce the modes of discrimination on which we focus, let us first present the basic contest that we study.[4]

2. The contest and its designer

In our contest there are two risk-neutral contestants, the high and low benefit contestants, 1 and 2.[5] The initial prize valuations of the contestants are denoted by n_1 and n_2 and, with no loss of generality, we assume that $n_1 \geq n_2$ or $k = \frac{n_1}{n_2} \geq 1$ and that the contest designer has full knowledge of the contestants' prize valuations. In the vast contest literature, the lotteries proposed by Tullock (1980) are most commonly assumed as the contest success functions (CSFs), see Konrad (2009) and references

[3] In contrast to our study that focuses on a simultaneous game under complete information, there are papers that deal with a sequential contest setting and incomplete information. See, for example, Moldovanu and Sela (2006), Segev and Sela (2014a, 2014b) and the literature mentioned in the latter strudy.

[4] A comparison between Tullock lotteries and an APA has been recently presented in Mealem and Nitzan (2015), assuming a given form of discrimination (see Table 4 in the Conclusion). The emphasis in the current paper is, however, different because it focuses on effort maximization under different forms of permissible discrimination and the family of logit CSFs.

[5] Specific cases of *N*-player contests will be dealt with in the sequel, e.g., in Section 10.1.

therein. In two-player contests, for $x_1 \geq 0$, $x_2 \geq 0$, $\alpha > 0$ and $\delta > 0$, these logit functions take the form:[6]

(1) $$p_1(x_1, x_2) = \begin{cases} \dfrac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha} & \text{if} \quad x_1 + x_2 > 0 \\ 0.5 & \text{if} \quad x_1 = x_2 = 0 \end{cases}$$

Usually, x_1 and x_2 are interpreted as the contestants' efforts. However, p_1 has two possible interpretations. It can be interpreted as contestant 1's winning probability of an indivisible prize or as his share in a divisible prize, see Corchon (2007). In turn, the winning probability of contestant 2 or his share in the prize is equal to $p_2 = 1 - p_1$. The exponent α is a parameter that represents the effect of a real unit of investment on the winning probability of a contestant[7] while the asymmetry between the impact of the contestants' efforts is captured by the parameter δ, $\delta > 0$. One reason for the popularity of this CSF is that it has appealing axiomatization, see Clark and Riis (1998), Corchon and Dahm (2010), Jia (2008, 2010), Skaperdas (1996).[8] The special attention given to the simple lottery CSF, where $\alpha = 1$, can be justified, as recently argued by Franke et al. (2013), on the grounds that it lends itself to a very appealing competitive-market interpretation.

Given the contestants' fixed prize valuations and the CSF, the function that specifies the contestants' winning probability given their efforts, $p_i(x_1, x_2)$, the expected net payoff (surplus) of contestant *i* is:

(2) $$E(u_i) = p_i(x_1, x_2)n_i - x_i, \quad (i=1,2)$$

In the optimal contest design setting, the objective function of the contest designer is:

(3) $$G = x_1 + x_2$$

[6] Later on we relate to "head starts-discrimination" that was examined under APA and under a lottery. In the latter case the assumption is that in (1), $\alpha = 1$ and, therefore, the form of the contest success functions corresponding to this case is presented separately, see equation (5).
[7] For the most widely studied APA, $\alpha = \infty$.
[8] Munster (2009) has recently generalized the axiomatic approach to group CSFs.

3. Discrimination

The designer of the contest can resort to direct or structural discrimination. In the former case, he (directly) affects the prize valuations of the contestants. In the latter case, he (structurally) affects the winning probabilities (or shares of the prize) of the contestants by controlling the parameters of the CSF. Dependent on the controlled parameter, we distinguish between overt, covert and head starts-discrimination.

3.1 Direct discrimination: Differential taxation

Direct discrimination via differential taxation of the contested prize that affects the contestants' actual prize valuations, n_1 and n_2, is a pair of (positive or negative) amounts, ε_1 and ε_2 that changes the prize valuations to $(n_1+\varepsilon_1)$ and $(n_2+\varepsilon_2)$. A contest designer who applies such a taxation scheme must ensure that the transformed prize valuations are positive. Otherwise the contestants will not voluntarily take part in the contest and the designer's revenue will be equal to zero. We also assume that the contest designer faces a balanced-budget constraint, that is, ε_1 and ε_2 must also satisfy the requirement that $p_1\varepsilon_1 + p_2\varepsilon_2 = 0$.[9]

The contest that we study is amenable to two types of interpretations. Under the first and more common one, the contest determines the contestants' winning probabilities of an indivisible prize, Konrad (2009). In this case, the balanced-budget constraint means that the designer's expected expenditures are equal to zero. Under the second interpretation, the contest determines the shares of a divisible prize won by the contestants, Corchon and Dahm (2010), Lee and Lee (2012), Warneryd (1998). In this case, the balanced-budget constraint means that the designer's certain expenditures are equal to zero. The balanced budget constraint can be justified under both interpretations, but it is more plausible under the second interpretation.

Under the first interpretation of the CSF, this constraint requires that the designer's expected expenditures are equal to zero. In this case, ε_1 (ε_2) is the tax (subsidy) levied on (given to) contestant 1 (2) if he wins the contest. This ex-ante balanced-budget constraint is reasonable when the designer is "risk neutral" in the sense that he does not mind to face an ex-post deficit situation after the outcome of

[9] The possibility of a balanced-budget constraint faced by the contest designer has not been dealt with in the contest literature. The possibility of caps on the contestants' efforts has been examined, for example, by Che and Gale (1998), Ujhelyi (2009).

the contest has been revealed. Alternatively, in our setting, the designer might wish to delegate the responsibility of financing the contest he designs by controlling the ex-post values of the prize.[10] In such a case, assuming that he can resort to the services of a perfectly competitive insurance market, the designer can purchase an insurance policy that enables him to implement his preferred contest transferring the funding responsibility while still being obligated to the balanced-budget constraint because otherwise in the equilibrium of the designed contest the insurance transaction will not be realized. In other words, in the insurance market equilibrium, the price of the preferred policy will be equal to the expected fair premium (zero). Note that the balanced-budget constraint assumption is plausible, even without the delegation of the funding responsibility, when the designer controls a series of identical contests that are held during a fixed period (typically weekly, monthly or quarterly contests that are held during the budget year) and assuming that he must apply the same differential taxation in all the contests (the same tax law must apply to all the contests in the relevant period). In such a case, the designer's taxation decision does not require a repeated game modeling; the single decision the designer makes applies to all the contests and he tries to ensure that during the relevant period the net transfers between the contestants are cancelled out such that his budget is balanced.

Under the second interpretation of the CSF, where the prize is shared by the competitors, the balanced-budget constraint requires that the (certain) sum of the positive and negative taxes applied by the discriminating designer is equal to zero. In this case, ε_1 (ε_2) is the tax (subsidy) levied on (given to) contestant 1 (2) if he wins the whole prize (his share in the prize is equal to 1). Under this interpretation, direct discrimination subject to a balanced-budget constraint is of particular relevance in certain applications. In these applications asymmetric agents compete for a divisible prize; one agent (contestant 1) with a well established reputation competes against another agent (contestant 2) who is relatively unknown. Because of contestant 1's reputation, a larger significance is given to the contested prize (project) and therefore his prize valuation is larger than the prize value of his rival, contestant 2. Two such applications are presented below where the balanced-budget constraint is plausible. In the first example, the contest designer typically engages in a certain activity (some well-defined task or project) restricted to a certain budget. Although the budget is

[10] For a study of delegation of the rent-seeking activity by the contestants, see Baik and Kim (1997).

earmarked only for this activity, it can be used to manipulate and affect the incentives of the contestants (the contractors) who compete for the outsourced project. But a designer who engages in such manipulations and in particular, in discrimination, must satisfy the contest balanced-budget constraint that we assume in order to ensure the overall budget constraint is satisfied:

The example of municipal projects: A municipal authority is conducting a tender for a divisible project such as urban development including development of a sewage system, roads, sidewalks and gardening. Two companies compete for a share in the project. The municipal authority is restricted to a budget allocated, for example, by the federal government. Although the budget is allocated only to urban development, it can be used to influence the incentives of the competing contestants by applying the two possible modes of discrimination. In order to satisfy the overall budget constraint, a designer who resorts to structural and direct discrimination must also satisfy the assumed balanced budget constraint.

