Variations in Transaction Costs and Monetary Policy

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Abstract

In this paper the New Keynesian model is modified in order to include an explicit transaction technology. This technology integrates an endogenous foundation to money into the model. The resulting model is used to analyze the effects of shocks to the transaction technology. Additionally, a comparative analysis of monetary policy shocks for two version of the model distinguished by different total factor productivities in the transaction technology is conducted. Thus the impact of transaction cost variations on the monetary transmission mechanism is studied. Specifically, the model analyzes changes in the relative contribution of different transmission channels to the overall transmission process due to the mention variations in transaction costs..

Keywords: transaction costs, monetary transmission mechanism, New Keynesian model

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1 Introduction

In this paper an extended variation of the New Keynesian macroeconomic model is used in order to analyze the consequences of lower transaction costs on the monetary transmission mechanism. In contrast to traditional approaches to integrate transaction costs into macroeconomic models by using some artificial constructs as in the literature of money-in-utility model of Sidrauski (1967), cash-in-advance or shopping-time models such as in Clower (1967) or Brock (1974), transactions costs are modelled by means of spatial segmentation and an explicit and resource-intensive transaction technology. Herein we follow the models in Starr (2002, 2003) which accounts for the existence of a payments system and allows us to vary efficiency parameters within the according technology.
In order to limit the complexity of the model while still providing a transmission model not too far from the current status quo in policy making nor the real-world transmission process, the analysis is mainly concentrated on three specific transmission channels: the interest rate channel, the credit channel and the exchange rate channel. Hence the model is a two-country model, which includes some asymmetric information in the credit markets. Because monetary policy not only cares about the stabilization of output and inflation, but also about efficiency within the payments system and about the development of capital stocks, it is endogenously derived by assuming a benevolent central bank which optimizes the welfare of the aggregate economy.
The paper starts by depicting the structure of the model. Thereafter optimal behavior for both households and firms is derived in sections 3 and 4. After presenting conditions for market equilibria a New Keynesian Phillips Curve is derived in section 6. The private sector of the economy is linearized in sections 7 and 8. Section 9 introduces the central bank in an explicit manner. In section 10 the model is calibrated, while in section 11 the simulation results are presented. The discussion of the results follows in section 12, while in section 13 derivations of intermediate results are added as an appendix.
Concerning notation it should be noted that the model is presented for the domestic economy, while the foreign economy is modelled symmetrically. Hence references to the referring economy will be only made when necessary in form of the following superscripts: d (f) stands for variables of the domestic (foreign) economy, whereby dd (ff) stands for variables related to domestic (foreign) agents acting on domestic (foreign) markets, while df (fd) denotes variables related to domestic (foreign) subjects acting on foreign (domestic) markets. Note also that the superscripts exclusively accompany aggregate variables.

2 Structure of the Economy

The new feature of resource-consuming transaction media is implemented by assuming spatial segmentation. The domestic (foreign) economy consists of an interval of market-places $[0, \frac{1}{2}\iota]$ ($[\frac{1}{2}\iota, \iota]$), of which each is defined as a distinct location in the corresponding economy. Within these marketplaces financial transactions do not impose any costs. Between the marketplaces wealth can only be transferred by the use of some transaction or payment media. Generally each good of the economy may serve as such an instrument. But the low production costs of distinct goods specially designed as payment media prevent the usage of any other good for this purpose. Within both economies there exists a continuum of firms, of which each produces a distinct consumption good and is located in a specific marketplace. For the homeland this is the continuum $[0, \varsigma]$, while firms belonging to $[\varsigma, 1]$ are located abroad. Thus the parameter ς is essentially a measure for the relative weight between domestic and foreign production sectors. To assure the simultaneous production of a variety of distinct goods in the same marketplace we assume $\frac{1}{2}\iota < \varsigma$, respectively $\frac{1}{2}\iota < 1 - \varsigma$. Households, of which there is also a continuum of $[0, \varsigma]$ in the domestic and $[\varsigma, 1]$ in the foreign economy, are able to access all marketplaces for purchasing purposes. But regarding its labor income each household is tied to a distinct marketplace, in its homeland, by means of a labor contract with duration of one period. For all households the localization of labor contracts in an arbitrary future period is governed by the same idiosyncratic, uniform density function across all marketplaces of the country of origin. Besides the firms producing consumption goods there is a second type of firms, which supply financial intermediation, and of which a finite set $M(k)$ is located in a distinct marketplace k, and the central bank, which offers payment services and conducts monetary policy. Moreover there are suppliers of capital goods which, like households, have access to all marketplaces in the economy. Riskless bonds are traded on a distinct marketplace in either economy. Besides private individuals there are two public authorities. The government collects taxes and uses them for consumption purposes. By assumption both of these activities are exposed to exactly the same marginal transaction costs as any transaction of an arbitrary household. As already mentioned monetary policy is conducted by the central bank.

3 Households

Households are modelled as heterogeneous agents, who differ in their contractual ties to a specific marketplace with regard to supply of labor, but not in preferences. Thus, each domestic household supplies its labor only in a single marketplace, which in each period is drawn newly from the same uniform i.i.d. distribution across the set of all domestic marketplaces. The realization of its stochastic localization is experienced by the household after its decision concerning its demand and supply behavior. Such a modelling on one hand induces the necessity to shift purchasing power to marketplaces where no income is earned, and can be interpreted as infinite transaction costs of labor between marketplaces or severely limited mobility of households, while on the other hand it generates a certain degree of consumer heterogeneity in the economy.[1] Households consume domestic and for-

[1] Note that the stochastic nature of localization is necessary because a deterministic localization would imply asymmetric behavior in steady state, which in turn would destroy the possibility of exact aggregation within the linear approximated model. Indeed it might also be able to find a solution using the

eign goods and invest in riskless assets within both economies, whereby it is assumed that their money holdings are restricted to the economy of their origin. Households maximize the expected net present value of their current and future utilities from consumption and leisure given a budget for each period, which is contingent on the realizations drawn from the public distributions of the various sources of uncertainty in the model. Expectations are assumed to be formed rationally, while the household's consumption bundle in each period,

$$c_t^i = \left(\int_0^{\varsigma} x^i_{tm_{int}}{}^{\frac{\theta-1}{\theta}} dm_{int} + \int_{\varsigma}^1 x^i_{tm_{int}}{}^{\frac{\theta-1}{\theta}} dm_{int} \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

is characterized by imperfect substitutability between different varieties m_{int} of consumption goods, for each of which the household's demand $x^i_{tm_{int}}$ can be decomposed into purchases B and sales S in the different marketplaces, i.e. $x^i_{tm_{int}} = \int_0^{\iota} (x^{ikB}_{tm_{int}} + x^{ikS}_{tm_{int}}) dk$. The functional form of the consumption bundle is identical across all households.
In summary, the differences between the households are exclusively due to the different localization of their employment contracts.

3.1 Expenditure Minimization

In period t each household minimizes the net value of her total current and future expenditures for given time paths of consumption, labor supply l_t^i and savings, which may either be in the form of domestic and/or foreign bonds or payment media, a given income distribution across marketplaces and given nominal opportunity costs of money holdings determined by the nominal interest rate r_{t-1} between periods $t-1$ and t. For analytical matters we split the household i in a continuum $[0, \iota]$ of subunits connected by an externality in utility (respectively expenditures) following the proposal in Starr (2002). Each of these subunits ik is located within a specific marketplace k, in which either specific consumption goods or domestic or rather foreign bonds are traded exclusively. Each subunit minimizes the family's common expenditure function

$$E_t \left(\sum_{q=0}^{\infty} \prod_{s=t}^{t+q-1} \frac{1}{1+r_s} \int_0^{\iota} \left((x^{ikB}_{(t+q)m} + x^{ikS}_{(t+q)m}) + \int_0^{\varsigma} p^{m_{int}}_{(t+q)} x^{ik}_{(t+q)m_{int}} dm_{int} + t^{ik}_{t+q} \right.\right.$$
$$\left.\left. + \int_{\varsigma}^1 e_{t+q} p^{m_{int}}_{(t+q)} x^{ikf}_{(t+q)m_{int}} dm_{int} + r_{t+q-1} x^i_{(t+q-1)m'} - (x^{ikB}_{(t+q-1)m} + x^{ikS}_{(t+q-1)m}) \right) dk \right) \tag{2}$$

by choosing its optimal local demands for consumption goods $x^{ik}_{(t+q)m_{int}}$ and central bank money $x^{ikB}_{(t+q)m} + x^{ikS}_{(t+q)m}$.[2] While the former are traded at price $p^{m_{int}}_{(t+q)}$ or rather, in the case

concept of approximate general equilibrium, e.g. Krusell et al. (1998), but this would complicate the analysis significantly. Moreover the stochastic localization process might offer the potential for future work to integrate some unemployment features by a search model structure.

