Contest effort

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Abstract

This chapter compares "effort" with contest success functions of the Tullock logit-lottery type and the all-pay-auction. We describe the circumstances in which combinations of the parameters in the general contest give rise to greater effort. Our presentation encompasses in principle two types of contests. In rent-seeking contests, the social objective is to minimize effort because effort is used unproductively. In the contest-design literature, the objective of the designer of the contest can be to maximize effort because the effort is privately beneficial to the contest designer, as when "effort" takes the form of a monetary bribe and the objective of the designer of the contest is to maximize bribe revenue. The relation to rent seeking in that case is that the bribes become rents to be contested.

1. Introduction

Bribes and rent seeking are related: bribes are monetary transfers that redistribute income without the necessity of efficiency losses but bribe revenue attracts rent seeking when the position of bribe recipient is contestable (Hillman and Katz 1987). A contest-design literature (Gradstein and Konrad 1999) has considered how a contest might be designed to maximize effort of contestants when the effort is beneficial to the contest designer, as in the case of bribes. We can view the bribes as rent creation and rent extraction (Hillman 2013).

In this chapter, we describe how choice of the contest-success function (CSF) affects the total "effort" in a contest or the value of bribes. We compare outcomes with the two most common contest-success functions, which are a Tullock logit-lottery type (Tullock 1980) and an all-pay auction or APA (Hillman and Samet 1987; Hillman and Riley 1989). We shall use the general terminology "effort" in expositing the models, where effort includes bribes made to influence the outcome of a contest.

Fang (2002) initiated the study of the comparison between efforts in contests based on the CSFs. He focused on a simple lottery and clarified the role of asymmetry between the contestants' valuations of prizes on the outcome of the comparison regarding social loss. In this chapter, we describe generalizations of the comparison between the contest-success functions allowing for asymmetry in valuations of prizes; any Tullock-type lottery; any overt structural bias among contestants (Lien 1986, 1990; Clark and Riis 2000; Epstein et. al. 2011, 2013; Nti 2004) and any pattern of differential prize taxation satisfying the balanced-budget constraint (Mealem and Nitzan 2012, 2014). In some of the cases, we also allow for any number of contestants N (Franke et al. 2013, 2014; Mealem and Nitzan 2012).

The conclusions of the comparison between a Tullock lottery and the APA depend on whether the lottery is simple or not; on the asymmetry between the contestants' valuations; on the number of the contestants; and on the form of the allowed bias between the contestants. The comprehensive clarification of the role of these parameters in effort evoked or bribe revenue provided in contests is the purpose of this chapter.

A detailed summary of the conclusions is presented in the final section. A less specific brief summary of the primary conclusions is presented below, assuming asymmetric prize valuations.

- In a fair (un-biased) two-player contest, the APA yields larger total efforts at persuasion than a *simple* lottery, if the contestants' prize valuations are not too asymmetric.

- In a fair two-player contest, the effort-maximizing lottery for the contest designer yields larger total efforts at persuasion than the APA.

- In a fair contest allowing overt discrimination as well as differential prize taxation of the prize subject to a balanced-budget constraint, the effort-maximizing lottery for the contest designer yields larger total efforts at persuasion than an APA, for any number of contestants.

- In an unfair contest allowing overt discrimination, but not differential prize taxation, the effort-maximizing *simple* lottery for the contest designer yields smaller total efforts at persuasion than the effort-maximizing APA, for any number of contestants.

- When overt discrimination is allowed, but differential prize taxation is not allowed, there is a lottery that yields certain total efforts that are equal to the expected total efforts under the effort-maximizing APA.

2. The extended contest and the CSFs

2.1. The setting

In the basic one-stage contest setting, there are two risk-neutral contestants, the high and low benefit contestants, 1 and 2.[1] The valuations of the prize of the contestants are denoted by v_i, $v_1 > v_2$ or $k = \dfrac{v_1}{v_2} > 1$. Assuming common knowledge of the contestants' prize valuations, we are concerned with the conditions that give rise to maximal contest effort, which is a rational objective for the contest designer who is the beneficiary of the efforts of contenders in maximizing the value of his position. Given the contestants' fixed prize valuations and the CSF, which is the function that specifies the contestants' winning probability given their efforts $\Pr_i(x_1, x_2)$, the expected net payoff (surplus) of contestant i is:

(1) $$E(u_i) = \Pr_i(x_1, x_2)v_i - x_i, \quad (i=1,2)$$

where $x_1 \geq 0$ and $x_2 \geq 0$ denote the contestants' efforts (if $x_1 = x_2 = 0$, then $\Pr_i = 0.5$). The expected efforts of the contestants in equilibrium are denoted by:

(2) $$G = E(x_1 + x_2)$$

That is, the function (2) specifies the Nash-equilibrium efforts of the contestants in the equilibrium of the contest where the applied CSF is $\Pr_i(x_1, x_2)$ and the payoff functions of the contestants are given by (1). We focus on the widely studied logit CSFs that include APAs and Tullock's lotteries.[2]

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[1] The general case of an N-player contest will be discussed later.

