Prize Sharing in Collective Contests

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Abstract

The characteristics of endogenously determined sharing rules and the group-size paradox are studied in a model of group contest with the following features: (i) The prize has mixed private-public good characteristics. (ii) Groups can differ in marginal cost of effort and their membership size (iii) Every group decides how to share the prize without knowing the sharing rules of the other groups. We provide simple characterizations of the relationship between group characteristics, performance of the competing groups (winning probability and per capita expected utility) and the type of sharing rules they select. The role of the nature of the prize is also considered.

1. Introduction

This study considers collective contests for group-specific benefits. Examples of such contests include a competition by local governments for subsidies, an R&D race by research groups or a competition for resources by countries. It is well-known that the competing groups in such contests have to cope with the collective-action or the free-rider problem, as very clearly argued by Olson (1965). Given the prevalence of collective contests and the significant efficiency and distributional implications of the collective action problem, the question what kind of groups are advantageous in such contests has been of major concern in the relevant literature in economics and political science.

The studies attempting to clarify the relationship between the characteristics of a group and its performance viz., contest winning probability and per capita utility, can be classified by the nature of the contested prize. The case where the prize is a pure public good for each group is treated by Katz, Nitzan and Rosenberg (1990) and Riaz, Shogren and Johnson (1995).[3] It has been concluded that in this setting a group with larger membership is advantageous, namely, its winning probability is larger than or equal to that of a smaller group. Esteban and Ray (2001) study collective contests with prizes that have mixed public-private characteristics, as in Chamberlin (1974). They have been able to derive a simple sufficient condition relating to the elasticity of the marginal cost of effort that ensures that a larger group attains a higher winning probability.

In the model of Esteban and Ray, however, the private part of the prize is assumed to be divided equally among the members of the winning group. Other

[3] Baik (1993) also considers this kind of contests, but his main concern is the effect of group-members asymmetry on the winning probability of the group. An extension of his analysis to more general contests and to an all-pay auction appears, respectively, in Nti (1988) and Baik, Kim and Na (2001).

possibilities of division are disregarded. In contrast, studies of collective contests on private-good prizes do consider alternative ways of prize division among the members of the winning group. One possibility is that, as in Katz and Tokatlidu (1996) and Warneryd (1998), the division of the prize is also determined non-cooperatively, subsequent to its award to the winning group. That is, in a first stage the groups compete on the prize and then, in a second stage, the prize is contested again by the members of the winning group. The studies following this line of research, (see Hausken (2005) and Konrad (2006) for a comprehensive survey) treat explicitly the intra and inter- group conflicts. But in this hierarchical structure of contests the private-good prize is not really shared by the members of the winning group. In fact, such two-stage competition implies that any pre-agreement regarding the prize sharing among the group members is impossible and the partition of the total population of the individuals into groups only determines the initial inter-group and the possible final intra-group contests.

Another possibility is that, prior to the contest on the private-good prize, members of a group agree on the sharing rule of the prize. As argued by Olson (1965), such a rule can be interpreted as a device to provide an adequate "selective incentive". Nitzan (1991) parameterizes the sharing rule applied by a group as a linear combination between the equalitarian and the relative effort sharing rules. Under such rules part of the prize is divided equally among the group members and the rest is divided proportionally to the members' efforts. Lee (1995) proposes an extended two-stage contest, adding a stage where each group selects the prize sharing rule so that it maximizes its welfare. The complete characterization of equilibria in these models is provided in Ueda (2002).[4]

Allowing the possibility of endogenously selected sharing rules, former studies of collective contests assume that each group's sharing rule is observable by members of the other groups. The plausibility of this assumption is arguable. An irreversible choice of an observable sharing rule works as a commitment which is affected by strategic considerations. If the agreed sharing rule in a group is unobservable by members of the other groups, such strategic considerations can be disregarded[5]. Very recently, Baik and Lee (2006) point out the problem and analyze a contest with private sharing rules selected by two groups of equal memberships. Within a rather specific model, they derive an interesting result that the more efficient group assigns more weight to the equalitarian principle, namely, it divides a larger part of a private-good prize equally.

In this paper, as in Esteban and Ray (2001), we consider groups competing for a mixed private-public-good prize. However, we do allow the private part to be distributed by an endogenous, unobservable sharing rule that is applied by the group winning the contest. In addition, unlike Esteban and Ray, we permit imperfect substitutability between the public and the private components of the contested prize. The collective contest for a pure private-good prize among m possibly asymmetric groups ($m \geq 2$) is a special case of our extended collective contest. In our model, groups can differ both in their membership and in their efficiency. The efficiency differences are represented by variable marginal costs of effort made by the individual group members.

[4] The class of group sharing rules has an alternative interesting interpretation. As argued by Baik (1994) and Baik and Lee (2001), it can be interpreted as a "winner-help-loser" agreement, or a self insurance device applied by the groups.

[5] See Katz (1991) for general arguments on this problem.

Our rather general collective contest that allows membership and efficiency asymmetries renders possible the analysis of three kinds of divisions. The division of the prize into public and private parts, the division of the private-good component of the prize among the group members (which is determined endogenously and privately), and the division of the players among the groups, that determines the size of the groups. Our setting thus provides a very rich and flexible basis to analyze the effect of group and prize characteristics on the performance of groups in collective contests and on their unobservable, endogenously chosen sharing rules.

