The Market for Information Intermediaries and Its Effects

Ran Weksler*

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Abstract

Consider an agent that wants to sell an asset. The agent has only soft partial information about the value of the asset, but he has the option of hiring the services of an appraiser. Once hired, the appraiser finds out the real value of the asset and provides an appraisal that the agent can later voluntarily use in his interaction with a buyer. In the main model we are analyzing, the seller's utility from holding on to an asset with common value v is v while the buyer's utility is $v + \triangle$. In this adverse selection environment we show that if the price of the appraiser's service is above $\triangle$, the market for the asset can collapse entirely as a result of the existence of the appraiser in the market. To be precise, we show that markets that function reasonably well in the absence of appraisers can become totally dysfunctional in response to the entry of an appraiser into the market. We also demonstrate that even if the market does not collapse the appraiser can only harm the efficiency of the market and in some cases can even reduce the utility of the seller from an ex-post perspective.

1 Introduction

Rating agencies such as Standard & Poor's, Moody's, and Fitch Group play a major role in financial markets in general and credit markets in particular. The leading rating agencies typically use a business model in which the issuer of a financial asset is the customer; that is, the issuer approaches the rating agency, asks for a rating, and pays the rating agency for it. It is obvious that such a business model suffers from an inherent incentive problem, but, arguably, this problem is solved due to the reputation concerns of rating agencies. Rating agencies in that respect function as a channel through which an issuer of a financial asset can credibly communicate information about the asset to the investors. Another service that rating agencies provide to investors in the financial market is their expertise in analyzing assets; that is, their ratings reflect not only the

*Department of Economics and The Federmann Center for the Study of Rationality, The Hebrew University of Jerusalem.

information that the issuer provides but also the insights of the rating agency staff. These insights may include information that was not previously known to the asset issuer. Rating agencies market many products with different business models, one of which is corporate governance ratings (CGS). These ratings are made public free of extra charge at the company's discretion. The rating agency even commits not to reveal whether a particular company has approached them for a rating: "assessments can be provided to companies on a confidential basis" (see Standard & Poor's Corporate Governance Services, 2005).[1] There are in fact many other instances where ratings are provided on similar terms to the ones described in the CGS example. The common procedure for getting a credit rating includes a preliminary rating, at which point in the process the issuer can decide, on the basis of the preliminary rating, to opt out of the full credit rating process. From a theoretical point of view, this kind of procedure is almost equivalent to one in which the issuer can decide whether or not to disclose a credit rating after he gets it confidentially from the rating agency. Another product that is marketed by most rating agencies on similar terms to a credit rating is the private rating. According to Standard & Poor's website, "distributed via a secure website for distribution to up to 75 named third-parties." Examples of products similar to ratings can be found also outside the financial world: Students can take a test to measure their academic abilities or their language skills, pay some upfront fee and, once they get their results, have the option to reveal them or not to prospective universities. An owner of a painting or a house can get it appraised before selling it; the owner chooses whether or not to disclose the appraisal to a potential buyer. In these examples it is very natural to think that the seller has some imperfect private information about the asset he is selling and also that he is getting some reserve utility, which depends on his private information, in case he decides not to sell the asset. In this paper we analyze a common value environment with adverse selection and endogenous information acquisition. Specifically, a seller who is trying to sell an asset has some partial and soft information about its unknown common value, while the existence of gains from trade and their magnitude are common knowledge. The seller can choose to get his asset appraised at a cost; if he decides to do so he can choose whether or not to disclose the appraisal to the market. The main objective of the present paper is to get some understanding of the effect that the presence of information intermediaries (such as rating agencies or appraisers) has on the market for the asset. These information intermediaries, as in the corporate governance rating example, add information relative to the seller's initial signal and also certify this information so that the seller can present it credibly to the market. The main result of the paper is that these implications can be very negative, a market that without an information intermediary suffers from a minor efficiency loss due to a standard adverse selection problem, might completely collapse as a result of the entry of the intermediary into the market[2].

[1] This information regarding CGS was first mentioned by Faure-Grimaud et al. (2009).

[2] We will show that there are even instances where the market without information intermediary is efficient, while the market after an entry of the appraiser collapses entirely (see Corollary 5)

This result is quite surprising since one could expect that the option to hire an appraiser and by that produce verifiable evidence would improve the efficiency of the market as it removes some of the friction that arise due to the inability of the seller to communicate his private information in a verifiable manner. In fact, we show in the paper that this intuition is indeed correct in a model where the initial signal of the seller is perfect.

The basic intuition of this result is as follows. The option to buy an appraisal and then withhold it is lowering the attractiveness of the strategy of selling the asset without hiring the appraiser because types that do hire the appraiser would conceal unflattering appraisals. It follows that in this aspect the presence of an appraiser in the market worsens the adverse selection problem.

There are a few other papers that deal with endogenous information acquisition in various contexts. One line of research in this literature looks at the economic scenario mainly from the perspective of the intermediary and asks what is the optimal service he can offer if he wants to maximize his profits. Lizzeri (1999) shows that a monopolistic intermediary will commit to never reveal any rating and will optimally make a simple announcement to the effect that a given firm has hired its services. In a related setup but with risk-averse buyers or competitive sellers, Peyrache and Quesada (2004) show that the equilibrium will entail partial disclosure of information. Faure-Grimaus et al. (2009) ask a similar question but in a setup with initial partial private information, as in our model, and renegotiation-proof contracts. Their result is opposed to Lizzeri's: they find that the optimal renegotiation-proof contract in this setting is a contract in which the intermediary discloses all information he has on the rated firm. The present paper takes a different approach from these contributions: we assume that reputation concerns are strong enough to make the intermediary always choose to give to the seller an accurate appraisal, and then we ask how does the presence of such intermediaries affect the market for the asset. Shavell (1994) analyzes one-sided information acquisition and the disclosure of information prior to the sale of an asset in a competitive market. He compares the equilibrium information acquisition with socially efficient information acquisition in the four constellations in which (i) information has social value versus no social value and (ii) disclosure is mandatory versus voluntary. The main result of Shavell's paper is that the mandatory (voluntary) disclosure regime gives incentives for optimal (excessive) acquisition of information. The voluntary disclosure model in Shavell's paper as some similarities to the model in the present paper but is also fundamentally different. In Shavell's setup the seller of the asset does not get utility from holding on to the asset and is initially uninformed. We consider two models, in the first model we preserve Shavell's assumption that the seller does not get utility from holding on to the asset, while in the second model we relax this assumption and allow the seller to receive substantial utility in case he keeps possession of the asset. In both models we assume that the seller has initial partial information about the common value of the asset. Another paper that takes a similar approach to information acquisition comes from the bargaining literature: Dang (2008) analyzes two-sided information acquisition in an ultimatum bargaining model with common values. In his model both the buyer and the seller are initially uninformed. The buyer can learn the common value at a cost before making a take-it-or-leave-it offer and the seller can also learn the common value at the same cost after he observes the offer but before he needs to respond. Dang finds that the fact that the seller (responder) can acquire information after seeing the offer gives him a credible speculative threat that can collapse the market under some circumstances. Dang's captures bilateral trade situations or decentralized over-the-counter markets while our result applies to centralized markets. In that sense our result can be viewed as complementary to dang's. The literature on voluntary disclosure of information was originated from the well-known unraveling result. Grossman and Hart (1980) ,Grossman (1981) and Milgrom (1981) show that whenever a seller is perfectly informed and can certify at no cost the quality of the good sold, the only equilibria will result in all the information being disclosed. The force driving these fundamental results is that the market interprets withheld information as information that is unfavorable about the firm's value. The unraveling result stood in the center of many subsequent contributions such as Milgrom and Roberts (1986), Farrell (1985), and Okuno-Fujiwara, Postlewaite, and Suzumura (1990). The accounting literature builds on this idea in analyzing under what circumstances a manager will choose to withhold information. Verrecchia (1983) shows how the existence of disclosure-related costs provides an explanation to the well documented phenomena of managers that chose to withhold private information. Dye (1985) looks for a different explanation for the same phenomena, his model assumes that the manager is informed only with some exogenous probability and the investors do not observe whether or not the manager is actually informed .The idea behind these articles is that the adverse selection might be less severe when information is withheld, if uninformed agents doubt the motives of informed ones, leading to partial disclosure in equilibrium. In contrast to these contributions, in the first part of the paper we prove a generalization of the unraveling theorem in a one-sided information acquisition environment. We show that in the model where the seller gets negligible payoff if he decides to hold on to the asset, if the cost of hiring the intermediary is below some positive threshold then the unique equilibrium of the game is the unraveling equilibrium. That is, all seller's types hire the intermediary and disclose any possible evidence they receive from the intermediary. In this model we derived few other important results, we mention here two of them. First, we derive a seemingly surprising result that the inverse demand for the service of the appraiser is not necessarily monotonic. Specifically, we demonstrate in our structured model that if the probability of hiring the appraiser is high enough the inverse demand is monotonic increasing. That is, in this region the unique price that corresponds to a probability of hiring the appraiser grows as the hiring probability grows. The second result relates to a model in which the price for the appraiser service is not determined exogenously but rather at the intersection between the inverse demand curve and a fix supply curve. We show that in this model, under pretty general conditions, all types of the seller are strictly better-off under a regulation that force the seller to disclose the appraisal in case he has one (mandatory disclosure), relative to the regulation that gives the disclosure discretion to the seller (voluntary disclosure). The article is organized as follows. Section 2 presents the model. Section 3 presents an important benchmark model where the seller's utility if he decides to hold on to the asset is negligible. Section 4 presents the complete model of one-sided information acquisition with adverse selection. Section 5 discusses the robustness of the main results and section 6 concludes.

2 The Model

There are three characters in our story. First we have a seller who owns one asset and he has private information about its common value v. This private information is partial and soft; that is, the seller starts the game with a noisy signal about the value of his asset and he cannot credibly communicate this information. We denote the set of possible signals/types by θ and assume that this set is a closed interval, specifically, $\theta = [\varepsilon, 1-\varepsilon]$ for some $\varepsilon \in \left(0, \frac{1}{2}\right)$. The seller's type is chosen by nature according to the uniform distribution, i.e., $\vartheta \sim U[\varepsilon, 1-\varepsilon]$. We denote the set of possible values by $V = [0,1]$. We assume that v_ϑ, the common value of the asset conditional on the seller being of type $\vartheta \in \theta$, is distributed uniformly on the interval[3] $[\vartheta-\varepsilon, \vartheta+\varepsilon]$, i.e., $v_\vartheta \sim U[\vartheta-\varepsilon, \vartheta+\varepsilon]$. The buyer values the asset more than the seller does. Namely, if the seller decides to sell an asset with value v the buyer gets a payoff of $v + \triangle$ for some non-negative $\triangle$,while if the seller decides to hold on to an asset with value v he gets a payoff of $\alpha \cdot v$ for some $\alpha \in [0,1]$. The second character in our story is a competitive buyer (or the market). This character plays a pretty passive role in our story. We assume that the market is fully competitive and that the buyers are risk neutral. Accordingly, if the seller decides to sell the asset, the buyers will just update their beliefs about the value of the asset according to whatever information they can deduce from the play of the game and then pay the seller the expected value of the asset conditional on this information plus $\triangle$. The third character is an appraiser who has the ability to conduct an inspection of the asset; after the inspection the appraiser learns the asset's true common value and he also has the ability to certify this information. In our baseline model the price $q > 0$ for the appraiser's service is determined exogenously. The market does not observe whether or not the seller hired the appraiser.

The game has 4 stages:

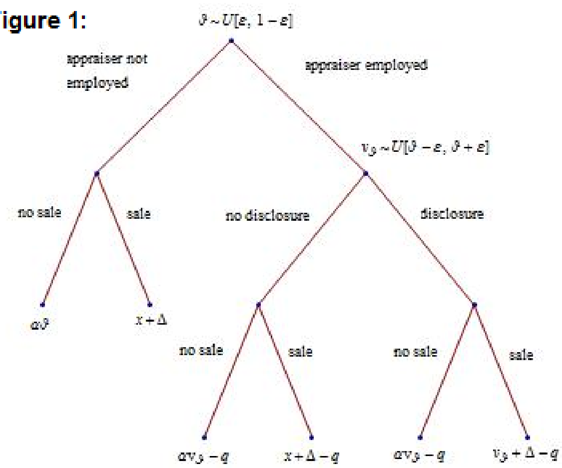
1. Nature chooses the type of the seller and the common value of the asset.
2. The seller, after observing only his signal $\vartheta \in \theta$, decides whether to hire the appraiser at a price $q > 0$ (recall that the market does not observe this decision).
3. If the seller hired the appraiser at stage 2, the seller learns the common value of the asset v and decides whether or not to verifiably disclose this information to the market.

[3]Most of the result hold also in a less restrictive model, we show that in section 5.

4. The seller decides whether to sell the asset or to hold on to the asset.

Note that the market is unable distinguish between a seller who does not hire the appraiser and a seller who does hire the appraiser but does not disclose the information obtained from him. (see Figure 1 for a visual depiction of the game.)

Figure 1:



3 The Seller always Wants to Sell[4]

In this part of the paper we will analyze an important special case of our model in which the seller does not gain any utility from holding on to the asset.[5] In this case the seller will always choose to sell the asset and therefore the efficient outcome will always be obtained. The objective of this exercise is twofold: first, there are many economic scenarios that correspond to this kind of preference of the seller.[6] Second, we want to get some basic understanding of the demand for the service that the appraiser is providing using a relatively simple model.

3.1 The Demand For the Appraiser Service

In this sub-section we want to develop the demand function for the appraiser service.

[4]This case corresponds to a special case of the model where $\alpha = 0$ and $\triangle = 0$.

[5]One interpretation that is consistent with this model is a private value environment in which the highest possible value for the seller is lower than the lowest possible value for the buyer.

[6]A very popular model in economic theory literature is a private value model where the highest possible valuation of the seller is lower than the lowest possible valuation of the buyer. This model is practically equivalent to the model we consider in this section.

3.1.1 Double cutoff structure

We start by proving two basic lemmas. First we want to prove that in equilibrium the strategy of the seller in the disclosure stage must have a cutoff structure. The argument behind this proof is simple: according to the sequential equilibrium notion it must be the case that in equilibrium all types of the seller agree on the anticipated price the market will pay for an unappraised asset.[7] It is clear that given such a price p, a seller with an appraisal at his disposal will disclose it if and only if it is above p. Before we state the lemma formally we add new notation; Denote by $\sigma_3^{\hat{v}}$ the disclosure strategy $\sigma_3(v) = \begin{cases} 1 & v > \widehat{v} \\ 0 & otherwise \end{cases}$, where $\sigma_3(v) = r$ indicates that a seller with appraisal v discloses it with probability r.

Lemma 1. ***(cutoff structure in the disclosure stage).*** *For every $\sigma_2 \in \{\sigma \mid \sigma : \theta \to [0,1]\}$, denote by $G(\sigma_2)$ the subgame that starts at stage 3, when it is common knowledge that the seller played according to σ_2 at stage 2. For every $\sigma_2 \in \{\sigma \mid \sigma : \theta \to [0,1]\}$, there exists some threshold value $\widehat{v}(\sigma_2) \in V$ such that in the unique equilibrium of the subgame $G(\sigma_2)$ all the seller's types that have the option of disclosing their value will disclose it if and only if their value is above $\widehat{v}(\sigma_2)$, i.e., all relevant types of the seller use the same strategy $\sigma_3^{\widehat{v}(\sigma_2)}$.*

Proof. In every subgame $G(\sigma_2)$, the market has some belief about the value of the asset in the case where no information is disclosed. In a sequential equilibrium it must be the case that the market will best respond correspondingly to its belief; i.e., the buyer will pay the expected value of the asset with respect to his belief. It also follows from the sequential equilibrium notion that it must be the case that the seller will anticipate this behavior. It follows that the seller (if he has the option according to σ_2) will disclose his appraisal if and only if it exceeds the anticipated payment in case of no disclosure; i.e., an equilibrium of the subgame must have a cutoff structure. □

The second lemma establishes that in the setup we are considering in this section we have a single-crossing property also in the hiring decision stage; that is, in equilibrium the seller's strategy must be to, hire the appraiser if and only if his type is above some cutoff. The intuition behind this result is as follows. As we mentioned earlier, it must be that in equilibrium all types of the seller agree on the anticipated price the market will pay for an unappraised asset. In this setup, given such a price p, the seller has only two alternatives: he can sell his asset at price p or hire the appraiser and then disclose the appraisal if and only if it exceeds p. It follows that every seller's type can calculate the expected incremental payoff it will receive in the case where he chooses to hire the appraiser: if the appraisal is below p the incremental payoff is zero and if the appraisal is above p the incremental payoff is the difference between the appraisal

[7] By unappraised asset, we mean an asset that the seller does not disclose any information about. This may be because the seller did not hire the appraiser in the first place or because he did hire the appraiser but chose not to disclose the appraisal.

and p. Given that if $\vartheta_1 > \vartheta_2$ then F_{ϑ_1} first order stochastic dominates F_{ϑ_2}, we can deduce that for every p the expected incremental payoff is monotonically increasing as a function of the seller's type.[8] It follows that given that it costs q to hire the appraiser, it must be the case that the expected incremental payoff function will cross q at most once. In order to state the lemma it will be useful the add one notation: denote by $\sigma_2^{\hat{\vartheta}}$the strategy $\sigma_2(\vartheta) = \begin{cases} 1 & \vartheta > \hat{\vartheta} \\ 0 & \vartheta < \hat{\vartheta} \end{cases}$, where $\sigma_2(\vartheta) = r$ indicates that the seller of type ϑ hires the appraiser with probability r.

Lemma 2. ***(cutoff structure in stage 2).*** *In every equilibrium of the game there exists a threshold type $\hat{\vartheta} \in \theta$ such that the equilibrium strategy of the seller in stage 2 is $\sigma_2^{\hat{\vartheta}}(\vartheta)$.*

Proof. In stage 2 the seller plays as if he knows the price he will get in case of no disclosure (this is true because in equilibrium the belief of the seller must be correct). In other words, the seller knows that if he decides to hire the appraiser then he will disclose its value if and only if it exceeds this price. Let us denote this price by $P_{ND} \geq 0$. A seller of type $\vartheta \in \theta$ will decide to hire the appraiser if and only if $\int_{P_{ND}}^{1} (\widetilde{v} - P_{ND}) f_\vartheta(\widetilde{v})\, d\widetilde{v} \geq q$. It follows that in order to prove the lemma it will be sufficient to show that the expression $\int_{P_{ND}}^{1} (\widetilde{v} - P_{ND}) f_\vartheta(\widetilde{v})\, d\widetilde{v}$ is nondecreasing in ϑ for every $P_{ND} \in [0,1]$[9] Note that the integrand in the expression is nonnegative in the range of the integral and so our desired property just follows as a feature of first-order stochastic dominates for nonnegative random variables.