[2] In this expenditure function any expenses for purchasing interest-bearing assets are not included, because the decision with respect to those is part of the second optimization stage and does not influence

of consumption goods produced abroad, at $e_t p^{m_{int}}_{(t+q)}$, i.e. after conversion with the nominal exchange rate e_t, the latter is traded at par value, but additionally generates the cost of inflation determined by the nominal interest rate r_{t+q-1}. The variable t^{ik}_{t+q} denotes taxes levied on household i in marketplace k. In each period the household has to respect an array of constraints. The first two of these comprise a given common consumption bundle (1) and a given bundle and composition of savings in payment media

$$\int_0^{\frac{1}{2}\iota} x^{ikB}_{(t+q)m} + x^{ikS}_{(t+q)m} - x^{ikB}_{(t+q-1)m} - x^{ikS}_{(t+q-1)m} dk \geq x^i_{(t+q)m} - x^i_{(t+q-1)m} \ \forall\, m \in M([0, \frac{1}{2}\iota]), \tag{3}$$

which may be interpreted as a prohibition of short sales. The second subset of constraint constitute the local income constraints

$$\begin{aligned}\int_0^{\varsigma} p^{km_{int}}_{(t+q)} x^{ik}_{(t+q)m_{int}} dm_{int} + \int_{\varsigma}^{1} e_{t+q} p^{m_{int}}_{(t+q)} x^{ikf}_{(t+q)m_{int}} dm_{int} + b^{ik}_{t+q} + e_{t+q} b^{ikf}_{t+q} \\ + x^{ikB}_{(t+q)m} + x^{ikS}_{(t+q)m} + r_{t+q-1}(x^{ikB}_{(t+q-1)m'} + x^{ikS}_{(t+q-1)m'}) + t^{ik}_{t+q} \\ \leq x^{ikB}_{(t+q-1)m} + x^{ikS}_{(t+q-1)m} + W^k_{t+q} l^{ik}_{t+q} + (1 + r_{t+q-1}) b^{ik}_{t+q-1} + (1 + r_{t+q-1}) e_{t+q} b^{ikf}_{t+q-1} \\ = I^{ik}_{t+q} \qquad \forall\, k \in [0, \iota]^3.\end{aligned} \tag{4}$$

Finally the local household has to take into account the probability distribution for its family to sign a labor contract across marketplaces. As already noted this distribution is uniform over the domain of all domestic marketplaces except those in which only bonds b_{t+q} are traded. This implies that households of domestic origin are never employed in the foreign economy. To simplify analysis it is assumed that using the cheapest available transaction media between an arbitrary marketplace and those in which bonds are traded, is at least as expensive as the usage of transaction media between the marketplace, in which the household is bound to a labor contract, and any other marketplace. Hence there is no incentive to avoid direct transactions between marketplaces by processing transactions via the financial markets. Thus the volume of transactions in financial markets is independent of the chosen structure of the consumption bundle and the savings within a period and vice versa. Consequently the local income constraints take a very simple form, since the single remaining income sources in local marketplaces are labor income, which is denoted by local wages W^k_{t+q} times labor supply l^{ik}_{t+q} and accrues only in a stochastic, yet unknown marketplace. Profits are excluded, because, as will be discussed below in more detail, profits of consumption good producers are never redistributed. The expenditures in the different markets also comprise the interest loss implied by holding central bank money, which earns no return. Since the household knows that transactions induce resource costs and that the central bank does not accept negative profits, it also pays

the first one. Adding the according terms would not change the results. Hence those are omitted for the sake of simplicity.

[3] One should bear in mind that consumption goods and bonds are only traded in distinct marketplaces and hence the demand for them is zero by definition in all other marketplaces.

attention to the budget restriction of this institution, i.e.

$$\int_0^{\varsigma} r_{t+q-1} x^i_{(t+q-1)m} di \quad = \quad \int_0^1 \int_0^1 s^k_{(t+q)m} \left(\frac{x^{ikB}_{(t+q)m} - x^{ikS}_{(t+q)m}}{\lambda^k_{t+q}} + \phi^m \right) dkdi. \tag{5}$$

An explicit interpretation of this expression shall be postponed until the profit function of the issuers of payment media is introduced. At this point it should be only mentioned that λ_{t+q} denotes the domestic general price level, and that money issued by the domestic central bank is held only in the domestic economy.

Using the definition from (1), substituting the resulting fraction of (5) into the objective function we obtain the following first order conditions (FOCs) for each subunit ik:

$$A \equiv -\o p^{m_{int}}_{(t+q)}(1+\nu^{ik}_{t+q})+\lambda^i_{t+q}(c^i_{t+q})^{\frac{1}{\theta}} \left(\int_0^{\iota} x^{ik}_{(t+q)m_{int}} dk \right)^{\frac{-1}{\theta}} \leq 0 \quad \o = \begin{cases} 1 & m_{int} \in [0, \varsigma] \\ e_{t+q} & m_{int} \in [\varsigma, 1] \end{cases} \tag{6}$$

$$x^{ik}_{(t+q)m_{int}} \geq 0 \qquad A x^{ik}_{(t+q)m_{int}} = 0 \qquad \forall m_{int} \in [0,1] \qquad \forall q \in \mathbb{N}$$

$$B \equiv \hat{\o} \left(-(1+\nu^{ik}_{t+q}) \left(1 + \hat{\o} \frac{s^k_{(t+q)m}}{P^k_{t+q}} \right) + \frac{1+\nu^{ik}_{t+q+1}}{1+r_{t+q}} + \mu_{(t+q)m} - \frac{\mu_{(t+q+1)m}}{1+r_{t+q}} \right) \leq 0 \tag{7}$$

$$B x^{ikj}_{(t+q)m} = 0 \quad \forall q \in \mathbb{N} \quad \hat{\o} = \begin{cases} 1 & x^{ikB}_{(t+q)m} \geq 0 \\ -1 & x^{ikS}_{(t+q)m} \leq 0 \end{cases} \qquad j = \begin{cases} B & \hat{\o} = 1 \\ S & \hat{\o} = -1 \end{cases}$$

Additionally in any optimum each of the constraints (1), (3) and (4) must either hold as equality, if the according shadow price is positive, or the shadow prices must be zero in case of inequalities.

Thus shadow price of the local income constraint ν^{ik}_{t+q} must be nonzero in all marketplaces in which the income constraint is binding. Hence, in the case of efficiency all markets are distorted compared to a model without transactions costs, since no income is wasted. Economically this implies that not only the demand for goods or assets, whose purchases imply additional transaction costs, but also the consumption in the familiar marketplace is distorted, since transaction costs have both both income and substitution effects. Furthermore, in an interior solution, efficiency requires that the current value of the distinct shadow prices of income constraints coincide across time. This is due to the fact that intertemporal differences in the distortion induced by transaction costs call for an intertemporal reallocation in order to minimize overall individual losses. Mathematically, optimality requires the constancy of ν^{ik}_{t+q} over time.
From (6) the total demand of an individual for a given type of a consumption good can be directly derived as

$$x^i_{(t+q)m_{int}} \equiv \int_0^{\iota} x^{ik}_{(t+q)m_{int}} dk = c^i_{t+q} \left(\frac{\o p^{m_{int}}_{t+q}(1+\nu^{ik}_{t+q})}{\lambda^i_{t+q}} \right)^{-\theta} \qquad \forall m_{int} \in [0,1]. \tag{8}$$