We have been able to establish a simple relationship, in equilibrium, between the characteristics of a group, its selected sharing rule, its winning probability and per capita expected utility. We first find that a group attaining a higher winning probability chooses a more equalitarian sharing rule that divides a larger part of the private-good component of the prize equally, independent of each member's performance in the contest. This is a significant generalization of the result obtained by Baik and Lee (2006), and we can use it to examine the effect of asymmetry in the efficiency of the contestants, in the valuation of the prize, in income or in lobbying capability on the sharing rules selected by the competing groups. We then obtain sufficient conditions for a group with larger membership to be advantageous (to attain a higher winning probability or a higher per capita utility) and to select a more equalitarian sharing rule. It turns out that in our extended setting a larger group always attains a higher winning probability unless the prize is purely private. This means that the main result of Esteban and Ray (2001) is significantly strengthened when the sharing rules are allowed to be endogenous; whereas under the equalitarian sharing rule Olson's group size paradox is not necessarily satisfied, in our extended contest with groups that control their sharing rules the paradox is never satisfied, provided that the contested prize is not a purely private good. Thirdly, focusing on the effect of the nature (composition) of the prize on the equilibrium in a two-group contest, we derive the conditions that ensure an inverse or direct relationship between a change in the share of the private-good component and the performance of the groups and the sharing rules they apply. Although the analysis is confined to a special two-group contest with quadratic costs, the results are indicative as to role played by the various parameters in this context.

In all the results, the *elasticity of the benefit from the private part of the prize,* which is denoted by η, plays an important role. The clarification of the role played by this elasticity on the outcome of group contests is an essential part of the contribution of the paper. Esteban and Ray (2001) have pointed out and clarified the significance of the elasticity of the marginal cost of effort in determining the relationship between group size and its performance in the contest. We stress and explain that in more general contests the former elasticity η can play a more crucial role.

The next section presents our extended model of group-specific contests. Section 3 contains the equilibrium analysis and the first main result. Section 4 presents the comparative statics analysis, four more results and some illustrations that are based on the assumptions of quadratic cost functions and CES benefit functions. Some concluding remarks appear in the last Section 5.

2. The Extended Group Contest

(a) Prize, Benefit and Cost.

Let us consider a contest in which m groups compete for a prize. The membership of group i is denoted by N_i ($i = 1, \ldots, m$). We assume that the prize is a mixture of public and private goods. That is, a winning group gets some group-specific public goods and private goods that can be shared among its members. Such a mixed prize can be found, for example, in R&D contests. In such contests, the prize won by one of the competing groups consists of improved reputation (the status and recognition associated with winning the R&D race, which can be equally shared by all members of the winning firm) and of monetary benefits (the profit associated with winning the contest, that can be shared equally or not-equally by some or by all group members). In regional, community or government division contests the prize is often some budget, part of which can take the form of monetary transfers while the rest must be used to supply some local public goods, see Nitzan (1994). When a local government wins a contested subsidy earmarked for some public undertaking, part of it can be provided as an extra margin for the employed local people. Even an electoral competition can be conceived as a contest on a prize with mixed private-public good components, because a winning candidate is typically committed to the provision of both public and private benefits to his supporters.

For simplicity, we assume that every member of every group applies the same benefit function $B(q, G)$ to evaluate the prize, where q is the amount of the private good distributed to the individual and G is the amount of group-specific public good provided to the group to which the individual belongs. This function is twice differentiable, and $B(q, G) > 0$ unless $(q, G) = (0, 0)$. Furthermore, $B_q > 0$, $B_G \geq 0$, and $B_{qq} \leq 0$ hold for all $q > 0$, $G > 0$. The CES benefit function $B(q, G) = \left(b_1 q^\rho + b_2 G^\rho\right)^{\frac{1}{\rho}}$ with $0 < b_1 < 1$, $0 < b_2 < 1$ and $\rho \leq 1$, satisfies these conditions. We will often refer to this useful case.

We normalize the total prize to unity, and denote the ratio of the private-good part by γ $(0 < \gamma \leq 1)$. That is, the model covers all prize compositions but the pure public-good case. The ratio is given exogenously. As already mentioned, we assume that, prior to the contest, members of each group can make a binding agreement on the rule they apply for sharing the private part of the prize. This rule is assumed to be chosen from the class of sharing rules that are linear combinations of the equalitarian and the relative-effort sharing rules. Denote the agreed weight of the relative-effort rule in group i by δ_i. Then, if group i wins the contest, a member of the group having put effort $a \geq 0$ receives the benefit

$$B\left(\gamma \cdot (\delta_i \cdot \frac{a}{A_i} + (1-\delta_i) \cdot \frac{1}{N_i}), 1-\gamma\right),$$

where A_i is the aggregate amount of effort put by the members of group i.

A member of group i incurs the cost $v_i(a)$ when making an effort equal to a while trying to win the prize. The cost function is symmetric within a group, but it can differ across the competing groups. For every i, let $v_i(0) = 0$, $v_i'(a) > 0$ and $v_i''(a) \geq 0$ for all $a > 0$. To guarantee that every individual chooses a positive effort in equilibrium, we also assume that $\lim_{a \to 0} v_i'(a) = 0$.

(b) The Structure of the Contest.

The extended group contest proceeds in two stages. In the second stage, a member of a group chooses his effort level individually, given the value of δ_i, i.e., the sharing rule of the private-good part of the prize agreed in the first stage. As we will see, symmetry of the members in a group results in symmetric expected utility in the second stage. So we can simply assume that the decision on δ_i in the first stage is made to maximize per capita utility in the group.

Although the decision on δ_i is made taking into consideration the contest in the second stage, we still need to distinguish between the two cases of observable and non-observable sharing rules. If the agreed upon sharing rule is observable from outside, it works as a commitment in the group contest and has a strategic effect on the other competing groups. The existence of such strategic effects must affect the sharing rule agreed upon in the first stage. However, the presumption of observable group agreement on the sharing rule may be difficult to justify. As recently argued by Baik and Lee (2006), if the agreed sharing rules are not observable, the equilibrium sharing rule of a group is determined not given the other groups' sharing rules, but given the efforts they make. The contest can therefore be conceived as a single-stage game where each group chooses the group effort via the weight assigned to the relative-effort rule. In other words, this weight is the control variable of the groups. We will follow this approach.

Let us assume that the contest winning probability of group *i* is given by

$$\pi_i = \frac{A_i}{\sum_{k=1}^{m} A_k}, \tag{1}$$

where A_k is the total amount of effort made by the members of group *k*. Although we apply the common simple lottery contest success function, notice that our model allows heterogeneity in the contestants' effectiveness by allowing differences in the cost functions of the groups.