□

3.1.2 The Inverse Demand Function

The objective of this subsection is to introduce the notion of inverse demand function into our context and to show that this function is well defined. In classical economics models the inverse demand function maps prices to quantities; that is, the function $P(Q)$ answers the question of what the price should be if exactly Q units are sold. In our context $D^{-1}(\beta) := q(\beta)$ is the cost of the appraiser's services that induces the seller to hire the appraiser with probability $\beta \in [0,1]$. This function is well defined if for every $\beta \in [0,1]$ there exists a unique price $q(\beta)$ that answers the above question. We now proceed to a lemma that states that given a cutoff seller's type the threshold value (disclosure stage) is unique, the lemma offers in addition a characterization of this threshold value.

Lemma 3. *Assume an equilibrium where, at stage 2, the seller uses the strategy $\sigma_2^{\hat{\vartheta}}$.*

[8]Note that in section 5 we show that most of the important result in paper are also true in a more general environment. In order to avoid reproving all these results, we try, when it is reasonable, to prove them by using only the structure we assume at section 5 at the first place.

[9]We should also assume that all seller types break ties in the same way.

If $E\left[v \mid \vartheta \leq \widehat{\vartheta}\right] \leq \underline{v_{\hat{\vartheta}}} \left(\frac{\hat{\vartheta}+\varepsilon}{2} \leq \hat{\vartheta} - \varepsilon\right)$ *then the threshold value is simply* $v^*\left(\hat{\vartheta}\right) = E\left[v \mid \vartheta \leq \widehat{\vartheta}\right] = \frac{\hat{\vartheta}+\varepsilon}{2}$*; that is, on the equilibrium path all the seller's types that hire the appraiser at stage 2 will go on to disclose all possible value realizations.*

If $E\left[v \mid \vartheta \leq \widehat{\vartheta}\right] > \underline{v_{\hat{\vartheta}}} \left(\frac{\hat{\vartheta}+\varepsilon}{2} > \hat{\vartheta} - \varepsilon\right)$ *then the threshold value will be the unique solution to the following fixed-point problem:*

$$v^*\left(\widehat{\vartheta}\right) = E\left[v \mid \left(\vartheta \leq \widehat{\vartheta}\right) \vee \left(\left(\vartheta \geq \widehat{\vartheta}\right) \wedge \left(v \leq v^*\left(\widehat{\vartheta}\right)\right)\right)\right]$$

That is, on the equilibrium path all the seller's types that have the option of disclosing their value will do so if and only if their realized value exceeds $v^*\left(\widehat{\vartheta}\right)$*; i.e., all the seller's types that have an appraisal in their disposal will use the strategy* $\sigma_3^{v^*\left(\widehat{\vartheta}\right)}$ *at stage 3.*

Proof. See Appendix A. □

Corollary 1. *The inverse demand function for the appraiser's service is well defined.*

Proof. The probability of hiring the appraiser $\beta \in [0,1]$ is uniquely mapped to a cutoff type $\vartheta(\beta) \in \theta = [\varepsilon, 1-\varepsilon]$ according to $\beta = \frac{1-\varepsilon-\vartheta}{1-2\varepsilon}$. We showed in the previews lemma that a cutoff type $\vartheta(\beta)$ is uniquely mapped to a cutoff value $v^*(\vartheta(\beta))$. Finally, a cutoff type $\vartheta(\beta)$ and a cutoff value $v^*(\vartheta(\beta))$ is uniquely mapped to a price q of the appraiser service according to;

$$q(\beta) = \int_{v^*(\vartheta(\beta))_{ND}}^{1} \left(\widetilde{v} - v^*(\vartheta(\beta))\right) f_{\vartheta(\beta)}\left(\widetilde{v}\right) d\widetilde{v}$$

□

The intuition behind the uniqueness result in Lemma 3 is as follows: in equilibrium it must be the case that the price that the market pays for an unappraised asset is correct; that is, it must be equal to the expected value of the asset conditional on it being sold undisclosed in equilibrium. In addition we already saw that this price determines the disclosure strategy of the seller. Therefore, given that only types above some cutoff type $\hat{\vartheta} \in \theta$ have an appraisal at their disposal, we are looking for a fixed point of the following description: a price p such that the expected value of all types below $\hat{\vartheta}$ and all types above type $\hat{\vartheta}$ given that their value is below p is exactly equal to [10] p. That is, the seller's types that have an appraisal at their disposal will hide bad appraisals, since in equilibrium the market anticipates this behavior, these seller's types can only hide bad appraisals that are below the fixed point. As shown by Guttman

[10] This description includes the possibility that all the possible value realizations of types above $\hat{\vartheta}$ exceeds the expected value given that the seller's type is below $\hat{\vartheta}$, in that case the fix point is simply the expected value given that the seller's type is below $\hat{\vartheta}$.

et al. (2014) this fixed point is unique. Given a cutoff type $\hat{\vartheta}$ we can introduce a function that gets as an input a threshold value $\hat{v}$ and returns the expected value of all types below $\hat{\vartheta}$ and all types above type $\hat{\vartheta}$ given that their value is below $\hat{v}$. Guttman et al. (2014) proved that a fixed point must be a minimal point of this function and that this function has a unique minimal point.[11] We now proceed to our first theorem.

Theorem 1. *For every $\varepsilon \in \left(0, \frac{1}{2}\right)$, the following properties hold:*

1. $D^{-1}(\beta) = \int_{v^*(\vartheta(\beta))}^{\vartheta(\beta)+\varepsilon} (v - v^*(\vartheta(\beta))) f_{\vartheta(\beta)}(v)\, dv$, where $\vartheta(\beta) = 1 - \varepsilon - \beta(1 - 2\varepsilon)$.

2. There exist $\hat{\beta}(\varepsilon) \in [0, 1)$ such that D^{-1} is decreasing on the segment $\left(0, \hat{\beta}(\varepsilon)\right)$ and increasing the segment $\left(\hat{\beta}(\varepsilon), 1\right) \neq \emptyset$.

3. $D^{-1}\left(\hat{\beta}(\varepsilon)\right) > \frac{\varepsilon}{4}$.

Proof. See Appendix A. □

Since this theorem and its proof are quite involved, we will focus on two main points from the theorem and try to explain the idea behind them. First, the theorem states that the inverse demand function is bounded from below by[12] $\frac{\varepsilon}{4} > 0$. The second point is that the inverse demand function is U-shaped; that is, the function is downward sloping for low probabilities of hiring the appraiser and upward sloping for high probabilities of hiring the appraiser. The idea behind the first property is pretty simple. First we can bound the price for the unappraised asset from above with the expected value of the asset conditional on the type being below the cutoff type. The seller hides his appraisal only if it is below the price of the unappraised asset, and this behavior can only lower the equilibrium price for unappraised asset relative to the case where the same types hire the appraiser and disclose all the appraisals. It follows that we can bound the probability that the cutoff type will disclose its appraisal from below and this gives us a lower bound on the expected incremental payoff the cutoff type gets from hiring the appraiser. The expected incremental payoff the cutoff type gets from hiring the appraiser must be equal to the cost of the appraiser's service in equilibrium. We now move to try to explain the idea behind the U-shape of the inverse demand function. As we already established, the equilibrium cost of the appraiser's service is equal to the incremental payoff the cutoff type gets from hiring the appraiser. It is easy to see that the most important aspect that determines this incremental payoff is the probability of disclosure of the cutoff type.[13] When we lower the cutoff type two things happen simultaneously

[11] This result is known in the literature as the "minimum principle".

[12] The theorem states that $D^{-1}\left(\hat{\beta}(\varepsilon)\right) > \frac{\varepsilon}{4}$,an immediate consequence of this property is that the Inverse Demand Function is bounded from below by $\frac{\varepsilon}{4}$ this is because $D^{-1}\left(\hat{\beta}(\varepsilon)\right) \leq D^{-1}(\beta)$ for all $\beta \in [0, 1]$.

[13] In the proof of theorem 1 we show that in our structured model the incremental payoff of the cutoff type is a function only of his probability of disclosure, i.e., the type influences this payoff only through its effect on the probability of disclosure.

that affect the probability of disclosure of the cutoff type. First, obviously the cutoff type becomes a worse type; that is, for any given disclosure threshold the new cutoff type will disclose its appraisal with lower probability. Second, the fact that now more types hire the appraiser in equilibrium causes a drop in the disclosure threshold, which clearly effects the probability of disclosure of the cutoff type positively. The theorem states that when a relatively small fraction of the seller's types hire the appraiser in equilibrium the former effect dominates and when the fraction of types that hire the appraiser is relatively large the latter effect dominates. The argument in the proof is more subtle but this is the rough intuition behind it.

3.2 Generalization of the Unraveling Result

The unraveling result (based on work by Grossman & Hart (1980), Grossman (1981), Milgrom (1981)) is the most fundamental and important result in the growing literature on strategic/voluntary disclosure of verifiable information. The result establishes that if it is known that the agent has information that he can credibly disclose and the disclosure process is costless, then the unique equilibrium of the communication game is one where all the types disclose their private information. As shown in Dye (1985) and Jung and Kwon (1988), if there is a possibility that the agent has no information or has no possibility of disclosing his information, the equilibrium is only partially revealing, with the low types choosing to pool with the uninformed agents and not to disclose information. Verrecchia (1983) showed that also in an environment where information disclosure is costly the equilibrium will be only partially revealing. Our result generalizes the unraveling result in the sense that the result may hold even in a model where the agent initially cannot credibly disclose information but can acquire the option to do so. The difference between our environment and Verrecchia's is that in our environment the agent is initially only partially informed and the appraiser'a service includes more refined information in addition to the certification.

Theorem 2. *Fix $\varepsilon \in \left(0, \frac{1}{2}\right)$; for every price $q \in \left(0, D^{-1}\left(\hat{\beta}(\varepsilon)\right)\right) \supseteq \left(0, \frac{\varepsilon}{4}\right)$ the unique equilibrium of our game is an unraveling equilibrium; i.e., all the seller's types hire the appraiser and disclose every appraisal.*

Proof. We have that for every $q \in \left[0, D^{-1}(1) = \varepsilon\right]$ the unraveling equilibrium exists. This is true because the price for unappraised asset under the unraveling equilibrium is zero, i.e., the worst possible value of the asset. Therefore the lowest type $\underline{\vartheta} = \varepsilon$, which is also the cutoff type in the unraveling equilibrium, is willing to pay at most $D^{-1}(1) = \varepsilon$ for the appraiser's service and all the other types are willing to pay even more. From that we can deduce that the unraveling equilibrium exists for every $q \leq D^{-1}(1) = \varepsilon$. In addition, from Theorem 1 we have that for every $\beta \in [0, 1]$, $D^{-1}(\beta) > \frac{\varepsilon}{4}$. From these two facts we can conclude that for every $q \in \left(0, D^{-1}\left(\hat{\beta}(\varepsilon)\right)\right) \supseteq \left(0, \frac{\varepsilon}{4}\right)$, the unique equilibrium is the unraveling equilibrium. □

The main idea behind the proof is that there is a difference between the worst possible asset value and the worst possible type. In the unraveling equilibrium it must be the case that the equilibrium price for an unappraised asset is zero, i.e., the worst possible asset value. Therefore, the worst possible seller's type have a positive willingness to pay for the appraiser service, he can guarantee for himself a net payoff of $E\left[\tilde{v} \mid \tilde{\vartheta} = \underline{\vartheta} = \varepsilon\right] = \varepsilon$ instead of zero. Therefore, for every $q \in (0, \varepsilon]$ the unraveling equilibrium exists. The results then follows from Theorem 1.

3.3 Mandatory Disclosure vs. Voluntary Disclosure from the Point of View of the Regulator

Consider a regulator whose objective is that as much information as possible flows to the market. The regulator can choose between two regulations: **voluntary disclosure** and **mandatory disclosure**.

- **Voluntary disclosure:** The seller decides whether or not to disclose the appraisal (the model analyzed above).
- **Mandatory disclosure:** The seller must disclose the appraisal.

In this part we assume that there is a cost $c > 0$ for the appraiser's effort.

3.3.1 Mandatory Disclosure

In this subsection we analyze the mandatory disclosure game. We start by proving a theorem that characterizes all possible equilibria of this game. In the next subsection we will compare between the two regulations.

Theorem 3. *Fix $\varepsilon \in \left(0, \frac{1}{2}\right)$, Define: $\overline{q} := E\left[v \mid \widetilde{\vartheta} = \overline{\vartheta}\right] - E\left[v \mid \widetilde{\theta} \leq \overline{\vartheta}\right] = 1 - \varepsilon - \frac{1}{2}$. For every $q \in [0, \overline{q}]$ there is a unique equilibrium with the next simple structure: there exist a cutoff type $\vartheta_m(q) \in \theta$such that every seller's type $\vartheta \geq \vartheta_m(q)$ hire the appraiser and every seller type $\vartheta < \vartheta_m(q)$ does not. The function $\vartheta_m(q)$ is strictly increasing and we have that $\vartheta_m(0) = \underline{\vartheta} = \varepsilon$, $\vartheta_m(\overline{q}) = \overline{\vartheta} = 1 - \varepsilon$. For every $q > \overline{q}$ there is a unique equilibrium in which none of the seller's types hire the appraiser.*

Proof. See Appendix A. □

The intuition behind this result is pretty straightforward. Given a price q for the service of the appraiser, the cutoff type $\vartheta_m(q)$ will be the type that is indifferent between selling without hiring the appraiser at price $E\left[\tilde{v} \mid \tilde{\vartheta} \leq \vartheta_m(q)\right]$ and hiring the appraiser and getting a payoff of $\vartheta_m(q) - q$. If there exists such a type the corresponding equilibrium will be the unique equilibrium. If such a type does not exist, the unique equilibrium will the one where none of the seller's type hire the appraiser; instead they will all sell at price[14] $E\left[\tilde{v} \mid \tilde{\vartheta} \leq \overline{\vartheta} = 1 - \varepsilon\right] = \frac{1}{2}$. In the rest of the section we will refer to the outcome in which all the possible information is transmitted to the market as the "first best" outcome. The next corollary is a direct consequent of the previous theorem.

Corollary 2. *The mandatory disclosure policy cannot implement the "first best" outcome.*

Proof. The corollary follows from the fact that under the mandatory disclosure policy the "first-best" outcome can be implemented if and only if there exists an equilibrium in which all the seller's types hire the appraiser. This can happen if and only if $q = 0$, which stands in a contradiction to $q \geq c > 0$. □

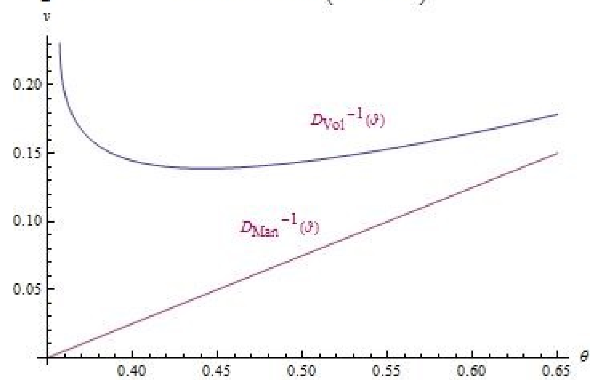
3.3.2 Comparison between Mandatory and Voluntary Disclosure

The next corollary compares the two regulation under the criterion of implementing the "first best". The idea behind the difference between the regulations can be easily seen by considering the case where all the seller's types hire the appraiser. Under the voluntary disclosure regulation it must be that the off-the-equilibrium-path-belief in case a seller tries to sell without disclosing appraisal is that the asset has the worst possible common value. But under the mandatory disclosure regulation the story is different, it is not consistent that the off-the-equilibrium-path-belief will be the same because there is no option to hire the appraiser and then withhold the appraisal. It follows that the worst possible consistent belief is that the seller that deviated is the worst possible seller's type. It is clear that under such an off-the-equilibrium-path-belief and the requirement to disclose the appraisal whatever it is, the worst type willingness to pay for the service of the appraiser is zero.

Corollary 3. *The voluntary disclosure regulation dominates the mandatory disclosure regulation in the following sense: under the voluntary disclosure regulation the "first best" outcome can be implemented (for $c \in (0, \varepsilon]$) and if the cost is low enough (for $c \in \left(0, D^{-1}\left(\hat{\beta}(\varepsilon)\right)\right) \supseteq \left(0, \frac{\varepsilon}{4}\right)$) even as a unique equilibrium. Under the mandatory disclosure regulation the "first best" outcome can not be implemented at all (see Figure 2 for the comparison between the inverse demand in the case of $\varepsilon = 0.35$).*

[14] This result is identical to the result in Verrecchia (1983).

Figure 2: Man Vs Vol (ε=0.35)



Proof. Follows directly from Theorems 2 and 3. $\square$

3.4 Fixed Supply Model

Until now we assumed that the price for the appraiser service q is determined exogenously, in this subsection we are considering the case where the price is determined endogenously but in a very specific way. Fix some $\varepsilon \in \left(0, \frac{1}{2}\right)$. There is a mass of size 1 of ex-ante identical sellers. The appraiser has a fixed supply; that is, the appraiser can provide his service to at most proportion $\lambda \in [0, 1]$ of the sellers population. The cost of the service of the appraiser is the market-clearing price. Denote by $\vartheta_{fs}(\lambda)$ the cutoff type that corresponds to the fixed supply $\lambda \in [0, 1]$, i.e. $\lambda = \frac{1-\varepsilon-\vartheta_{fs}(\lambda)}{1-2\varepsilon}$. Recall that we showed that under both the mandatory regulation and voluntary regulation the inverse demand function is well defined. Let $U^{\lambda}_{Vol}(\vartheta)$, $U^{\lambda}_{Man}(\vartheta)$ be the expected utility of type ϑ and let $q_{Vol}(\lambda)$, $q_{Man}(\lambda)$ be the cost of the appraiser's service in the unique equilibrium under the voluntary disclosure regulation and the mandatory disclosure regulation respectively, when the fixed supply is λ. We first want to state a result that is a direct consequence of the U-shaped result in this fixed supply model.