The second application describes a situation where the balanced-budget constraint is due to a different reason. The constraint is no longer related to a fixed budget which is at the disposal of the designer for the purpose of carrying out a particular project. It is due to the fact that the two competing contestants are (at least partly) controlled by a parent company. The parent company may not prevent competition between its two subsidiaries by custom or by the law. However, despite the existing competition, the parent company still has the ability to enforce some overall financial discipline as well as the power to ensure that the designer's strength in manipulating the companies is limited. The control of the parent company on its two subsidiaries and its power in dealing with the designer, given the conflict of interests between them, explains its success in enforcing the balanced-budget constraint.

The example of portfolio distribution between two investment houses: In the capital market, the commission rate charged by an investment house is usually inversely related to the size of the customer’s investment. Suppose that the average commission rate in the industry is $\bar{z}$ and that a large client (e.g., a pension fund of some large employer) is interested in distributing its portfolio between two investment houses that are subsidiaries of some parent company. Despite the affinity between the two investment houses, they compete in the market.[11] The first investment house has an established reputation while the other smaller house is relatively unknown. Given the importance of the large customer, the preservation of the reputation of the first investment house (contestant 1) implies that it assigns a higher value to the investment of the large customer (the employer's pension fund). Often, such a pension fund prefers to invest a large share of its portfolio in the reputable and usually larger investment house and this enables obtaining a commission rate lower than $\bar{z}$. This implies that the pension fund actually "taxes" the larger investment house (contestant 1) relative to $\bar{z}$. On the other hand, the unknown investment house (contestant 2) usually receives a smaller share of the portfolio. However, the commission rate it charges is higher than $\bar{z}$. The balanced-budget constraint is satisfied because of the market forces; The pension fund is interested in reducing the commission rate, and the parent company of the two investment houses is interested in increasing the commission rate.

3.2 Overt discrimination: Built in structural partiality

In this case we enable the contest designer to control (see equation (1)) the parameter δ. This means that he can apply overt structural discrimination that affects the contestants' winning probabilities (the same efforts may yield different winning probabilities, depending on the value of this parameter). By (1), a reduction in δ increases the bias in favor of contestant 1, who is assumed to be, with no loss of generality, the more motivated contestant (the one with the higher prize valuation). Furthermore, $0 < \delta < 1$ implies a bias in favor of contestant 1. When $\delta = 1$ the contest is fair, there is no bias. When $\delta > 1$ the bias is in favor of contestant 2. Axiomatization of the logit CSF with asymmetric efforts appears in Clark and Riis (1998). Applications of this CSF allowing discrimination via the control of δ appear in Epstein et al. (2011) and Franke (2012). Epstein et al. (2013) have recently shown that structural discrimination is effective; it is useful as a means of increasing the contestants' efforts when applied independently.

[11] Two such competing investment houses, Psagot and Ofek, were subsidiaries of the leading Bank in Israel, Bank Leumi. Another example of two such companies is Gadish and Tagmulim,that were two subsidiaries of another major Israeli bank, Bank Hapoalim.

Franke et al. (2014a) studied *N*-player contests ($N \geq 2$) focusing on just two cases where $\alpha = 1$ or $\alpha = \infty$, assuming that the designer can discriminate contestant *i* by controlling the effect of his effort by multiplication of x_i by δ_i. In other words, his "modified" effective effort due to such discrimination becomes $\delta_i x_i$ (for the case of two contestants, δ in equation (1) is given by $\delta = \frac{\delta_2}{\delta_1}$). In this case the winning probability of contestant *i* for the lottery contest is:

(4) $$p_i(x_i, x_{-i}) = \begin{cases} \frac{\delta_i x_i}{\sum_{j=1}^{N}(\delta_j x_j)} & if \ \sum_{j=1}^{N}(\delta_j x_j) \neq 0 \\ 0 & if \ \sum_{j=1}^{N}(\delta_j x_j) = 0 \end{cases}$$

3.3 Covert discrimination: Differential relative effectiveness

In this case we enable the contest designer to control the parameter α (given that $\delta = 1$). This allows the ratio between the contestants' winning probabilities to differ from the ratio between their efforts. The meaning of covert discrimination is clear when the contestants' efforts are different. If $0 < \alpha < 1$ ($\alpha > 1$), then there is a bias in favor of the contestant who exerts a smaller (larger) effort in the sense that his relative winning probability is larger than his relative effort. That is, if $x_2 < x_1$, then $\frac{p_2}{p_1} > \frac{x_2}{x_1}$ ($\frac{p_1}{p_2} > \frac{x_1}{x_2}$). However, $\alpha = 1$ implies "covert impartiality". In terms of partiality, the difference between covert and overt discrimination is fundamental. Whereas control of δ means discrimination based on the identity of the contestants, control of α does not imply ex-ante discrimination between the contestants, but discrimination between different amounts of effort exerted by any contestant – discrimination between the relative effectiveness of a unit of effort, dependent on whether the exerted effort is small or large.

3.4 Head starts-discrimination

Franke et al. (2014b) studied *N*-player contests ($N \geq 2$) focusing, again, on just two cases where $\alpha = 1$ or $\alpha = \infty$, assuming that the designer can discriminate contestant *i* by controlling the effect of his effort by adding β_i to x_i. In other words, the "modified" effective effort of contestant *i* due to such discrimination now becomes $x_i + \beta_i$. In this case the winning probability of contestant *i* for the lottery contest is:[12]

$$(5) \qquad p_i(x_i, x_{-i}) = \begin{cases} \dfrac{x_i + \beta_i}{\sum_{j=1}^{N}(x_j + \beta_j)} & \text{if } \sum_{j=1}^{N}(x_j + \beta_j) \neq 0 \\ 0 & \text{if } \sum_{j=1}^{N}(x_j + \beta_j) = 0 \end{cases}$$

Notice that a contestant who enjoys head starts-benefits from his advantageous position in the sense that his rivals must first pass the head starts to be able to compete on equal footing.

4. Impartiality: The benchmark case

In the basic and most commonly studied case, $\alpha = \delta = 1$, $\beta = 0$ and $\varepsilon_1 = \varepsilon_2 = 0$. In this case, in the asymmetric equilibrium contestant 1 has an advantage because his winning probability is larger, $p_1 = \dfrac{k}{k+1}$. The initial payoff advantage of contestant 1 reduces the intensity of the competition and, in turn, the contestants' incentives to exert efforts are reduced and the total efforts are equal to $G = \dfrac{n_1 n_2}{n_1 + n_2}$. These efforts will be compared to the efforts obtained in different environments that allow discrimination - with the constrained environments that allow just one type of discrimination or only two modes of partiality.

5. One-mode partiality

We now examine environments that allow just one mode of discrimination. When discrimination can be overt, the designer’s preferred overt discrimination is $\delta = k$, Epstein et al. (2013). This optimal bias accomplishes the designer's attempt to maximize the extent of competition between the contestants. In fact, such a bias eliminates the advantage of contestant 1 and creates actual equality between the competitors (case 1 in Table 1). Also, as shown in Epstein et al. (2011), when the logit

[12] For head starts-discrimination, Franke et al. (2014b) deal with the comparison between the expected efforts of the contestants under APA and the lottery. This comparison is presented in the sequel. Our notation for head starts-discrimination differs from that of Franke et al. (2014b) in order to distinguish between this type of discrimination which is denoted by β and overt discrimination which is denoted by δ.

contest success function does not exhibit increasing returns to scale, an all-pay auction is always preferred to a logit contest success function from the point of view of the contest designer (case 2 in Table 1). The explanation of this result is due to the fact that although discrimination creates actual equality between the competitors both under the all-pay-auction and under Tullock's lottery, in the latter case the assumption that the marginal effect of effort on the winning probability cannot exceed 1 ($\alpha \leq 1$) induces smaller efforts relative to those obtained under the all-pay-auction. A similar result has been obtained by Franke et al. (2014a) in an *N*-player contest, ($N \geq 2$) for the special lottery case where $\alpha = 1$. That is, when covert discrimination is not allowed the APA is preferred to the simple lottery (see case 3 in Table 1).