3. Equilibrium.

In the second stage of the extended group contest, given the pre-agreed upon group sharing rule of the private part of the prize, i.e., the value of δ_i, each member of group i determines the effort level *a* by solving the problem:

$$\max_{a \geq 0} \pi_i \cdot B\left(\gamma \cdot (\delta_i \cdot \frac{a}{A_i} + (1-\delta_i) \cdot \frac{1}{N_i}), 1-\gamma\right) - v_i(a),$$

In equilibrium, the optimal positive effort the individual chooses satisfies the equality:

$$\frac{A-A_i}{A^2}\cdot B\left(\gamma\cdot(\delta_i\cdot\frac{a}{A_i}+(1-\delta_i)\cdot\frac{1}{N_i}),1-\gamma\right)+\frac{1}{A}\cdot\gamma\delta_i\cdot\frac{\partial B\left(\gamma\cdot(\delta_i\cdot\frac{a}{A_i}+(1-\delta_i)\cdot\frac{1}{N_i}),1-\gamma\right)}{\partial q}\cdot\frac{A_i-a}{A_i}-v_i'(a)=0$$

where $A=\sum_{k=1}^{m}A_k$. It can be easily verified that every member of group *i* chooses a symmetric effort level. We therefore obtain that in equilibrium,

$$(1-\frac{A_i}{A})\cdot B\left(\frac{\gamma}{N_i},1-\gamma\right)+\gamma\delta_i\cdot\frac{\partial B\left(\frac{\gamma}{N_i},1-\gamma\right)}{\partial q}\cdot(1-\frac{1}{N_i})-v_i'\left(\frac{A_i}{N_i}\right)\cdot A=0\cdot \tag{2}$$

From this equation one can derive the relationship between A_i, the total amount of effort in group *i*, and the pre-agreed upon sharing rule δ_i and the effort made in the other groups, $A_{-i}=A-A_i$. The left-hand-side of equation (2) is strictly decreasing with A_i. So we can define the function $A_i(\delta_i, A_{-i})$ giving group *i*'s total effort satisfying (2).

Given the effort made by the other groups, A_i increases in δ_i . In the first stage of the extended contest, the members of group *i* affect the total amount of effort in their group by determining δ_i, to maximize the per capita utility (notice that they can predict that every group member attains a symmetric utility). Since the members cannot observe the sharing rules agreed upon in the other groups, they select their sharing rule given the A_j's and not the δ_j's, ($j \neq i$). That is, in the first stage of the game δ_i is the solution of the problem:

$$\max_{0\le\delta_i\le 1}\frac{A_i(\delta_i,A_{-i})}{A_{-i}+A_i(\delta_i,A_{-i})}\cdot B\left(\frac{\gamma}{N_i},1-\gamma\right)-v_i\left(\frac{A_i(\delta_i,A_{-i})}{N_i}\right).$$

Since $\frac{\partial A_i}{\partial\delta_i}>0$, we get that a necessary condition to the solution of the problem faced by group *i* is:

$$\frac{A-A_i(\delta_i,A_{-i})}{A}\cdot B\left(\frac{\gamma}{N_i},1-\gamma\right)-v_i'\left(\frac{A_i(\delta_i,A_{-i})}{N_i}\right)\cdot\frac{A}{N_i}\ \ge(\le)\ 0 \text{ if } \quad \delta_i>0\ (<1). \tag{3}$$

Equation (2) implies that $\delta_i = 0$ is impossible. Notice that the left-hand side of (3) is strictly decreasing in A_i. Therefore, if (3) holds as an equality for some $\delta_i < 1$, it must be the unique solution. Otherwise, the per capita utility is strictly increasing with respect to δ_i on [0,1], and $\delta_i = 1$ is the unique solution.

Before considering the existence of equilibrium, suppose, firstly, that in equilibrium group *i* chooses $0 < \delta_i < 1$. Since $\pi_i = \frac{A_i}{A}$, in this case condition (3) has the form:

$$(1-\pi_i)\cdot B\left(\frac{\gamma}{N_i}, 1-\gamma\right) - v_i'\left(\frac{A}{N_i}\cdot\pi_i\right)\cdot\frac{A}{N_i} = 0 \cdot \tag{4}$$

Substitution of (4) into equation (2) yields our first main result.

Proposition 1. *An interior equilibrium sharing rule of the private-good component of the prize among the N_i members of group i, i = 1, …, m , is given by*

$$\delta_i = \frac{1-\pi_i}{\eta(N_i,\gamma)}, \tag{5}$$

where

$$\eta(N_i,\gamma) = \frac{\partial}{\partial q}B\left(\frac{\gamma}{N_i}, 1-\gamma\right)\cdot\frac{\frac{\gamma}{N_i}}{B\left(\frac{\gamma}{N_i}, 1-\gamma\right)} \tag{6}$$

is the elasticity of the benefit from the private part of the prize.

The proposition establishes that, in interior equilibria, there exists a direct relationship between the winning probability and the endogenously determined share of the private part of the prize that is equally distributed among the group members. This finding considerably simplifies the comparative statics analysis on group sharing rules in the next section. Notice that since the benefit function is concave with respect to the private part of the prize, we get that $\eta(N_i,\gamma)\leq 1$, with strict inequality unless the prize is purely private ($\gamma=1$) and the benefit function has the form $B(q,0)=bq$, where b>0.