[2] In Epstein et al. (2011a), when the weight assigned to the expected welfare of the contestants is sufficiently high, in equilibrium there is no contest (and so no efforts are made) and the winner is the contestant with the higher valuation. In contrast, in our model, such an equilibrium cannot emerge because the weight assigned to the expected welfare of the contestants is zero. Therefore, any equilibrium is an interior one: there is real competition and "meaningful" winning (each contestant makes an effort with a positive winning probability).

2.2 Differential prize taxation under a balanced-budget constraint

Direct discrimination via differential taxation of the contested prize that affects the contestants' actual prize valuations, v_1 and v_2, is a pair of (positive or negative) amounts, ε_1 and ε_2 that changes the prize valuations to $(v_1 + \varepsilon_1)$ and $(v_2 + \varepsilon_2)$. A contest designer who applies such a taxation scheme must ensure that the transformed prize valuations are positive; otherwise the contestants will not voluntarily take part in the contest and the designer's revenue will be equal to zero. We also assume that the contest designer faces a balanced-budget constraint, that is, ε_1 and ε_2 must satisfy the requirement that the designer's expected expenditures are equal to zero, that is, $p_1\varepsilon_1 + p_2\varepsilon_2 = 0$.[3] This ex-ante balanced-budget constraint is reasonable when the designer is "risk neutral" in the sense that he does not mind facing an ex-post deficit situation after the outcome of the contest has been revealed. The balanced-budget constraint is more plausible when the designer controls a series of identical contests that are held during a fixed period (typically weekly, monthly or quarterly contests that are held during the budget year). In such a case, the designer tries to ensure that during the relevant period the net transfers between the contestants are cancelled out such that his budget is balanced.

2.2 All-pay auctions

In our setting, winning with certainty means that the CSF leaves no residual winning uncertainty after the revelation of the contestants' efforts. In such a case, the certain winner is the contestant who makes the largest effective effort, where a unit of effort by one contestant is not necessarily equally effective as a unit of effort of his rival, as first suggested in the context of a bribery game by Lien (1986), (1990) and later by Clark and Riis (2000). That is, the CSF for $\delta > 0$ is an APA given by:

$$(3) \qquad p_1(x_1, x_2) = \begin{cases} 1 & \text{if} & x_1 > \delta x_2 \\ 0.5 & \text{if} & x_1 = \delta x_2 \\ 0 & \text{if} & x_1 < \delta x_2 \end{cases}$$

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[3] The possibility of a balanced-budget constraint faced by the contest designer was first considered in Mealem and Nitzan (2012, 2014). The possibility of caps on the contestants' efforts has been examined, for example, by Che and Gale (1998) and Ujhelyi (2009).

and for $\delta = 0$, $p_1(x_1, x_2) = 1$. By (3), a reduction in δ increases the overt structural bias in favor of the more motivated contestant 1. Furthermore, $0 \leq \delta < 1$ implies a bias in favor of contestant 1. When $\delta = 1$ the contest is fair, there is no bias. When $\delta > 1$ the bias is in favor of contestant 2.[4]

2.3 The logit lotteries

When winning is uncertain, the CSF leaves some residual winning uncertainty after the revelation of the contestants' efforts. In such a case, in equilibrium every contestant has some positive winning probability. Sufficiently large investment of effort can secure a high probability of winning, but not certain winning.

Our analysis is confined to the well studied logit Tullock-type lotteries. For $\delta > 0$, these lotteries are given by:[5]

(4)
$$p_1(x_1, x_2) = \frac{x_1^{\alpha}}{x_1^{\alpha} + (\delta x_2)^{\alpha}}$$

$0 < \alpha < \infty$ and for $\delta = 0$, $p_1(x_1, x_2) = 1$, where α and δ are selected by the contest designer. The interpretation of δ is as in sub-section (c).