When $k > 1$, the designer can choose α in the range $0 < \alpha \leq 2$ and there exists a value in this range that yields certain efforts that are larger than the expected efforts obtained under the APA, Epstein et al. (2013) (case 4 in Table 1). In this context, Fang (2002, section 5.3) has shown that with asymmetric prize valuations, $k > 1$, and $\alpha = 1$, in equilibrium $G_L = \frac{n_1 n_2}{n_1 + n_2}$ is larger than G_A, provided that the gap between the contestants' stakes is sufficiently large, that is, $k > 1 + \sqrt{2}$. Epstein et al. (2013) considerably strengthens this finding by establishing that in a fair contest ($\delta = 1$), if $k > 1$, then a designer who can select the exponent α, *always* prefers a logit lottery because it yields larger efforts relative to the APA, even when the sufficient condition $k > 1 + \sqrt{2}$ *is not* satisfied. As explained in Epstein et al. (2013), the economic intuition behind the result in case 4 is the following. For $k > 1$, a designer setting an optimal α for a pure-strategy equilibrium must choose an exponent α which is smaller than 2. Furthermore, in a pure-strategy equilibrium the designer may choose an exponent that is smaller than that α which causes contestant 2's utility to be equal to zero. In other words, in equilibrium contestant 2's utility can be positive. This has been shown by Nti (2004) for a sufficiently large value of *k* (see Section 4 in his paper). However, for lower values of *k*, but still $k > 1$, α reduces contestant 2's utility to zero (The critical *k* is $k = 3.509$). In any case, for $k > 1$ the optimal α for a pure-strategy equilibrium is smaller than 2, $0 < \alpha(k) < 2$. Under the APA where $\alpha = \infty$, the contest equilibrium is in mixed strategies, the total expected efforts are equal to $\frac{n_2(n_1 + n_2)}{2n_1}$ and the surplus of contestant 2 is completely eliminated. It remains to explain why the designer prefers the optimal pure-strategy equilibrium to the mixed-strategy equilibrium. Notice that in the move from equilibrium in pure strategies to equilibrium in mixed strategies α is increased, however, $x_1^* / x_2^* = k$. The increase in α increases the winning probability of contestant 1 and reduces the winning probability of contestant 2. However, contestant 1 is induced to reduce his efforts in the mixed-strategy equilibrium corresponding to the larger α because this further increases his expected utility. Such reduction is possible because contestant 2 whose utility is reduced or remains equal to zero in the mixed-strategy equilibrium is also induced to reduce his effort.

When the designer applies just direct discrimination, if $k > 1$, then the optimal taxation scheme under any Tullock-type lottery with $0 < \alpha < 2$ does not equalize the contestants' final stakes, but still preserves their relative magnitude. That is, $(\varepsilon_1, \varepsilon_2) \neq (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$ and $\varepsilon_1 + \varepsilon_2 > 0$. Furthermore, the total efforts of the contestants corresponding to the optimal taxation scheme under the APA are equal to the average prize valuation, $G_A = 0.5(n_1 + n_2)$ and the optimal taxation scheme equalizes the contestants final stakes, that is, $(\varepsilon_1, \varepsilon_2) = (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$. These total efforts are larger than or equal to those obtained under any Tullock-type lottery with $0 < \alpha \leq 2$, Mealem and Nitzan (2014a) (case 5 in Table 1).

When the designer applies just head starts-discrimination, Franke et al. (2014b) have shown that the total efforts of N contestants, $N \geq 2$, are larger in an APA than in a simple lottery (with no covert discrimination, $\alpha = 1$). For two contestants the explanation of this result is as follows (for a generalization, see Li and Yu (2012) and the discussion following the proof of Proposition 3.1 in Franke et al. (2014b)): When the designer chooses $\delta_2^* - \delta_1^* = n_1 - n_2$ he transfers to his own control all the expected benefit of contestant 1, which is obtained when there is no head starts-discrimination. In this case, $G_A = \frac{n_2^2}{2n_1} + n_1 - \frac{n_2}{2}$. For more than two contestants, the expected efforts can be even higher (see case 6 in Table 1).

Table 1: Application of one type of discrimination

Type of environment	**Optimal discrimination and the corresponding efforts**	**Reference**
Case 1: Overt discrimination ($0 < \alpha \le 2$ is given)	$\delta = k$ (in this case $p_1 = p_2 = 0.5$, $x_i^* = 0.25\alpha n_i$, $G = 0.25\alpha(n_1 + n_2)$)	Appendix B in Epstein et al. (2013)
Case 2: Restricted overt discrimination ($\alpha \le 1$ or $\alpha = \infty$ are given)	If $\alpha \le 1$ then $\delta = k^{\alpha}$ (in this case $p_1 = p_2 = 0.5$, $x_i^* = 0.25\alpha n_i$, $G = 0.25\alpha(n_1 + n_2)$). If $\alpha = \infty$ then $\delta = k$ (in this case, $x_i^* = 0.5 n_i$, $G = 0.5(n_1 + n_2)$).	Propositions 1, 2 and 5 in Epstein et al. (2011)
Case 3: Restricted overt discrimination, N contestants, where $n_1 \ge n_2 \ge ... \ge n_N$ ($\alpha = 1$ or $\alpha = \infty$ are given)	If $\alpha = 1$ then $G_L = 0.25\left[\sum_{j \in K^*} n_j - \frac{(k^* - 2)^2}{\sum_{j \in K^*} 1/n_j}\right]$ where $K^* = \left\{ i \in N \middle\vert \frac{k^* - 2}{n_i} < \sum_{j \in K^*} 1/n_j \right\}$ (see Proposition 4.1 in Franke et al. (2014a)). If $\alpha = \infty$ then $G_A \ge 0.5(n_1 + n_2)$ (see Theorem 3.2 in Franke et al. (2014a)) $G_A > G_L$ (see Theorem 4.3 in Franke et al. (2014a)).	Proposition 4.1 and Theorems 3.2 and 4.3 in Franke et al. (2014a)
Case 4: Covert discrimination where $0 < \alpha \le 2$ ($\delta = 1$ is given)	If $k = 1$ then $\alpha = 2$ and $G = G_A$. If $k > 1$ then $\alpha = \alpha(k) < 2$ and $G > G_A$ ($G_A = \frac{n_2(n_1 + n_2)}{2n_1}$, as is well known, e.g., see Konrad (2009))	Proposition 1 in Epstein et al. (2013)
Case 5: Direct discrimination with	If $k = 1$, then $\varepsilon_1 = \varepsilon_2 = 0$ and $G = 0.25\alpha(n_1 + n_2) = 0.5\alpha n$ where	Propositions 1-3 in Mealem and

balanced-budget constraint ($\delta = 1$ and $0 < \alpha \le 2$ or $\alpha = \infty$ are given)	$n_1 = n_2 = n$. If $k > 1$, then for $0 < \alpha < 2$, $\varepsilon_1 < 0 < \varepsilon_2$, $\varepsilon_1 + \varepsilon_2 > 0$ and $0.25\alpha(n_1 + n_2) < G < 0.5(n_1 + n_2)$. For $\alpha = 2$ ($p_1 = p_2 = 0.5$) or $\alpha = \infty$ $(\varepsilon_1, \varepsilon_2) = (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$, $x_i^* = 0.25(n_1 + n_2)$, $G = 0.5(n_1 + n_2)$.	Nitzan (2014a)
Case 6: Head starts-discrimination, N contestants, where $n_1 \ge n_2 \ge ... \ge n_N$ ($\alpha = 1$ or $\alpha = \infty$ are given)	If $\alpha = 1$, then $G_L = \max\left\{\frac{n_1}{4}, \frac{k_0 - 1}{\sum_{i \in K_0} 1/n_i}\right\}$ where $K_0 = \left\{ j \in N \middle| \frac{j-1}{n_j} < \sum_{i=1}^{j} 1/n_i \right\}$ (see Proposition 4.7 in Franke et al. (2014b)). If $\alpha = \infty$ then $G_A \ge \max\left\{ \frac{n_2^2}{2n_1} + n_1 - \frac{n_2}{2}, \frac{n_N^2}{2n_1} + n_1 - \frac{n_N}{2} \right\}$ (see Proposition 3.1 in Franke et al. (2014b)). $G_A > G_L$ (see Proposition 5.2 in Franke et al. (2014b)).	Propositions 3.1, 4.7 and 5.2 in Franke et al. (2014b)