We are ready to use the same technique as in Esteban and Ray (2001) and Ueda (2002) to establish the existence and uniqueness of equilibrium in the extended group contest. Consider, hypothetically, equation (4) as the condition implicitly defining π_i as a function of A, γ, and the membership N_i. Then, π_i is continuous and strictly decreasing in A. Also, $\lim_{A\to 0}\pi_i = 1$ and $\lim_{A\to\infty}\pi_i = 0$. As A increases, the value of δ_i derived from equation (5) approaches 1. When $\eta(N_i, \gamma)$ is less than 1, δ_i can exceed 1 for any A larger than some level, say A_R. For any such equilibrium value of the total effort A, group i sets $\delta_i = 1$, and π_i is determined by the equation

$$(1-\pi_i)\cdot B\left(\frac{\gamma}{N_i},1-\gamma\right)+\gamma\cdot\frac{\partial B\left(\frac{\gamma}{N_i},1-\gamma\right)}{\partial q}\cdot(1-\frac{1}{N_i})-v_i'\left(\frac{A}{N_i}\cdot\pi_i\right)\cdot A=0, \qquad (7)$$

which is derived from (2), setting $\delta_i = 1$. Notice that π_i is still continuous and strictly decreasing in A and $\lim_{A\to\infty}\pi_i = 0$.

Now, consider the "pseudo" winning probability function of group i that depends on A, $\pi_i^P(A)$: $(0\ \infty)\to\mathbb{R}$, which is defined as follows: for any A in $(0, A_R]$, this function assigns the value of π_i given by equation (4), and for any A larger than A_R, it assigns the value of π_i determined by equation (7). The derived function is continuous and strictly decreasing, with $\lim_{A\to 0}\pi_i^P(A) = 1$ and $\lim_{A\to\infty}\pi_i^P(A) = 0$. If A is an equilibrium total amount of effort, group i's effort level A_i must satisfy $\frac{A_i}{A}=\pi_i^P(A)$.

In equilibrium, however, the sum of the winning probabilities of the m groups is equal to 1. There exists a unique value A^* such that $\sum_{i=1}^m \pi_i^P(A^*)=1$. When the total amount of effort is A^*, $A_i^* = \pi_i^P(A^*)\, A^*$ satisfies the best response conditions characterizing the choice of the sharing rule δ_i. Hence our extended group contest has a unique (pure-strategy) equilibrium.

4. Comparative Statics

Henceforth we concentrate on interior equilibria in which every group chooses a "mixed" sharing rule of the private component of the prize, i.e., $0 < \delta_i < 1$,for all $i = 1, \ldots, m$. We first discuss the implications of Proposition 1 regarding the relationship between the endogenously determined group equalitarianism and the winning probability of the group and the relationship between group equalitarianism and the elasticity of the benefit from the private good component of the prize. We then examine how differences in the cost function, income, benefit evaluation and group membership affect the winning probabilities, per capita utility and the selected group sharing rules. Focusing on the special case of two-group contests with quadratic cost functions, our final concern is with the effect of changes in the nature of the prize (the composition of the private and public-good components of the prize) on the equilibrium efforts, performance and sharing rules of the competing groups. The special contest where cost functions are quadratic and individual utilities are of the CES form is used to illustrate some of the results.

(a) The Selected Sharing Rule, Performance and Benefit Elasticity

An interior equilibrium sharing rule satisfies equation (5). From this surprisingly simple relationship, we can conclude that a group attaining a higher winning probability chooses a lower δ_i, or a more equalitarian sharing rule, other things being equal. This property was recently pointed out by Baik and Lee (2006), who studied a specific model of two-group contest with equal group membership and a pure private-good prize. Our first result implies that the property is actually robust. It also entails that δ_i is negatively related to $\eta(N_i, \gamma)$. That is, a group with a higher elasticity of benefit from the private part of the prize tends to be more equalitarian.

We can interpret this relationship by referring to the classical argument made by Sen (1966) in the context of producer cooperatives. In an interior equilibrium, groups use mixtures of the relative-effort rule and the equalitarian rule to share the private part of the prize. The reason is that complete reliance on the former rule induces the members to make excessive efforts that prohibit the attainment of Pareto optimum, while reliance just on the latter equalitarian rule also results in an inefficient outcome, because the individuals are induced to make insufficient efforts. A group that can secure a higher winning probability has room to loosen up its members' incentive to make efforts. Similarly, a higher $\eta(N_i, \gamma)$ induces more effort from the group members and makes room to reduce δ_i, that is, to apply a more equalitarian sharing rule.

(b) Different Marginal Costs.

Assuming that all the groups have the same number of members, say N, we now allow variability in the efficiency of the groups that takes the form of different marginal costs of effort. The first comparative statics result is

Proposition 2. *Let the members of group k have lower marginal costs than those of members of group l. That is, $v_k'(a) < v_l'(a)$ for all $a > 0$. Then, in equilibrium, $\pi_k > \pi_l$ and $\delta_k < \delta_l$. Also, per capita utility is larger in group k than in group l.*

Proof. Equation (4) implies that

$$\pi_k \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right)\frac{N}{A^*} + v_k'\left(\frac{A^*}{N}\cdot\pi_k\right) = \pi_l \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right)\frac{N}{A^*} + v_l'\left(\frac{A^*}{N}\cdot\pi_l\right).$$

Suppose that $\pi_k \le \pi_l$. We therefore obtain the strict inequality: $v_k'\left(\frac{A^*}{N}\cdot\pi_k\right) < v_l'\left(\frac{A^*}{N}\cdot\pi_l\right)$,

which makes the above equation impossible. This means that $\pi_k > \pi_l$. Since $\eta(N_k, \gamma) = \eta(N_l, \gamma)$, we get that $\delta_k < \delta_l$. Finally, notice that A_k which maximizes the per capita utility of group k, is larger than A_l. Denoting the per capita utility of group i by u_i, and noticing that $v_k(a) < v_l(a)$ for all $a > 0$, we get that

$$u_k = \frac{A_k}{A^*}\cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_k}{N}\cdot\right) \ge \frac{A_l}{A_{-k}+A_l}\cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_l}{N}\right)$$

$$> \frac{A_l}{A^*}\cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_l}{N}\right) > \frac{A_l}{A^*}\cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_l\left(\frac{A_l}{N}\right) = u_l$$

Q.E.D.