3. Unbiased APA vs. the simple unbiased lottery

With asymmetric prize valuations, $k > 1$, and $\alpha = 1$, Fang (2002) (see section 5.2), has shown that a simple lottery can yield more effort than an APA. Specifically, in equilibrium, the total effort in a contest based on a simple lottery, $G_L = \frac{v_1 v_2}{v_1 + v_2}$, is larger

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[4] For alternative additive modeling of the bias, see Konrad (2002) and Li and Yu (2012).
[5] As is well known (see Konrad 2009 and references therein), when $0 < \alpha \leq 2$, the contest has a pure-strategy equilibrium. Since

$$p_1(x_1, x_2) = \frac{x_1^{\alpha}}{x_1^{\alpha} + (\delta x_2)^{\alpha}} = \frac{1}{1 + (\delta x_2 / x_1)^{\alpha}},$$

one can easily see that for $\alpha = \infty$ the logit lottery takes the form (3). The form in (4) is slightly different from the form of the logit lottery in Epstein et al. (2011) because in the present chapter, as will become clear, it is important to make a meaningful comparison between the degree of discrimination under the two types of CSFs. Such a comparison requires that discrimination is defined in a similar way in the two cases.

than the expected efforts in a contest based on an APA, $G_A = \dfrac{v_2(v_1 + v_2)}{2v_1}$,[6] provided that

the gap between the contestants' stakes is sufficiently large, that is, $k > 1 + \sqrt{2}$.[7] The intuitive explanation of this result is as follows: The intensity of the rent-seeking competition in the all-pay auction game is largely determined by the gap between the valuations of the contestants. As this gap becomes more substantial, the competition becomes weaker and the politician gains less in expected effort or revenue. In contrast, the politician's revenue from the lottery is less dependent on this gap. Furthermore,

since $G_A = \dfrac{v_2(v_1 + v_2)}{2v_1}$ is the expected revenue for the all-pay auction, while for the

lottery $G_L = \dfrac{v_1 v_2}{v_1 + v_2}$ is a certain revenue, assuming risk neutrality, the politician will

strictly prefer lottery even when $k = 1 + \sqrt{2}$. Undoubtedly, the significance of this result is that it paved the way for the comparison between the two prototypical CSFs that are sometimes called (see Hillman and Riley, 1989) the discriminating and the simple non-discriminating CSFs.

3. Unbiased APA vs. the unbiased lottery

As mentioned above, Fang (2002) has shown that with asymmetric prize valuations, a simple lottery can be more wasteful than an APA (yielding larger total efforts) provided that the gap between the contestants' stakes is sufficiently large. Epstein et. al. (2013) have considerably strengthened this finding by establishing that in a fair contest ($\delta = 1$

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[6] In equilibrium of the unbiased APA ($\delta = 1$) (Hillman and Riley 1989; Konrad 2009), the expected efforts are $x_1^* = 0.5v_2$ and $x_2^* = \dfrac{v_2^2}{2v_1}$. In turn, the expected net payoff of the player with the lower prize valuations is zero, namely, when $k > 1$, only the player with the higher prize valuations enjoys some surplus. The expected net payoff of the player with the higher prize valuations is equal to $E(u_1^*) = (v_1 - v_2)$. In equilibrium, the value of the designer's objective function (the expected aggregate efforts of the contestants in the mixed-strategy equilibrium) is equal to $G_A = \dfrac{v_2(v_1 + v_2)}{2v_1}$.

[7] This condition is derived from the general condition obtained by Fang (2002) for any number of contestants N. Matros and Duffy (2012) study a model of public-good provision assuming that the good is provided if the total amount of contributions made in a contest exceeds the common value of the public good. In this different setting, the authors present conditions for the superiority of the unbiased simple lottery relative to the unbiased APA (the good is supplied under the simple lottery but not under an APA).

), if $k > 1$, then a designer who can select the exponent α, *always* prefers a logit lottery because it yields larger efforts relative to the APA, even when the sufficient condition $k > 1 + \sqrt{2}$ *is not* satisfied. In fact, the exponent of the preferred lottery satisfies $0 < \alpha < 2$, which means that the corresponding contest game has a unique pure-strategy equilibrium.

The result of Epstein et. al. (2013) implies that, in a fair contest, if $k > 1$, then a Tullock-type lottery is always preferred to the APA for a risk neutral or a risk averse designer.[8] Hence, the lottery is socially more wasteful than the APA. Alcalde and Dahm (2010) have shown that for any $\alpha \geq 2$ there exists an equilibrium in mixed strategies that is equivalent to the equilibrium of the APA. However, so far a characterization of the complete set of mixed-strategy equilibria is not available. Since Epstein et. al. (2013) show that there exists α, $0 < \alpha < 2$, that yields efforts that are larger than those obtained under the APA, it is clear that this conclusion remains valid (at least partly) when the parameter α satisfies the requirement $0 < \alpha < \infty$.