These equations demonstrate that the individual demand for a given consumption good is determined by the overall consumption of the household, the real price of the good,

which consists of the ratio of its price to the shadow price for an additional unit of consumption, λ^i_{t+q}, and the transaction cost. The latter represents the fact that in most cases the purchase of the consumption good needs to be financed by funds generated in another marketplace. Because the demand for consumption goods in marketplace k not only depends on the prices of these goods, but also on the shadow price of local income constraints, different transaction costs might drive a wedge between the demands for distinct goods, even if prices are symmetric.[4] Recalling the assumption that consumption goods will be never used as payment media, these equations also show that the demand for each consumption good is concentrated in the market in which it is produced.
Substituting the demand functions into the definition of the consumption index in (1) and solving the result for λ^i_{t+q} yields the cost-of-consumption index for the household i

$$\lambda^i_{t+q} = \left(\int_0^{\varsigma} \left(p^k_{(t+q)s}(1+\nu^{ik}_{t+q})\right)^{1-\theta} ds + \int_{\varsigma}^{1} \left(e_{t+q}p^{kf}_{(t+q)s}(1+\nu^{ik}_{t+q})\right)^{1-\theta} ds \right)^{\frac{1}{1-\theta}}. \tag{9}$$

The general price level is the aggregation of this index across all domestic households and thus coincides with the expectation value for individual indices for all households.
In an efficient solution the expenditures for savings in money exactly equal the target value, i.e. (3) holds as equlity. Hence the shadow price of savings in the central bank money $\mu_{(t+q)m}$ is positive and implicitly defined by

$$(1+\nu^{ik}_{t+q})(1+\hat{\varnothing}\frac{s^k_{(t+q)m}}{\lambda^i_{t+q}}) - \frac{1+\nu^{ik}_{t+q+1}}{1+r_{t+q}} = \frac{\mu_{(t+q+1)m}}{1+r_{t+q}} - \mu_{(t+q)m} \tag{10}$$

which demonstrates that principally the shadow prices of local income constraints would diverge from each other depending on whether central bank money is bought ($\hat{\varnothing} = 1$) or sold ($\hat{\varnothing} = -1$) in the markets in question by household i. But since the household is not sure about the placement of her labor contract, the shadow prices for domestic marketplaces are calculated as expectation value and thus coincide across marketplaces. On the other hand the shadow prices for local income in foreign marketplaces are non-stochastic, since ex ante the household knows that she will not earn anything in these marketplaces. Hence the related shadow prices are again identical to each other, but different to those in domestic marketplaces.

3.2 The Intertemporal Maximization Problem

With the structure of consumption and payment media bundles already determined from cost minimization the household chooses the optimal allocation of time between leisure and labor, its intertemporal consumption pattern and its portfolio of savings, which can be distributed across bonds and the various payment media of the economy, in order

[4] Transaction costs have a similar influence on the differences between demands for different goods as the home-bias in Warnock (2000).

to maximize its expected utility. Doing this it has to respect its intertemporal budget restriction

$$\begin{aligned}
B_t^i \quad &\equiv \quad \int\limits_0^{\iota} \bigg(\Big(\int_0^{\varsigma} p_t^{m_{int}} x_{tm_{int}}^{ik} dm_{int} + \int_{\varsigma}^{1} e_t p_t^{m_{int}} x_{tm_{int}}^{ik} dm_{int} + (x_{tm}^{ikB} + x_{tm}^{ikS}) \\
&\quad -(x_{(t-1)m}^{ikB} + x_{(t-1)m}^{ikS}) + r_{t+q-1}(x_{(t-1)m'}^{ikB} + x_{(t-1)m'}^{ikS}) + b_t^i - (1+r_{t-1})b_{t-1}^i \\
&\quad +e_t \left(b_t^{if} - (1+r_{t-1}^f) b_{t-1}^{if} \right) + t_t^{ik} \bigg) dk = \lambda_t^i c_t^i + \mu_{tm}^i \left(x_{tm}^i - x_{(t-1)m}^i \right) \\
&\quad +t_t^i + (1+\nu_t^{ib})(b_t^i - (1+r_{t-1})b_{t-1}^i) + (1+\nu_t^{ibf}) e_t \left(b_t^{if} - (1+r_{t-1}^f) b_{t-1}^{if} \right) \\
&\leq \quad \int_0^{\iota} W_t^k l_t^i dk = I_t^i,
\end{aligned} \tag{11}$$

and, again, equation (5) assuring zero profits for the central bank. The budget restriction can be interpreted as follows. Left of the equation sign and inside the integral, the first and second terms are the nominal sum of consumption expenditures, the third and fourth denote the nominal savings in the form of payment media, the fifth represents the costs of holding central bank money and the remaining terms are the period's net savings in domestic and foreign bonds and local lump-sum taxes, which can be aggregated across all marketplaces to global taxes t_t^i. In front of the inequality sign this expression is restated as the sum of expenditure on consumption and nominal savings, whereby consumption and the different saving instruments are valued with the referring cost indices implicitly defined by the integration of transaction costs into prices, while lump-sum taxes are retained in explicit form. For bonds the cost indices are equal to the shadow prices of income in the relevant marketplaces within the cost minimization problem. The stock of any payment medium m held by a household at the end of the period is denoted as $x_{(t+q)m}^i = \int_0^{\iota} (x_{t+s}^{ikB} + x_{t+s}^{ikS}) dk$. The available income follows can be seen the right side of the inequality and consists in nominal labor income. For foreign households, symmetric constraints must hold. The structure of the budget restriction implies that foreign currency can not be used in the domestic economy. This might be interpreted as a legislative entry barrier, or prohibitive entry costs for the foreign central bank.[5]

Assuming that central bank money is used as a saving instrument by each household and thus its shadow price is nonzero, the budget restriction can be reformulated as a law of motion for the state variable holdings of central bank money, i.e. x_{tm}^i,

$$\begin{aligned}
x_{tm}^i - x_{(t-1)m}^i = \frac{1}{\mu_{tm}^i} &\left(\bar{W}_t l_t^i - \lambda_t^i c_t^i + \Pi_t^i - (1+\nu_t^{ib}) \left(b_t^i - (1+r_{t-1}) b_{t-1}^i \right) \right. \\
&\left. -(1+\nu_t^{ibf}) e_t \left(b_t^{if} - (1+r_{t-1}^f) b_{t-1}^{if} \right) - t_t^i \right),
\end{aligned} \tag{12}$$

where $\bar{W}_t$ denotes the expected nominal wage of period t across all domestic marketplaces. Respecting this constraint in all periods the household maximizes her discounted expected

[5] Of course this assumption is closely related to the assumption that only domestic currency generates utility to domestic residents. Cf. Obstfeld et al. (1995), p.627.

utility

$$E_t\left(\sum_{q=0}^{\infty}\beta^j U_{t+q}(c_{t+q}, l_{t+q})\right)$$

by choosing optimal paths for consumption, labor and holdings of payment media and bonds. Note that with respect to assets, only net holdings are chosen in this stage, because the gross volume which determines transaction costs, is determined by cost minimization. The household has to meet the additional constraint of no short sales in payment media

$$x^i_{(t+q)m} \geq 0, \tag{13}$$

since its borrowing is restricted to interest-bearing bonds. Furthermore, given an infinite time horizon, in such a problem for each household there is the incentive to build up negative savings in each period, because the resulting payment obligations can be postponed into an uncertain future. But on the aggregate level such behavior is not possible, because there are no potential lenders. Therefore a no-Ponzi-game condition has to be incorporated, which prevents households from building up debt with a negative net present value throughout the whole infinite time horizon. On the other side there is no incentive to build up a positive net present value, since this is identical to wasting income. Formalizing these two arguments one is left with the transversality condition

$$\lim_{t\to\infty}\left(\frac{1}{1+r_t}\right)^t x^{ik}_{tm} + b^i_t + e_t b^{if}_t = 0. \tag{14}$$