In light of the existing results in the contest literature, Franke (2012), Lien (1990), one may intuitively expect that equalization of stakes is *always* the optimal strategy for a revenue-maximizing contest designer. At first glance, such expectation is plausible because equal stakes imply maximal competition that apparently induces the largest efforts. It turns out that when the designer applies just direct discrimination the fulfillment of this expectation under an APA is the exception rather than the rule. The extreme nature of the APA results in an extreme optimal taxation scheme. In contrast, optimal taxation under the widely studied lottery CSFs proposed by Tullock (1980) (see equation 1 where $\delta = 1$), is not extreme; it reduces the gap between the contestants' stakes, but does not eliminate it, as established in Mealem and Nitzan (2014a, Proposition 2). That is, when $k > 1$, *the optimal taxation scheme under any Tullock- type lottery with* $0 < \alpha < 2$ *does not equalize the contestants' final stakes, but preserves their relative magnitude. That is,* $(\varepsilon_1, \varepsilon_2) \neq (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$ *and* $\varepsilon_1 + \varepsilon_2 > 0$. This means that if the contestants' initial stakes are different, then a designer who chooses a taxation scheme subject to a balanced-budget constraint does not have an incentive to eliminate the gap between the contestants' prize valuations and the reduced initially higher stake is still larger than the increased initially lower stake.[13]

It seems to us that the result rationalizing the preserved asymmetry in the contest has significant implications in public economics. In particular, it can be used to explain why contingent taxation of the prize won in a lottery contest between two lobbyists representing two interest groups, such as the "rich" and the "poor" or "consumers" and a "monopoly", tends to preserve the initial ex-ante inequality between the interest groups represented by the lobbyists. In general, efficient contest design requires that the natural existing variety between the contestants' prize valuations should be counterbalanced by direct discrimination. That is, it necessitates reduction in the initial stake heterogeneity, but usually not its elimination. The exception is the extreme uniformity required under an APA. Variety is natural initially and usually also finally (after the application of optimal direct discrimination).

6. Pair-wise comparison of one-mode partiality

The benchmark of this section is the symmetric situation where $\alpha = \delta = 1$, $\beta = 0$ and $\varepsilon_1 = \varepsilon_2 = 0$. This case has been justified in several studies. Starting with this symmetric benchmark, our aim is to examine the designer's behavior, given that he can apply only a restricted, one-mode discrimination. That is, he can discriminate just through the control of one of the parameters α ($0 < \alpha \leq 2$ or $\alpha = \infty$), δ, $\delta > 0$, $(\varepsilon_1, \varepsilon_2)$ ($p_1\varepsilon_1 + p_2\varepsilon_2 = 0$) or β, having the option of selecting one of two

[13] For further discussion of the reason for this policy, see Case 8 in Table 2.

parameters. The question is which parameter the designer will prefer to control when facing the possible pair-wise comparisons of two parameters. We specify in Table 2 the conditions that give rise to overt or covert discrimination, overt or direct discrimination, direct or covert discrimination and head starts or overt discrimination.

Comparing overt and covert discrimination, the preferred type of discrimination depends on *k*, the contestants' relative stakes (Case 7 in Table 2). Specifically, overt (covert) discrimination is expected when *k* is sufficiently large (small), see Proposition 1 in Mealem and Nitzan (2012b). Let us explain the intuition behind this result for a relatively small *k*. When *k* is relatively small, already in the benchmark situation there is a relatively intense competition between the contestants (recall that $k \geq 1$) and, consequently, their efforts are relatively large. Choosing $\alpha = 1$ and setting $\delta = k$ (Case 1 in Table 1, where $\alpha = 1$) does not result in a significant increase in the intensity of the competition and, in turn, in the exerted efforts despite the fact that in this case the designer increases the intensity of competition to its maximal level. In contrast, choosing $\delta = 1$ and setting a relatively high degree of covert discrimination $\alpha = \alpha(k)$ (Case 4 in Table 1) increases the intensity of competition and induces a more substantial increase in the contestants' efforts. The reason is that in this case, if *k* is sufficiently low, $\alpha(k)$ gets closer to 2. The constraint on a further increase in $\alpha(k)$ is due to the requirement of keeping contestant 2 in the "game" by ensuring that his utility does not become negative. Since $\alpha(k)$ is relatively large, in equilibrium there is a relatively high bias in favor of the contestant who exerts the larger effort. Both of the contestants, being aware of the potential advantage (the stronger effect of effort on the probability of winning) of a more active contestant, have an incentive to make large efforts. Consequently, the equilibrium aggregate efforts are larger than those obtained when $\delta = k$.

Table 2: Pair-wise comparison between different modes of discrimination

Type of environment	**Optimal discrimination and the corresponding efforts**	**Reference**
Case 7: Comparison between overt and covert discrimination - if overt discrimination is chosen,	$\delta = k$ and $\alpha = 1$ (Case 1 in Table 1, where $\alpha = 1$), if $k > 3^{0.75}$.	Proposition 1 in Mealem and Nitzan (2012b)

then $\alpha = 1$ is given, and if covert discrimination is chosen, then $\delta = 1$ is given	$\delta = 1$ and $\alpha = \alpha(k) > 1\frac{1}{3}$ (Case 4 in Table 1), if $1 < k < 3^{0.75}$.	
Case 8: Comparison between overt and direct discrimination ($0 < \alpha < 2$ is given)	As in case 1 in Table 1, with overt discrimination we get $G = 0.25\alpha(n_1 + n_2)$. With direct discrimination, total efforts are larger, but smaller than $G = 0.5(n_1 + n_2)$.	Overt discrimination is studied in Epstein et al. (2013), Appendix B. Direct discrimination is studied in Mealem and Nitzan (2014a) (see Proposition 2).
Case 9: Comparison between covert and direct discrimination	Direct discrimination yields larger (smaller) efforts than covert discrimination for high (low) values of *k*.	Covert discrimination - Proposition 1 in Epstein et al. (2013). Direct discrimination - Propositions 1-3 in Mealem and Nitzan (2014a).
Case 10: Comparison between head starts and overt discrimination, *N* contestants, where $n_1 \geq n_2 \geq ... \geq n_N$ ($\alpha = 1$ is given)	Overt discrimination yields at least the same efforts as head starts-discrimination. If $n_1 > n_2$, then overt discrimination yields larger efforts than head starts-discrimination.	Propositions 5.2 in Franke et al. (2014b)
Case 11: Comparison between head starts and overt discrimination, *N* contestants, where $n_1 \geq n_2 \geq ... \geq n_N$ ($\alpha = \infty$ is given)	Head starts-discrimination yields at least the same efforts as overt discrimination. If $n_1 > n_2$, then head starts-discrimination yields larger efforts than overt discrimination.	Propositions 5.2 in Franke et al. (2014b)

Comparing overt to direct discrimination, it turns out that for every α, $0 < \alpha < 2$ (and therefore, in particular for $\alpha = 1$), direct discrimination is superior to overt discrimination (Case 8 in Table 2). This result is due to the fact that applying only overt discrimination, the designer will secure complete symmetry between the contestants and, in turn, equal winning probabilities and total efforts that are equal to $G = 0.25\alpha(n_1 + n_2)$, see Appendix B, Epstein et al. (2013).[14] By Mealem and Nitzan (2014a), the application of direct discrimination could enable the designer to eliminate/neutralize the initial difference in the contestants' stakes and thus increase the intensity of competition and attain efforts that are equal to those obtained under optimal overt discrimination. But the designer can almost always (for $0 < \alpha < 2$) increase further the contestants' efforts - the move from equalized stakes to a situation where the gap between the contestants' stakes is reduced but not eliminated enables the designer to further increase the contestants' efforts by fully taking advantage of the potential "income effect" associated with a scheme that increases the sum of the final stakes. This positive income effect dominates the negative effect on total efforts due to the reduced competition associated with the creation of a gap between the contestants' final stakes.