4. APA vs. lottery with $\alpha \leq 1$: the case of endogenous δ

Epstein et al. (2011) have shown that, with overt discrimination, the effort-maximizing APA is the designer's preferred CSF (it yields larger efforts than the effort-maximizing lottery) as long as $0 < \alpha \leq 1$. For $\alpha = 1$ and any number of contestants, this result remains valid, as shown by Franke et al. (2014), in the general *N*-player contest. This contest has a two-stage structure: In the first stage the administrator decides on the specific personal prices that determine the winning probabilities or the weights assigned to the agents that might result in a biased contest rule. In the second stage, the contestants choose effort, taking as given the specified price/weight structure of the administrator and the efforts of opponents. Although the underlying two-stage game is highly stylized and simple, its analysis is rather complex: it requires a solution of a bilevel program or, more precisely, a mathematical program with equilibrium constraints.

5. APA vs. lottery with $0 < \alpha < \infty$: the case of endogenous δ and $N = 2$

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[8] This result differs from that obtained in Epstein and Nitzan (2006, section 4.2.2). In fact it corrects that result, which did not take into account the non-negativity constraints of the contestants' utilities.

In a Nash equilibrium of the contest, every contestant determines effort, given the parameters of the contest success function. In the unconstrained environment that allows overt structural discrimination, anticipating the effort of the contestants, the contest designer sets the parameters of the contest success functions such that efforts are maximized. When considering the APA, we take into account the degree of discrimination δ set by the designer. When considering the logit lottery, we take into account both δ and the exponent α selected by the designer. Note that, in a rent-seeking context, the preferred parameters for the designer are the socially most wasteful ones.

Most of the literature studying Tullock lotteries and the APA disregarded deliberate discrimination between the contestants and control of the exponent α. However, in the context of an APA, Lien (1986, 1990) and Clark and Riis (2000) studied a bribery game in which an official (designer of the contest) exercises overt structural discrimination in a multiplicative form. Nti (2004) examined the effect of the exponent α on the contestants' efforts in Tullock lotteries disregarding discrimination between the contestants and focusing on pure-strategy equilibria. He thus initiated the study of effort-maximizing contest design by control of the exponent α for Tullock CSFs. As noted, Epstein et al. (2011) presented a complete analysis of the designer's effect on contestants' efforts by control of the degree of discrimination both for the APA and the logit lotteries, assuming that in the lotteries' case the exponent α is given and restricted to the range $0 < \alpha \leq 1$. They thus re-shifted the emphasis in the study of effort-maximizing contest design to control through overt discrimination.

In this section the two approaches are combined allowing the control of both the exponent α and the degree of discrimination between the contestants. In addition, the analysis is generalized by allowing any value of the exponent α in the logit CSFs that gives rise to a Tullock-type lottery associated with a pure-strategy or a mixed-strategy equilibrium, $0 < \alpha < \infty$, or to an APA that is associated with a mixed-strategy equilibria where $\alpha = \infty$.[9]

5.1. The effort-maximizing APA

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[9] Note that under the logit CSF, where $0 < \alpha \leq 2$, the exerted efforts are certain whereas, under the APA, the meaning of effort is expected effort.

Under the APA, aggregate efforts (2) are maximized by determining the effort-maximizing value of δ. The effort-maximizing δ is equal to k (for a proof, see Epstein et al., 2011, proposition 1, assuming that no weight is assigned to the expected welfare of the contestants). This effort-maximizing bias maximizes the extent of competition between the contestants. In fact, such bias eliminates the advantage of contestant 1, creates actual equality between the competitors, and completely eliminates their surplus. The corresponding value of the expected aggregate efforts of the contestants in the mixed-strategy equilibrium of the contest is $G_A = 0.5(v_1 + v_2)$. Thus, in equilibrium, the expected total efforts are equal to the average prize valuation.