Finally, the household knows that the central bank needs to balance its budget in all periods, and hence accounts for that by meeting equation (5).
Without presenting the detailed solution, which can be found in appendix 12.1, we simply indicate that this control problem is solved by a Hamiltonian approach, in which the three variables z^i_t, ρ_t and v_{tm} are defined as shadow prices for the constraints (12), (5) and (13). Moreover it should be noted that in order to simplify notation from now on the operator for expectation value E_t is be omitted except for special occasions. Thus all current and future variables except for predetermined variables denote in fact expectation values on the basis of the information in period t. Furthermore, in order to derive more explicit results, we assume a special form for the utility function. Like the major part of the literature using these kinds of models we suppose a linear separable utility function with a unitary elasticity for both arguments, consumption and leisure. Utility is given by the function

$$E_t(U) = E_t\left(\sum_{j=0}^{\infty}\beta^j\left(\frac{1}{1-\sigma}c^{1-\sigma}_{t+j} + \frac{1}{1-\eta}(1-l_{t+j})^{1-\eta}\right)\right), \tag{15}$$

which ensures constant elasticities of substitution between leisure and consumption within a period and between periods. Moreover, the function displays the risk aversion of households, which depends negatively on the elasticity of intertemporal substitution with respect to changes in the interest rate.[6] From (94), (95) and (99) one obtains the equations

$$
\begin{aligned}
c_t^{-\sigma} &= p_t^c \left(\beta z_{t+1} - z_t + \frac{1 + \frac{r_t}{\hat{s}_{t+1}^m}}{p_{t+1}^c} \beta \left(c_{t+1}^{-\sigma} - \frac{z_{t+1}}{\mu_{(t+1)m}} \right) + \frac{z_t}{\mu_{tm}} \right) && (16) \\
(1 - l_t^i)^{-\eta} &= \frac{z_t}{\mu_{(t)m}} \bar{W}_t && (17)
\end{aligned}
$$

of which the first resembles the Euler condition for consumption in the traditional New-Keynesian macroeconomic model, but comprises additional summands indicating the distortions caused by transaction costs, while the second equation is the standard condition for labor supply. Solving the first equation forward one can find that current consumption is a function of current and future costs of consumption, nominal interests, resource costs for transaction services, shadow prices and cost indices of money savings. Note that while deriving the Euler condition, we implicitly used an equation to be presented later, which states that the transaction volume of each household is identical to the sum of its consumption purchases in marketplaces without income plus net savings in bonds and money. Therefore the new variable p_t^c appears, which is the aggregation of consumption good prices across all goods.

Additionally, (94) and (95) together determine the intratemporal choice between consumption and leisure.

$$
\frac{(c_t^i)^{-\sigma}}{(1 - l_t^i)^{-\eta}} = \frac{\lambda_t^i}{\bar{W}_t} + \rho_t \frac{\mu_{tm}^i}{z_t^i} \frac{s_t^{km}}{\bar{W}_t} \frac{p_t^c}{\lambda_t^i} \qquad (18)
$$

Transaction costs displace the ratio between consumption and leisure towards the latter, since leisure is free of any extra costs. The model's version of the uncovered interest parity

$$
1 - \beta \frac{z_{t+1}\mu_{tm}}{z_t \mu_{(t+1)m}} \frac{1 + \nu_{t+1}^{ik}}{1 + \nu_t^{ik}} (1 + r_t) = \left(1 - \beta \frac{z_{t+1}\mu_{tm}}{z_t \mu_{(t+1)m}} \frac{1 + \nu_{t+1}^{ik}}{1 + \nu_t^{ik}} \frac{e_{t+1}}{e_t} (1 + r_t^f) \right) e_t \qquad (19)
$$

holds due to (96) and (97). Hence, international arbitrage in riskless assets is not possible. For an efficient use of income the reformulated budget restriction (12) gives directly the optimal net demand for central bank money, which depends on the household's future demand for this asset, its expected future income from labor and bonds and its expected future expenditures for consumption, savings and taxes. This equation has no similarities with the familiar demand for money derived in more standard money-in-utility models such as in Sidrauski (1967) or cash-in-advance models as in Lucas (1980), since the appearance of (5) destroys the direct connection between the shadow price of the intertemporal budget constraint and the nominal interest rate and implies that money demand is not solely determined due to the intertemporal allocation, but also due to transaction needs. The money demand no longer determines the bond demand, but both are chosen simultaneously, whereby bonds fill the gap between income and the expenditures for consumption

[6]For the given utility function the Arrow-Pratt measure of relative risk aversion coincides with the elasticity of intertemporal substitution and is equal to σ. Cf. Obstfeld et al. (1996), p.278f.

and payment media, while their composition between domestic and foreign bonds assure an equilibrium in the bond markets and the uncovered interest parity.
If the nominal interest rate r_t, and therefore the opportunity costs of holding money is zero and thus identical to the optimal rate of interest in Friedman (1969), optimal usage of money is no longer determined, since in this case there have to be zero production costs. Thus arbitrary amounts of the money can be held. In this case the neoclassical neutrality of nominal money holds in the short run.

In general there are two reasons for transactions across marketplaces. Firstly households have to move financial wealth across marketplaces, if individual income is concentrated in one or a few marketplaces. Secondly, each consumption good is supplied exclusively in a distinct marketplace. Therefore, spatial specialization and labor immobility substantiates the need for resource-intensive transactions within the model.[7] As a potential third reason arbitrage can be cancelled out due to the fact that the price of the money is legally fixed to its denomination value. For efficiently used payment the sign of the net demand in a marketplace k can be deduced from (**??**) as

$$x_{mt}^{ik} \lesseqgtr 0 \quad \text{iff} \quad I_t^{ik} \lesseqgtr r_{t-1} x_{t-1}^{ik} + c_t^i \left(\frac{p_t^{m_{int}}(1+\nu_t^{ik})}{\lambda_t^i} \right)^{-\theta}. \tag{20}$$

Equation (20) determines the optimal use of money across marketplaces. To generate the equivalents for bond markets the demand for goods has to be exchanged for the current bond holdings
Of course, symmetric conditions can be derived for the foreign economy, which is modelled equivalently to the domestic economy.

4 Firms

There are two kind of firms. Firstly there is a continuum of firms with a technology characterized by fixed costs, and thus increasing returns to scale, producing consumption goods and behaving as monopolists. These firms theoretically can choose to serve up to two fields of business, which are the physical production of the good m_{int} or the production of payment services in this good by acting as a trading post. Both business fields are characterized by distinct technologies. But due to two arguments the supply of payment services by consumption good producers can be ruled out. Firstly, as long as consumption good producers compete in the payment sector with a non-profit-maximizing, but cost-covering central bank, they can not expect any profits from being trading posts. Secondly, their technology tends to be inferior as that of the central bank due to the advantages of specialization. For both reasons it is assumed that the supply of transactions services in any consumption good is not profitable. In terms of Starr (2002) this implies that, for

[7] Both ideas have a long tradition and date back to classical theory. They are associated with such famous names as Jevons, Menger and Smith. Cf. Ostroy et al. (1990), p.26ff.

each agent, the costs of any transaction using a technology of a producer of consumption goods would be higher than that implied by the technology of the central bank. Hence, for these firms only the production of goods for consumer markets is of interest. Therefore the relevant production function for each supplier of consumption goods is

$$y^k_{m_{int}} = f_{m_{int}}(k_{tm_{int}}, l_{tm_{int}}) = \max\{a_t l^{1-\alpha_1}_{tm_{int}} k^{\alpha_1}_{tm_{int}} - \phi_1, 0\}, ^{8} \tag{21}$$

where a_t is an aggregate stochastic productivity shock with $E(a_t) = 1$, the density function $F(a_t)$ and the domain $\mathbb{R}_+$, and $k_{tm_{int}}$ and $l_{tm_{int}}$ denote firm-specific capital and labor input, respectively. Following Bernanke et al. (1998) we assume that these firms do not reallocate their profits to households, but accumulate profits resulting in a firm specific net wealth position $N^{m_{int}}_t$ built up from former and current profits. This equity is used to partially finance their capital needs for the next period's production. This serves to integrate a credit channel for monetary policy into the model.
As already introduced, a fraction $1 - \varsigma$ of the producers of consumption goods are located in foreign marketplaces, while a fraction ς are located in those of the domestic economy. Within the foreign economy exactly a fraction $\frac{1-\varsigma}{0.5\iota}$ of goods are produced in every marketplace, whereby the intersection of the produced goods between marketplaces is empty. Similarly, at home the fraction of goods produced exclusively within a marketplace is $\frac{\varsigma}{0.5\iota}$. Hence every producer of a consumption good is a monopolist in the whole world. Due to legal restrictions or prohibitively high entry costs foreign technologies are locked out of domestic marketplaces, while domestic technologies are locked out of the foreign economy.