Theorem 4. *Under the voluntary disclosure regulation, for any* $\varepsilon \in \left(0, \frac{1}{2}\right)$ *the function* $q_{Vol}(\cdot)$ *is strictly monotonically decreasing on the segment* $\left(0, \hat{\beta}(\varepsilon)\right)$ *and strictly monotonically increasing on the segment* $\left(\hat{\beta}(\varepsilon), 1\right) \neq \phi$. *That is, in the segment* $\left(\hat{\beta}(\varepsilon), 1\right)$, *if the proportion of the population that the appraiser is able to serve increases, then the cost of the appraiser's service also increases.*

Proof. Follows from Theorem 1. $\square$

3.4.1 Mandatory Disclosure vs. Voluntary Disclosure from the Point of View of the Seller

The next theorem states that the seller is always weakly better off under the mandatory disclosure regulation and characterize the conditions under which all the seller's types are strictly better off. To be exact the theorem partitions the parameter space into two regions. In the first, the seller acts under the voluntary disclosure regulation exactly as he would under the mandatory disclosure regulation and so the seller is indifferent between these regulations. In the second region the difference between the regulations kicks in and we get the stark result that all the seller's types strictly prefer the mandatory disclosure regulation to the voluntary disclosure regulation. The intuition behind this result is as follows. First it is clear that if the seller is actually using the option to hide his appraisal the price for unappraised asset will be lower under voluntary disclosure. This is simply because the price for unappraised asset should now be calculated while taking into account the appraisals that are not being disclosed. Therefore, the seller's types that do not hire the appraiser (the same types under both regulations, including the cutoff type) are strictly better off under mandatory disclosure. From the observation about the price for unappraised asset we can deduce that the equilibrium price for the appraiser's service is higher under voluntary disclosure, this is true both because the price for unappraised asset is lower and because the seller discloses only favorable appraisals under voluntary disclosure. On the surface, it is not clear that the types that hire the appraiser are better-off under mandatory disclosure because on the one hand they indeed pay more to hire the appraiser but on the other hand they get better service from the appraiser due to the fact that they can choose to disclose only favorable appraisals. The reason that the former effect dominates the latter is that the payoff of the types that hire the appraiser is less sensitive to the type because of the option to hide appraisals. In other words, the division of the payoff between the types that hire the appraiser is more favorable to the lower types under voluntary disclosure because they use the option to hide appraisals more often. This argument completes the proof because if the lowest type that hires the appraiser (the cutoff type) is strictly better off under mandatory disclosure, then it is clear that this is true also for all higher types.

Theorem 5. *For every* $\lambda \in [0, 1]$*,*

- If $\vartheta_{fs}(\lambda) - \varepsilon \geq \frac{\vartheta_{fs}(\lambda)+\varepsilon}{2}$ then for every type $\vartheta \in \theta = [\varepsilon, 1-\varepsilon]$ it holds that $U^{\lambda}_{Man}(\vartheta) = U^{\lambda}_{Vol}(\vartheta)$.
- If $\vartheta_{fs}(\lambda) - \varepsilon < \frac{\vartheta_{fs}(\lambda)+\varepsilon}{2}$ then for every type $\vartheta \in \theta = [\varepsilon, 1-\varepsilon]$ it holds that $U^{\lambda}_{Man}(\vartheta) > U^{\lambda}_{Vol}(\vartheta)$.

Proof. See Appendix A. □

4 Fixed Gains from Trade[15]

In this section we analyze a richer environment that includes an initial adverse selection problem in addition to the other properties of the model. In this model, if the common value of the asset is $v \in V$ then the seller gets a utility of v if he decides to hold on to the asset while the buyer gets a payoff of $v + \triangle$ in case he purchases the asset; that is, the gains from trade are set to be $\triangle > 0$ and this fact is common knowledge. Before we get in to the analysis of the general model we present two important benchmarks.

4.1 The No-Appraiser Benchmark

This benchmark is a standard adverse selection model in the spirit of Akerlof's "Market for Lemons" (1970). The only difference from the standard adverse selection setup is that the seller's private information is partial, this fact makes no difference in the analysis if there is no possibility of gathering more precise information.

Theorem 6. *The unique equilibrium of the no-appraiser benchmark game is the following: all seller types with $\vartheta < \vartheta^* = \underline{\vartheta} + 2\triangle$ sell the asset and all other types do not, and the asset sells at price $p^* = \underline{\vartheta} + 2\triangle$.*

Proof. First observe that given a price p that the market pays for the asset we have a single-crossing property; that is, there exists at most one type $\vartheta(p)$ such that if $\vartheta < \vartheta(p)$ then it is optimal for type ϑ to sell at price p and if $\vartheta > \vartheta(p)$ then it is optimal for type ϑ to hold on to the asset. It is easy to see that $\vartheta(p) = p$; i.e., type $\vartheta = p$ is indifferent between selling at price p and holding on to the asset and getting a utility of p. In equilibrium it must be the case that the beliefs of the market are correct, from which it follows that the price that the market pays in equilibrium must satisfy the condition that $p = E\left[\tilde{v} \mid \tilde{\vartheta} \leq \vartheta = p\right] = \frac{\underline{\vartheta}+p}{2} + \triangle$. From this we may conclude that in equilibrium $\vartheta^* = p^* = \underline{\vartheta} + 2\triangle = \varepsilon + 2\triangle$. □

Corollary 4. *If $\triangle \geq \frac{1}{2} - \varepsilon$ then the unique equilibrium of the no-appraiser benchmark is efficient, i.e., all the seller's types sell in equilibrium and therefore all the gains from trade are realized.*

Proof. We need to show that if $\triangle \geq \frac{1}{2} - \varepsilon$ then $\vartheta^* \geq \overline{\vartheta}$:

$$\vartheta^* \geq \overline{\vartheta}$$

$$\iff \varepsilon + 2\triangle \geq 1 - \varepsilon$$

$$\iff 2\triangle \geq 1 - 2\varepsilon$$

$$\iff \triangle \geq \frac{1}{2} - \varepsilon$$

□

[15] This case corresponds to a special case of the model in which $\alpha = 1$ and $\triangle > 0$.

4.2 Perfect Initial Signal Benchmark

In this sub-section we want to quickly point out what happens if the initial signal of seller is perfect. To be clear, we are analyzing in this sub-section the model we described above in the specific case where $\varepsilon = 0$. The first simple observation we want to make is that in an equilibrium of this model a seller type will hire the appraiser if and only if he would later disclose the appraisal. This is clear because in equilibrium it must be that all the types have correct beliefs regarding the price of unappraised asset, in this model there is no uncertainty regarding the expected content of the appraisal, therefore the seller hires the appraiser if and only if his (perfect) signal is above the equilibrium price for unappraised asset. It follows that the incremental payoff a seller type can get from hiring the appraiser is at most the fix gain from trade $\triangle$, from this we can deduce that if $q > \triangle$ the unique equilibrium must be the equilibrium in the no-appraiser benchmark game. If $0 < q < \triangle$ there is a unique equilibrium with the following form; there exist a cutoff type $\dot{\vartheta}(q) < \vartheta^* = \underline{\vartheta} + 2\triangle = 2\triangle$ ($\underline{\vartheta} = \varepsilon = 0$), such that all types below this cutoff type sell without hiring the appraiser at price $\frac{\dot{\vartheta}(q)}{2} + \triangle$ and all types above the cutoff types hire the appraiser, disclose the appraisal and sell. The cutoff type $\dot{\vartheta}(q)$is the unique solution for the next equation; $\frac{\vartheta}{2} + \triangle = \vartheta + \triangle - q \implies \dot{\vartheta}(q) = 2q < 2\triangle = \vartheta^*$. The fact that for every $0 < q < \triangle$ it holds that $\dot{\vartheta}(q) < \vartheta^*$ means that the presence of an appraiser in the market creates a trade-off in terms of efficiency, on one hand all types above ϑ^*sell the asset and hire the appraiser, this is an efficiency gain of $(1 - 2\triangle)(\triangle - q)$ relative to the same model without an appraiser. On the second hand, types between $\dot{\vartheta}(q)$ and ϑ^* sold their asset also when there was no appraiser in the market, so this is an efficiency loss of[16] $(2\triangle - 2q)\,q$. It is easy to see that in this model if the initial adverse selection problem is severe enough, i.e., $\triangle < \frac{1}{4}$ then the former force always dominates the latter, that is, the presence of the appraiser helps the market even if we count the money spent on the appraiser as complete social waste. The general point we want to make in this sub-section is that if the initial signal of seller is perfect and the initial adverse selection problem is significant then the presence of an appraiser never harms the efficiency of the market, and if $q < \triangle$ it helps enhancing the efficiency in the market by removing some of the asymmetric information friction.

4.3 Model with Appraiser

In this subsection we analyze the richest environment so far. This environment includes an initial adverse selection problem, as in the previous benchmark, as well as costly endogenous private information. We start by stating the main result, a theorem that characterizes all possible equilibria of this model as a

[16] In this argument we think of the price for the appraiser service as a social cost, but this is not necessarily true in real world situation, it could be for example that the appraiser is a monopolist and q is just his monopolistic price. If this is the case then the social waste from the business transaction between the seller and the appraiser is smaller than what we account for in the present analysis.

function of the cost of hiring the appraiser q. After the formulation of the theorem we will present few lemmas in order to give some intuition for the proof and the economic forces that are in display in this model. First, we denote by q^* the incremental expected utility that type $\vartheta^* = \underline{\vartheta} + 2\triangle$ gains from deviating to hiring the appraiser in the equilibrium of the no-appraiser benchmark[17], i.e.,

$$q^* := \left(\frac{\vartheta^* + \varepsilon - p^*}{2\varepsilon}\right) \cdot \left(\frac{\vartheta^* + \varepsilon + p^*}{2} - p^*\right) =$$

$$= \triangle + \left(\frac{p^* - (\vartheta^* - \varepsilon)}{2\varepsilon}\right) \cdot \left(p^* - \frac{p^* + (\vartheta^* - \varepsilon)}{2}\right) = \left(\frac{1}{2} + \frac{\triangle}{2\varepsilon}\right)^2 \cdot \varepsilon$$

Theorem 7. *If $\triangle < \frac{\varepsilon}{4}$ then there exists $\mathring{q} \in \left(\frac{\varepsilon}{4}, q^*\right)$ such that:*

- *For every $q > q^*$, the unique equilibrium is the equilibrium from the no-appraiser benchmark.*
- *For every $q \in (\mathring{q}, q^*)$ all equilibria besides the no-trade equilibrium has the following structure: There exists $\underline{\vartheta} < \underline{\vartheta}(q) < \overline{\vartheta}(q)$ such that (1) every $\vartheta \in (\underline{\vartheta}, \underline{\vartheta}(q)) \neq \phi$ sells without hiring the appraiser, (2) every $\vartheta \in \left(\underline{\vartheta}(q), \overline{\vartheta}(q)\right) \neq \phi$ hires the appraiser and sells (without disclosing if $v \leq v^*\left(\underline{\vartheta}(q), \overline{\vartheta}(q)\right)$ and disclosing otherwise), (3) every type $\vartheta \in \left(\overline{\vartheta}(q), \overline{\vartheta}\right)$ does not hire the appraiser and does not sell.*
- *For every $q \in (\triangle, \mathring{q}) \neq \phi$ the unique equilibrium is the no-trade equilibrium; that is, all types do not sell the asset (and also do not hire the appraiser).*
- *For every $q \in (0, \triangle)$ the unique equilibrium is the unraveling equilibrium ,i.e., all seller's types hire the appraiser, disclose their appraisal and sell.*

Proof. See Appendix A. □

The theorem states that if the initial signal of the seller is not very accurate, i.e., $\triangle < \frac{\varepsilon}{4}$, the space of prices for the service of the appraiser is partitioned into four intervals. First, consider the interval of prices above q^*. The theorem states that if the price q is an element of this segment then the unique equilibrium of the game is effectively the equilibrium of the no-appraiser benchmark. Roughly speaking, the price is high enough to deter all the seller's types from hiring the appraiser and so the game is played as if the appraiser does not exist. The segment that is located at the other extreme is $(0, \triangle)$. If the price q is an element of this segment then the unique equilibrium is the unraveling equilibrium; i.e., all the seller's types hire the appraiser, disclose their appraisal, and sell the asset. The last two segments are $(\triangle, \mathring{q})$ and $(\mathring{q}, q^*)$ for some $\mathring{q} \in\in \left(\frac{\varepsilon}{4}, q^*\right)$ If the price q is an element of the latter segment then equilibria must have the property

[17] In the equilibrium of the no-appraiser benchmark type ϑ^* is indifferent between selling the asset at price $p^*$and holding on the asset. Therefore, the incremental expected utility type ϑ^*gets from deviating to hiring the appraiser is the same whether we calculate it relative to the "selling" strategy or the "hold on to the asset" strategy.

that the set of types that hire the appraiser is an interior segment.[18] In the former range of prices $(\triangle, \mathring{q})$ the unique equilibrium is the no-trade equilibrium; i.e., all the types do not sell the asset and do not hire the appraiser. We now turn to give a detailed overview of the proof that includes some of the main lemmas, starting with an informal proof of the result on the segment of prices $(0, \triangle)$. It is clear that the unraveling equilibrium exists when $q < \triangle$ because in this equilibrium the price for unappraised asset is 0. Therefore, all the types prefer holding on to the asset to selling it at price $p = 0$. From this and from the fact that in the unraveling equilibrium the payoff of every type $\vartheta \in \theta$ is exactly $E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta\right] = \vartheta$ we get that the willingness to pay of all the types is exactly $\triangle$, and so all the types will indeed hire the appraiser if his price is below $\triangle$. In order to understand way the unraveling equilibrium is also the unique equilibrium when $q \in (0, \triangle)$, we introduce the next two important lemmas.

Lemma 4. *Given a price p for an unappraised asset, the willingness to pay for the appraisal function is* $WTP^{p}(\vartheta) = \begin{cases} a^{p-\triangle}(\vartheta) & \vartheta < p \\ b^{p-\triangle}(\vartheta) + \triangle & \vartheta \geq p \end{cases}$, *where:*

$$a^{v}(\vartheta) := (1 - F_{\vartheta}(v)) \cdot \left(E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} > v\right] - v\right)$$

$$b^{v}(\vartheta) := F_{\vartheta}(v) \cdot \left(v - E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} < v\right]\right)$$

i.e., if $p \in (\varepsilon, 1-\varepsilon)$ then $WTP^{p}(\vartheta)$ is single-peaked.

Lemma 5. *Given prices q for hiring the appraiser and p for unappraised asset, the set of types that prefer to hire the appraiser is a segment or the empty set.*

Lemma 4 gives us the structure of the willingness to pay for the appraiser function. Given a price p for an unappraised asset, a seller's type $\vartheta \in \theta$ has three options: it can sell the asset at price p, it can hold on to the asset and get an expected payoff of ϑ, or it can hire the appraiser. If seller's type ϑ hires the appraiser it will disclose the appraisal if and only if the appraisal is above $p - \triangle$, in which case the expected payoff of type ϑ will be:

$$F_{\vartheta}(p-\triangle) \cdot p + (1 - F_{\vartheta}(p-\triangle)) \cdot \left(E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} > p - \triangle\right] + \triangle\right) =$$

$$= p + a^{p-\triangle}(\vartheta) = \vartheta + \triangle + b^{p-\triangle}(\vartheta)$$

It follows that given p the willingness to pay for the services of the appraiser of type ϑ is the difference between the payoff if it hired the appraiser;

$$p + a^{p-\triangle}(\vartheta) = \vartheta + \triangle + b^{p-\triangle}(\vartheta)$$

and the maximum between the two other options. It is clear that if $\vartheta \geq p$ $(resp.\, \vartheta < p)$ the maximum will be obtained if the seller's type ϑ holds on to the asset (resp. sells the asset). It follows that if $\vartheta \geq p$ $(\vartheta < p)$, then the willingness to pay for the appraiser will be:

$$\vartheta + \triangle + b^{p-\triangle}(\vartheta) - \vartheta = \triangle + b^{p-\triangle}(\vartheta) \;\left(p + a^{p-\triangle}(\vartheta) - p = a^{p-\triangle}(\vartheta)\right)$$

[18] The no-trade equilibrium also exists in this range of prices.

The main idea behind this lemma is that $a^{p-\triangle}(\vartheta)$ is a monotonically increasing function and $b^{p-\triangle}(\vartheta)$ is a monotonically decreasing function. Therefore, the willingness to pay for the appraiser's service is a single-peaked function if $p \in (\varepsilon, 1-\varepsilon)$, it is a monotonically increasing function if $p > \overline{\vartheta} = 1-\varepsilon$, and it is a monotonically decreasing function if $p < \underline{\vartheta} = \varepsilon$. From this observation Lemma 5 follows easily. By Lemma 5 we divide the possible segments of types that hire the appraiser in equilibrium into two groups: segments of the form $\left[\underline{\vartheta}, \hat{\vartheta}\right]$ for some $\hat{\vartheta} > \underline{\vartheta} = \varepsilon$ and segments of the form $[\vartheta_1, \vartheta_2]$ for some $\vartheta_2 > \vartheta_1 > \underline{\vartheta} = \varepsilon$. We now present another lemma that gives us a lower bound on the price for the appraiser's services in equilibrium where the segment of types that hire the appraiser are of the latter form.