5.2 The effort-maximizing lottery

Under the logit contest success function, we take into account the effort-maximizing level of α, $0 < \alpha < \infty$, and δ, $\delta \geq 0$. Let us partition the range of the parameter α into $0 < \alpha \leq 2$ and $2 < \alpha < \infty$. For $0 < \alpha \leq 2$, where a unique pure-strategy equilibrium exists, the effort-maximizing values of δ and α are equal to $\delta^* = k$ and $\alpha^* = 2$ (see Epstein et al., 2013). The corresponding value of the designer's objective function (the certain aggregate efforts of the contestants in the pure-strategy equilibrium of the contest) is $G_L = 0.5(v_1 + v_2) = G_A$. We therefore obtain:

Proposition 1: *If we take into account the effort-maximizing values of both the degree of discrimination δ, $\delta \geq 0$, and the exponent α, then the maximal contestants' efforts under a lottery are larger than or equal to the expected efforts obtained in the mixed-strategy equilibrium of the effort-maximizing APA.*

The combined effect of the designer's preferred discrimination $\delta = k$ and his selection of the exponent α in the logit CSF

$$p_1(x_1, x_2) = \frac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha} = \frac{1}{1 + (\delta x_2 / x_1)^\alpha}$$

that gives rise to a pure-strategy equilibrium when $0 < \alpha \leq 2$, results in equal (expected) efforts and elimination of the surplus of both of the contestants. This result implies that, from the designer's viewpoint, the effort-maximizing logit lottery (that need not satisfy the constraint $0 < \alpha \leq 2$) cannot be dominated by the APA. In other words, in terms of

contested bribes, the endogenously selected lottery cannot be socially less wasteful than the endogenously selected APA.

Proposition 1 establishes that the intuition that a larger exponent α would induce greater effort is not valid. Under discrimination, even the use of an effort-maximizing APA that is maximally receptive to the contestants' efforts, would not induce greater efforts than those obtained when one is less receptive to the contestants' efforts, $\alpha^* = 2$, allowing random winning. Recall that, in both cases, bids are subject to effort-maximizing discrimination, the effort-maximizing degree of discrimination being equal to k. Nevertheless, the valid part of the above intuition is that a larger exponent α indeed induces greater efforts under the logit lottery provided that the exponent α gives rise to a pure-strategy equilibrium.

6. APA vs. lottery with $0 < \alpha \leq 2$: the case of $\delta = 1$, $N = 2$ and direct discrimination
If the designer faces a balanced-budget constraint, then his preferred taxation under the APA yields total efforts that are equal to the average of the initial stakes. Mealem and Nitzan (2014) have shown that these efforts are larger than those obtained under almost any Tullock-type lottery and, in any event, they are always larger than or equal to those obtained under any lottery.

Proposition 2: *The total efforts of the contestants corresponding to the designer's preferred taxation scheme under the APA are equal to the average prize valuation,* $G_A = 0.5(v_1 + v_2)$. *These total efforts are larger than or equal to those obtained under any Tullock-type lottery with* $0 < \alpha \leq 2$.

The maximal efforts under an APA can also be secured under a Tullock-type lottery with the exponent α being equal to 2. In other words, the maximal efforts under differential taxation can be attained in the mixed-strategy equilibrium of the extreme logit CSF where $\alpha = \infty$ (the APA) or in the pure-strategy equilibrium of the extreme logit CSF where $\alpha = 2$. Note that such equivalence has the flavor of the neutrality result obtained in Alcalde and Dahm (2010). However, in Mealem and Nitzan's (2012) setting of contest design, the contestants' maximal efforts are larger than those obtained in the setting of Alcalde and Dahm (2010) because of the allowed discrimination between the

contestants via the application of the designer's most effective scheme of differential taxation of the prize.

7. The case of any N and the effort-maximizing lottery (α, δ and direct discrimination)

Examining direct and overt discrimination, Mealem and Nitzan (2012) clarify the effectiveness of discrimination when it can take the form of both differential prize taxation and structural discrimination. The first claim in their study is:

Claim 1: *For $k > 1$ and $0 < \alpha < 2$, the combination of direct and overt discrimination yields larger efforts than those obtained just by direct discrimination.*[10]

The proof of this claim uses the following idea: the designer increases the polarization between the contestants' stakes by reducing the stake of contestant 1 and increasing the stake of contestant 2. The increase in polarization is associated with an increase in the sum of the contestants' prize valuations. But to enable the increased polarization, the balanced-budget constraint requires creating a structural bias in favor of contestant 1, the contestant whose stake has been reduced, by selecting δ that is smaller than 1. In the proof of the claim, the required reduction in δ results in the preservation of the contestants' winning probabilities while increasing the sum of their stakes, and this causes the increase in their exerted efforts. This idea raises the following question: what happens to the total efforts if the designer "maximizes" the extent of polarization between the contestants' stakes by reducing the stake of contestant 1 almost to zero ($\varepsilon_1 \to -v_1^+$) and by increasing the stake of contestant 2 to a "very large" level. Clearly, to ensure that the balanced-budget constraint is satisfied, the designer must create an appropriate bias in favor of contestant 1 by selecting a very small δ. That is $p_1\varepsilon_1 + p_2\varepsilon_2 = 0$ or

(5)
$$\delta = \left(\frac{v_1 + \varepsilon_1}{v_2 + \varepsilon_2} \right) \left(-\frac{\varepsilon_1}{\varepsilon_2} \right)^{\frac{1}{\alpha}}.$$

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[10] In section 4 of Mealem and Nitzan (2012), it is shown that direct discrimination is more effective than overt discrimination. This implies that the combination of direct and overt discrimination yields larger efforts than those obtained just by overt discrimination.