Secondly there is the central bank, which produces goods money used for payment and saving purposes. By assumption the central bank is present in each domestic market-place. Its production function

$$\frac{y^{mkB}_{tm} - y^{mkS}_{tm}}{P_t} = f^m(k_{tm}, l_{tm}) = a_{2t} l^{1-\alpha_2}_{tkm} k^{\alpha_2}_{tkm} - \phi^m_1 \tag{22}$$

is be modelled as one producing a local real gross transaction value by using labor and capital as inputs. The total factor productivity a_{2t} is modelled as an AR(1) process

$$a_{2t} = a_{2(t-1)} + \epsilon^{a2}_t \tag{23}$$

with a stochastic term ϵ^{a2}_t modelled equivalently to the stochastic total factor productivity of consumption good producers. Obviously, both selling and purchasing generate resource efforts for the central bank, which therefore exclusively depend on the real overall transaction volume.[9]
Differing from Starr (2002) we add additional structure to the model economy by assuming, like Rotemberg (1984) or Williamson (1999), that there are companies located in reach market-place offering financial intermediation. In order to simplify the analysis, financial intermediation is assumed to generate no resource costs, except for monitoring costs, in the case of default, which will be discussed below.

[8]This production function seems to contain the risk of discontinuity. But in line with the existing literature this can be ruled out by the assumption that in steady state production is sufficiently far away from the place of discontinuity. Cf. Christinao et al. (2001), p.8.

[9]Cf. Starr (2002), p.12 and Starr (2003), p.461.

4.1 Optimal Corporate Debt Contracts

Credit markets are characterized by the possibility to evade debt serving and thus contain an incentive for risk pooling. Together with the transaction costs in the economy this is a strong economic argument for financial intermediation, since the potential to reduce the amount of financial transactions generates economies of scale.[10] The potential debt evasion is modelled along the slightly modified arguments of Galor et al. (1993), who assume that a borrower can evade serving debt by investing in a resource-consuming technology. On the other hand lenders can use a monitoring technology in order to observe the evading behavior of their borrowers. Thus they can prevent losses by precautionary measures. The evading technology uses units of the aggregate consumption bundle as inputs u_t and implies a linear cost function with slope χ and the fixed cost block $\bar{\chi}$. The monitoring technology is also linear in u_t with unitary marginal costs.[11] Hence the borrower chooses to comply with the loan contract, if

$$(1 + r^{m_{int}}_{(t-1)c})(Q_{t-1}k^{m_{int}}_t - N^{m_{int}}_{t-1}) \leq (\bar{\chi} + \chi u_t)\lambda_t \tag{24}$$

holds. Thus repaying the loan, whose size is the difference between the current capital stock $k^{m_{int}}_t$ valued by the capital price of the last period Q_{t-1} minus equity $N^{m_{int}}_{t-1}$ inherited from the last period, at the gross interest rate $1+r^{m_{int}}_{(t-1)c}$ is at least as cheap as the evasion.[12] On the other hand the lender will extend credit, if and only if its net return at least covers the monitoring costs, i.e.

$$(1 + r^{m_{int}}_{(t-1)c} - (1 + r_{(t-1)}))(Q_{t-1}k^{m_{int}}_t - N^{m_{int}}_{t-1}) \geq \lambda_t \chi u_t. \tag{25}$$

Since financial intermediation is a competitive business both inequalities hold in equilibrium as equalities. This implies that actually debt evasion does not happen, but lenders need to employ the costly monitoring technology in order to increase the cost of evading up to the point above which evasion is no longer worthwhile for borrowers. Substituting the first condition into the second one, solving for the effort u_t and substituting this back into (24) the risk premium of the contract can be restated as a function of loan size and riskless interest rate

$$\frac{1 + r^{m_{int}}_{(t-1)c}}{1 + r_{t-1}} = \frac{\chi}{\chi - 1} - \frac{\lambda_t \bar{\chi}}{(1 + r_{t-1})(\chi - 1)(Q_{t-1}k^{m_{int}}_t - N^{m_{int}}_{t-1})} > 1. \tag{26}$$

[10] In the argument employed resource costs of transactions play a similar role as the monitoring costs in the famous model of Diamond (1984).

[11] Of course there is a famous alternative method to integrate financial imperfections into the model, which is known under the term financial accelerator. While this model appeals due to its economic interpretation and its elegance, its integration into a computable general equilibrium framework has turned out to be problematic, because the attempts to calibrate the model failed to produce convincing results. This not only holds for the model presented in the main text, but even the calibration in the existing literature, e.g. Bernanke et al. (1998), seems to be inconsistent, since using the reported calibration values produces an inconsistent budget restriction within the optimization of the financial contract. Cf. Bernanke et al. (1998) p.33,49. For this reason the model presented relies on the specification above, which has the virtue of the existence of consistent calibrations.

[12] Capital is assumed to be homogenous in the sense of differences between the value of new and old, but not depreciated, capital (Bernanke et al. (1998)). But since we assume that firms are never fully financed, there is no market for already used capital. Hence capital is immobile between sectors (Woodford (2003)).

4.2 The Optimization of Consumption Good Producers

As in all models of the New-Keynesian type we assume some price rigidity, which we incorporate by following an inflation-adjusted variant of the modelling in Calvo (1983). Therefore firms sometimes adjust and sometimes do not adjust their prices depending on the probability $1-\varrho$ of adjusting. If prices are not adjusted, the price is equal to that of the last period times the unconditional expectation of gross inflation, which coincides with the gross inflation rate in steady state, i.e. $p_{tm_{int}} = p_{(t-1)m_{int}}(1+\pi)$.[13] If prices are to be adjusted, the profit maximizing level for the new price will be chosen. Doing this a consumption good producer has to respect the given demand function for its good

$$\begin{aligned} x_{(t+q)m_{int}} \equiv & \int_0^{\varsigma} \left(c^i_{(t+q)} + (y^{PCd}_{(t+q)} + y^{MCd}_{(t+q)} + y^{tc}_{(t+q)} + g_{t+q})\right) \left(\frac{p^{km_{int}}_{t+q}(1+\nu^{ik}_{t+q})}{\lambda^i_{t+q}}\right)^{-\theta} di \quad (27) \\ & + \int_{1-\varsigma}^{1} \left(c^i_{(t+q)} + (y^{PCf}_{(t+q)} + y^{MCf}_{(t+q)} + y^{tcf}_{(t+q)} + g^f_{t+q})\right) \left(\frac{p^{km_{int}}_{t+q}(1+\nu^{ik}_{t+q})}{e_{t+q}\lambda^i_{t+q}}\right)^{-\theta} di, \end{aligned}$$

which consists of the five components: households' demand, capital producers' demand, demand for input for monitoring services, demand of resources used in bequeathing capital from old firms to new, and public sector demand. The latter four components are of the same functional form as the first one. Hence, one can express the objective function of a firm, which is located in marketplace k, as the expected present value of all current and future net profits

$$\begin{aligned} E_t(\Pi^{m_{int}k}) = & E_t\left(\sum_{q=0}^{\infty}\prod_{s=0}^{q}\left(\frac{\gamma}{1+r_{t+s-1}}\right)(p^k_{(t+q)m_{int}}x_{(t+q)m_{int}} - W^k_{t+q}l^{km_{int}}_{t+q}\right. \quad (28) \\ & -(1+r_{t+q-1})N^{m_{int}}_{t+q-1} - (1+r^{m_{int}}_{c(t+q-1)}))(Q_{t+q-1}k_{t+q} - N^{m_{int}}_{t+q-1}) \\ & \left. +(1-\delta)Q_{t+q}k_{t+q-1})\right), \end{aligned}$$

whereby each period's net profit is weighed not only with the associated discount rate, but additionally with the relevant power of the exogenous probability for the firm to survive until next period denoted as γ. Profits in each period are equal to the net returns from selling the whole production of good m_{int}, because no producer of consumption goods offers payment services. Moreover the finance costs of the current period's investment expenditures pertain to the same period's profit, because investment is due to actual capital depreciation.[14] In choosing optimal sequences of adjusted prices $(p^{*k}_{(t+q)m_{int}})_{q\in\mathbb{Z}_+}$[15]

[13] We use the same modelling as Erceg et al. (2000). Cf. Erceg et al. (2000), p.288f.

[14] Two points can be made about this profit function. Firtsly the above profit function is not formulated as the discounted stream of future incomes in the case of perpetuating the currently chosen price, because, as discussed in Sveen et al. (2004b), capital dynamics imply an influence of future prices on today's investment decision. Secondly, the delay in the investment function incorporates the real world feature of long-run investment, e.g. R & D, into the model.