Lemma 6. *In an equilibrium in which the set of types that hire the appraiser is a segment where $[\vartheta_1, \vartheta_2]$ for some $\vartheta_2 > \vartheta_1 > \underline{\vartheta}$, it must be the case that $q > \frac{\varepsilon}{4}$.*

The idea behind this lemma is that when the set of types that hire the appraiser is a segment where $[\vartheta_1, \vartheta_2]$ for some $\vartheta_2 > \vartheta_1 > \underline{\vartheta}$, it must be the case that type $\vartheta_1 > \underline{\vartheta}$ prefers selling at the price for unappraised assets to holding on to the asset, i.e., $p > \vartheta_1$. It follows that:

$$WTP^p(\vartheta_1) = a^{p-\triangle}(\vartheta) = (1 - F_\vartheta(p-\triangle)) \cdot \left(E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} > p - \triangle\right] - (p-\triangle)\right)$$

We have that in such an equilibrium;

$$p < \frac{\vartheta_1 + \underline{\vartheta}}{2} + \triangle = \frac{\vartheta_1 + \varepsilon}{2} + \triangle < \vartheta_1 + \triangle$$

From which it follows that $(1 - F_\vartheta(p-\triangle)) > \frac{1}{2}$ and that:

$$\left(E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} > p - \triangle\right] - (p-\triangle)\right) > \frac{\varepsilon}{2}$$

From this lemma it follows that if $q < \triangle < \frac{\varepsilon}{4}$, then the only feasible equilibria in which a positive measure of seller's types hire the appraiser are equilibria in which the set of types that hire the appraiser is a segment where $\left[\underline{\vartheta}, \hat{\vartheta}\right]$ for some $\hat{\vartheta} \geq \underline{\vartheta} = \varepsilon$. We want to argue that it must be the case that $\hat{\vartheta} = \overline{\vartheta} = 1-\varepsilon$, the reason being that if $\hat{\vartheta} < \overline{\vartheta}$ then it must be that type $\hat{\vartheta}$ prefers holding on to the asset to selling at the price for unappraised assets p, but this is not possible because types from the segment $\left[\hat{\vartheta}, \overline{\vartheta}\right]$ have a profitable deviation to hiring the appraiser (this deviation guarantee a payoff of at least $\vartheta + \triangle - q$ and we have that $\vartheta + \triangle - q > \vartheta$ because $\triangle > q$). So far we have established that when $q < \triangle$, the only equilibrium in which a positive measure of seller's types hire the appraiser is the unraveling equilibrium (all the seller's types hire the appraiser). It is left to show that there cannot exist an equilibrium in which a set of measure zero of seller's types hire the appraiser. It is obvious that the only candidate is the no-appraiser benchmark equilibrium, but this cannot be an equilibrium because $q < \triangle < \frac{\varepsilon}{4} < q^*$. This argument concludes the proof of the uniqueness of the unraveling equilibrium when $q < \triangle$. The theorem also mentions two other segments of prices for the services of the appraiser; namely there exists $q^* > \mathring{q} > \frac{\varepsilon}{4}$ such that (1) for every $q \in (\mathring{q}, q^*)$ all equilibria besides the no-trade equilibrium has the structure $[\vartheta_1, \vartheta_2]$ for some $\vartheta_2 > \vartheta_1 > \underline{\vartheta}$, (2) for every

$q \in (\triangle, \mathring{q})$ the unique equilibrium is the no-trade equilibrium. The intuitions for this result are similar to the intuitions for the uniqueness of the unraveling result when $q < \triangle$. First, the equilibrium of the no-appraiser benchmark does not exist because $q < q^*$. Second, the no-trade equilibrium exists because $q > \triangle$. There cannot be an equilibrium where the set of seller's types that hire the appraiser is a segment with the structure $\left[\underline{\vartheta}, \hat{\vartheta}\right]$ for some $\hat{\vartheta} > \underline{\vartheta} = \varepsilon$ because in such a case there would be unraveling in the disclosure stage. However, this is impossible because the incremental payoff that each seller's type that hires the appraiser gets is exactly $\triangle$ and $q > \triangle$. As we saw earlier, if $q < \frac{\varepsilon}{4}$ there cannot be an equilibrium where the set of types that hire the appraiser is a segment with the structure $[\vartheta_1, \vartheta_2]$ for some $\vartheta_2 > \vartheta_1 > \underline{\vartheta}$. It follows that the unique equilibrium when $\triangle < q < \frac{\varepsilon}{4}$ is the no-trade equilibrium. When $q > \frac{\varepsilon}{4}$, then the equilibrium where the set of types that hire the appraiser is a segment with the structure $[\vartheta_1, \vartheta_2]$ for some $\vartheta_2 > \vartheta_1 > \underline{\vartheta}$ can exist, and indeed for every seller's type with $\vartheta < \vartheta^*$ there exists a unique $l(\vartheta) > \vartheta$ such that an equilibrium of this form exists and the types that hire the appraiser in this equilibrium are the types in the segment $[\vartheta, l(\vartheta)]$.

The next corollary argues that if the initial signal of the seller is highly not informative ($\varepsilon > 0.4$) then it is possible that the entrance of an appraiser to the market will move us from the best possible result to the worst possible result in terms of social welfare.

Corollary 5. *If $\varepsilon > 0.4$ and $\frac{1}{2} - \varepsilon < \triangle < q < \frac{\varepsilon}{4}$ then in the no-appraiser benchmark the unique equilibrium is the efficient equilibrium and in the model with appraiser the unique equilibrium is the no-trade equilibrium.*

Proof. If $\varepsilon > 0.4$ then $\frac{1}{2} - \varepsilon < \frac{\varepsilon}{4}$. According to Corollary 4 if $\frac{1}{2} - \varepsilon < \triangle$ the unique equilibrium in the no-appraiser benchmark is the efficient equilibrium, and According to Theorem 7 if $\triangle < q < \frac{\varepsilon}{4}$ the unique equilibrium in the model with appraiser is the no trade equilibrium. □

The next theorem states that if the initial signal is even less informative then the signal described in Theorem 7 and the cost of the appraiser's services is between $\triangle$ and q^*, then every seller's type is better off in the no-appraiser equilibrium than in any other equilibrium in our model.

Theorem 8. *If $\triangle \leq 0.6863 \cdot \frac{\varepsilon}{4}$ then for every $\vartheta \in (\underline{\vartheta}, \vartheta^*)$ $l(\vartheta) < \vartheta^*$, and so if $\triangle \leq 0.6863 \cdot \frac{\varepsilon}{4}$ and $q \in (\triangle, q^*)$ then the seller is better off ex-post in the no-appraiser equilibrium than in any equilibria in a market with an appraiser.*

Proof. See Appendix A. □

The idea behind this theorem is the following; First, the condition $\triangle \leq 0.6863 \cdot \frac{\varepsilon}{4}$ insures that for every $\vartheta \in (\underline{\vartheta}, \vartheta^*)$ $l(\vartheta) < \vartheta^*$. Second, given that we have that for every $\vartheta \in (\underline{\vartheta}, \vartheta^*)$ $l(\vartheta) < \vartheta^*$we can show that all the types are better-off in the no-appraiser equilibrium. Seller's types in the segment $(\underline{\vartheta}, \vartheta)$ are better-off in the no-appraiser equilibrium simply because the cost of the appraiser's service is higher in the no-appraiser equilibrium than in any equilibria in a market with an appraiser. It is clear that type $l(\vartheta)$ is the type whose payoff is the highest among all the types in the segment $[\vartheta, l(\vartheta)]$,this type is indifferent in our equilibrium between holding on to the asset and hiring the appraiser, so his equilibrium payoff is $l(\vartheta)$. But $l(\vartheta) < \vartheta^*$ so it must be the case that $l(\vartheta) < \frac{\vartheta^*+\varepsilon}{2} + \triangle$ and $\frac{\vartheta^*+\varepsilon}{2} + \triangle$ is exactly its payoff in the no-appraiser equilibrium.

4.4 Competitive Appraisers vs. Monopolistic Appraiser

In this subsection we relax the assumption that the price q that the seller needs to pay for the appraiser's service is determined exogenously. We compare two mechanisms in which the price q is determined: competition between appraisers and a monopolistic appraiser.

4.4.1 Competitive Appraisers

The model we have in mind is the following: two (or more) appraisers compete in prices for the demand of the seller. Both appraisers offer exactly the same service; if hired they inspect the asset, find out its common value v ,and give the seller verifiable evidence that certify this information (appraisal). We assume that both appraisers face the same cost when they supply their service, and we denote this cost by $c > 0$. The timeline of the game is as follows. First, each appraiser chooses a price q_i $i \in \{1, 2\}$, and these prices are observed by all the participants in the game. The game continues exactly the same as in the original model except that the buyer can choose which appraiser (if any) to hire; note that the market does not observe whether or not the seller hires an appraiser.

Lemma 7. *If an appraiser $i \in \{1, 2\}$ is hired in equilibrium by a positive measure of seller's types then $q_i = c$.*

Proof. It is easy to see that this is a standard price competition with a homogeneous good. The result follows from the well-known result by Bertrand (1883). □

4.4.2 Monopolistic Appraiser and Comparison

In this model the game starts with the monopolistic appraiser choosing a price q for his services, and the game continues exactly the same as in the original model. We again assume that the appraiser faces a cost $c > 0$ when he supplies his service.

Remark 1. When $q > \triangle$, the equilibrium in which all the seller's types do not hire the appraiser and do not sell the asset, no-trade equilibrium, always exists. This equilibrium implements the worst possible outcome of the game, and so if for some $q > \triangle$ there exists another equilibrium in the game induced by q, then it must be that this equilibrium dominates the no-trade equilibrium ex-post. We will follow the convention that in such cases the total collapse of the market equilibrium is never the realized equilibrium (Grossman and Perry (1986) solution concept reinforces this convention).

Theorem 9. *If* $\frac{\varepsilon}{4} > c > \triangle$ *then the seller is ex post better off in the monopolistic appraiser's model than in the competitive appraisers model.*

Proof. We already showed that if $\frac{\varepsilon}{4} > q > \triangle$ then the unique equilibrium of the induced subgame is the no-trade equilibrium. By Lemma 7 we get that if $\frac{\varepsilon}{4} > q > \triangle$ then the unique equilibrium in the competitive appraisers model is the no-trade equilibrium. As we mentioned earlier, this equilibrium is the worst possible result for the seller. As part of Theorem 7 we established that if $q^* = \left(\frac{1}{2} + \frac{\triangle}{2\varepsilon}\right)^2 \cdot \varepsilon > q > \mathring{q} > \frac{\varepsilon}{4}$ then there exists other equilibria in the induced subgame besides the no-trade equilibrium. All these equilibria (besides the no-trade equilibrium) involve a positive measure of seller's types that hire the appraiser, a positive probability of selling the asset, and therefore a better result ex post for the seller. From this we can deduce that if $\frac{\varepsilon}{4} > c > \triangle$, then a monopolistic appraiser will choose a price q with $\left(\frac{1}{2} + \frac{\triangle}{2\varepsilon}\right)^2 \cdot \varepsilon > q > \mathring{q} > \frac{\varepsilon}{4}$, and this completes the proof. □

The intuition for this result is very simple given our previous results. By Lemma 7 we have that in the competition model it must be the case that $q_1 = q_2 = c$, and so if $\frac{\varepsilon}{4} > c > \triangle$ the market will collapse according to Theorem 7. A monopolistic appraiser can use the fact that according to Theorem 8 there are equilibria in which a positive measure of seller's types hire the appraiser when $q^* > q > \mathring{q} > \frac{\varepsilon}{4}$ to set a price in the segment $(\mathring{q}, q^*)$. With such a price the market will not collapse (under the assumption in Remark 1) and so all the seller's types will be weakly better off under a monopolistic appraiser and a positive measure of seller's types will be strictly better off.

4.5 Mandatory Disclosure vs. Voluntary Disclosure

4.5.1 Mandatory Disclosure

We now turn to analyze the same model but with one change: the seller must disclose the evidence he got from the appraiser in case he hired him (another way to think of this model is that the appraiser must make his appraisal public). This change creates a big difference in terms of the incentives of the seller to hire the appraiser, as demonstrated in the following theorem.[19]

Theorem 10. *If* $q > \triangle$ *the unique equilibrium is the benchmark no-appraiser equilibrium. If* $q < \triangle$ *there exists a unique* $\vartheta_m(q) < \vartheta^*$ *such that in the unique equilibrium types with* $\vartheta > \vartheta_m(q)$ *hire the appraiser and sell and types with* $\vartheta < \vartheta_m(q)$ *do not hire the appraiser and sell.*

[19]This model is effectively equivalent to the perfect initial signal benchmark.

Proof. The benchmark no-appraiser equilibrium continues to exist if $q > \triangle$ because the maximal extra payoff a seller can get from a deviation to hiring the appraiser is $\triangle$ (this happens in the case where $\vartheta \geq \vartheta^*$). The reason for the difference from the voluntary disclosure model is that the seller has no way to get extra payoff by concealing the appraisal in case of a bad one. This is also the unique equilibrium because from the same reason there cannot be an equilibrium in which the seller hire the appraiser and there is no reason that a new equilibrium, in which the seller does not hire the appraiser, will emerge if it did not emerge in the no appraiser benchmark. If $q < \triangle$ every type $\vartheta \in \theta$ must get a payoff of at least $\vartheta + (\triangle - q) > \vartheta$, because every type can get this payoff by hiring the appraiser. It is clear that given a price p that a seller gets for the asset in the case where no appraisal is disclosed, we have a single crossing-property; that is, the willingness to pay for the appraiser's service is monotonically increasing in ϑ. In equilibrium it must be the case that $p = \frac{\vartheta_m(q)+\underline{\vartheta}}{2} + \triangle$ and that the cutoff type $\vartheta_m(q)$ is indifferent between hiring the appraiser and selling without disclosing at price p. It follows that it must be the case that $p = \vartheta_m(q) + (\triangle - q)$. From these two equations we get that:

$$\vartheta_m(q) + (\triangle - q) = \frac{\vartheta_m(q) + \underline{\vartheta}}{2} + \triangle \Rightarrow \vartheta_m(q) = \underline{\vartheta} + 2q < \underline{\vartheta} + 2\triangle = \vartheta^*$$

□

4.5.2 Comparison

From the two characterization theorems (Theorem 7 and theorem 10) we can see that there is a clear trade-off between these two disclosure regimes. If $0 < q < \triangle$, then all the seller's types hire the appraiser and disclose their appraisal under the voluntary regime, while only types above $\vartheta(q) = \underline{\vartheta} + 2q > \underline{\vartheta}$ hire the appraiser under the mandatory regime. In this range of appraiser prices we get that under both disclosure regimes the seller always sells his asset, but more information is disclosed and more money is spent on the appraiser under the voluntary regime than under the mandatory regime. We now want to continue the comparison under the assumption that $\triangle < \frac{\varepsilon}{4}$. We showed in theorem 7 that if $\triangle < q < \frac{\varepsilon}{4}$ then the unique equilibrium under the voluntary regime is the no-trade equilibrium. It follows that in this region the seller does not hire the appraiser under both disclosure regimes, but all types below $\vartheta^* = \underline{\vartheta} + 2\triangle$ sell their asset under the mandatory regime. In this region of parameters the difference between the two regimes is very stark, and the mandatory regime is much more efficient. If $\left(\frac{1}{2} + \frac{\triangle}{2\varepsilon}\right)^2 \cdot \varepsilon > q > \frac{\varepsilon}{4}$, then the trade-off between information disclosure and efficiency is manifested once again. Under the voluntary regime some positive measure of types will hire the appraiser (to be precise, there exists $\mathring{q} \in \left(\frac{\varepsilon}{4}, q^*\right)$ such that for every $q \in (\mathring{q}, q^*)$ there exists an equilibrium in which a positive measure of types hire the appraiser) and information will be disclosed to the market. On the other hand, efficiency is again lost relative to the mandatory regime because even if more types sell in equilibrium it must be the case that these types hire the appraiser at a price that is above the gain from trade[20] $\triangle$. When $q > \left(\frac{1}{2} + \frac{\triangle}{2\varepsilon}\right)^2 \cdot \varepsilon = q^*$ there is no difference between the regimes.

5 Robustness

In this part of the paper we want to establish that the main results of the paper hold also when a more general and unstructured model is assumed. The reason we have chosen to work with a structured model is twofold. First, such a model is more tractable and, second, some of the more subtle results such as Theorem 8 require a specific structure in order to be proved. Let us now consider the following general framework. The set of seller's types is a segment $\theta = \left[\underline{\vartheta}, \overline{\vartheta}\right]$. Every type $\vartheta \in \theta$ corresponds to a signal that the seller has about his asset; for every $\vartheta \in \theta$ denote by F_ϑ the distribution of the common value of the asset conditional on observing the signal ϑ. For two seller's types $\vartheta_1, \vartheta_2 \in \theta$ it holds that $\vartheta_2 > \vartheta_1 \implies F_{\vartheta_2}\, fosd\, F_{\vartheta_1}$. At the beginning of the game nature draws a signal $\vartheta \in \theta$ according to some continuous distribution G, and then nature draws the common value v of the asset from the distribution F_ϑ. We assume that the common value v is nonnegative and that $v = 0$ is in the support of at least type $\underline{\vartheta}$. The seller observes only the signal ϑ that is unverifiable, and he can hire an appraiser by paying a fee $q > 0$. If he hires the appraiser he learns the true common value v, and he gets verifiable evidence (appraisal) that he can voluntarily disclose to the market. The market does not observe whether or not the seller hires the appraiser. The rest of the model is the same as the model presented in Section 2.

We want to formulate a theorem in this model that corresponds to Theorem 7, but in order to do so we need more notation. Denote $\mu_\vartheta := E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta\right]$, $\underline{\mu_\vartheta} := E\left[\tilde{v} \mid \tilde{\vartheta} < \vartheta\right]$, $S := min_{\vartheta \in \theta}\left(1 - F_\vartheta\left(\underline{\mu_\vartheta}\right)\right) \cdot \left(E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} > \underline{\mu_\vartheta}\right] - \underline{\mu_\vartheta}\right)$.

Theorem 11. *If $\triangle < S$ then for every $q \in (\triangle, S) \neq \phi$ the unique equilibrium is the no-trade equilibrium; that is, all types do not sell the asset (and also do not hire the appraiser). For every $q \in (0, \triangle)$ in the unique equilibrium all types hire the appraiser, disclose their appraisal and sell.*

Proof. See Appendix A. □

Theorem 12. *(Theorem 2) Fix $\varepsilon \in \left(0, \frac{1}{2}\right)$; for every price $q \in (0, S)$ the unique equilibrium of our game is an unraveling equilibrium; i.e., all the seller's types hire the appraiser and disclose every appraisal.*

Proof. In the model where the seller gets a utility of zero in the case where he holds on to the asset, it follows from a previous lemma that the single-crossing

[20] If we think of the price q for the appraiser's service as a cost this statement is precise, but if this is the price the appraiser chose to, say, optimize his payoff and the cost he has to bear is lower than $\triangle$ it may be that efficiency will be gained relative to the mandatory regime. Theorem 8 ensures that this is not the case; that is, if $\triangle \leq 0.6863 \cdot \frac{\varepsilon}{4}$, then it must be the case that the mandatory regime is more efficient.

property holds. That is, given a price p that the market pays for an unappraised asset and a price q for hiring the appraiser, there exists at most one cutoff type $\vartheta(p,q) \in \theta$ such that all types below this cutoff type find it optimal to sell at price p (without hiring the appraiser) and all types above this cutoff type find it optimal to hire the appraiser and disclose the appraisal if and only if it is above p. In this model the incremental payoff the cutoff type gets from hiring the appraiser must be the equilibrium price of the appraiser q. We want to show that if the cutoff type is strictly above $\underline{\vartheta}$ then its incremental payoff must be above S. This can be shown by exactly the same argument we used at the end of the proof of theorem 1. It is left to show that for every price $q \leq \mu_{\underline{\vartheta}}$ there exists an equilibrium where the cutoff type is $\underline{\vartheta}$. This is clear because when the cutoff type is $\underline{\vartheta}$ we must have unraveling in the disclosure stage of the game, and so $p = 0$ and the incremental payoff of the cutoff type $\underline{\vartheta}$ is $\mu_{\underline{\vartheta}}$. □

6 Conclusion

In this paper we considered a model in which an agent wants to sell an asset. The agent has only soft partial information about the common value of the asset, but he has the option of hiring the services of an appraiser. Once hired, the appraiser finds out the real common value of the asset and provides hard evidence (an appraisal) that the agent can later voluntarily use in his interaction with a buyer. This framework captures many economic scenarios, including financial market situations such as equity selling and debt issuing. In the financial market story the role of the appraiser is played by a rating agency. As we argued in the Introduction, the business relationship between a rating agency and a debt issuer is very similar to the relationship we modeled between a seller and an appraiser. Although rating agencies state that they publish ratings regardless of their content, it is known that in practice the rating agency gives a heads-up to the issuer regarding the content of the upcoming rating and that, in so doing, it effectively gives the issuer the option of withholding the rating by opting out of the credit rating procedure before it ends. In that sense we argue that our main result on the possible collapse of the market for the asset in response to the entry of an appraiser into the market can be interpreted as a possible explanation for credit crunches in financial markets. This paper is a first step in a research agenda that calls for taking a deeper look into the role that rating agencies play in financial markets and specifically the potential damage they can cause.