The two types of discrimination exerted in this case are somewhat different than those described in the proof of claim 1, because the designer does not confine himself to preserving the winning probabilities of the contestants. It turns out that, for $\alpha = 1$, this combined discrimination with maximal polarization, that is, $\varepsilon_1 \to -v_1^+$ and $\varepsilon_2 \to \infty$, is an effort-maximizing strategy yielding efforts that are almost equal to v_1, the initial higher prize valuation of contestant 1. For example, for $\alpha = 1$, $v_1 = 100$ and $v_2 = 2$, if the designer considerably increases the polarization between the contestants' stakes by selecting $(\varepsilon_1, \varepsilon_2) = (-99.9, 100000)$ and δ according to (3), $\delta \cong 9.9898 \cdot 10^{-10}$, the total efforts are equal to $G = 99.7$. This example illustrates the more general finding obtained in the first part of Proposition 1 presented in Mealem and Nitzan (2012). That is, when $0 < \alpha \leq 1$, combined discrimination with maximal stake polarization and selection of δ that satisfies (5) yields efforts that are almost equal to αv_1. When $\alpha = 1$ these efforts are equal to those obtained under the most effective "take it or leave it" mechanism. However, in our setting v_1 is almost obtained using a standard simple lottery that allows structural bias between the contestants, without setting a minimal effort for contestant 1, without disregard for the balanced-budget constraint and without deterring the participation of one of the contestants.

Undertaking the extreme combined discrimination that maximizes polarization, $\varepsilon_1 \to -v_1^+$ and $\varepsilon_2 \to \infty$, while choosing δ according to (5), such that the balanced-budget constraint is satisfied, is possible for $0 < \alpha \leq 1$. But it is not possible for $1 < \alpha \leq 2$. The reason is that the designer's selection of $(\varepsilon_1, \varepsilon_2, \delta)$ must ensure that the utility of the contestants is not negative, to prevent their abandonment of the competition and, in turn, the decline of the contestants' efforts to zero. In this case ($1 < \alpha \leq 2$), when $k > 1$, $\varepsilon_1 \to -v_1^+$, $\varepsilon_2 = \dfrac{\varepsilon_1}{1 - \alpha}$ and δ is set according to (5). The corresponding total efforts converge to $\dfrac{1}{\alpha} v_1 + \left(1 - \dfrac{1}{\alpha}\right) v_2$, which is smaller than v_1. This result appears in the second part of Proposition 1 in Mealem and Nitzan (2012).

The special appeal of the dual polarized discrimination strategies presented in the above discussion is highlighted by the following result.

Proposition 3: *For any $0 < \alpha \le 2$, the dual polarized discrimination strategies applied in the discussion above yield the maximal equilibrium total efforts of the contestants.*

The relationship between the exponent α of a lottery and the maximal attainable efforts G is presented in Figure 1. It turns out that under any lottery exhibiting constant or increasing returns to scale, $1 \le \alpha < 2$, and under lotteries exhibiting decreasing returns to scale, such that $0.5 + \frac{1}{2k} < \alpha < 1$, the combined effects of the extreme polar modes of discrimination increase effort or the designer's revenue beyond the average value of the initial prize valuations, $0.5(v_1 + v_2)$, which is the maximal effort obtained by either direct or overt discrimination under *any* possible lottery. Proposition 3 implies that when the designer applies two modes of discrimination, overt and direct discrimination, each type of discrimination has a positive "added value" that enhances the exertion of efforts relative to the situation in which the designer resorts to just one mode of discrimination. That is, the two modes of discrimination are supportive or "complementing" - the combination of overt and direct discrimination yields larger efforts than obtained by separate application of one of these modes of discrimination for almost any given level of covert discrimination ($0 < \alpha < 2$). Furthermore, under lotteries with increasing or constant returns to scale, as well as under some lotteries with decreasing returns to scale, such combined discrimination yields efforts that are larger than the average prize valuation (see ABC in figure 1), which is the largest possible total effort under separate application of overt or of direct discrimination. The advantage of combining these two types of discrimination relative to the use of a single mode of discrimination is due to the distinctive features of the contribution of each of these modes of discrimination to the exerted efforts as described below.