[15] Instead of the current optimal adjusted price the firm chooses a whole sequence of current and future optimally adjusted prices, because future optimal adjusted prices influence price expectations and therefore its investment policy as indicated in the previous footnote.

and the optimal sequences of labor $(l_{t+q}^{m_{int}})_{q\in\mathbb{Z}_+}$ and capital $(k_{t+q+1}^{m_{int}})_{q\in\mathbb{Z}_+}$ the firm has to respect additionally the terms of the financial contract given in (26), its technology given in (21), the development of the firm-specific capital stock

$$k_{t+q}^{m_{int}} = (1-\delta)k_{t+q-1}^{m_{int}} + I\left(\frac{k_{t+q}^{m_{int}}}{k_{t+q-1}^{m_{int}}}\right)k_{t+q-1}^{m_{int}}, \tag{29}$$

the Calvo-style development of expected future prices

$$p_{(t+q+1)m_{int}}^{k} = \begin{cases} p_{(t+q+1)m_{int}}^{*k} & \text{with prob.} \quad 1-\varrho \\ p_{(t+q)m_{int}}^{k}(1+\pi) & \text{with prob.} \quad \varrho \end{cases} \tag{30}$$

and finally the development of its equity. The latter is determined by the profit level according to

$$N_{t+q}^{m_{int}} = \gamma\left((1+r_{t+q-1})N_{t+q-1}^{m_{int}} + \Pi_{t+q}^{m_{int}}\right) + (1-\gamma)(1-tc)\left((1+r_{t+q-1})\bar{N}_{t+q-1}^{m_{int}} + \bar{\Pi}_{t+q}^{m_{int}}\right), \tag{31}$$

and can be decomposed into individual gross profits for the case of surviving and average gross profits minus transfer costs for the case of the firm's death. The last equation reveals that in the case of a firm's demise the resulting market niche is occupied by a successor, which inherits a distinct percentage $1-tc$ of the aggregate capital of all firms which have ceased to operate in the current period. The residual capital is consumed by the transfers from old to new firms and establishes the costs of bequests.

The investment function $I(.)$ defines the real expenditures needed in period $t+q$ to generate the capital stock $k_{t+q+1}^{m_{int}}$ for the next period.[16] By assumption the function is globally convex and satisfies the properties $I(1) = \delta$ and $I'(1) = 1$, where δ denotes the depreciation rate, which is identical across all firms. Hence, in steady state, the investment balances the capital depreciation caused by physical losses and there are no adjustment costs in the investment process. Outside of steady state the convex shape of the investment function generates adjustment costs, because investment expenditures do not transform into future capital one-to-one.[17]

With respect to the origin of new capital the model follows a simplified version of Bernanke et al. (1998). There are competitive, global firms, which produce new physical capital goods by combining final outputs to material input y_{t+q}^{PC}. Their common production function is

$$I_{t+q}^{S} = \Phi\left(\int_0^1 (y_{(t+q)m_{int}}^{PC})^{\frac{\theta-1}{\theta}} dm_{int}{}^{\frac{\theta}{\theta-1}}\right). \tag{32}$$

Hence, in optimum, one gets

$$Q_{t+q} = \frac{\lambda_{t+q}}{\Phi'} \tag{33}$$

[16] The capital stock is measured at the beginning of each period.

[17] This formulation of the capital stock dynamics follows the approach in chapter 5 of Woodford (2003) or the corrected version in Woodford (2005), respectively.

and an optimal investment demand for input factors, which is of exactly the same form as aggregate consumption demand, if, for simplicity, it is assumed that producers of new capital incur the same marginal transaction costs as the average of all households. Additionally, in the steady state, $\Phi(.) = \delta k$ and $\Phi' = 1$ hold by assumption, i.e. the production function is normalized to ensure, on the one hand, an equilibrium in the market for investment goods and, on the other hand, no adjustment costs of investment and the equivalence of the general price level and the price for new capital. Nevertheless, as already mentioned, outside of the steady state there are adjustment costs in capital stock and due to the individual evolvement of a firm's capital stock it is not possible to immediately shift capital from one sector to another without costs despite the fact that there is a single rental market for capital within the model.[18]

Substituting the incentive compatibility constraint, the investment function and the law of motion for prices into the profit function and using the fact that the single firm is too small to influence the aggregate consumption price level or the aggregate consumption expenditures, the solution is characterized by the following conditions, which hold for each $q \geq 0$.

$$E_t\left[-W^k_{t+q}(1-\psi^{m_{int}}_{1(t+q)}) + \psi^{m_{int}}_{2(t+q)}(1-\alpha_1)\left(\frac{k^{m_{int}}_{t+q}}{l^{m_{int}}_{t+q}}\right)^{\alpha_1}\right] = 0 \quad (34)$$

$$\begin{aligned} E_t\Bigg[\Bigg(\Bigg(I'\left(\frac{k^{m_{int}}_{t+q+2}}{k^{m_{int}}_{t+q+1}}\right)\frac{k^{m_{int}}_{t+q+2}}{k^{m_{int}}_{t+q+1}} - I\left(\frac{k^{m_{int}}_{t+q+2}}{k^{m_{int}}_{t+q+1}}\right) - (1-\delta)\Bigg)\frac{(1-\psi^{m_{int}}_{1(t+q+2)})\gamma}{(1-\psi^{m_{int}}_{1(t+q+1)})}\cdot \quad (35)\\ \left(\frac{\chi}{\chi-1}Q_{t+q+1} - \frac{(1-\delta)Q_{t+q+2}}{1+r_{t+q+1}}\right) - I'\left(\frac{k^{m_{int}}_{t+q1}}{k^{m_{int}}_{t+q}}\right)\cdot \\ \left(\frac{\chi}{\chi-1}Q_{t+q}(1+r_{t+q}) - (1-\delta)Q_{t+q+1}\right) + \frac{\psi^{m_{int}}_{2(t+q+1)}\alpha_1}{(1-\psi^{m_{int}}_{1(t+q+1)})}\left(\frac{l_{t+q+1}}{k^{m_{int}}_{t+q+1}}\right)^{1-\alpha_1}\Bigg] = 0 \end{aligned}$$

$$E_t\left[\sum_{n=q}^{\infty}\prod_{s=0}^{n}\frac{\gamma}{1+r_{s-1}}\left(\frac{x_{(t+n)m_{int}} + p^{m_{int}}_{(t+s)}\frac{\partial x_{(t+n)m_{int}}}{\partial p^{m_{int}*}_{(t+q)}}}{(1-\psi^{m_{int}}_{1(t+n)})^{-1}} - \psi^{m_{int}}_{2(t+n)}\frac{\partial x_{(t+n)m_{int}}}{\partial p^{m_{int}*}_{(t+q)}}\right)\right] = 0 \quad (36)$$

$$E_t\left[\psi^{m_{int}}_{1(t+q+1)} - \psi^{m_{int}}_{1(t+q)}\right] = E_t\left[(1-\frac{\chi}{\chi-1})(1-\gamma\psi^{m_{int}}_{1(t+q+1)}) - (1-\gamma)\psi^{m_{int}}_{1(t+q)}\right] \quad (37)$$

The variables $\psi^{m_{int}}_{1(t+q)}$ and $\psi^{m_{int}}_{2(t+q)}$ denote the shadow prices of the law of motion of equity and the technological constraint, which are both adjusted for the discounting term. For optimality these two constraints must hold.

Combining (34) and (35) yields

$$\frac{k^{m_{int}}_{t+q}}{l^{m_{int}}_{t+q}} = \frac{1}{h^{1m_{int}}_{(t+q)}}, \quad (38)$$

[18] Cf. Bernanke et al. (1998), p.18f., Woodford (2003), p.354.

where

$$h^{1m_{int}}_{(t+q)} \equiv \frac{(1-\alpha_1)Q_{t+q+1}}{\alpha_1 W^k_{t+q+1}} \left(I'\left(\frac{k^{m_{int}}_{t+q+1}}{k^{m_{int}}_{t+q}}\right) \left(\frac{\chi(1+r_{t+q})}{\chi-1}\frac{Q_{t+q}}{Q_{t+q+1}} - (1-\delta)\right) - \frac{(1-\psi^{m_{int}}_{1(t+q+2)})\gamma}{(1-\psi^{m_{int}}_{1(t+q+1)})} \cdot \right. \tag{39}$$
$$\left. \left(\frac{\chi}{\chi-1} - \frac{(1-\delta)Q_{t+q+2}}{(1+r_{t+q+1})Q_{t+q+1}}\right) \left(I'\left(\frac{k^{m_{int}}_{t+q+2}}{k^{m_{int}}_{t+q+1}}\right)\frac{k^{m_{int}}_{t+q+2}}{k^{m_{int}}_{t+q+1}} - I\left(\frac{k^{m_{int}}_{t+q+2}}{k^{m_{int}}_{t+q+1}}\right) - (1-\delta)\right)\right),$$

is the optimal ratio of the two input factors capital and labor.[19] This ratio does not need to be efficient, because, as argued in Woodford (2003), the supply of firms might be restricted by demand. Hence firms do not adjust the marginal product of capital to its price but to the reduction in labor costs generated by a marginal substitution of capital for labor. Sveen et al. (2004) add that future marginal labor savings depend on future prices, if the firm is a price setter, because future prices influence future demand, which feeds back into the current investment decision. Hence investment, or capital, dynamics integrate an array of effects, which might impede efficiency in the model, especially outside of steady state.