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Appendix A

Lemma. *(3)*

Proof. In a sequential equilibrium the requirement is that the beliefs would end up to be correct, so it must be the case that given the price that the buyer pay in case of no disclosure , the seller best response would make this price choice correct. We will divide our analysis according to two kinds of seller types. The first kind are seller types have the next property:

$$E\left[v \mid \vartheta \leq \widehat{\vartheta}\right] \leq \underline{v_{\hat{\vartheta}}} \left(\frac{\hat{\vartheta}+\varepsilon}{2} \leq \hat{\vartheta}-\varepsilon\right)$$

Denote the set of this kind of seller types by ; i.e.,

$$A := \left\{\hat{\vartheta} \in \theta \mid E\left[v \mid \vartheta \leq \widehat{\vartheta}\right] \leq \underline{v_{\hat{\vartheta}}}\right\}$$

We start by proving that in an equilibrium where the threshold type $\widetilde{\vartheta}$ is an element of the set A all the seller types that hired the appraiser in stage 2 ($\left\{\vartheta \mid \vartheta \geq \widetilde{\vartheta}\right\}$) would also disclose their realized value whatever it is. First we will show that this behavior is an equilibrium in the subgame where only seller types in the set $\left\{\vartheta \mid \vartheta \geq \widetilde{\vartheta}\right\}$ have the option to disclose their value. This is true because if the strategy of the seller types, that have the option to disclose, is to disclose every value then the price that the market would pay in case of no disclosure must be $E\left[v \mid \vartheta \leq \widetilde{\vartheta}\right]$. Now if this is the price then indeed the best replay of every seller type in the set $\left\{\vartheta \mid \vartheta \geq \widetilde{\vartheta}\right\}$ is to disclose every value because:

$$\forall\vartheta \in \left\{\vartheta \mid \vartheta \geq \widetilde{\vartheta}\right\}\ if\ E\left[v \mid \vartheta \leq \widetilde{\vartheta}\right] \leq \underline{v_{\widetilde{\vartheta}}} = \widetilde{\vartheta}-\varepsilon\ then\ E\left[v \mid \vartheta \leq \widetilde{\vartheta}\right] \leq \underline{v_{\vartheta}} = \vartheta-\varepsilon$$

Now we prove that this equilibrium is the unique equilibrium in the subgame we are considering. From Lemma 1 it is sufficient to show that there is no equilibrium with a threshold structure in the subgame. Assume by contradiction that there is such an equilibrium, that is, there exist $v^* > \underline{v_{\widetilde{\vartheta}}}$ such that the strategy of the seller types that have the option to disclose their value -$\left\{\vartheta \mid \vartheta \geq \widetilde{\vartheta}\right\}$- is $\sigma_3^{v^*}$. Therefore, the market would pay in case of no disclosure:

$$E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right]$$

but because,

$$\forall\vartheta \in \left\{\vartheta \mid \vartheta \geq \widetilde{\vartheta}\right\}\ E\left[v \mid \vartheta \leq \widetilde{\vartheta}\right] \leq \underline{v_{\widetilde{\vartheta}}} \leq \underline{v_{\vartheta}}$$

it must hold that:

$$v^* > E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right]$$

This is a contradiction because the seller should disclose every value with:

$$v \in \left(E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right], v^*\right) \neq \Phi$$

That is, the strategy of the seller is not a best replay to the market play. We proceed by analyzing the compliment set of firm types:

$$A^c = \left\{\hat{\vartheta} \in \theta \mid E\left[v \mid \vartheta \leq \widehat{\vartheta}\right] > \underline{v_{\hat{\vartheta}}}\right\}$$

Our claim is that in a subgame where $\widetilde{\theta} \in A^c$ is the threshold type the threshold value is the unique solution of the next equation:

$$v^* = E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right]$$

First note that in this case disclosing every value is not an equilibrium. This is because if this is the equilibrium strategy of the firm types that have the option to disclose their value -$\left\{\vartheta \mid \vartheta \geq \widetilde{\vartheta}\right\}$ - then in equilibrium the price the market would pay in case of no disclosure must be $E\left[v \mid \vartheta \leq \widetilde{\vartheta}\right]$, but $\widetilde{\theta} \in A^c$, so it holds that $E\left[v \mid \vartheta \leq \widetilde{\vartheta}\right] > \underline{v_{\widetilde{\vartheta}}}$, it follows that for the seller to best respond it must withhold values in the segment $\left(\widetilde{\vartheta} - \varepsilon, E\left[v \mid \vartheta \leq \widetilde{\vartheta}\right]\right)$, i.e., the strategy of disclosing every value is not an equilibrium. Now will show that in an equilibrium of our subgame the threshold value $v^* \in (\underline{v_{\widetilde{\vartheta}}}, 1]$ must be a solution to the next equation:

$$v^* = E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right]$$

Assume by contradiction that:

$$v^* < E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right]$$

It follows that the seller has a profitable deviation to withhold values in the next segment:

$$(v^*, E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right])$$

Alternatively, if $v^* > E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right]$, it follows that the seller has a profitable deviation to disclose values with:

$$v \in (E\left[v \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right], v^*)$$

It is easy to verify that if v^*solves the equation, then the strategy:

$$\forall \vartheta \in \theta \; \sigma_3^{v^*} = \begin{cases} 1 & v > v^* \\ 0 & else \end{cases}$$

constitute an equilibrium, and that for every $\widehat{\vartheta} \in A^c$ there exists a unique solution to the equation:

$$v^* = E\left[v_\vartheta \mid \left(\vartheta \leq \widetilde{\vartheta}\right) \vee \left(\left(\vartheta \geq \widetilde{\vartheta}\right) \wedge (v \leq v^*)\right)\right]$$

□

Theorem. *(1)*

Proof. The first point in theorem follows from the fact that given a price p the incremental payoff the cutoff type will receive from hiring the appraiser is exactly $\int_p^{\vartheta(\beta)+\varepsilon}(v-p)\,f_{\vartheta(\beta)}(v)\,dv$ (if the appraisal turned out to be lower than p then the cutoff seller's type would not disclose it, in this cases its incremental payoff is 0, if the appraisal turned out to be higher than p then the cutoff seller's type would disclose it, in this cases its incremental payoff is the difference between the value of the asset and p). In equilibrium where the cutoff type is $\vartheta(\beta)$ it must be that $p=v^*(\vartheta(\beta))$.We prove the second point of the theorem with the help of a series of lemmas.

Lemma 8. $\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\frac{\partial\left(E\left[\tilde{v}\mid\tilde{\vartheta}\le\vartheta\ or\ \left(\tilde{\vartheta}>\vartheta\ and\ \tilde{v}\le v\right)\right]\right)}{\partial\vartheta}\mid_{\vartheta=\hat{\vartheta},v=v^*\left(\vartheta=\hat{\vartheta}\right)}$.

Proof. Denote $Exp(\vartheta,v):=E\left[\tilde{v}\mid\tilde{\vartheta}\le\vartheta\ or\ \left(\tilde{\vartheta}>\vartheta\ and\ \tilde{v}\le v\right)\right]$. First we already established that given any $\varepsilon\in\left(0,\frac{1}{2}\right)$ for every $\vartheta\in\theta=[\varepsilon,1-\varepsilon]$ it holds that:

$$v^*(\vartheta)=Exp(\vartheta,v^*(\vartheta))=E\left[\tilde{v}\mid\tilde{\vartheta}\le\vartheta\ or\ \left(\tilde{\vartheta}>\vartheta\ and\ \tilde{v}\le v^*(\vartheta)\right)\right]$$

It follows that for every $\hat{\vartheta}\in\theta$ $\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\frac{dExp(\vartheta,v^*(\vartheta))}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}$. From the calibrated "minimum principle" we have that for every $\vartheta\in\theta$ $\frac{\partial Exp(\vartheta,v)}{\partial v}\mid_{v=v^*(\vartheta)}=0$, it follows that:

$$\frac{dExp(\vartheta,v^*(\vartheta))}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\frac{\partial Exp(\vartheta,v)}{\partial\vartheta}\mid_{\vartheta=\hat{\vartheta},v=v^*(\vartheta)}+\frac{\partial Exp(\vartheta,v)}{\partial v}\mid_{\vartheta=\hat{\vartheta},v=v^*(\vartheta)}\cdot\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=$$

$$=\frac{\partial Exp(\vartheta,v)}{\partial\vartheta}\mid_{\vartheta=\hat{\vartheta},v=v^*(\vartheta)}$$

This ends the proof because we get that:

$$\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\frac{dExp(\vartheta,v^*(\vartheta))}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\frac{\partial Exp(\vartheta,v)}{\partial\vartheta}\mid_{\vartheta=\hat{\vartheta},v=v^*(\vartheta)}$$

□

Lemma 9. *For every $\varepsilon\in\left(0,\frac{1}{2}\right)$ The function $v^*$is concave.*

Proof. First is easy to see that for every $\varepsilon\in\left(0,\frac{1}{2}\right)$ There exist at most one type $\dot{\vartheta}(\varepsilon)\in\theta=[\varepsilon,1-\varepsilon]$ such that $\frac{\dot{\vartheta}(\varepsilon)+\varepsilon}{2}=\dot{\vartheta}(\varepsilon)-\varepsilon$, and it is clear that for all $\vartheta>\dot{\vartheta}(\varepsilon)$ $\left(\vartheta<\dot{\vartheta}(\varepsilon)\right)$ it holds that $\frac{\vartheta+\varepsilon}{2}<\vartheta-\varepsilon$ $\left(\frac{\vartheta+\varepsilon}{2}>\vartheta-\varepsilon\right)$. We already saw in the proof of lemma 3 that if $\frac{\vartheta+\varepsilon}{2}<\vartheta-\varepsilon$ then $v^*(\vartheta)=\frac{\vartheta+\varepsilon}{2}$. It follows that for every $\varepsilon\in\left(0,\frac{1}{2}\right)$ the function $v^*$is concave on the segment $\left[\dot{\vartheta}(\varepsilon),1-\varepsilon\right]$ because a linear function is concave. It is left to show that v^* is also concave on the segment $\left[\varepsilon,\dot{\vartheta}(\varepsilon)\right]$, Note that on this segment we have that $\frac{\vartheta+\varepsilon}{2}>\vartheta-\varepsilon$

and so $v^*(\vartheta) < \frac{\vartheta+\varepsilon}{2}$ for every type in this segment. In order to prove this lemma we need to show that the derivative of $v^*$is monotonic non-decreasing. From lemma 4 we have that $\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\frac{\partial Exp(\vartheta,v)}{\partial\vartheta}\mid_{\vartheta=\hat{\vartheta},v=v^*(\vartheta)}$ so we will show that $\frac{\partial Exp(\vartheta,v)}{\partial\vartheta}\mid_{\vartheta=\hat{\vartheta},v=v^*(\vartheta)}$is monotonic non-decreasing. This derivative has the following form for every $\hat{\vartheta}\in\left[\varepsilon,\dot{\vartheta}(\varepsilon)\right]$:

$$\frac{\partial Exp(\vartheta,v)}{\partial\vartheta}\mid_{\vartheta=\hat{\vartheta},v=v^*(\vartheta)}=$$

$$=lim_{\delta\to 0}\frac{\frac{2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}}{\left(2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\delta\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+\delta^2+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}\right)}\cdot v^*\left(\hat{\vartheta}\right)+\frac{\left(\delta\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+\delta^2\right)\cdot\left(\frac{\hat{\vartheta}+\varepsilon+v^*\left(\hat{\vartheta}\right)}{2}\right)}{\left(2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\delta\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+\delta^2+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}\right)}-v^*\left(\hat{\vartheta}\right)}{\delta}=$$

$$=lim_{\delta\to 0}\frac{\frac{2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}}{\left(2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\delta\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+\delta^2+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}\right)}\cdot v^*\left(\hat{\vartheta}\right)+\frac{\left(\delta\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+\delta^2\right)\cdot\left(\frac{\hat{\vartheta}+\varepsilon+v^*\left(\hat{\vartheta}\right)}{2}\right)}{\left(2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\delta\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+\delta^2+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}\right)}-v^*\left(\hat{\vartheta}\right)}{\delta}=$$

$$=lim_{\delta\to 0}\frac{\left(\delta\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+\delta^2\right)\cdot\left(\left(\frac{\hat{\vartheta}+\varepsilon+v^*\left(\hat{\vartheta}\right)}{2}\right)-v^*\left(\hat{\vartheta}\right)\right)}{\delta\left(2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\delta\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+\delta^2+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}\right)}=\frac{\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)^2}{2\left(2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}\right)}$$

We want to show that this expression is monotonic non-decreasing in $\hat{\vartheta}$ it sufficient to show that the numerator is increasing slower than the denominator. In order to show this we differentiate both the numerator and the denominator and show that the derivative of numerator is lower.

$$\left(\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)^2\right)'=2\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)\left(1-\left(v^*\left(\hat{\vartheta}\right)\right)'\right)$$

$$\Longleftrightarrow 2\left(2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}\right)=4\varepsilon+2\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)\left(\left(v^*\left(\hat{\vartheta}\right)\right)'-1\right)$$

It follows that:

$$4\varepsilon+2\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)\left(\left(v^*\left(\hat{\vartheta}\right)\right)'-1\right)\geq 2\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)\left(1-\left(v^*\left(\hat{\vartheta}\right)\right)'\right)$$

$$\Longleftrightarrow 4\varepsilon\geq\left(1-\left(v^*\left(\hat{\vartheta}\right)\right)'\right)\left(2\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)+2\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)\right)$$

$$\Longleftrightarrow 4\varepsilon\geq\left(1-\left(v^*\left(\hat{\vartheta}\right)\right)'\right)(4\varepsilon)$$

$$\Longleftrightarrow\left(v^*\left(\hat{\vartheta}\right)\right)'\geq 0$$

It is clear that for every $\varepsilon\in\left(0,\frac{1}{2}\right)$ and every $\vartheta\in[\varepsilon,1-\varepsilon]$ it holds that $\left(v^*\left(\hat{\vartheta}\right)\right)'\geq 0$. □

Lemma 10. *For every* $\varepsilon\in\left(0,\frac{1}{2}\right)$ $lim_{\hat{\vartheta}\to\varepsilon}\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\infty$.

Proof. We saw in the previews lemma that $\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\frac{\left(\hat{\vartheta}+\varepsilon-v^*\left(\hat{\vartheta}\right)\right)^2}{2\left(2\varepsilon\left(\hat{\vartheta}-\varepsilon\right)+\frac{\left(v^*\left(\hat{\vartheta}\right)-\left(\hat{\vartheta}-\varepsilon\right)\right)^2}{2}\right)}$.

It is clear that in the limit where the cutoff type $\hat{\vartheta}$ is approaching the lowest type $\underline{\vartheta}=\varepsilon$ we will have all most unraveling in the disclosure stage of the game, i.e., $lim_{\hat{\vartheta}\to\varepsilon}v^*\left(\hat{\vartheta}\right)=0$. It is now easy to see that the limit of the numerator is ε while the limit of the denominator is 0, so we can derive that for every $\varepsilon\in\left(0,\frac{1}{2}\right)$ $lim_{\hat{\vartheta}\to\varepsilon}\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=\infty$. □

Lemma 11. *For every $\varepsilon\in\left(0,\frac{1}{2}\right)$ there exist a type $\hat{\vartheta}\left(\varepsilon\right)\in\left(\varepsilon,1-\varepsilon\right]$ such that $\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}(\varepsilon)}=1$, D^{-1} is decreasing on the segment $\left(\varepsilon,\hat{\vartheta}\left(\varepsilon\right)\right)\neq\emptyset$ and increasing the segment $\left(\hat{\vartheta}\left(\varepsilon\right),1-\varepsilon\right)$.*

Proof. The willingness to pay function of the cutoff type is a function only of the probability of disclosure, for the type $\hat{\vartheta}$ with $\frac{dv^*}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}=1$ it holds that the derivative of the probability of disclosure equals to 0 because the disclosure threshold increases exactly with the same rate as the rate of the improvement of the distribution of values. The rate of improvement of the values distribution is always equal to 1 (the rate in which the expectation is increasing), so we get the last part of the lemma as a consequence of lemma 5. The existence part of the lemma follows from lemma 5 and lemma 6 together, i.e., if $v^*$is continues and concave and its derivative approaches infinity when the cut of type approaches the lowest type then it must be the case that the derivative is bigger than 1at least on some open neighborhood of the lowest type. □