By Proposition 2, a group with more efficient contestants attains a higher winning probability, applies a more equalitarian group sharing rule and secures a higher per capita utility. In addition to these straightforward implications, the proposition can be easily applied to shed light on the role of differences in the valuation of the prize, in income and in lobbying capability.

(b-i) Differences in valuation of the prize

To study the effect of variability in the evaluation of the prize, we modify the model by letting members of group i have the benefit function $w_iB(q, G)$, where $w_i > 0$ is the augmenting factor. If $w_k > w_l$, members of group k value the contested prize more than members of group l, without affecting the value of the elasticity $\eta(N, \gamma)$. Letting a member's cost function have the same form v in all groups, equation (4) takes the form:

$$(1-\pi_i)\cdot B\left(\frac{\gamma}{N},1-\gamma\right)-\frac{1}{w_i}\cdot v'\left(\frac{A}{N}\cdot\pi_i\right)\cdot\frac{A}{N}=0,$$

In this case $\frac{v'(a)}{w_i}$ can be conceived as the marginal cost in group i. By applying Proposition 2, we can conclude that $w_k > w_l$ implies that $\pi_k > \pi_l$, $\delta_k < \delta_l$, and $u_k > u_l$. That is, increased valuation of the prize and increased efficiency that takes the form of reduced marginal cost have the same effect on the group winning probability, sharing rule and per capita utility.

(b-ii) Differences in income.

To study the effect of income variability, we modify the model by assuming that an individual's preferences are represented by an additively separable utility function of his benefit from the prize and of his income I. Specifically, the individual's utility is

$$B(q, G) + V(I),$$

where $V' > 0$ and $V'' < 0$ for all $I > 0$. Interpreting the effort level a as money expenditure, we can define the cost function of a member in group i as

$$v_i(a) = -V(I_i - a),$$

where I_i is the common income of the members of group i.[6] Let $I_k > I_l$, that is, the members of group k are richer than those of group l. Then, by assumption,

$$v_k'(a) = V'(I_k - a) < V'(I_l - a) = v_l'(a),$$

and so, by Proposition 2 we get that $\pi_k > \pi_l$, $\delta_k < \delta_l$, and $u_k > u_l$.

That is, increased income is interpreted as reduced marginal cost of effort and therefore it has the same effect on the group winning probability, sharing rule and per capita utility.

[6] Rigorously speaking, this cost function does not satisfy the assumption $\lim_{a\to 0} v_i'(a) = 0$, which assures that in equilibrium every group makes a positive effort. In this version of our extended contest, therefore, our arguments are valid, provided that the equilibrium effort of every group is positive.

(b-iii) Differences in political influence

To study the effect of variability in the political influence or lobbying power of the individuals, we have to modify the model by introducing asymmetry into the contest success function. Namely, allow members of different groups to differently affect their group winning probability. Essentially, in such asymmetric version of the contest it is possible to formulate differences in political capabilities such that they are equivalent to differences in the marginal cost of effort across groups. In turn, we could establish, again by applying Proposition 2, that increased political power, which is interpreted as reduced marginal cost of effort, increases the group winning probability, increases the group equalitarianism and increases the group per capita utility.

(c) Different Membership Size

The advantage of membership size is the main topic of Esteban and Ray (2001). They provide a sufficient condition for a group with larger membership to attain a higher winning probability in equilibrium. They also prove that per capita utility increases (decreases) with membership size when the prize is purely public (private). In this sub-section we generalize and sharpen their results using the extended contest that allows the endogenous determination of the group sharing rules and imperfect substitution between the private and the public components of the prize.

Let all members of the competing groups share the same cost function v. Following Esteban and Ray (2001), we denote by $\alpha(a)$ the elasticity of the marginal cost,

$$\alpha(a)=\frac{a\cdot v''(a)}{v'(a)}.$$

Also, let us pretend that the membership N_i in (6) is a continuous variable and view the benefit elasticity η as its continuous function. The membership size viewed as a continuous variable will be denoted by *n*. Our third proposition generalizes Esteban and Ray's result on the relationship between the size of the competing groups and their winning probability.

Proposition 3. *Suppose that all group members share the same cost function v. Then the winning probability of an N'-member group is larger than that of an N-member group, N < N', if*

$$1 - \max_{n \in [N,\, N']} \eta(n, \gamma) + \inf_{a \geq 0} \alpha(a) > 0. \tag{8}$$

Proof. The basic idea of the proof is similar to the one applied by Esteban and Ray in the proof of their Proposition 1. Keeping *A* unchanged at its equilibrium value and examining the behavior of π_i, while pretending that in equation (4) N_i is a continuous variable, we obtain that

$$\left.\frac{\partial \pi_i}{\partial n}\right|_{n=N} = \frac{N}{\pi_i} \cdot \frac{1 - \eta(N, \gamma) + \alpha\left(\frac{A^*}{N} \cdot \pi_i\right)}{\pi_i + \alpha\left(\frac{A^*}{N} \cdot \pi_i\right)}. \tag{9}$$

If inequality (8) holds, this derivative is positive at all values of *n* in the closed interval [*N*, *N* ']. This establishes the validity of Proposition 3.

Q.E.D.

Proposition 3 establishes that in the extended contest, a larger group will have a higher winning probability, if the difference between $\max_{n \in [N,\, N']} \eta(n, \gamma)$ and $\inf_{a \geq 0} \alpha(a)$ is sufficiently small. More specifically, if this difference is smaller than 1. Economically, this condition makes sense. An increase in membership reduces the per capita private-good component of the prize. This reduction of the individual's benefit, induces a member to reduce effort. The extent of this first incentive can be measured by $\eta(N_i, \gamma)$. On the other hand, the same increase in membership reduces the individual's marginal cost at a given level of group effort, inducing a member to increase effort. The extent of this second incentive can be measured by $\alpha(\frac{A_i}{N_i})$. If the extent of the difference between these two incentives, which can be measured by $(\max_{n\in[N,N']} \eta(n,\gamma) - \inf_{a\geq 0} \alpha(a))$, is sufficiently small, then the increase from N to N' will result in an increase in the effort made by the group and, consequently, in its winning probability.