Comparing covert and direct discrimination, the preferred type of discrimination depends on *k*, the contestants' relative stakes (Case 9 in Table 2). Since direct discrimination is always preferred to overt discrimination, assuming that $\alpha = 1$, and for high values of *k*, the latter is superior to covert discrimination, we get that in these instances, direct discrimination is also preferred to covert discrimination. In contrast, when *k* is sufficiently small, covert discrimination can be superior to direct discrimination. For example, when $k \to 1$, that is, $n_1 \to n_2$, under covert discrimination, $\alpha \to 2$ and, therefore, total efforts converge to the average prize valuation (to the stake of one of the contestants). However, under direct discrimination, when $k \to 1$, total efforts converge to one half of the average stake. Clearly then covert discrimination dominates direct discrimination.

Head starts-discrimination can be an effective instrument to induce higher contest efforts. However, its effectiveness depends on the contest success function. In

[14] Notice that the outcome of complete symmetry between the contestants is the equalization of the contestants' winning probabilities. Under overt discrimination, this result is reached despite the fact that the contestants' prize valuations differ, whereas under direct discrimination, this result is reached when the designer equates the contestants' prize valuations.

the case of an all-pay auction (Case 11 in Table 2) head starts-discrimination can always be designed such that the level playing field among the two active contestants is balanced reducing to zero the equilibrium payoff of the strongest contestant. This payoff-loss due to increased bidding increases the revenue of the contest designer even more than under the optimal overt discrimination (multiplicative bias). Hence, an all-pay auction with optimal head starts-discrimination does not only yield higher revenue than a symmetric all-pay auction without head starts, but also larger revenue relative to the optimal overt discrimination under an all-pay auction or a simple lottery ($\alpha = 1$). Under a simple lottery, however, head starts-discrimination is ineffective in generating additional revenue (Case 10 in Table 2). Active contestants who receive a head starts reduce effort to the same degree as the amount of the received head starts; from their point of view, head starts are perfect substitutes for own effort. As the ’effective’ effort (effort plus head starts) of directly affected contestants remains constant, non-affected contestants will not change their strategic response either. Total effort under head starts-discrimination tends to be lower such that no head starts-discrimination is the preferred option. The only possible exception is the case where exactly one contestant remains active having to compete against the sum of head starts granted to all other non-active contestants. In this situation, head starts-discrimination might in some cases yield a larger revenue than no head starts-discrimination. However, its potential to generate additional revenue is severely limited due to the fact that all other contestants refrain from exerting any effort. To sum up, under a simple lottery, optimal overt discrimination is superior to head starts-discrimination.

7. Two-mode discrimination

In this section we examine environments that allow the use of two-mode discrimination. As shown in Epstein et al. (2013) and Mealem and Nitzan (2014a), if the designer cannot combine direct and overt discrimination, but can combine each of these modes of discrimination with covert discrimination, then he can attain maximal efforts that are equal to the average prize valuation, $G = 0.5(n_1 + n_2)$, see cases 12 and 13 in Table 3. These efforts are obtained either by selecting the optimal lottery where $0 < \alpha \leq 2$ or the APA where $\alpha = \infty$. In the former case the maximal efforts are obtained in a pure-strategy equilibrium of the contest where the optimal α is $\alpha = 2$.

In the latter case, the efforts are obtained in a mixed-strategy equilibrium of the contest. The optimal overt discrimination in Case 12 requires that δ is equal to *k*, which accomplishes the designer's attempt to maximize the extent of competition between the contestants. We suggest the following intuition (see footnote 8 in Epstein et al. (2013)) regarding the equilibrium (in pure strategies) outcome and its sensitivity to α ($0 < \alpha \leq 2$). The designer tends to support the contestant with the lower prize valuation ($\delta^* = k$) to attain complete "balance" between the contestants' wining probabilities (ensure that $p_i = 0.5$), for any given α, $0 < \alpha \leq 2$. An increase in α induces the contestants to increase their effort at the same rate in order to increase their winning probability. Since at the same time the designer favorably discriminates the contestant with the lower prize valuation, the equilibrium winning probability is unchanged, $p_i = 0.5$. But by raising α the designer increases the aggregate efforts, and in turn his payoff. The designer therefore prefers the highest possible α, $\alpha = 2$, that enables him to extract the maximal possible surplus from the contestants which reduces their net payoff to zero. As mentioned above, the corresponding value of the designer's objective function is $G = 0.5(n_1 + n_2)$.

Table 3: Combinations of two types of discrimination

Type of environment	**Optimal discrimination and the corresponding efforts**	**Reference**
Case 12: Overt and covert	$\delta = k$ and $\alpha = 2$ ($p_1 = p_2 = 0.5$) or $\delta = k$ and $\alpha = \infty$ (in this case, $x_i^* = 0.5n_i$, $G = 0.5(n_1 + n_2)$)	Proposition 1 in Epstein et al. (2011) and Proposition 2 in Epstein et al. (2013)
Case 13: Direct discrimination with balanced-budget constraint and covert discrimination ($\delta = 1$ is given)	$\alpha = 2$ ($p_1 = p_2 = 0.5$) or $\alpha = \infty$ $(\varepsilon_1, \varepsilon_2) = (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$ (in this case, $x_i^* = 0.25(n_1 + n_2)$, $G = 0.5(n_1 + n_2)$)	Propositions 1 and 3 in Mealem and Nitzan (2014a)

8. Pair-wise comparison of two-mode discrimination

Does larger flexibility that allows the designer to use two modes of discrimination rather than one mode induce larger contestants' efforts? For the two cases in Table 3, the answer to this question is positive and the reason has to do with the distinctive features of the separate application of the alternative modes of discrimination. This means that when the designer applies two modes of discrimination, each type of discrimination has a positive "added value" that enhances the exertion of efforts relative to the situation where the designer resorts to just one mode of discrimination. That is, two modes of discrimination are supportive or "complementing" as clarified below.

1. The combination of overt and covert discrimination yields larger efforts than those obtained by separate application of covert discrimination and even by any combined application of overt and covert discrimination, as long as the latter discrimination is restricted to the range $0 < \alpha < 2$. The designer's optimal strategy of this two-mode overt-covert discrimination is to equalize the contestants' prize valuations by applying overt discrimination while choosing the extreme mode of covert discrimination, that is, $\alpha = 2$. This optimal strategy induces efforts that are equal to the average initial prize valuation. The contribution of the two modes of discrimination is the following:

- **(i)** Overt discrimination attains complete symmetry between the contestants and, in turn, equality between their winning probabilities, as in the case where overt discrimination is applied separately.
- **(ii)** Given the symmetry attained by the application of overt discrimination, the designer selects the maximal α, that is, $\alpha = 2$. The designer has an incentive to select this extreme form of covert discrimination because an increase in α increases the effect of a contestants' effort on his winning probability and this stimulates the total exerted efforts.

2. The combination of direct and covert discrimination yields larger efforts than those obtained by separate application of covert discrimination and even by any combined application of direct and covert discrimination, as long as the latter discrimination is restricted to the range $0 < \alpha < 2$. The designer's optimal strategy of this two-mode direct-covert discrimination is to equalize the contestants' prize valuations by applying direct discrimination while choosing the extreme mode of covert discrimination, that is, $\alpha = 2$. This optimal strategy induces efforts that are equal to the average initial prize valuation. To clarify this result, notice that the application of a non-extreme covert discrimination, that is, $0 < \alpha < 2$, creates a "constraint" that does not allow direct discrimination to eliminate the gap between the contestants' prize valuations. The reason is that the contestants' utility in equilibrium must be non-negative, and this restriction becomes less demanding as α is increased (up to the extreme value $\alpha = 2$). This means that an increase in α has, in fact, two effects on the total efforts.