We now move to prove the last point of the theorem, we have that for every $\vartheta\in\theta$ $v^*\left(\vartheta\right)\leq E\left[\tilde{v}\mid\tilde{\vartheta}\leq\vartheta\right]=\frac{\vartheta+\varepsilon}{2}\leq\vartheta=E\left[\tilde{v}\mid\tilde{\vartheta}=\vartheta\right]$ and we also have that for every $\vartheta\in\theta$ $1-F_{\vartheta}\left(\vartheta\right)=\frac{1}{2}$. From that we can derive that in every equilibrium the cutoff type discloses his appraisal at least with probability $\frac{1}{2}$. In addition we can deduce that the expected incremental payoff the cutoff type gets is at least $\frac{1}{2}\cdot\left(E_{\vartheta}\left[\tilde{v}\mid\tilde{\vartheta}=\vartheta\,and\,\tilde{v}\geq v^*\left(\vartheta\right)\right]-v^*\left(\vartheta\right)\right)\geq\frac{1}{2}\cdot\left(\frac{\vartheta+\varepsilon+\vartheta}{2}-\vartheta\right)=\frac{\varepsilon}{4}$ let us look at the expression for $q\left(\beta\right)=D^{-1}\left(\beta\right)$;

$$q=\int_{v^*\left(\vartheta\left(\beta\right)\right)_{ND}}^{1}\left(\widetilde{v}-v^*\left(\vartheta\left(\beta\right)\right)\right)f_{\vartheta(\beta)}\left(\widetilde{v}\right)d=$$

$$=\left(1-F_{\vartheta(\beta)}\left(v^*\left(\vartheta\left(\beta\right)\right)\right)\right)\cdot\left(E_{\vartheta(\beta)}\left[\tilde{v}\mid\tilde{v}\geq v^*\left(\vartheta\left(\beta\right)\right)\right]-v^*\left(\vartheta\left(\beta\right)\right)\right)\geq$$

$$\geq\left(1-F_{\vartheta(\beta)}\left(E_{\vartheta(\beta)}\left[\tilde{v}\right]\right)\right)\cdot\left(\frac{\vartheta\left(\beta\right)+\varepsilon-v^*\left(\vartheta\left(\beta\right)\right)}{2}\right)\geq\frac{1}{2}\cdot\left(\frac{\vartheta\left(\beta\right)+\varepsilon-\vartheta\left(\beta\right)}{2}\right)=\frac{\varepsilon}{4}$$

□

Theorem. *(3)*

Proof. Let us start with establishing the single crossing property in this setup, i.e., equilibrium must have a cutoff structure. In a sequential equilibrium there must be a common belief regarding the price the seller would get from the market in the case where no information has been disclosed, denote it by p_{nd}. It is easy to see that every seller type $\vartheta \in \theta$ with $E\left[v \mid \widetilde{\vartheta} = \vartheta\right] > p_{nd}$ would choose to hire the appraiser and every firm type $\vartheta \in \theta$ with $E\left[v \mid \widetilde{\vartheta} = \vartheta\right] < p_{nd}$ would choose not to. This follows from the fact that under the mandatory disclosure policy the expected price a seller type would get from the sellers in case it chooses to hire the appraiser is simply the expected value of the asset[21]. Note that the function $g(\vartheta) := E\left[v \mid \widetilde{\vartheta} = \vartheta\right] - E\left[v \mid \widetilde{\theta} \leq \vartheta\right]$describes the incremental expected payoff of seller type ϑ (net of the price q), in the subgame where the belief of the market is that ϑ is the cutoff type. For every price $q \in [0, \overline{q}]$there is a unique solution to the $g(\vartheta) = q$. define $\vartheta_m(q) := g^{-1}(q)$. Equilibrium of the game is completely characterized by the strategy of the firm, We argue that for $q \in [0, \overline{q}]$the unique equilibrium (strategy) is $\sigma_m(q, \vartheta) = \begin{matrix} 1 & \vartheta \geq \vartheta_m(q) \\ 0 & \vartheta < \vartheta_m(q) \end{matrix}$. First we show that this is indeed an equilibrium. If this is the seller strategy then the market must update their beliefs in case of no disclosure to the belief that the firm type is lower than $\vartheta_m(q)$, and so they will pay in this case $E\left[v \mid \widetilde{\theta} \leq \vartheta_m(q)\right] = \frac{\vartheta_m(q)+\varepsilon}{2}$. Because we have that $g(\vartheta_m(q)) = q$ and the single crossing property we get that $\vartheta_m(q)$must be the threshold type, so $\sigma_m$is the best response of the seller which means that σ_m is an equilibrium. The uniqueness follows easily from the single crossing property and the unique solution of $g(\vartheta) = q$. For $q > \bar{q}$ it holds that for every $\vartheta \in \theta$ $g(\vartheta) < q$ it follows that there is a unique equilibrium in which non of the seller types would choose to hire the appraiser. □

Theorem. *(5)*

Proof. We start with the first part of the theorem, if $\vartheta_{fs}(\lambda) - \varepsilon \geq \frac{\vartheta_{fs}(\lambda)+\varepsilon}{2}$ then it must be that the price for unappraised asset is the same under both regulations and equal to $E\left[\tilde{v} \mid \tilde{\vartheta} \leq \vartheta_{fs}(\lambda)\right] = \frac{\vartheta_{fs}(\lambda)+\varepsilon}{2}$. This is duo to the fact that types that have an appraisal in their disposal ,i.e., types with $\vartheta \geq \vartheta_{fs}(\lambda)$, would never choose to hide it under the voluntary regulation because the worst possible appraisal for these types is still above the price for unappraised asset. It follows that the behavior of the seller's types is exactly the same under both regulations and so the utility of each type is the same under both regulations. We now move to the proof of the second and more interesting part of the theorem. First it is clear that if $\vartheta_{fs}(\lambda) - \varepsilon < \frac{\vartheta_{fs}(\lambda)+\varepsilon}{2}$ then in the equilibrium of the subgame, under the voluntary disclosure regulation, where all types above

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Note that for the single crossing property to hold in the mandatory disclosure setup we do not need the firm types value distribution to be ordered according to first order stochastic domination relation, it suffice that the distributions will be ordered by their expectation.

$\vartheta_{fs}(\lambda)$ have an appraisal in their disposal and all types below $\vartheta_{fs}(\lambda)$ do not, the cutoff type $\vartheta_{fs}(\lambda)$ do not disclose his appraisal with positive probability. It follows that the price for unappraised asset under the voluntary disclosure regulation $v^*(\vartheta_{fs}(\lambda))$ is strictly below the price for unappraised asset under the mandatory regulation $\frac{\vartheta_{fs}(\lambda)+\varepsilon}{2}$. The price for unappraised asset is the utility of the types that do not hire the appraiser in equilibrium, so we get that these types strictly prefer the mandatory regulation It is left to show that also the types that do hire the appraiser in equilibrium prefer the mandatory regulation. From the fact that the price for unappraised asset is lower under the voluntary disclosure regulation we can deduce that the price for hiring the appraiser is strictly higher under the voluntary disclosure regulation (the cutoff seller type could have chose to disclose every realization, in that case the incremental payoff he get is strictly higher under the voluntary disclosure regulation simply because the price for unappraised asset under the voluntary disclosure is lower). It is clear that $U^{\lambda}_{Man}(\vartheta_{fs}(\lambda)) > U^{\lambda}_{Vol}(\vartheta_{fs}(\lambda))$, this is because type $\vartheta_{fs}(\lambda)$ is indifferent between hiring the appraiser and selling at the price for unappraised. In order to finish the proof we need first to prove the next lemma.

Lemma 12. *For every* $\hat{\vartheta} \in \theta$ $\frac{dU^{\lambda}_{Vol}(\vartheta)}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}} \leq \frac{dU^{\lambda}_{Man}(\vartheta)}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}$.

Proof. First it is clear that the payoff of all the types below the cutoff type $\vartheta_{fs}(\lambda)$ is a constant under both disclosure regulation, i.e., these types sell the asset at the price for unappraised asset and therefore their payoff does not depend on their type. The expected payoff of types above the cutoff type $\vartheta_{fs}(\lambda)$under the mandatory regulation is simply the expected value of their asset minus the price of the appraiser, i.e., for all $\vartheta > \vartheta_{fs}(\lambda)$ $U^{\lambda}_{Man}(\vartheta) = E_{\vartheta}[\tilde{v}] - q_{Man} = \vartheta - q_{Man}$. It follows that For every $\hat{\vartheta} > \vartheta_{fs}(\lambda)$ $\frac{dU^{\lambda}_{Man}(\vartheta)}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}= 1$. It is left to show that For every $\hat{\vartheta} > \vartheta_{fs}(\lambda)$ $\frac{dU^{\lambda}_{Vol}(\vartheta)}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}\leq 1$. Remember that we have that $\vartheta_{fs}(\lambda) - \varepsilon < v^*(\vartheta_{fs}(\lambda)) < \frac{\vartheta_{fs}(\lambda)+\varepsilon}{2}$ and so it must be that types$\vartheta >\vartheta_{fs}(\lambda)$ such that $\vartheta - \varepsilon \geq v^*(\vartheta_{fs}(\lambda))(\vartheta \geq v^*(\vartheta_{fs}(\lambda)) + \varepsilon)$ disclose any appraisal they get in the voluntary disclosure regulation equilibrium so their expected utility is simply $U^{\lambda}_{Vol}(\vartheta) = E_{\vartheta}[\tilde{v}] - q_{Vol} = \vartheta - q_{Vol}$, it follows that for every $\hat{\vartheta} > v^*(\vartheta_{fs}(\lambda)) + \varepsilon$ it holds that $\frac{dU^{\lambda}_{Vol}(\vartheta)}{d\vartheta}\mid_{\vartheta=\hat{\vartheta}}= 1$. Remember that we have that $\vartheta_{fs}(\lambda) - \varepsilon < v^*(\vartheta_{fs}(\lambda)) < \frac{\vartheta_{fs}(\lambda)+\varepsilon}{2}$ and so there are types $\vartheta > \vartheta_{fs}(\lambda)$ with $\vartheta - \varepsilon < v^*(\vartheta_{fs}(\lambda))$ their expected utility is:

$$U^{\lambda}_{Vol}(\vartheta) = \int_{\vartheta-\varepsilon}^{v^*(\vartheta_{fs}(\lambda))} v^*(\vartheta_{fs}(\lambda)) \cdot \frac{1}{2\varepsilon} dv + \int_{v^*(\vartheta_{fs}(\lambda))}^{\vartheta+\varepsilon} v \cdot \frac{1}{2\varepsilon} dv - q_{Vol} =$$

$$= \frac{1}{2\varepsilon}\left(\int_{\vartheta-\varepsilon}^{v^*(\vartheta_{fs}(\lambda))} v^*(\vartheta_{fs}(\lambda))\, dv + \int_{v^*(\vartheta_{fs}(\lambda))}^{\vartheta+\varepsilon} v dv\right) - q_{Vol} =$$

$$= \frac{1}{2\varepsilon}\left(v^*(\vartheta_{fs}(\lambda))(v^*(\vartheta_{fs}(\lambda)) - (\vartheta - \varepsilon)) + \frac{(\vartheta+\varepsilon)^2 - (v^*(\vartheta_{fs}(\lambda)))^2}{2}\right)$$

From that we can deduce that for every $\hat{\vartheta} < v^* (\vartheta_{fs} (\lambda)) + \varepsilon$:

$$\frac{dU^{\lambda}_{Vol} (\vartheta)}{d\vartheta} |_{\vartheta=\hat{\vartheta}} = \frac{1}{2\varepsilon} \left(\frac{1}{2} + \frac{\hat{\vartheta} - v^* (\vartheta_{fs} (\lambda))}{2\varepsilon} \right) < \frac{1}{2\varepsilon} \left(\frac{1}{2} + \frac{\hat{\vartheta} - \left(\hat{\vartheta} - \varepsilon \right)}{2\varepsilon} \right) = 1$$

This completes the proof of the lemma because we portioned the types space in to three regions and and showed that the desired property holds in each one them. The lemma ends the proof of the theorem because we already showed that $U^{\lambda}_{Man} (\vartheta_{fs} (\lambda)) > U^{\lambda}_{Vol} (\vartheta_{fs} (\lambda))$ and it is clear that if a function is larger than another function at the starting point and the derivative of the leading function is weakly larger at every point than the leading function will continue to lead and even weakly expand this lead. □

□

Theorem. *(7)*

Proof. We start the proof by taking a closer look at the equilibrium from the no appraiser benchmark. We first want to show that the willingness to pay function for the appraiser service is single-peaked with the pick at type ϑ^*. That is, for every $\vartheta_1 < \vartheta_2 \leq \vartheta^* (\vartheta_1 > \vartheta_2 \geq \vartheta^*)$ the incremental expected utility type ϑ_2 would get from deviating to hiring the appraiser is bigger than what type ϑ_1 would gain from the same deviation. The reason is the following, types bellow ϑ^*sell the asset at price p^*in the equilibrium, so they gain from hiring the appraiser in case of realizations that are above p^*.In these cases their behavior would effectively change to disclosure of their new information and selling their asset in the corresponding price. It follows that in this range higher types would have a higher willingness to pay for the appraiser service given this equilibrium. Types above ϑ^* consume their asset themselves in the equilibrium so they gain more relative to their equilibrium payoff in case of realization that are lower than p^* , this is because in these cases they gain in addition to $\triangle$ also rents duo to the fact that they manage to sell at price that is above the value of the asset for the buyers. It follows that in this range lower types would have a higher willingness to pay for the appraiser service. From continuity of the willingness to pay function we get that for every price $q < q^*$there exist a neighborhood around type ϑ^*such that all the types in this neighborhood has a profitable deviation to hiring the appraiser. One conclusion from this exercise is that when $q < q^*$ the equilibrium from the no appraiser benchmark ceases to exist. Another conclusion is that if $q > q^*$then the equilibrium from the no appraiser benchmark exists. This equilibrium is also unique (in some sense) in this region because $q^*$is the maximal willingness to pay in every potential equilibrium and so if the price is higher it must be that no type is hiring the appraiser in equilibrium. There are only two possible such equilibria the benchmark equilibrium and the total collapse of the market equilibrium, the benchmark equilibrium yields weakly higher payoff for every type and strictly higher for types below ϑ^*. Our convention would be that in such cases the dominated equilibrium is not counted as an equilibrium (Grossman & Perry solution concept reinforce this convention), in that sense we have the uniqueness result. From a more general perspective we can state the following: given a price $p > 0$ that the market is paying for the asset in case the seller did not disclose any information we can calculate the willingness to pay for the appraiser service as a function of the seller type. The willingness to pay for the appraiser service for some type $\vartheta \in \theta$ is equal to the incremental expected utility type ϑ would receive from using the appraiser service relative to the binding outside option, that is, the maximum between the expected utility from consuming the asset himself and selling it at price p. From the argument above we get that given such price $p > 0$, the willingness to pay for the appraiser service function is single-picked with a pick at the type $\vartheta \in \theta$ that solves the next equation $\vartheta - \triangle = p$;i.e., the expected utility of this type is the same whether he chooses to sell the asset at price p or consume the asset himself. For $\vartheta \in \theta$ Define $D(\vartheta, p) := \begin{cases} 1 & p < \vartheta - \varepsilon \\ \frac{\vartheta+\varepsilon-p}{2\varepsilon} & p \in (\vartheta-\varepsilon, \vartheta+\varepsilon) \\ 0 & p > \vartheta + \varepsilon \end{cases}$. Note that the incremental expected utility type ϑ would receive from using the appraiser service relative to selling at price $\vartheta - \varepsilon < p < \vartheta + \varepsilon$ is:

$$\left(\frac{\vartheta+\varepsilon-p}{2\varepsilon}\right) \cdot \left(\frac{\vartheta+\varepsilon+p}{2} - p\right) = \left(\frac{\vartheta+\varepsilon-p}{2\varepsilon}\right) \cdot \left(\frac{\vartheta+\varepsilon-p}{2}\right) =$$

$$= \frac{(\vartheta+\varepsilon-p)^2}{4\varepsilon} = \frac{(\vartheta+\varepsilon-p)^2}{4\varepsilon^2} \cdot \varepsilon =$$

$$= \left(\frac{\vartheta+\varepsilon-p}{2\varepsilon}\right)^2 \cdot \varepsilon = D(\vartheta, p)^2 \cdot \varepsilon$$

Note also that the incremental expected utility type ϑ would receive from using the appraiser service relative to consuming the asset himself is:

$$\triangle + \left(\frac{p-(\vartheta-\varepsilon)}{2\varepsilon}\right) \cdot \left(p - \left(\frac{p+(\vartheta-\varepsilon)}{2}\right)\right) = \triangle + \left(\frac{p-(\vartheta-\varepsilon)}{2\varepsilon}\right) \cdot \left(\frac{p-(\vartheta-\varepsilon)}{2}\right) =$$

$$= \triangle + \frac{(p-(\vartheta-\varepsilon))^2}{4\varepsilon} = \triangle + \frac{(p-(\vartheta-\varepsilon))^2}{4\varepsilon^2} \cdot \varepsilon =$$

$$= \triangle + \left(\frac{p-(\vartheta-\varepsilon)}{2\varepsilon}\right)^2 \cdot \varepsilon = \triangle + \left(1 - \frac{\vartheta+\varepsilon-p}{2\varepsilon}\right)^2 \cdot \varepsilon = \triangle + (1 - D(\vartheta, p))^2 \cdot \varepsilon$$

We get that the willingness to pay for the appraiser service function is the maximum of this two expressions, that is:

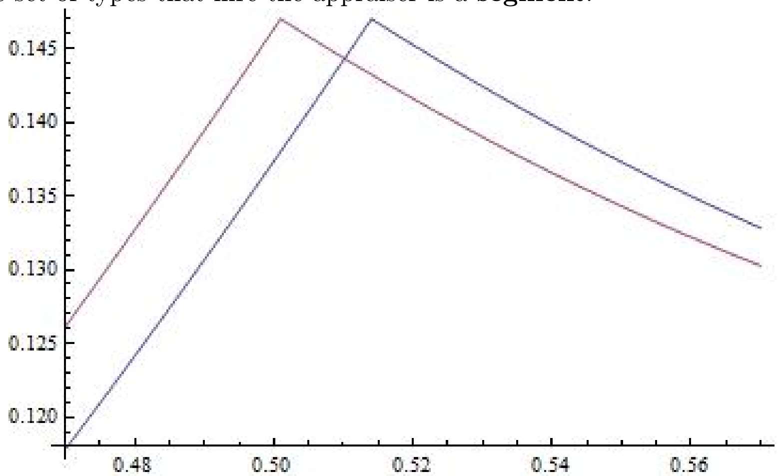
$$WTP(\vartheta, p) = max\left\{\left(\frac{\vartheta+\varepsilon-p}{2\varepsilon}\right) \cdot \left(\frac{\vartheta+\varepsilon+p}{2} - p\right), \triangle + \left(\frac{p-(\vartheta-\varepsilon)}{2\varepsilon}\right) \cdot \left(p - \left(\frac{p+(\vartheta-\varepsilon)}{2}\right)\right)\right\} =$$

$$= max\left\{D(\vartheta, p)^2 \cdot \varepsilon, \triangle + (1 - D(\vartheta, p))^2 \cdot \varepsilon\right\}$$

It follows that the willingness to pay function is a function of ϑ and p only via $D(\vartheta, p)$; That is, $WTP(\vartheta, p) = WTP(D(\vartheta, p))$. We know that $D(\vartheta, p)^2 \cdot \varepsilon = \triangle + (1 - D(\vartheta, p))^2 \cdot \varepsilon$ when the seller is indifferent between selling at price p and consuming himself; i.e., when $p = \vartheta - \triangle$. Note that $D(\vartheta, \vartheta - \triangle) = \frac{\vartheta+\varepsilon-(\vartheta-\triangle)}{2\varepsilon} = \frac{\varepsilon+\triangle}{2\varepsilon} = \frac{1}{2} + \frac{\triangle}{2\varepsilon}$. It follows that:

$$WTP(D) = \begin{cases} \triangle + (1-D)^2 \cdot \varepsilon & D > \frac{1}{2} + \frac{\triangle}{2\varepsilon} \\ D^2 \cdot \varepsilon & D \leq \frac{1}{2} + \frac{\triangle}{2\varepsilon} \end{cases}$$

A useful conclusion from this exercise is that a change from p to $p' > p$ ($p' < p$) corresponds simply to a shift to the right (left) of the willingness to pay function. A simple conclusion from the conclusion is that in equilibrium it must be that the set of types that hire the appraiser is a **segment**.