As we have already pointed out, $\eta(n,\gamma)$ is always less than 1, unless the prize is purely private. By Propositions 3 and 1, this implies

Corollary 1.

(a) *In a contest for a mixed private-public good where* $\gamma \neq 1$, *a larger group always attains a higher winning probability.*

(b) *Assume that* $\eta(n,\gamma)$ *is non-decreasing with respect to n. Then, in a contest for a mixed private-public good, the sharing rule applied by a larger group is more equalitarian.*

Part (a) of the Corollary implies that in our extended contest, allowing the endogenous determination of group sharing rules eliminates the ambiguity regarding the effect of group size on its winning probability; a larger size always increases the winning probability of the group, provided that the prize is not a pure private good. This result considerably strengthens Esteban and Ray's (2001) main claim that Olson's group size paradox is not necessarily satisfied. By part (b) of the corollary, if $\eta(n,\gamma)$ is non-decreasing with respect to *n,* then a larger size also results in increased equalitarianism.

The next proposition sheds new light on how prize evaluation, viz., the elasticity of the benefit from the private-good component of the prize, affects the relationship between membership size and per capita utility.

Proposition 4. *Suppose that all group members share the same cost function v. Then the per capita utility in an N'-member group is larger than that in an N-member group, N < N', if*

$$\max_{n\in[N,\,N']}\eta(n,\gamma)\le\frac{1}{2}, \qquad (10)$$

Proof. Again, keeping A unchanged at its equilibrium value and viewing N_i as a continuous variable, we can examine the behavior of the per capita utility

$$u_i=\pi\left(A^*,N_i\right)\cdot B\left(\frac{\gamma}{N_i},1-\gamma\right)-v_i\left(\frac{A}{N_i}\cdot\pi\left(A^*,N_i\right)\right),$$

where $\pi(A^*, N_i) = \pi_i$ is the value of the winning probability given by equation (4). By using equations (4) and (9), we get that

$$\left.\frac{\partial u_i}{\partial n}\right|_{n=N}=\frac{\pi_i}{N}\cdot B\left(\frac{\gamma}{N},1-\gamma\right)\cdot\left\{1-\eta(N,\gamma)+\frac{\pi_i}{\pi_i+\alpha\left(\frac{A^*}{N}\cdot\pi_i\right)}\left(1-\eta(N,\gamma)-\pi_i\right)\right\}, \qquad (11)$$

If $1-\eta(N,\gamma)-\pi_i\ge 0$, then the right hand side of the equation is positive.

If $1-\eta(N,\gamma)-\pi_i<0$, then

$$\left.\frac{\partial u_i}{\partial n}\right|_{n=N} > \frac{\pi_i}{N}\cdot B\left(\frac{\gamma}{N},1-\gamma\right)\cdot\{1-\eta(N,\gamma)+(1-\eta(N,\gamma)-\pi_i)\} = \frac{\pi_i}{N}\cdot B\left(\frac{\gamma}{N},1-\gamma\right)\cdot(2-2\eta(N,\gamma)-\pi_i)\cdot$$

Thus, $\max_{n\in[N,N']}\eta(n,\gamma)\le\frac{1}{2}$ implies that the per capita utility is increasing with respect to membership size on the interval [*N*, *N'*].

Q.E.D.

At this stage, it is instructive to illustrate the results by examining the CES family of benefit functions, $B(q,G)=\left(b_1q^{\rho}+b_2G^{\rho}\right)^{\frac{1}{\rho}}$, with $0 < b_1 < 1$, $0 < b_2 < 1$ and $\rho \le 1$.[7] With the CES form, we get that

$$\eta(n,\gamma)=\frac{b_1\cdot\left(\frac{\gamma}{n}\right)^{\rho}}{b_1\cdot\left(\frac{\gamma}{n}\right)^{\rho}+b_2\cdot(1-\gamma)^{\rho}}, \tag{12}$$

which is always less than 1. That is, condition (8) is always satisfied, so a larger group indeed has the advantage of having a higher winning probability. Also, note that when $\rho < 0$, $\eta(n,\gamma)$ becomes non-decreasing with respect to n, so we can apply Corollary 1(b). Allowing the endogenous choice of group sharing rules in the extended contest with CES benefit functions, therefore strengthens Esteban and Ray's result because a larger group *always* attains a higher winning probability. As to the use of Proposition 4, we can see that with the CES specification, the condition $\eta(n,\gamma)\le\frac{1}{2}$ can be written as $\left(\frac{b_2}{b_1}\right)^{-\frac{1}{\rho}}\cdot\frac{\gamma}{1-\gamma}\le n$. If $\left(\frac{b_2}{b_1}\right)^{-\frac{1}{\rho}}\cdot\frac{\gamma}{1-\gamma}\le N_{\min}$, where $N_{\min} = \min\{N_1, \ldots, N_m\}$, or $\gamma\le\frac{N_{\min}}{N_{\min}+\left(\frac{b_2}{b_1}\right)^{-\frac{1}{\rho}}}$ holds, then per capita utility is increasing with respect to

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[7] Notice that the linear specification of the benefit function adopted by Esteban and Ray (2001), i.e., $B(q,G)=Mq+PG$, is a special case of the CES family.

membership size. That is, increased membership is advantageous not only because it increases the group winning probability, but also because it increases the utility of its members. This result holds true not only in the case of a pure public-good prize, but also in the intermediate cases where the public part of the prize is sufficiently large.[8]

(d) Composition of the Prize

A change of γ, the composition of the prize, alters the value of the prize for the individual contestants. When all groups have the same membership size, such a modification of composition uniformly changes the benefit from the prize for all of the contestants. Even in this case, however, the change of effort in each group is equivocal. An increase in the benefit induces the groups to put more effort in trying to win the contest. This means that the total amount of effort goes up, which could discourage the groups. Due to this strategic effect, some groups may ultimately reduce their effort.