(i) An increase in α increases the effect of effort on a contestant's winning probability and, therefore, it increases the incentive to exert efforts

(ii) The increase in α requires that the gap between the contestants' stakes is reduced via direct discrimination. The reduction in the gap between the contestants' prize valuations has two opposite effects on total efforts. On one hand, the reduced asymmetry between the contestants stimulates competition and, in turn, increases the exerted efforts. On the other hand, it reduces the sum of the contestants' prize valuations and, therefore, moderates the corresponding income effect on the exerted efforts.

It turns out that the two effects that increase efforts (the one described in (**i**) and the first effect described in (**ii**)) are stronger (together) than the moderating income effect that reduces efforts (the second effect described in (**ii**)). This explains why the designer has an incentive to increase covert discrimination as much as possible, that is, choose $\alpha = 2$ and, in this extreme case, the applied direct discrimination must equalize the contestants' prize valuations.

The comparison between these two-mode discrimination implies that the direct-covert and overt-covert modes of discrimination induce the same total efforts: the average initial prize valuations of the contestants. That is, these two two-mode discrimination are perfect substitutes.

Relating to the possibility of combining head starts with another mode of discrimination, Franke et al. (2014b) point out in their conclusion that a natural extension of their analysis would be to consider simultaneous head starts and overt discrimination. They conjecture that their results would also hold under this generalization. For the all-pay auction the lower bound derived in their Proposition 3.1 presumably cannot be further increased by allowing additional biases. Likewise under the (simple) lottery contest it might not be possible to improve on the revenue generated under the optimal bias by allowing additional head starts. Moreover, affine transformations of the considered type can potentially be generalized even further. For the homogeneous *N*-player and the heterogeneous two-player case Dasgupta and Nti (1998), and Nti (2004), show that a contest game involving any positive and increasing transformation is strategically equivalent to a contest game with an appropriately specified affine transformation as above. Whether their results can be extended to the heterogeneous *N*-player case is left for future research".

9. Conclusion

With the exception of head starts-discrimination in a simple lottery, in all other contests we have examined, reduction of the asymmetry between the contestants enhances competition and, in turn, the exerted efforts. To attain the maximal efforts, in some of the cases, the designer has to ensure complete symmetry by implementing a discrimination policy that equalizes the winning probabilities of the contestants. In other cases, he has to reduce the asymmetry, but not to eliminate it, such that the winning probability of the contestant with the higher stake is reduced, but is still larger than 0.5. The reduction in asymmetry can be obtained by changing the effect of the contestants' efforts on the winning probability, by changing the contestants' prize valuations or by combining these modes of discrimination. We have focused on four modes of discrimination. Three of these modes are intended to affect the parameters of the contest success function that yields the contestants' winning probabilities. Overt discrimination and head starts-discrimination allow different winning probabilities that correspond to equal efforts. Covert discrimination allows the ratio between the contestants' winning probabilities to differ from the ratio between their efforts. Under the fourth type of direct discrimination, the designer does not affect the contest success function, but changes the contestants' prize valuations via (positive and negative) transfer payments. Usually, such changes in the contestants' stakes involve some cost/benefit to the designer. A plausible requirement for direct discrimination is that this expected cost/benefit is bounded. We have therefore examined direct discrimination which is subjected to a balanced-budget constraint, see the examples in section 3.1.

The objective of the designer is to maximize the exerted efforts of the contestants (his revenue). Starting with the benchmark situation where $\alpha = \delta = 1$, $\beta = 0$ and $\varepsilon_1 = \varepsilon_2 = 0$, suppose that the designer can apply just a single mode of discrimination. That is, applying overt discrimination he can control just δ, exercising direct discrimination he can control $(\varepsilon_1, \varepsilon_2)$, ensuring that the balanced budget constraint is satisfied, implementing covert discrimination he can control α and implementing head starts-discrimination he can control β. Exploiting the heterogeneity between the contestants, and having the option of selecting one of two parameters as a means of increasing his revenue, let us summarize the results of the possible pair-wise comparisons.

Let us start with the comparison between overt and direct discrimination (Case 8 in Table 2). Under overt discrimination, complete symmetry is a necessary condition for effort maximization (Epstein et al. (2013)). Direct discrimination enables the designer to neutralize the initial difference in the contestants' stakes and thus increase the intensity of competition and, in turn, yield efforts that are equal to those obtained under overt discrimination. But starting from this situation, direct discrimination for any α, $0 < \alpha < 2$, enables a further increase in the exerted efforts (Mealem and Nitzan (2014a)).

Turning to the comparison between overt and covert discrimination (Case 7 in Table 2 - Mealem and Nitzan (2012b)), notice that when *k* is relatively small, already in the benchmark situation there is a relatively intense competition between the contestants and, consequently, their efforts are relatively large. Choosing optimal overt discrimination ($\delta = k$ and $\alpha = 1$) does not result in considerable increase in the intensity of the competition and, in turn, in the exerted efforts, despite the fact that in this case the designer increases the intensity of competition to its maximal level. In contrast, setting a relatively high degree of covert discrimination[15] (high α and $\delta = 1$) increases the intensity of competition and induces a more substantial increase in the contestants' efforts.

The comparison of covert and direct discrimination (Case 9 in Table 2), implies that the preferred type of discrimination depends on *k*, the contestants' relative

[15] This will occur for $1 \le k < 3^{0.75}$ when the optimal α, $\alpha > 1\frac{1}{3}$, satisfies $(\alpha - 1)k^{\alpha} = 1$, see Mealem and Nitzan (2012b).

stakes. Direct discrimination yields larger (smaller) efforts than covert discrimination for high (low) values of *k* (Mealem and Nitzan (2012b) and Epstein et al. (2013)).

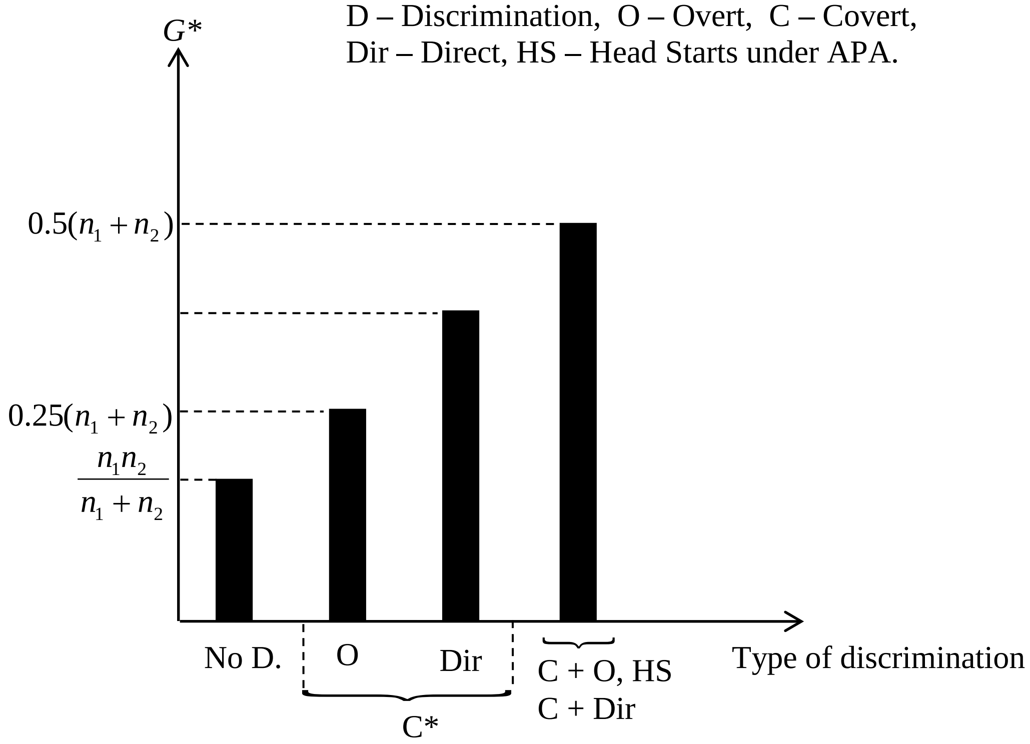
Comparison between head starts and overt discrimination in a simple lottery where $\alpha = 1$ (Case 10 in Table 2) shows that overt discrimination yields efforts that are at least equal to those obtained under head starts-discrimination. The intuition behind this finding has been presented in Franke et al. (2014b), see last paragraph of Section 6.