We now want to show that for every type $\hat{\vartheta}$ with $\underline{\vartheta} < \hat{\vartheta} < \vartheta^*$there exist a **unique** $l\left(\hat{\vartheta}\right) \in \left(\hat{\vartheta}, \overline{\vartheta} = 1 - \varepsilon\right]$ and a price for the appraiser service q with $q^* > q > \frac{\varepsilon}{4} > \triangle$ such that every $\vartheta \in \left(\underline{\vartheta}, \hat{\vartheta}\right)$ sell without hiring the appraiser, every $\vartheta \in \left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)$ hire the appraiser and sell (disclose and sell when it is optimal), every $\vartheta \in \left(l\left(\hat{\vartheta}\right), \overline{\vartheta}\right) \neq \phi$ does not sell, is an equilibrium. Take $\hat{\vartheta} < \vartheta^*$, it holds that for small enough $\delta > 0$,$v^*\left(\hat{\vartheta}, \hat{\vartheta} + \delta\right) \approx E\left[\tilde{v} \mid \left(\vartheta \leq \hat{\vartheta}\right)\right] \approx \frac{\hat{\vartheta} + \underline{\vartheta}}{2} > \hat{\vartheta} + \delta - \triangle$. It follows that:

$$D\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, \hat{\vartheta} + \delta\right)\right) < D\left(\hat{\vartheta} + \delta, v^*\left(\hat{\vartheta}, \hat{\vartheta} + \delta\right)\right) < \frac{1}{2} + \frac{\triangle}{2\varepsilon}$$

From that we can deduce that:

$$WTP\left(\hat{\vartheta} + \delta, v^*\left(\hat{\vartheta}, \hat{\vartheta} + \delta\right)\right) > WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, \hat{\vartheta} + \delta\right)\right)$$

Define two functions $f, g \colon \left(\hat{\vartheta}, \overline{\vartheta}\right] \to \mathbb{R}$, $f(\vartheta) := WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, \vartheta\right)\right)$ and $g(\vartheta) := WTP\left(\vartheta, v^*\left(\hat{\vartheta}, \vartheta\right)\right)$, it easy to see that both f, g are continues. We already established that for small enough $\delta > 0$ it holds that $g\left(\hat{\vartheta} + \delta\right) > f\left(\hat{\vartheta} + \delta\right)$. If they do not cross we get that $WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, \overline{\vartheta}\right)\right) < WTP\left(\overline{\vartheta}, v^*\left(\hat{\vartheta}, \overline{\vartheta}\right)\right)$, it follows that the price $q = WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, \overline{\vartheta}\right)\right)$ would induce only types above $\hat{\vartheta}$ to hire the appraiser and it would constitute an equilibrium. In addition we can see that in such a case there can not be an equilibrium with any other $\vartheta \in \left(\hat{\vartheta}, \overline{\vartheta}\right)$; That is, because it must be that $WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, \vartheta\right)\right) < WTP\left(\vartheta, v^*\left(\hat{\vartheta}, \vartheta\right)\right)$ for all $\vartheta \in \left(\hat{\vartheta}, \overline{\vartheta}\right)$, so in order to induce type $\hat{\vartheta}$ to hire the appraiser it must be the case that $q \leq WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, \vartheta\right)\right)$. But then we get a contradiction because for small enough $\delta > 0$ type $\vartheta + \delta$ would want to deviate to hiring the appraiser (in such an equilibrium it must be that types above the cut off type ϑ would find it optimal to consume their asset given q and $p = v^*\left(\hat{\vartheta}, \vartheta\right)$). If f, g cross only once we get that this cross is the unique equilibrium, that is, if we have a unique $l\left(\hat{\vartheta}\right) \in \left(\hat{\vartheta}, \overline{\vartheta}\right]$ such that $f\left(l\left(\hat{\vartheta}\right)\right) = g\left(l\left(\hat{\vartheta}\right)\right)$ it is easy to see that the price $q = WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)\right)$ would induce an equilibrium. From the fact that they cross only once we can derive that $D\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)\right) < \frac{1}{2} + \frac{\triangle}{2\varepsilon}$ and $D\left(l\left(\hat{\vartheta}\right), v^*\left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)\right) > \frac{1}{2} + \frac{\triangle}{2\varepsilon}$. It follows that given the price $q = WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)\right)$ for the appraiser service and a price $p = v^*\left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)$ that the seller receives for the asset in case he does not disclose information, a type $\vartheta \in \theta$ would hire the appraiser if and only if $\vartheta \in \left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)$.$q = WTP\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)\right) \geq \frac{\varepsilon}{4}$ because $D\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)\right) > \frac{1}{2}$ (and also remember that $D\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, l\left(\hat{\vartheta}\right)\right)\right) < \frac{1}{2} + \frac{\triangle}{2\varepsilon}$). The uniqueness of the equilibrium in the case where f , g cross only once follows from the next argument. First recall that the set of types that acquire the appraiser service in any equilibrium must be a segment. Denote by $l\left(\hat{\vartheta}\right)$the type that solves $f\left(\vartheta\right) = g\left(\vartheta\right)$ and denote by $\tilde{\vartheta} \in \left(\hat{\vartheta}, \overline{\vartheta} = 1 - \varepsilon\right]$ a potential candidate for equilibrium; That is, we want to show that if $\tilde{\vartheta} \neq l\left(\hat{\vartheta}\right)$ then every type $\vartheta \in \left(\underline{\vartheta}, \hat{\vartheta}\right)$ sell without hiring the appraiser, every $\vartheta \in \left(\hat{\vartheta}, \tilde{\vartheta}\right)$ hire the appraiser and sell (disclose and sell when it is optimal), every $\vartheta \in \left(\tilde{\vartheta}, \overline{\vartheta}\right)$ does not sell, is **not** an equilibrium.. If $\tilde{\vartheta} < l\left(\hat{\vartheta}\right)$ then it must be that $g\left(\tilde{\vartheta}\right) > f\left(\tilde{\vartheta}\right)$, in order to induce type $\hat{\vartheta}$ to acquire the appraiser it must be that $q \leq f\left(\tilde{\vartheta}\right)$, but then for continuity of the willingness to pay function we get that for small enough δ type $\tilde{\vartheta} + \delta$ would find it optimal to deviate and acquire the appraiser. If $\tilde{\vartheta} > l\left(\hat{\vartheta}\right)$ then it must be that $g\left(\tilde{\vartheta}\right) < f\left(\tilde{\vartheta}\right)$, in order to induce type $\tilde{\vartheta}$ to acquire the appraiser it must be that $q \leq g\left(\tilde{\vartheta}\right)$, but then for continuity of the willingness to pay function we get that for small enough δ type $\hat{\vartheta} - \delta$ would find it optimal to deviate and acquire the appraiser. If f , g cross more than once, it is easy to see that the only way that this can occur is if there exist $\check{\vartheta} \in \left(\hat{\vartheta}, \overline{\vartheta}\right]$ such that $D\left(\hat{\vartheta}, v^*\left(\hat{\vartheta}, \vartheta\right)\right) = D\left(\vartheta, v^*\left(\hat{\vartheta}, \vartheta\right)\right) = 1$ for every $\vartheta > \check{\vartheta}$. But this can not be an equilibrium because $\underline{\vartheta} < \hat{\vartheta} < \vartheta^*$. If this was an equilibrium it must be that $v^*\left(\hat{\vartheta},\vartheta\right)=\frac{\hat{\vartheta}+\varepsilon}{2}>\hat{\vartheta}-\triangle$. $\triangle<\varepsilon$ and so for small enough δ type $\hat{\vartheta}+\delta$ must have realizations below $v^*\left(\hat{\vartheta},\vartheta\right)=\frac{\hat{\vartheta}+\varepsilon}{2}>\hat{\vartheta}-\triangle$ in his support, it follows that such types would find it optimal to deviate to not disclosing realizations below $v^*\left(\hat{\vartheta},\vartheta\right)=\frac{\hat{\vartheta}+\varepsilon}{2}>\hat{\vartheta}-\triangle$, this is a contradiction to $D\left(\hat{\vartheta},v^*\left(\hat{\vartheta},\vartheta\right)\right)=1$. One more thing we have to show is that there can not be an equilibrium in which the segment of types that hire the appraiser is contained in the segment $\left(\vartheta^*,\overline{\vartheta}\right]$ and that $q>\triangle$. Assume by contradiction that there exist such an equilibrium, and denote by $(\vartheta_1,\vartheta_2)$ the segment that hire the appraiser in that equilibrium $(\vartheta_1>\vartheta^*)$ and by $\tilde{p}$ the price a seller gets for the asset in case he sells without disclosing any information. If $q>\triangle$ it must be that $D(\vartheta_1,\tilde{p})<\frac{1}{2}+\frac{\triangle}{2\varepsilon}$ (otherwise equilibrium must have the property that all types that hire that appraiser disclose with probability one, but then it must be that $q\leq\triangle$). It follows that the outside option that binds for type $\vartheta_1$is to sell at price $\tilde{p}$. From that we can derive that $\tilde{p}=v^*(\vartheta_1,\vartheta_2)>\vartheta_1-\triangle$. But this is impossible because on one hand $v^*(\vartheta_1,\vartheta_2)<\frac{\vartheta_1+\varepsilon}{2}$, and on the second hand because $\vartheta_1>\vartheta^*$it must be that $\vartheta_1-\triangle>\frac{\vartheta_1+\varepsilon}{2}$. We established that in any equilibrium in which the segment of types that hire the appraiser does not include $\underline{\vartheta}=\varepsilon$ the price for the appraiser service must be at least $\frac{\varepsilon}{4}$. If the segment of types that acquire the appraiser does include $\underline{\vartheta}=\varepsilon$ it is easy to see that we must have unraveling in equilibrium; i.e., $v^*(\underline{\vartheta},\cdot)=0$. It follows that the price for the appraiser service must be below $\triangle$, that is, $q\leq\triangle$. We can conclude this part of the proof and state the following result, in every equilibrium in which a positive measure of types acquire the appraiser the price for the appraiser service q must be below $\triangle$ or above $\frac{\varepsilon}{4}$ $\left(q<\triangle \text{ or } q>\frac{\varepsilon}{4}\right)$. It remains to show that if $q\leq\frac{\varepsilon}{4}$ the only equilibrium in which no type acquire the appraiser is the one where all types consume their asset themselves. Denote the price a seller gets for the assets in case of selling without disclosing any information by v^*. It must be that equilibrium would have a cutoff structure, that is, all types below type $\vartheta=v^*+\triangle$ sell and all types above this type consume their asset themselves. In an equilibrium in which positive measure of types sell their asset it must be that $\frac{v^*+\varepsilon}{2}=v^*-\triangle$. It follows that the only candidate for equilibrium in which no type acquire the appraiser and there is a positive measure of types that sell their asset is the benchmark equilibrium; i.e., all types below ϑ^*sell and all types above ϑ^* consume their asset themselves. But we already saw that if $q<q^*=\left(\frac{1}{2}+\frac{\triangle}{2\varepsilon}\right)^2\cdot\varepsilon$ this equilibrium does not exist, and it is clear that $q^*>\frac{\varepsilon}{4}$. The conclusion is that the only equilibrium with $q\leq\frac{\varepsilon}{4}$ in which no type acquire the appraiser is the equilibrium in which all types consume their asset themselves. In summary we proved that the only equilibrium with $\triangle<q\leq\frac{\varepsilon}{4}$ is the equilibrium in which no type acquire the appraiser and all types consume their asset themselves; i.e., a total collapse of the market for the asset. □

Theorem. *(8)*

Proof. We prove the theorem with the help of a series of lemmas.

Lemma 13. *Define* $END(\vartheta_1,\vartheta_2,\tilde{v}) := E\left[v \mid \vartheta \le \vartheta_1 \text{ or } (\vartheta_1 < \vartheta \le \vartheta_2 \text{ and } v \le \tilde{v})\right]$. *The next equality holds for every* $\vartheta_1 \le \vartheta_2$: $\frac{\partial[v^*(\vartheta_1,\vartheta_2)]}{\partial\vartheta_1} = \frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))}$.

Proof. We have that $v^*(\vartheta_1,\vartheta_2) = END(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))$, it follows that:

$$\frac{\partial[v^*(\vartheta_1,\vartheta_2)]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2)} = \frac{\partial[END(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))} =$$

$$= \frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))} + \frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\tilde{v}}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))} * \frac{\partial[v^*(\vartheta_1,\vartheta_2)]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2)}$$

.

From the "Minimum Principle" we have that $\frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\tilde{v}}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))} = 0$.

We can conclude that $\frac{\partial[v^*(\vartheta_1,\vartheta_2)]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2)} = \frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))}$. □

Lemma 14. $lim_{\vartheta_1\to\vartheta_2}\frac{\partial[v^*(\vartheta_1,\vartheta_2)]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2)} = 1 \iff \vartheta_2 = \left(7 - 4\cdot\sqrt{2}\right)\cdot\varepsilon$.

Proof. From lemma 13 we have that:

$$lim_{\vartheta_1\to\vartheta_2}\frac{\partial[v^*(\vartheta_1,\vartheta_2)]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2)} = lim_{\vartheta_1\to\vartheta_2}\frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))}$$

$$\frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))} =$$

$$= \frac{1/6(\vartheta_2-\vartheta_1)^2 + \varepsilon(-\varepsilon+\vartheta_1) + \varepsilon(\varepsilon+\vartheta_1) - (\vartheta_2-\vartheta_1)(-\varepsilon+(2(\vartheta_2-\vartheta_1))/3+\vartheta_1) - 1/2(\varepsilon-\vartheta_2+v^*(\vartheta_1,\vartheta_2))(-\varepsilon+\vartheta_2+v^*(\vartheta_1,\vartheta_2))}{1/2(\vartheta_2-\vartheta_1)^2 + 2\varepsilon(-\varepsilon+\vartheta_1) + (\vartheta_2-\vartheta_1)(\varepsilon-\vartheta_2+v^*(\vartheta_1,\vartheta_2))} -$$

$$- \frac{(\varepsilon+\vartheta_1-v^*(\vartheta_1,\vartheta_2))(\varepsilon(-\varepsilon+\vartheta_1)(e+\vartheta_1) + 1/2(\vartheta_2-\vartheta_1)^2(-\varepsilon+(2(\vartheta_2-\vartheta_1))/3+\vartheta_1) + 1/2(\vartheta_2-\vartheta_1)(\varepsilon-\vartheta_2+v^*(\vartheta_1,\vartheta_2))(-\varepsilon+\vartheta_2+v^*(\vartheta_1,\vartheta_2)))}{(1/2(\vartheta_2-\vartheta_1)^2 + 2\varepsilon(-\varepsilon+\vartheta_1) + (\vartheta_2-\vartheta_1)(\varepsilon-\vartheta_2+v^*(\vartheta_1,\vartheta_2)))^2}$$

It follows that:

$$lim_{\vartheta_1\to\vartheta_2}\frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))} = \frac{(\varepsilon+\vartheta_2-v^*(\vartheta_1,\vartheta_2))\,v^*(\vartheta_1,\vartheta_2)}{4\cdot\varepsilon(\vartheta_2-\varepsilon)}$$

We have that $lim_{\vartheta_1\to\vartheta_2}v^*(\vartheta_1,\vartheta_2) = \frac{\vartheta_2+\varepsilon}{2}$, so we get that:

$$lim_{\vartheta_1\to\vartheta_2}\frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))} = \frac{(\vartheta_2+\varepsilon)^2}{16\cdot\varepsilon\cdot(\vartheta_2-\varepsilon)^2}$$

We solve $\frac{(\vartheta_2+\varepsilon)^2}{16\cdot\varepsilon\cdot(\vartheta_2-\varepsilon)^2} = 1$ and find that the relevant solution is $\hat{\vartheta}_2 = \left(7 - 4\cdot\sqrt{2}\right)\cdot\varepsilon$. □

Lemma 15. *If* $\hat{\vartheta}_2 \ge \vartheta^*$ *(*$\left(7-4\cdot\sqrt{2}\right)\cdot\varepsilon \ge \varepsilon + 2\triangle \iff (12-8\sqrt{2})\cdot\frac{\varepsilon}{4} \ge \triangle$*) then* $lim_{\vartheta_1\to\vartheta^*}\frac{\partial[v^*(\vartheta_1,\vartheta^*)]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta^*)} \ge 1$

Proof. This result follows directly from the proof of lemma 14; i.e., for $\vartheta_2 \le \hat{\vartheta}_2$ it holds that $lim_{\vartheta_1\to\vartheta_2}\frac{\partial[END(\vartheta_1,\vartheta_2,\tilde{v})]}{\partial\vartheta_1}\mid_{(\vartheta_1,\vartheta_2,v^*(\vartheta_1,\vartheta_2))} = \frac{(\vartheta_2+\varepsilon)^2}{16\cdot\varepsilon\cdot(\vartheta_2-\varepsilon)^2} \ge 1$. □

Lemma 16. *If* $\vartheta_1 < \vartheta^* \leq \hat{\vartheta}_2$ *then* $\frac{\partial[v^*(\vartheta_1,\vartheta^*)]}{\partial\vartheta_1} |_{(\vartheta_1,\vartheta^*)} \geq 1$.

Proof. We already established that for every $\tilde{\vartheta} \in \theta$ The function $v^*\left(\cdot, \tilde{\vartheta}\right)$ is concave, it follows that if $\vartheta_1 < \vartheta^* \leq \hat{\vartheta}_2$ then:

$$\frac{\partial\left[v^*\left(\vartheta_1, \vartheta^*\right)\right]}{\partial\vartheta_1} |_{(\vartheta_1,\vartheta^*)} \geq lim_{\vartheta_1 \to \vartheta^*} \frac{\partial\left[v^*\left(\vartheta_1, \vartheta^*\right)\right]}{\partial\vartheta_1} |_{(\vartheta_1,\vartheta^*)} \geq 1$$

□

Lemma 17. *If* $\vartheta^* \leq \hat{\vartheta}_2$ *then for every* $\vartheta_1 < \vartheta^*$ *it holds that* $WTP\left(\vartheta_1, v^*\left(\vartheta_1, \vartheta^*\right)\right) > WTP\left(\vartheta^*, v^*\left(\vartheta_1, \vartheta^*\right)\right)$.