When the competing groups have different size, a change of γ differently affects each group per capita benefit from the private part of the prize (and uniformly affects the per capita benefit from the public component of the prize). It may have a positive effect on the value of the prize in groups of some membership size, but negative effect in groups of a different membership size. Due to the strategic nature of the contest and the different impact on the benefit of different groups, the complicated overall effect of a change in the private component of the prize on the effort of the groups is ambiguous.

To resolve the ambiguity of the effect of changes in the composition of the prize, let us reduce the generality of our extended contest by focusing on the more

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[8] With the notation of Esteban and Ray (2001) and their linear specification $B(q, G) = Mq + PG$, $\theta_{\min} \geq \frac{1}{2}$, or $\frac{\frac{M}{N_{\min}}}{P + \frac{M}{N_{\min}}} \leq \lambda$ is a sufficient condition that a larger group attains a higher per capita utility, where $\theta_{min} = 1 - \eta(N_{min}, \gamma)$, and $\lambda = 1 - \gamma$ is the ratio of the public part of the prize .

manageable special case of t*wo-group contests with quadratic cost functions*. Suppose then that the quadratic cost function of each member in the first group, group 1, is $v_1(a) = \frac{h_1}{2}a^2$, while the quadratic cost function of every member in the second group, group 2, is $v_2(a) = \frac{h_2}{2}a^2$ (h_1, h_2 > 0). Notice that in this case the marginal cost elasticity is always equal to one in both of the groups, i.e., $\alpha(a) = 1$.

In this case, we can explicitly derive the equilibrium total effort A^* from equation (4) and the equilibrium condition $\pi_1 + \pi_2 = 1$, as follows:

$$(A^*)^2 = N_1 \cdot N_2 \cdot \left(\frac{B\left(\frac{\gamma}{N_1}, 1-\gamma\right)}{h_1} \cdot \frac{B\left(\frac{\gamma}{N_2}, 1-\gamma\right)}{h_2} \right)^{\frac{1}{2}}. \tag{13}$$

In turn, we can derive

$$\pi_i = \frac{N_i \cdot \left(h_j \cdot B\left(\frac{\gamma}{N_i}, 1-\gamma\right)\right)^{\frac{1}{2}}}{N_i \cdot \left(h_j \cdot B\left(\frac{\gamma}{N_i}, 1-\gamma\right)\right)^{\frac{1}{2}} + N_j \cdot \left(h_i \cdot B\left(\frac{\gamma}{N_j}, 1-\gamma\right)\right)^{\frac{1}{2}}}, \quad i, j = 1,2,\ i \neq j, \tag{14}$$

To state our last proposition, we denote the elasticity of the benefit from the public part of the prize, for a group with N members, by

$$\kappa(N,\gamma) = \frac{\partial}{\partial G} B\left(\frac{\gamma}{N}, 1-\gamma\right) \cdot \frac{\frac{\gamma}{N}}{B\left(\frac{\gamma}{N}, 1-\gamma\right)}. \tag{15}$$

Proposition 5. *Consider the equilibrium of a two- group contest with quadratic cost functions. Without loss of generality, let* $N_1 \leq N_2$. *Then, the winning probability of group 1 is increasing, decreasing, or constant with respect to* γ*, depending, respectively, on whether* $\eta(N_1,\gamma) - \frac{\gamma}{1-\gamma}\kappa(N_1,\gamma)$ *is larger than, smaller than, or equal* *to* $\eta(N_2,\gamma)-\frac{\gamma}{1-\gamma}\kappa(N_2,\gamma)$. (Notice that $\frac{\partial\pi_1}{\partial\gamma}+\frac{\partial\pi_2}{\partial\gamma}=0$, so the winning probabilities of group 2 and 1 change in opposite directions).

Proof. Dividing the denominator and the numerator in the right-hand-side of equation (14) by $B(\frac{\gamma}{N_i},1-\gamma)^{\frac{1}{2}}$, we get that

$$\pi_i=\frac{N_i h_j^{\frac{1}{2}}}{N_i h_j^{\frac{1}{2}}+N_j h_i^{\frac{1}{2}}\cdot\left(\frac{B(\frac{\gamma}{N_j},1-\gamma)}{B(\frac{\gamma}{N_i},1-\gamma)}\right)^{\frac{1}{2}}}$$

This means that the signs of the derivative of π_i and of $\log\frac{B(\frac{\gamma}{N_i},1-\gamma)}{B(\frac{\gamma}{N_j},1-\gamma)}$ with respect to γ are equal. That is, the sign of $\frac{\partial\pi_1}{\partial\gamma}$ is the same as that of

$$\frac{\partial}{\partial\gamma}\left(\log B\left(\frac{\gamma}{N_1},1-\gamma\right)-\log B\left(\frac{\gamma}{N_2},1-\gamma\right)\right)=\frac{1}{\gamma}\cdot\left\{\left(\eta(N_1,\gamma)-\frac{\gamma}{1-\gamma}\kappa(N_1,\gamma)\right)-\left(\eta(N_2,\gamma)-\frac{\gamma}{1-\gamma}\kappa(N_2,\gamma)\right)\right\},$$

which completes the proof Q.E.D.

Again, the CES specification of the benefit function is convenient to illustrate the usefulness of this proposition. With this form, $\eta(n, \gamma) + \kappa(n, \gamma) = 1$ and $\eta(n,\gamma)-\frac{\gamma}{1-\gamma}\kappa(n,\gamma)$ is positively related to $\eta(n, \gamma)$. Therefore, by (12), it is increasing or decreasing with respect to n, if ρ is positive or negative, respectively. Proposition 5 therefore implies that π_1 decreases (increases) with γ, if ρ is negative (positive). An increased share of the private-good component of the prize seems to aggravate the free-rider problem, and as a result a smaller group is advantageous relative to a larger group. Increasing the private-good component of the prize, however, can adversely affect a smaller group, if this component is not a good substitute for the public-good component of the prize.