Substituting the optimal ratio back into (34) yields a solution for $-\psi^{m_{int}}_{2(t+q)}/1-\psi^{m_{int}}_{1(t+q)}$. Since the production decision takes place prior to the realization of the productivity shock, nominal marginal costs in period $t+q$ accrue independently of the occurrence of positive profits. Moreover, they comprise the effect of the investment decision on future equity. Hence they coincide with the ratio mentioned and can be expressed as

$$s^{m_{int}}_{t+q} = \frac{1}{1-\alpha_1} W^k_{t+q} (h^{1m_{int}}_{(t+q)})^{\alpha_1}. \tag{40}$$

Substituting marginal costs into (37), replacing the expected price for period $t+q$ by the price optimally chosen in period t, after it has been adjusted for steady state inflation between these two periods, and yields an implicit expression for this variable.

$$E_t \left[\sum_{n=q}^{\infty} \left(\prod_{s=0}^{n} \frac{\varrho\gamma}{1+r_{t+s-1}} \right) \left(p^{*m_{int}}_t (1+\pi)^n - \frac{\theta}{\theta-1} s^{m_{int}}_{t+n} \right) x_{(t+n)m_{int}} \right] = 0. \tag{41}$$

Similarly to the standard New Keynesian model with endogenous capital formation as presented in e.g. Sveen et al. (2004a), the optimal price chosen today depends on current and expected future nominal costs, which are dependent amongst others on future optimal prices.[20] The financial contract does not alter the mark-up, since it is not connected to any form of additional market power of the producer, but only reflects the risk of debt

[19]Ignoring capital dynamics by setting the depreciation rate to one and the exit probability for the firm to zero yields the traditional result of models with an exogenously determined capital stock. This is still the most common assumption in the New Keynesian literature. Endogenous dynamics of the capital stock are discussed in Ireland (1996) and its successors.

[20]To understand the substitution of the optimal price chosen in period t for the expected prices of subsequent periods note that it is justified by the fact that the influence of the current optimal price on prices to be chosen optimally in the future is limited to its influence on capital dynamics, because in the case of reoptimization the individual will only consider its subsequent profit. Cf. Woodford (2004), p.9. Nevertheless, according to Sveen et al. (2004b) future marginal costs and aggregate demand depend on the prices optimally chosen in future. Hence, the current price decision depends on future price decisions

evasion. Hence, mark-ups remain equal across all firms.[21]
In contrast to the competitive structure in Bernanke et al. (1998) in the steady state there is a positive mark-up within the production sector. Moreover, dynamics outside the steady state do alter the equity development not only via the saving and portfolio decisions of (some) households but also by influencing the firm's profits.

Replacing the ratio $-\psi_2^{m_{int}}/(1-\psi_1^{m_{int}})$ through nominal marginal costs in (34)-(35) and log-linearizing[22] the results alongside with (37), (21), (31) and (41) leaves one with a system of six equations, including four difference equations, in the endogenous (exogenous) variables $\dot{s}_{t+q}^{m_{int}}$, $\dot{h}_{t+q}^{1m_{int}}$, $\dot{p}_{t+q}^{*m_{int}}$, $\dot{N}_{t+q}^{m_{int}}$, $\dot{k}_{t+q}^{m_{int}}$, $\dot{\psi}_{1(t+q)}^{m_{int}}$, ($\dot{W}_{t+q}^{k}$,$\dot{r}_{t+q}$,$\dot{Q}_{t+q}$,$\dot{c}_{t+q}$,$\dot{\nu}_{t+q}$,$\dot{\lambda}_{t+q}$, $\dot{y}_{t+q}^{PC}$, $\dot{y}_{t+q}^{MC}$) from the perspective of the firm. Herein $\dot{\nu}_{t+q}=\dot{\nu}_{t+q}^{dd}+\dot{\nu}_{t+q}^{df}+\dot{\nu}_{t+q}^{fd}+\dot{\nu}_{t+q}^{ff}$ denotes the aggregation of the deviation of the shadow prices of local income from steady state across all marketplaces and all households. It can be decomposed into the aggregates across all domestic (foreign) marketplaces and households or rather across domestic (foreign) households and foreign (domestic) marketplaces. The linearized versions of (21) and (34) can be used to eliminate the variables $\dot{h}_{(t+q)}^{1m_{int}}$ and $\dot{s}_{t+q}^{m_{int}}$ within the linearized versions of (35), (37) and (31). These form a subsystem of three difference equations in several exogenous and the four endogenous variables $\dot{p}_t^{m_{int}}, \dot{N}_{t+q}^{m_{int}}, \dot{k}_{t+q}^{m_{int}}, \dot{\psi}_{1(t+q)}^{m_{int}}$. Arranging the expected price and the exogenous variables for the moment on the right side this subsystem can be denoted as

$$E_t\left(\mathbf{\Upsilon} L^3\left((\dot{k}_{t+q+2}^{m_{int}}, \dot{N}_{t+q+1}^{m_{int}}, \dot{\psi}_{1(t+q+1)}^{m_{int}})^T\right)\right) = E_t\left((\varphi_{m_{int}}-(1-\varphi_{m_{int}}))\mathbf{\Xi}_e L^3(\dot{e}_{t+q+2})+\right. \quad (42)$$
$$\left.\mathbf{\Xi} L^3\left(\dot{p}_{t+q+2}^{m_{int}}, \dot{r}_{t+q+1}^{l}, \dot{c}_{t+q+2}, \dot{y}_{t+q+2}^{PC}, \dot{y}_{t+q+2}^{MC}, \dot{\nu}_{t+q+2}, \dot{\lambda}_{t+q+2}, \dot{W}_{t+q+2}^{k}, \dot{Q}_{t+q+1}^{l}, \dot{a}_{(t+q+2)}\right)^T\right),$$

where the matrices $\mathbf{\Upsilon}$, $\mathbf{\Xi}$ and $\mathbf{\Xi}_e$ are displayed in appendix 12.2, $L^3(x_{t+2})$ denotes the lag operator mapping an arbitrary variable x_{t+2} into (x_{t+2}, x_{t+1}, x_t), and $\varphi_{m_{int}}$ indicates whether the firm producing m_{int} is located in a domestic, i.e. $\varphi_{m_{int}}=1$, or a foreign marketplace, i.e. $\varphi_{m_{int}}=0$.[23] Moreover the superindex l can take the values d or f indicating whether a domestic or a foreign firm is observed. Averaging across all consumption producers in the same marketplace and subtracting the result from the last equation while remembering that the stochastic shock is known at the time of optimization only in expectation, one is left with

$$E_t\left(\mathbf{\Upsilon}\cdot L^3\left((\dot{\tilde{k}}_{t+q+2}^{m_{int}}, \dot{\tilde{N}}_{t+q+1}^{m_{int}}, \dot{\tilde{\psi}}_{1(t+q+1)}^{m_{int}})^T\right)\right) = E_t(\tilde{\mathbf{\Xi}}\cdot(\dot{\tilde{p}}_{t+q+2}^{km_{int}}, \dot{\tilde{p}}_{t+q+1}^{km_{int}}, \dot{\tilde{p}}_{t+q}^{km_{int}})^T).$$

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and feedback to the eventual future price decision results. But the influence of future prices on the current decision is limited to the indirect influence transmitted through other variables. This analysis could be challenged, since clearly future optimal prices influences the current net worth of the firm. But despite this argument the analysis in the main text treats the case of Sveen et al. (2004b), because the more general case without substitution reveals such a degree of complexity that a general analysis within this research project is not been possible.