Proof. Take $\vartheta_1 < \vartheta^*$, we have that $v^*\left(\vartheta^*, \vartheta^*\right) = \vartheta^* - \triangle$ and that for every $\vartheta' \in \left(\vartheta_1, \vartheta^*\right)$ it holds that $\frac{\partial\left[v^*\left(\vartheta', \vartheta^*\right)\right]}{\partial\vartheta_1} |_{(\vartheta',\vartheta^*)} \geq 1$, it follows that $v^*\left(\vartheta_1, \vartheta^*\right) \leq \vartheta_1 - \triangle$. From that we can derive that the pick of the function $WTP\left(\cdot, v^*\left(\vartheta_1, \vartheta^*\right)\right)$ is weakly smaller than ϑ_1.

Note that we also have that $v^*\left(\vartheta_1, \vartheta^*\right) > \vartheta_1 - \varepsilon$. It follows, From these two facts, that $WTP\left(\vartheta_1, v^*\left(\vartheta_1, \vartheta^*\right)\right) > WTP\left(\vartheta^*, v^*\left(\vartheta_1, \vartheta^*\right)\right)$. □

The conclusion from the last equation is that the first crossing of the functions $f\left(\vartheta\right) := WTP\left(\vartheta, v^*\left(\vartheta_1, \vartheta\right)\right)$, $g\left(\vartheta\right) := WTP\left(\vartheta_1, v^*\left(\vartheta_1, \vartheta\right)\right)$ must be smaller than ϑ^*, that is, $l\left(\vartheta_1\right) < \vartheta^*$.

From the first part of the theorem and from the fact that $l\left(\vartheta^*\right) = \vartheta^*$we infer that for every $\vartheta \in \left(\underline{\vartheta}, \vartheta^*\right)$ $l\left(\vartheta\right) < \vartheta^*$. We already established that if $q^* > q > \triangle$ the next set exhaust all possible equilibria:

$$\left\{\left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right), \tilde{p} = WTP\left(\tilde{\vartheta}, \left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right)\right)\right)\right) \mid \tilde{\vartheta} \leq \vartheta^*\right\} \cup \{MCE\}$$

Where MCE denotes the Market Collapse equilibrium and $\left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right), WTP\left(\tilde{\vartheta}, \left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right)\right)\right)\right)$ denotes the equilibrium in which every seller type $\vartheta \in \left(\underline{\vartheta}, \tilde{\vartheta}\right)$ sell without hiring the appraiser, every $\vartheta \in \left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right)\right)$ hire the appraiser and sell (sell without disclosing if $v \leq v^*\left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right)\right)$ and with disclosing otherwise), every type $\vartheta \in \left(l\left(\tilde{\vartheta}\right), \overline{\vartheta}\right)$ does not hire the appraiser and does not sell. First, it is clear that the benchmark equilibrium is better than the Market Collapse equilibrium. Second, it is also clear that every type $\vartheta \in \left[\vartheta^*, \overline{\vartheta}\right]$ is indifferent between the benchmark equilibrium and any equilibrium from the set. Given any equilibrium from the set, $\left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right), WTP\left(\tilde{\vartheta}, \left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right)\right)\right)\right)$ for some $\tilde{\vartheta} \leq \left(\underline{\vartheta}, \vartheta^*\right)$, every type $\vartheta \in \left(l\left(\tilde{\vartheta}\right), \vartheta^*\right)$ strictly prefers the benchmark equilibrium. This because in the $\tilde{\theta}$ equilibrium these types payoff is their payoff from consuming their asset themselves and in the benchmark equilibrium they sell in the price $p^* = \vartheta^* - \triangle$. Types $\vartheta \in \left(\underline{\vartheta}, \tilde{\vartheta}\right)$also strictly prefers the benchmark equilibrium because in both equilibrium these types sell without hiring the appraiser, but in the benchmark equilibrium they sell in a higher price:

$p^* = \vartheta^* - \triangle = \frac{\vartheta^*+\varepsilon}{2} > \frac{\tilde{\vartheta}+\varepsilon}{2} \geq v^*\left(\tilde{\vartheta}, l\left(\tilde{\vartheta}\right)\right) = \tilde{p}$

Type $l\left(\tilde{\vartheta}\right)$ gets a payoff of $l\left(\tilde{\vartheta}\right) - \triangle$ in the $\tilde{\vartheta}$ equilibrium, and because we have that $l\left(\tilde{\vartheta}\right) < \vartheta^*$we can deduce that $l\left(\tilde{\vartheta}\right) - \triangle < \vartheta^* - \triangle = p^*$. It follows that type $l\left(\tilde{\vartheta}\right)$ strictly prefers the benchmark equilibrium because his payoff in this equilibrium is p^*.

All types $\vartheta < l\left(\tilde{\vartheta}\right)$ also get a payoff of p^*in the benchmark equilibrium, so it remain to show that the payoff of these types is lower than the payoff of type $l\left(\tilde{\vartheta}\right)$ in the $\tilde{\vartheta}$equilibrium. This is true because the payoff of types that hire the appraiser is strictly monotonic increasing in the type. □

Theorem. *(11)*

Proof. We will prove this theorem with the help of a series of lemmas: first let us introduce to functions; for every possible common value $v \in V$ Define two functions: $a^v(\vartheta) := (1 - F_\vartheta(v)) \cdot \left(E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} > v\right] - v\right)$ $b^v(\vartheta) := F_\vartheta(v) \cdot \left(v - E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} < v\right]\right)$.

Lemma 18. *For every $\vartheta_1 < \vartheta_2$ if $F_{\vartheta_2}(v) < 1$ then $a^v(\vartheta_2) > a^v(\vartheta_1)$(the function a is the constant zero function as long as $F_\vartheta(v) = 1$ and then it is strictly monotonic increasing), and if $F_{\vartheta_1}(v) > 0$ then $b^v(\vartheta_1) > b^v(\vartheta_2)$ (the function b is strictly monotonic decreasing until it reaches zero and then it is constant zero function).*

Proof. It is well known that if $F_1\, fosd\, F_2$ then for every common value $v \in V$ if $F_2(v) < 1$ then $E_{F_1}[\tilde{v} \mid \tilde{v} > v] > E_{F_2}[\tilde{v} \mid \tilde{v} > v]$ and if $F_1(v) > 0$ then $E_{F_1}[\tilde{v} \mid \tilde{v} < v] > E_{F_2}[\tilde{v} \mid \tilde{v} < v]$. It follows that for every $\vartheta_1, \vartheta_2 \in \theta$ such that $\vartheta_1 > \vartheta_2$ if $F_{\vartheta_1}(v) = 1$ then $a^v(\vartheta_1) = a^v(\vartheta_2) = 0$, if $F_{\vartheta_1}(v) < 1$ then if $F_{\vartheta_2}(v) = 1$ then $a^v(\vartheta_1) > a^v(\vartheta_2) = 0$ but if $F_{\vartheta_2}(v) < 1$ then $E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta_1, \tilde{v} > v\right] > E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta_2, \tilde{v} > v\right]$ it follows that $a^v(\vartheta_1) > a^v(\vartheta_2)$. If $F_{\vartheta_2}(v) = F_{\vartheta_1}(v) = 0$ then $b^v(\vartheta_1) = b^v(\vartheta_2) = 0$, if $F_{\vartheta_2}(v) > 0, F_{\vartheta_1}(v) = 0$ then $b^v(\vartheta_2) > b^v(\vartheta_1) = 0$, and if $F_{\vartheta_2}(v) > F_{\vartheta_1}(v) > 0$ then $E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta_1, \tilde{v} < v\right] > E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta_2, \tilde{v} < v\right]$ it follows that $v - E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta_1, \tilde{v} < v\right] < v - E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta_2, \tilde{v} < v\right]$ and we get that $b^v(\vartheta_2) > b^v(\vartheta_1)$. □

Lemma 19. *Given a price p for unappraised asset, the willingness to pay for the appraisal function as the next structure: $WTP^p(\vartheta) = \begin{cases} a^p(\vartheta) & \mu_\vartheta < p \\ b^p(\vartheta) + \triangle & \mu_\vartheta \geq p \end{cases}$,i.e., if there exist $\vartheta \in \theta$ such that $\mu_\vartheta = p$ then $WTP^p(\vartheta)$ is single-peaked.*

Proof. The willingness to pay for the appraisal function is the difference between the expected payoff in case the seller has the appraisal and the expected payoff from the favorite alternative option. It is easy to see that if $\mu_\vartheta < p$ the favorite alternative option is to sell at price p, but if $\mu_\vartheta > p$ the favorite alternative option is to hold on to the asset. Note that $a^p(\vartheta) := (1 - F_\vartheta(p)) \cdot \left(E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} > p\right] - p\right)$ is exactly the incremental expected payoff a seller would get from hiring the appraiser in the case that is favorite alternative option is to sell at price p, and that $b^p(\vartheta) + \triangle = F_\vartheta(p) \cdot \left(p - E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} < p\right]\right) + \triangle$ is exactly the incremental expected payoff a seller would get from hiring the appraiser in the case that is favorite alternative option is to hold on to the asset. □

Lemma 20. *Given prices q for hiring the appraiser and p for unappraised asset the set of types that prefers to hire the appraiser is a segment or the empty set.*

Proof. If $p < min_{\vartheta\in\theta}\mu_\vartheta$ then $WTP^p(\vartheta) = b^p(\vartheta) + \triangle$. We have from Lemma 10 that the function b is strictly monotonic decreasing until it reaches zero and then it is constant zero function, so it must be that for any positive q the set of types that find it optimal to hire the appraiser is a segment with the next structure $[\underline{\vartheta}, \vartheta]$ for some $\vartheta \in \theta$ or the empty set. If $p > max_{\vartheta\in\theta}\mu_\vartheta$then $WTP^p(\vartheta) = a^p(\vartheta)$. We have from Lemma 10 that the function a is the constant zero function as long as $F_\vartheta(v) = 1$ and then it is strictly monotonic increasing, so it must be that for any positive q the set of types that find it optimal to hire the appraiser is a segment with the next structure $\left[\vartheta, \overline{\vartheta}\right]$ for some $\vartheta \in \theta$ or the empty set. If there exist $\vartheta \in \theta$ such that $p = \mu_\vartheta$ then $WTP^p(\vartheta)$ is single-peaked and for any positive q the set of types that find it optimal to hire the appraiser is a segment that includes type ϑ or the empty set. □

Lemma 21. *If $q > \triangle$ there can not exist an equilibrium in which the set of types that find it optimal to hire the appraiser is a segment with the next structure $[\underline{\vartheta}, \vartheta]$ for some $\vartheta > \underline{\vartheta}$.*

Proof. Assume by contradiction the existence of such an equilibrium. if $\vartheta < \overline{\vartheta}$ it must be that the favorite alternative option for type ϑ is to hold on to the asset. It can not be that ϑ is indifferent between the alternative options because this means that ϑ is the type with maximum willingness to pay and so from continuity it must be that if there are types smaller than him that finds it optimal to hire the appraiser then it must be the case that also types bigger than him would find it optimal to hire the appraiser, this is a contradiction. It is also not possible that the favorite alternative option for type ϑ is to sell the asset without hiring the appraiser, this is because in this region $WTP^p = a^p$ and we saw that a^p is monotonic increasing. It follows that in such an equilibrium it must be that all type bigger than ϑ hold on to the asset, from that we can deduce that there must be unraveling in such an equilibrium. The reason for that is that all the types that choose to sell have verifiable evidence, in such environment it was already proved that the unique equilibrium is the unraveling equilibrium. Now we get a contradiction because if there is unraveling all the types the hire the appraiser get an incremental expected payoff of exactly $\triangle$ and so it is not possible that they chose to hire the appraiser and pay a fee $q > \triangle$. If $\vartheta = \overline{\vartheta}$ it is clear that the unique equilibrium is the unraveling equilibrium and we get the same contradiction. □

Lemma 22. *In an equilibrium in which the set of types that hire the appraiser is a segment with the next structure* $[\vartheta_1, \vartheta_2]$ *for some* $\vartheta_2 > \vartheta_1 > \underline{\vartheta}$ *it must be that* $q > S := min_{\vartheta \in \theta} \left(1 - F_{\vartheta}\left(\underline{\mu_{\vartheta}}\right)\right) \cdot \left(E\left[\tilde{v} \mid \tilde{\vartheta} = \vartheta, \tilde{v} > \underline{\mu_{\vartheta}}\right] - \underline{\mu_{\vartheta}}\right)$.

Proof. From the same kind of argument we used in the proof of the previous lemma, if $\vartheta_1 > \underline{\vartheta}$ it must be that the favorite alternative option for type ϑ_1 is to sell the asset in the price p that the market pays for assets without a disclosed appraisal, it follows that $q = a^p(\vartheta_1)$. What can we say about this price p? We know that this price must be bounded from above by $\underline{\mu_{\vartheta_1}} := E\left[\tilde{v} \mid \tilde{\vartheta} < \vartheta_1\right]$,this is because all the the types the hire the appraiser will conceal the appraisal if it is below p. In equilibrium it must be that $p = E\left[\tilde{v} \mid \tilde{\vartheta} < \vartheta_1 \; or \; \left(\tilde{\vartheta} \in [\vartheta_1, \vartheta_2] \; and \; \tilde{v} < p\right)\right] < \underline{\mu_{\vartheta_1}} := E\left[\tilde{v} \mid \tilde{\vartheta} < \vartheta_1\right]$. We get that $q = a^p(\vartheta_1) > a^{\underline{\mu_{\vartheta_1}}}(\vartheta_1)$. It is clear that $a^{\underline{\mu_{\vartheta_1}}}(\vartheta_1) \geq min_{\vartheta \in \theta} a^{\underline{\mu_{\vartheta_1}}}(\vartheta) = S$. Now we can deduce that q ,the price for the appraiser services, in bounded from below by S in such an equilibrium. □

The conclusion from the last Lemma is that if $\triangle < q < S$ there can not be an equilibrium in which positive measure of sellers types hire the appraiser. It is left to show that in this parameter region the only equilibrium in which no positive measure of sellers types hire the appraiser is the collapse of the market equilibrium. Let us first describe this equilibrium in greater details, in this equilibrium all seller's types choose to hold on to their asset, so if the market observes a seller that offer his asset for sell this is an off the equilibrium path behavior. The description of this no trade equilibrium includes the property that if the market observes such off the equilibrium path action it believes that this action was done by a seller that deviated to hiring the appraiser and got the worse appraisal possible, i.e., $v = 0$. If this is the belief of the market it follows that the market would pay $p = \triangle$ in case it observes this off the equilibrium path action. From this we get that it must be that if a seller chooses to deviate to hire the appraiser he would optimally disclose every appraisal it gets. It follows that the incremental payoff from hiring the appraiser is exactly $\triangle$ and so it can not be a profitable deviation if $q > \triangle$. This argument establishes that the collapse of the market equilibrium exists in this parameter region . It is clear that the only other possible equilibrium where no positive measure of sellers types hire the appraiser is the one from the model without an appraiser, given a price p types ϑ with $\mu_{\vartheta} \leq p$ would sell and types with $\mu_{\vartheta} > p$ would hold on to the asset. There is a unique type $\vartheta^* \in \theta$ with $\underline{\mu_{\vartheta^*}} + \triangle = E\left[\tilde{v} \mid \tilde{\vartheta} < \vartheta^*\right] + \triangle = \mu_{\vartheta^*}$, and so the only possible other equilibrium with the property that non of the seller types hire the appraiser is the one where all types below type ϑ^*sell the asset and all types above type ϑ^*hold on to the asset. In this equilibrium type ϑ^*has the maximal willingness to pay for the appraiser service and it is exactly $\left(1-F_{\vartheta}\left(\underline{\mu_{\vartheta^*}}\right)\right)\cdot\left(E\left[\tilde{v}\mid\tilde{\vartheta}=\vartheta,\tilde{v}>\underline{\mu_{\vartheta^*}}\right]-\underline{\mu_{\vartheta^*}}\right)$. It is clear that $\left(1-F_{\vartheta}\left(\underline{\mu_{\vartheta^*}}\right)\right)\cdot\left(E\left[\tilde{v}\mid\tilde{\vartheta}=\vartheta,\tilde{v}>\underline{\mu_{\vartheta^*}}\right]-\underline{\mu_{\vartheta^*}}\right)\geq min_{\vartheta\in\theta}\left(1-F_{\vartheta}\left(\underline{\mu_{\vartheta}}\right)\right)\cdot\left(E\left[\tilde{v}\mid\tilde{\vartheta}=\vartheta,\tilde{v}>\underline{\mu_{\vartheta}}\right]-\underline{\mu_{\vartheta}}\right)=S$, ans so this equilibrium does not exist if $q<S$ because there exist a measurable set of types that finds it optimal to deviate to hiring the appraiser. This ends the proof of the part of the theorem regarding the uniqueness of the collapse of the market equilibrium if $\triangle<q<S$. It is left to show that if $q<\triangle$ in the unique all the seller's types hire the appraiser and there is unraveling in the disclosure phase of the game. We all ready showed that an equilibrium where the set of types that hire the appraiser has the next structure $[\vartheta_1,\vartheta_2]$ for some $\vartheta_2>\vartheta_1>\underline{\vartheta}$ can not exist if $q<S$, and we have that $q<\triangle<S$. In addition we showed that the equilibrium from the no appraiser benchmark does not exists if $q<S$. Clearly the collapse of the market equilibrium does not exists if $q<\triangle$. So we get that an equilibrium must have the next property; the set of types that hire the appraiser has this structure $[\underline{\vartheta},\vartheta]$ for some $\vartheta>\underline{\vartheta}$. We already showed that if $\vartheta<\overline{\vartheta}$ it must be that the types in the segment $\left[\vartheta,\overline{\vartheta}\right]$ hold on to the asset, but this obviously can not happen in equilibrium when $q<\triangle$. It follows that it must be that $\vartheta=\overline{\vartheta}$, that is, the set of types that hire the appraiser in equilibrium must be the entire set of types $\theta=\left[\underline{\vartheta},\overline{\vartheta}\right]$. It is again clear that in such a case there must be unraveling in the disclosure stage. $\square$