For the CES specification of the benefit function, we can also derive a simple formula of the effect of γ on η,

$$\frac{\partial}{\partial\gamma}\eta(n,\gamma)=\frac{\eta(n,\gamma)\cdot(1-\eta(n,\gamma))}{\gamma\cdot(1-\gamma)}\cdot\rho\cdot(1-2\gamma)\,. \tag{16}$$

This equation implies that the effect of a change in γ on η is determined by the sign of ρ and by whether the value of γ is larger or less than one half. For example, if ρ is negative and γ is less than one half, η increases with γ. In such a case, δ_1 increases with γ, but the effect on δ_2 is ambiguous.[9]

5. Conclusion

We have examined an m-group contest for a mixed private-public-good prize, in which the private part is distributed by an endogenous, unobservable sharing rule that is applied by the group winning the contest. In our setting, asymmetry among the competing groups is allowed in terms of both efficiency and membership size. Imperfect substitutability between the public and the private components of the prize is also permitted. The group contest has a unique (pure-strategy) equilibrium. Its

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[9] A two- group contest with quadratic functions and a CES benefit function is also an example that warrants our concentration on interior equilibrium. In this example, equation (14) can be simplified to

$$\pi_i=\frac{N_i\cdot h_j^{\frac{1}{2}}\left(\left(\frac{\gamma}{N_i}\right)^{\rho}+\left(\frac{b_2}{b_1}\right)\cdot(1-\gamma)^{\rho}\right)^{\frac{1}{2\rho}}}{N_i\cdot h_j^{\frac{1}{2}}\left(\left(\frac{\gamma}{N_i}\right)^{\rho}+\left(\frac{b_2}{b_1}\right)\cdot(1-\gamma)^{\rho}\right)^{\frac{1}{2\rho}}+N_j\cdot h_i^{\frac{1}{2}}\left(\left(\frac{\gamma}{N_j}\right)^{\rho}+\left(\frac{b_2}{b_1}\right)\cdot(1-\gamma)^{\rho}\right)^{\frac{1}{2\rho}}}$$

By (12), $\eta(n,\gamma)$ converges to 1 as $\frac{b_2}{b_1}$ converges to 0, for any n. Hence, the above equation and equation (5) imply that an interior equilibrium actually holds when $\frac{b_2}{b_1}$ is small enough.

characterization enabled the derivation of simple fundamental formulas that are most useful for analyzing the relationship between the characteristics of a group, its selected sharing rule, its winning probability and the per capita expected utility. In those formulas, the elasticity of the benefit from the private part of the prize plays a key role. Our main findings are the following:

(i) A group securing a higher winning probability is more equalitarian. The extent of equalitarianism is positively related to the elasticity of the benefit from the private component of the contested prize (Proposition 1).

(ii) A group with more efficient contestants, applies a more equalitarian sharing rule and attains a higher per capita utility (Proposition 2). Increased efficiency can take the equivalent form of higher valuation of the prize, higher income or higher political capability.

(iii) Generalizing the main results of Esteban and Ray (2001), we clarify under what conditions membership size is advantageous in terms of winning probability and in terms of per capita utility. In the former case, this condition involves both the elasticity of the marginal cost of effort and the elasticity of the benefit from the private part of the contested prize (Proposition 3) and it implies that a larger group *always* has a higher winning probability, unless the prize is purely private. In the latter case a stronger condition is required by restricting the range of the elasticity of benefit from the private part of the contested prize (Proposition 4).

(iv) Applying a special two-group version of the extended contest with quadratic cost functions, we have clarified the role of the elasticities of benefit from the private and public components of the prize in determining the effect of a change in the nature of the prize on the winning probabilities of the competing groups (Proposition 5).

Since most of the results are valid in the case of the pure private-good prize, the significance of our findings is preserved even if we disregard the nature of the prize in the model. However, a significant merit of the proposed model is its ability to treat mixed public-private-good characteristics of the prize in a very general way. In many group contests the prize contains both public and private factors. This is typically the case when the nature of the prize is determined by a commitment to allocate the fixed budget won by a group to the provision of some mix of private and public goods. Such a commitment may be voluntarily chosen by the winning group. It may also be enforced by the government that grants the prize. Our model is a useful tool for studying such contests.

Our analysis enables the study of three kinds of divisions: (i) the division of the prize into public and private parts; (ii) the division of the private-good component of the prize among the group members and (iii) the division of the players among the groups. The first division determines the nature of the prize that may be endogenously chosen by the agent that awards the prize. We have studied the effect of changes in the nature of the prize on the performance and the sharing rules of the groups, but have not studied the endogenous determination of the nature of the prize. This latter task is left for future research. The second type of division has been of major direct concern in this paper, and we have studied its endogenous determination by the contesting groups. We have also clarified the effect of the third division on the contest equilibrium, but have not examined the endogenous determination of the size of the groups. Group size can be voluntarily determined by the individual group members or perhaps by the agent that grants the prize. Such endogenous determination of group size is also left for future research.

One noticeable weak feature of our model, as well as of other models that study group contests, is the symmetry assumption regarding the members that belong to the same group. Without this assumption it is not clear how the arguments supporting the advantage of larger groups are amended. Investigation of the effect of asymmetry among members of a group on the analysis is another worthwhile undertaking for future research.

Finally, in our setting the extent of equalitarianism (equal sharing of the prize) is not determined by moral values, religious commitments or social ideology. It is the outcome of rational strategic incentives that arise in the contest environment. Interestingly, we find that in this competitive environment, more efficient groups (groups with lower marginal cost of effort) or groups with higher valuation of the prize, higher income or larger lobbying capabilities tend to be more equalitarian. That is, share equally a larger part of the private-good component of the prize. In addition, under the sufficient conditions we have stated, larger groups also tend to be more equalitarian. Testing empirically these predictions is another important task which is beyond the scope of our study.

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