Comparison between head starts and overt discrimination in an all-pay auction where $\alpha = \infty$ (Case 11 in Table 2) shows that optimal head starts-discrimination yields efforts that are at least equal to those obtained under overt discrimination (again see last paragraph of Section 6).

The conclusion emerging from the above comparisons is that if the designer can choose just a single mode of discrimination, he would never select overt discrimination, because it is dominated by direct discrimination. If asymmetry between the contestants' stakes is sufficiently high, direct discrimination is also superior to covert discrimination. However, if the contestants' prize valuations are sufficiently symmetric, covert discrimination is preferred to direct discrimination.

Does larger flexibility that allows the designer to use two modes of discrimination rather than one mode induce larger contestants' efforts? As we have seen in Section 8, the answer to this question for the two cases, 12 and 13, in Table 3 is positive. The combinations of overt-covert and direct-covert modes of discrimination induce the same efforts: the contestants' average initial prize valuations.

Figure 1: Performance summary of the alternative discrimination strategies for two contestants



*If $k > 3^{0.75}$ ($k \to 1$) direct/overt (covert) discrimination is superior to covert (direct/overt) discrimination.

The relationship between total efforts under no discrimination, one-mode discrimination and two-mode discrimination is summarized in Figure 1. Of the seven possibilities presented in the figure, the covert-direct discrimination strategy, the covert-overt strategy and head starts-strategy (the latter case is for $\alpha = \infty$) are equivalent; they yield the same revenue.

Finally, let us present the conclusions regarding the comparison between the effectiveness of the two most widely studied CSFs: the extremely competitive all-pay auctions and the considerably less competitive lotteries. Fang (2002) showed that with asymmetric prize valuations, $k > 1$, and a fair and simple contest ($\delta = 1$ and $\alpha = 1$), the equilibrium efforts under the prototypical lottery are larger than those obtained by a fair APA, provided that the gap between the contestants' stakes is sufficiently large ($k > 1 + \sqrt{2}$). The intuition behind this result is due to the contrasting effects of the nature of these CSFs and of heterogeneity in the contestants' stakes under these CSFs on the competitive drive of the contestants. On the one hand, the (extremely) discriminating APA ignites more competition than the non-discriminating simple lottery CSF. On the other hand, a gap between the contestants' stakes significantly reduces the more competitive nature of an APA, but only moderately reduces the competitive drive under the simple lottery. Epstein et al. (2013) considerably strengthened this finding by allowing the designer to control the exponent α. They have shown that in a fair contest ($\delta = 1$), a designer who can select the exponent α (apply covert discrimination), *always* prefers a lottery because it yields larger efforts relative to the APA, even when Fang's sufficient condition *is not* satisfied (it can be shown that the exponent of the preferred lottery satisfies $0 < \alpha < 2$, which means that the corresponding contest game has a unique pure-strategy equilibrium). The reason is that the adjustment of α enables the attainment of the largest degree of competitiveness taking into account the above two contrasting effects (for a more detailed explanation, see the discussion of case 4 in Table 1 in Section 5).

Another factor that affects the extent of competition and, in turn, the contestants' efforts, is the designer's control of δ, that is, the application of overt discrimination. Epstein et al. (2011) have shown that with overt discrimination, the optimal APA is the preferred CSF (it yields larger efforts than the optimal lottery) as long as $0 < \alpha \leq 1$. For $\alpha = 1$ and any number of contestants, this result remains valid, as recently shown by Franke et al. (2014a). Overt discrimination enables the attainment of maximal competitiveness because it allows the reliance on the extreme discriminating nature of the APA as well as complete leveling of the playing field. The possible stimulating effect of an increased α under the lottery CSF is neutralized by the constraint $0 < \alpha \leq 1$. In the multi-player game, although there is more competition in the optimal simple lottery case because more than two players take an active part in the contest, under the optimal APA, the very tough competition between the two contestants with the highest stakes is the dominant effect. If the stimulating effect of increased α is not neutralized and the designer can choose any $0 < \alpha \leq 2$, then the APA and the lottery with $\alpha = 2$ yield the same revenue, Epsten et al. (2013). When $0 < \alpha < 2$, the superiority of the APA is preserved when overt discrimination is replaced by direct discrimination, Mealem and Nitzan (2014a). Lastly, when the designer can apply head starts-discrimination with any number of contestants, the APA yields larger efforts than the optimal fair and simple lottery. The results of the comparison between the effectiveness of the APA and the relevant lottery in the seven possible cases we have examined are summarized in Table 4.

Table 4: Comparison between the effectiveness of an APA and a lottery

Type of environment	The preferred CSF	Reference
Case 1: Two contestants, $\alpha = 1$ and $\delta = 1$.	APA yields larger efforts than the fair and simple lottery if and only if $k < 1 + \sqrt{2}$.	Fang (2002)
Case 2: Two contestants, $\delta = 1$ and the designer can apply covert discrimination in the lottery case; choose α such that $0 < \alpha \leq 2$.	The optimal fair lottery yields larger efforts than an APA.	Epstein et al. (2013)
Case 3: Two contestants, the designer can apply overt discrimination (choose δ) and, in the lottery case, $0 < \alpha \leq 1$.	The optimal APA yields larger efforts than the optimal lottery.	Epstein et al. (2011)
Case 4: Any number of contestants, the designer can apply overt discrimination (choose δ) and, in the lottery case, $\alpha = 1$.	The optimal APA yields larger efforts than the optimal simple lottery.	Franke et al. (2014a)
Case 5: Two contestants, the designer can apply overt discrimination (choose δ) and, in the lottery case, apply covert discrimination (choose α) such that, $0 < \alpha \leq 2$.	The optimal APA yields the same efforts as the optimal lottery.	Epstein et al. (2013)
Case 6: Two contestants, $\delta = 1$, the designer can apply direct discrimination subject to a balanced-budget constraint, and, in the lottery case, $0 < \alpha \leq 2$.	For $0 < \alpha < 2$, the optimal APA yields larger efforts than the optimal fair lottery. For $\alpha = 2$ the optimal APA and the optimal fair lottery yield the same efforts.	Mealem and Nitzan (2014a)
Case 7: Any number of contestants, the designer can apply head starts-discrimination and, in the lottery case, $\alpha = 1$.	The optimal APA yields larger efforts than the optimal fair and simple lottery.	Propositions 5.2 in Franke et al. (2014b)

10. Possible generalization

10.1 Multiple contestants

A potential interesting extension of our study is the analysis of the multiple-player case. Results for the all-pay auction should be robust with respect to the number of players because only two players will actively participate in equilibrium. Only few studies dealt with *N*-player contests assuming lotteries with asymmetric contestants. Stein (2002), Fang (2002), Franke (2012) and Franke et al. (2013, 2014a, 2014b), assumed, for *N*-players, that $\alpha = 1$, and Cornes and Hartley (2005) allowed any α. Stein (2002) extended the two-player contest to *N*-player contest and examined how changes in the contestants' prize valuations and in the measure of their prior relative chance of winning affect the equilibrium efforts. Fang (2002) compared the simple lottery model where $\alpha = 1$ and $\delta = 1$ to the All-Pay Auction and examined the conditions under which the total efforts corresponding to a lottery are larger than those corresponding to the All-Pay Auction. Franke (2012) compared these efforts under Affirmative Action (AA), where the designer affects the winning probabilities of the contestants (in our case, via the selection of δ) to the efforts obtained under Equal Treatment (ET). For two contestants, he extended his analysis to the case where $\alpha \leq 1$, but for *N* players he confined the analysis to $\alpha = 1$. For *N*-player contests, Franke et al. (2013, 2014a, 2014b) have recently allowed overt discrimination and head starts-discrimination, but still focusing on the simple lottery case ($\alpha = 1$). Franke et al. (2013) have shown that in this setting the designer will level the playing field by encouraging weak contestants, but he will not equalize the contestants' chances of winning the contest. Franke et al. (2014a, 2014b) have shown that the maximal efforts secured by the optimal APA are larger than those obtained by any lottery.