  (i)   Direct discrimination increases as much as possible the initially lower prize valuation while reducing the initially higher prize valuation almost to zero. This increases the sum of the contestants' prize valuations to infinity and makes the 'income effect' (associated with a scheme that increases the sum of the final stakes from $(v_1 + v_2)$ to $(v_1 + \varepsilon_1 + v_2 + \varepsilon_2)$) of direct discrimination the dominant effect.[11]

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[11] For a clarification of the meaning of the income effect, see the discussion following Proposition 2 in Mealem and Nitzan (2014).

(ii) The maximal possible increase in the sum of the contestants' prize valuations is not the result of direct discrimination alone. It is rendered possible by overt discrimination that ensures that the balanced-budget constraint is satisfied. Specifically, overt discrimination counterbalances the above 'income effect' by almost completely favorably discriminating contestant 1, ensuring that his winning probability converges to zero.

The moderating effect described in (ii) is necessary to attain the maximal efforts. While overt discrimination has a 'second order' effect on efforts that moderates the income effect of the direct discrimination, it also enables the dominance of this 'first order' income effect on efforts described in (i), namely, the increase in efforts due to the increase in the sum of the contestants' prize valuations. The dominance of the effect of direct discrimination means that the more extreme the direct discrimination, the higher the total efforts and this requires the extremity of overt discrimination.

The discussion before Proposition 3 also implies that if the designer can control δ, ε_1, ε_2 as well as α, he can secure almost the largest possible efforts v_1 by selecting $\alpha = 1$ (recall that according to part 2 of Proposition 3 the efforts exerted when $1 < \alpha \le 2$ converge to a value that is smaller than v_1). Any lottery with $\alpha \neq 1$ is therefore inferior to a simple lottery where $\alpha = 1$, when in both cases the designer applies the effort-maximizing discrimination strategy, viz, the dual polarized discrimination strategy. The superiority of $\alpha = 1$ is in marked contrast to its non-effort-maximizing consequence when the designer is not allowed to discriminate between the contestants overtly or directly, or when the designer is allowed to discriminate between the contestants, but just overtly or just directly.

***Figure 1*: The relationship between the exponent α and the maximal attainable efforts G**
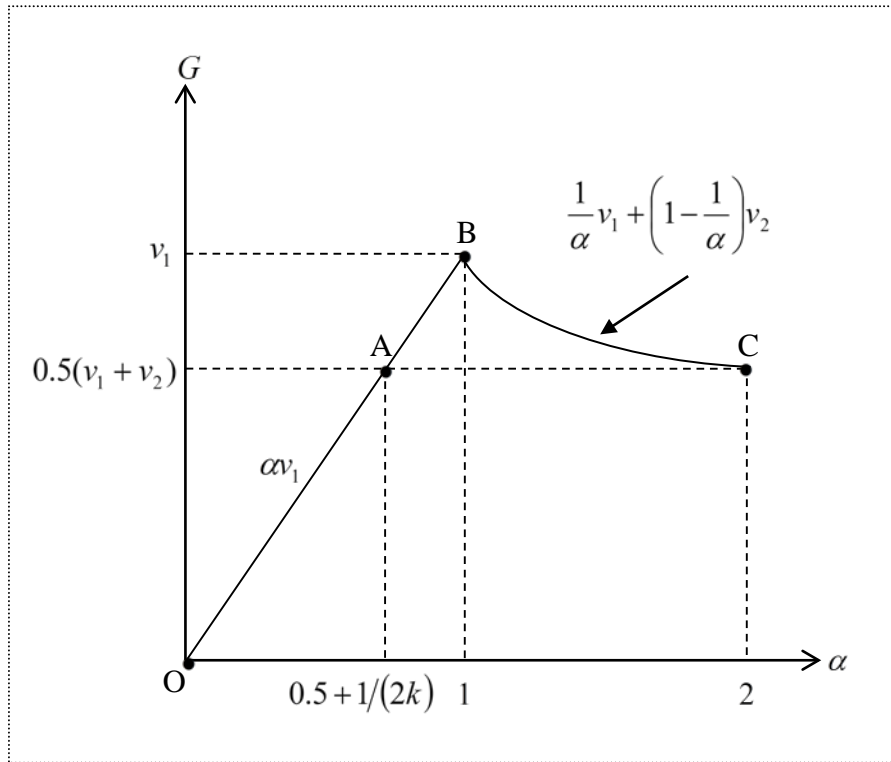
Two important conclusions can be drawn from the results. First, for $k > 1$, when the designer applies the (most effective) dual polarized discrimination strategy (the strategy that maximizes the contestants' efforts), an increase in α from $\alpha = 1$ to $\alpha = 2$ reduces efforts. Second, when $\alpha = 1$ and the designer applies a dual polarized discrimination strategy, he can attain the maximal efforts (almost v_1) given any number of contestants N. In the more general multi-player contest, the designer has to reduce the stakes of $N - 2$ contestants to zero, making sure that contestant 1 with the highest stake is not included among them. Applying the dual polarized discrimination strategy with respect to the two remaining contestants, the designer can induce efforts that are almost equal to v_1.

8. Summary

The outcome of a comparison between total effort or revenues received with the APA and a lottery is equivocal. The basic comparison by Fang (2002) for N-player contests implies that in a two-player fair simple contest, with $\alpha = 1$ and $\delta = 1$, the APA yields larger total effort than the simple lottery if and only if $k < 1 + \sqrt{2}$. Epstein et al. (2013) considerably strengthen this finding by establishing that, in a fair contest ($\delta = 1$), if $k > 1$, then a contest designer who can select the exponent α (regardless of whether

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the designer is risk neutral or risk averse), *always* prefers a logit lottery, which yields larger total effort relative to the APA, even when the sufficient condition $k > 1 + \sqrt{2}$ *is not* satisfied. In fact, the exponent of the preferred lottery satisfies $0 < \alpha < 2$, which means that the corresponding contest game has a unique pure-strategy equilibrium. Alcalde and Dahm (2010) have shown that for any $\alpha \geq 2$ there exists an equilibrium in mixed strategies that is equivalent to the equilibrium of the APA. However, so far, a characterization of the complete set of mixed-strategy equilibria is not available. Since Epstein et al. (2013) show that there exists α, $0 < \alpha < 2$, that yields total effort that are larger than obtained under the APA, it is clear that this conclusion remains valid when the parameter α has to satisfy the requirement $0 < \alpha < \infty$.

Che and Gale (1997) point out that intuition suggests that contest designers who are receptive to effort of contenders (a large exponent α) would induce greater effort, all else equal. Epstein et al. (2013) establish that this intuition is not valid when we move from the range $0 < \alpha \leq 2$ to $\alpha = \infty$. In particular, when $\delta = 1$ and $k > 1$, the parameter α in the logit CSF that yields the largest total effort is not $\alpha = \infty$, since there is some α that is smaller than 2 that yields larger total effort.

In Mealem and Nitzan (2012), we show that, for any number of contestants, if $\alpha = 1$ and dual overt and direct discrimination is allowed, then the effort-maximizing lottery yields larger total effort than the effort-maximizing APA. In such a case, total contest effort is maximal and equal almost to the highest prize valuation (the intuition behind this result was clarified in the preceding section).

Epstein et al. (2011) show that, for two contestants, under effort-maximizing overt discrimination (choice of δ) and a lottery with $0 < \alpha \leq 1$, the effort-maximizing APA yields larger total effort than the lottery. Franke et al. (2014) have generalized the result of Epstein et al. (2011), but only for a simple lottery with $\alpha = 1$, establishing that, for any number of contestants and effort-maximizing overt discrimination (choice of δ), the effort-maximizing APA yields larger total effort than the effort-maximizing simple lottery. The results of Epstein et al. (2011) and Franke et al. (2013, 2014) that establish the superiority of the effort-maximizing APA for the contest designer is not valid when the parameter α defining the lottery is allowed to satisfy $0 < \alpha \leq 2$. Epstein et al. (2013) show that, for two contestants, under effort-maximizing overt discrimination (choice of δ) and, in the lottery case, also choice of covert

discrimination (choice of α) such that $0 < \alpha \leq 2$, the effort-maximizing APA yields the same total effort as the effort-maximizing lottery.

Finally, Mealem and Nitzan (2014) show that for two contestants, $\delta = 1$ and effort-maximizing direct discrimination subject to a balanced-budget constraint, for $0 < \alpha < 2$, the effort-maximizing APA is yields larger total effort (is socially more wasteful in a rent-seeking context) than the effort-maximizing fair lottery; and for $\alpha = 2$ the effort-maximizing APA and the effort-maximizing fair lottery are equivalent (yield the same efforts).

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