As pointed out in the introduction, for *N*-player contests, a broader view of discrimination can also cover the control by the designer of the group of contestants and, in particular, their number and their optimal order in a simultaneous or in a multi-stage sequential contest. For such an analysis under an APA, see Moldovanu and Sela (2006), Segev and Sela (2014a).

For *N* players, Cornes and Hartley (2005) proposed an elegant way to examine the existence of equilibrium for any α. Among other things, they have shown that, for $\alpha = 1$, there exists a unique equilibrium in pure strategies. But, for $\alpha > 1$, there is no explicit presentation of equilibrium and, in fact, multiple equilibria are possible, which precludes the possibility of conducting comparative statics, see footnote 24 in Franke (2012). This implies that, to attain consistency of the results, we can choose $\alpha > 1$, for two players or $\alpha = 1$ for any number of players. Recently, for *N* players and $\alpha = 1$ Ewerhart (2014) has shown that in any rent-seeking contest with independent and continuous types, there exists a unique pure-strategy Nash equilibrium. In our study the focus is mainly on two-player contests (for some cases our results are also valid for the general *N*-player case, see cases 3 and 6 in Table 1) and, therefore, we can compare overt and direct discrimination also for lotteries with $\alpha \leq 2$, despite our inability to compute explicitly the equilibrium outcome under direct discrimination. This case is also more general than the one examined by Franke (2012), since he assumed for 2 players that $\alpha \leq 1$. The challenging question what happens when we move to an *N*-player contest for any α, $0 < \alpha \leq 2$, (note that in our study we have dealt only with the case $\alpha = 1$) seems an especially demanding challenge and is left for future research.

10.2 Intermediate values of the decisiveness parameter ($2 \leq \alpha < \infty$)

For a thorough study of equilibrium efforts in contests based on a lottery with $2 \leq \alpha < \infty$, Alcalde and Dahm (2010) have shown that there exists an equilibrium in mixed strategies that is equivalent to the equilibrium of the APA. Ewerhart (2015) has recently studied this case characterizing the equilibrium (see Theorem 4.1 in his paper). He established that "In any symmetric equilibrium of the two-player rent-seeking game with $2 < \alpha < \infty$ the support of the distribution of expenditure levels has the zero bid as an accumulation point." Furthermore, he also claims that his result can be extended to obtain an explicit characterization of the all-pay-auction equilibria constructed by Alcade and Dahm (2010) in contests with multiple players with heterogeneous valuations, providing a sufficient condition for such a generalization (see the discussion in section 7 of his paper).

10.3 Generalized objective function: Beyond revenue maximization

Most of the literature on optimal contest design has focused on the choice of contest characteristics assuming that the designer's objective function depends on the contestants' efforts. Few attempts have been made to study the relationship between the designed contest and more general objective functions that take into account not only the efforts incurred by the contestants.[16] Examining the endogenous determination of public policy that determines, in turn, the contestants' prizes or stakes, Epstein and Nitzan (2002), (2006a), (2006b), (2007), allow the designer's objective function to depend on the efforts of the contestants and on their expected aggregate utility. The weights assigned to these variables represent the political culture of the contest designers. Epstein et al. (2011) have assumed, as in Epstein and Nitzan (2006b, 2007), that the objective function of the contest designer is a weighted average of the expected social welfare and lobbying efforts:[17]

(6) $$G(\cdot) = \gamma\left[E(u_1) + E(u_2)\right] + (1-\gamma)(x_1 + x_2)$$

where the parameters γ and $(1-\gamma)$ are the weights assigned to the expected social welfare and the contestants' lobbying outlays. These weights represent the political culture; the culture reflected by the designer's genuine objectives or the culture that imposes this objective function on the designer. The designer is assumed to maximize the objective function (6) by setting the CSF, given the Nash equilibrium efforts of the contestants. His particular choice of the CSF together with the corresponding efforts of the contestants constitute the equilibrium of the extended contest.

Focusing on deterministic all-pay-auctions and logit CSFs (where $0 < \alpha \leq 1$), Epstein et al. (2011) have specified the relationship between discrimination in contests and the prevailing political culture (the weights assigned to the expected aggregate utility of the contestants and to their total efforts) as well as the asymmetry in the contestants' prize valuations. Under the logit CSF, they have derived the conditions that determine whether the optimal bias is in favor of the contestant with the larger or smaller prize valuation. It turns out that bias in favor of the more motivated contestant is driven by the assignment of sufficiently large weight to the expected utility of the contestants. In such a case, the contest designer wishes to increase the winning probability of the contestant with the larger prize valuation. Such an increase is sufficient to positively affect the total expected utility of the contestants.

[16] Studying the endogenous determination of the optimal prize, Runkel (2006) and Singh and Wittman (1998) consider a designer's payoff function that depends on the performance of the contestants and on the difference in their winning probabilities (the closeness of the contest). This difference represents the uncertainty of the contest outcome that affects the interest it arouses and, in turn, the size of the contest audience. Dasgupta and Nti (1998) consider the endogenous determination of the contest success function, assuming that the designer's objective function depends on aggregate efforts and on his own valuation of the prize that may induce him not to award the prize.

[17] As commonly assumed in the political economy literature, Grossman and Helpman (2001), Persson and Tabellini (2000).

Bias in favor of the contestant with the lower motivation is due to the assignment of sufficiently large weight to the contestants' efforts. In such a case, the contest designer wishes to equalize the "strength" of the contestants and increase the extent of competition in order to induce the contestants to make larger efforts. Under a logit CSF, the bias in favor of the contestant with the higher prize valuation is increasing in the weight assigned to the expected utility of the contestants. Under an all-pay-auction, since the equilibrium bias can take only two values (*k* or 0), the bias in favor of the more motivated contestant is almost always invariant to a change in the weight assigned to the contestants' expected utility. The effect of valuation asymmetry on the optimal bias is ambiguous. The bias in favor of the more motivated contestant is decreasing (non-decreasing) in valuation asymmetry provided that the weight assigned to the expected utility of the contestants is sufficiently small (large). Finally, in this setting, an all-pay auction is always preferred to a logit CSF from the point of view of the contest designer, provided that the logit CSF is of decreasing or constant returns to scale.

10.4 The unconstrained case

In the unconstrained case, where any mode of discrimination is possible, the designer can choose α, δ, β and $(\varepsilon_1, \varepsilon_2)$, such that $0 < \alpha \leq 2$, $\delta > 0$ and $p_1\varepsilon_1 + p_2\varepsilon_2 = 0$. The maximal flexibility the designer enjoys in the selection of the discrimination parameters enables him to increase the contestants' efforts almost to the higher prize valuation of contestant 1, n_1. This can be attained by giving up the possibility of applying covert discrimination, that is, select $\alpha = 1$, $\beta = 0$ and by resorting to direct and overt discrimination, setting $\varepsilon_1 \to -n_1^+$, $\varepsilon_2 \to \infty$ and $\delta = \left(\frac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2}\right)\left(-\frac{\varepsilon_1}{\varepsilon_2}\right)^{\frac{1}{\alpha}}$, Mealem and Nitzan (2012a). In this case the winning probability of contestant 1 or his prize share converges to 1, but his effort converges to zero and the winning probability or the prize share of contestant 2 converges to zero, but his effort converges to n_1. Total efforts therefore converge to n_1, $E(u_1) \to 0$ and $E(u_2) \to 0$. Even-though the designer can secure efforts that are almost equal to the highest prize valuation, we have not included this case among the other cases because of the difficulty to formally establish the equilibrium nature of the designer's strategy yielding this outcome.

Alternative mechanisms based on the "take it or leave it" principle can yield maximal efforts that are equal to the highest prize valuation, see Proposition 2 part B in Nti (2004). In one simple version of this mechanism, the designer introduces a minimal effort requirement which is equal to the highest prize valuation. This enables participation of the individual with the highest valuation and precludes participation of the other individuals.

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