[21]In a financial accelerator model mark-up would depend on risk premia and would differ across firms.

[22]An explicit exposition of the technique of log-linearizing can be found in Uhlig (2001).

[23]Note that for an individual firm m_{int} the vector $\mathbf{\Xi}_e$ multiplied by its scalar coefficient can be integrated within the matrix $\mathbf{\Xi}$. The notation above is chosen to generalize the expression for all firms.

This equation restates the same subsystem in terms of deviations of the distinct firm from the average level of all firms, where the difference between a variable $\dot{x}$ of a distinct firm and its average across all firms in the same marketplace is denoted as $\dot{\tilde{x}}$.
Following the arguments in Woodford (2004) local convexity of the profit function implies a locally unique equilibrium and therefore, for a given price $\dot{p}_t^{km_{int}}$ and exogenous variables, assumes that the solution is defined by the expression

$$(\dot{\tilde{k}}_{t+1}^{m_{int}}, \dot{\tilde{N}}_t^{m_{int}}, \dot{\tilde{\psi}}_{1t}^{m_{int}})^T = \mathbf{\Phi} \cdot (\dot{\tilde{k}}_t^{m_{int}}, \dot{\tilde{N}}_{t-1}^{m_{int}}, \dot{\tilde{\psi}}_{1(t-1)}^{m_{int}})^T + \mathbf{\Psi} \dot{\tilde{p}}_t^{m_{int}}, \tag{43}$$

for which the coefficients remaining will be determined. Similarly the optimally price deviation of period t can be supposed to be a linear function of the differences of all other endogenous variables from their average local values and the average local steady state deviation of the optimally chosen price across consumption good producers, i.e. $\dot{p}_t^{kc*}$.

$$\dot{p}_t^{m_{int}*} = \mathbf{\Omega}((\dot{\tilde{k}}_t^{m_{int}}, \dot{\tilde{N}}_{t-1}^{m_{int}}, \dot{\tilde{\psi}}_{1(t-1)}^{m_{int}})^T + \dot{p}_t^{kc*} \tag{44}$$

Using $\dot{p}_{t+q}^{m_{int}} = \varrho \dot{p}_{t+q-1}^{m_{int}} + (1-\varrho)\dot{p}_{t+q}^{*m_{int}}$ and updating (43) one period forward yields

$$E_t((\dot{\tilde{k}}_{t+2}^{m_{int}}, \dot{\tilde{N}}_{t+1}^{m_{int}}, \dot{\tilde{\psi}}_{1(t+1)}^{m_{int}})^T) = E_t\left((\mathbf{\Phi} + \mathbf{\Psi}(1-\varrho)\mathbf{\Omega})\,(\dot{\tilde{k}}_{t+1}^{m_{int}}, \dot{\tilde{N}}_t^{m_{int}}, \dot{\tilde{\psi}}_{1t}^{m_{int}})^T + \mathbf{\Psi}\varrho\dot{\tilde{p}}_t^{m_{int}}\right). \tag{45}$$

Substitution of (45) and (44) into (43) and comparison between the coefficients of the result and those of the original version of (43) shows that the supposed solution fits the problem for a given $\mathbf{\Omega}$, iff the equations

$$\begin{aligned} ((\tilde{\mathbf{\Xi}}_1\mathbf{\Omega}(\mathbf{1}-\varrho) - \mathbf{\Upsilon}_1)(\mathbf{\Phi} + (1-\varrho)\mathbf{\Psi\Omega}) - \mathbf{\Upsilon}_2 + (\tilde{\mathbf{\Xi}}_1\mathbf{\Omega\Psi}\varrho + \tilde{\mathbf{\Xi}}_2)(1-\varrho)\mathbf{\Omega})^{-1}\mathbf{\Upsilon}_3 &= \mathbf{\Phi} \\ ((\mathbf{\Upsilon}_1 - \tilde{\mathbf{\Xi}}_1\mathbf{\Omega}(1-\varrho))(\mathbf{\Phi} + (1-\varrho)\mathbf{\Psi\Omega}) + \mathbf{\Upsilon}_2 - (\tilde{\mathbf{\Xi}}_1\mathbf{\Omega\Psi}\varrho + \tilde{\mathbf{\Xi}}_2)(1-\varrho)\mathbf{\Omega})^{-1}\cdot & \\ \varrho(\tilde{\mathbf{\Xi}}_3 + \tilde{\mathbf{\Xi}}_1\mathbf{\Omega\Psi} + \tilde{\mathbf{\Xi}}_2 - \mathbf{\Upsilon}_1\mathbf{\Psi}) &= \mathbf{\Psi} \end{aligned} \tag{46}$$

hold.[24] Therein the matrix $\mathbf{\Upsilon} = (\mathbf{\Upsilon}_1, \mathbf{\Upsilon}_2, \mathbf{\Upsilon}_3)$ is partitioned using three quadratic submatrices. Since the vector $\tilde{\mathbf{\Xi}}_1$ is a zero vector, i.e. $\tilde{\mathbf{\Xi}}_1 = \mathbf{0}$, this system reduces to a much simpler one.[25] Treating $\mathbf{\Phi}$ as a parameter for the moment, the new system can be solved algebraically for the coefficient vectors $\mathbf{\Omega}$ and $\mathbf{\Psi}$ yielding

$$\mathbf{\Psi} = \left(\left(\mathbf{\Phi\Upsilon}_3^+ - \mathbf{Z}_1^T\left(\mathbf{I} - \mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right)\right)\varrho\mathbf{\Upsilon}_1 - \mathbf{I}\right)^{-1}\left(\mathbf{\Phi\Upsilon}_3^+ - \mathbf{Z}_1^T\left(\mathbf{I} - \mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right)\right)(\tilde{\mathbf{\Xi}}_3 + \varrho\tilde{\mathbf{\Xi}}_2) \tag{47}$$

$$\mathbf{\Omega} = \mathbf{L}^+\left(\left(-\mathbf{\Phi\Upsilon}_3^+ + \mathbf{Z}_1^T\left(\mathbf{I} - \mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right)\right)^{-1} - \mathbf{\Upsilon}_2 - \mathbf{\Upsilon}_1\mathbf{\Phi}\right) + \left(\mathbf{I} - \mathbf{L}^+\mathbf{L}\mathbf{Z}_2\right),^{26} \tag{48}$$

where

$$\begin{aligned} \mathbf{L} \equiv & -\left(\mathbf{\Upsilon}_1(\mathbf{I} - \left(\mathbf{\Phi\Upsilon}_3^+ - \mathbf{Z}_1^T\left(\mathbf{I} - \mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right)\right)\varrho\mathbf{\Upsilon}_1)^{-1}\left(\mathbf{\Phi\Upsilon}_3^+ - \mathbf{Z}_1^T\left(\mathbf{I} - \mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right)\right)\cdot\right. \\ & \left.(\tilde{\mathbf{\Xi}}_3 + \varrho\tilde{\mathbf{\Xi}}_2) + \tilde{\mathbf{\Xi}}_2\right)(1-\varrho). \end{aligned}$$

[24]The use of the inverse is possible due to the assumption of full rank for the relevant matrix. Since this assumption does not necessarily hold any solution must be checked for this feature.

[25]This reduction is presented in more detail in appendix 12.6.

Combining (43) and the dynamics in the difference of individual and average expected price deviations implied by (44) leaves one with

$$\begin{pmatrix} (\dot{\tilde{k}}^{m_{int}}_{t+1}, \dot{\tilde{N}}^{m_{int}}_{t}, \dot{\tilde{\psi}}^{m_{int}}_{1t})^T \\ \dot{\tilde{p}}^{m_{int}}_{t+1} \end{pmatrix} = \begin{pmatrix} \mathbf{\Phi} & \mathbf{\Psi} \\ (1-\varrho)\mathbf{\Omega} & \varrho \end{pmatrix} \begin{pmatrix} (\dot{\tilde{k}}^{m_{int}}_{t}, \dot{\tilde{N}}^{m_{int}}_{t-1}, \dot{\tilde{\psi}}^{m_{int}}_{1(t-1)})^T \\ \dot{\tilde{p}}^{m_{int}}_{t} \end{pmatrix}. \tag{49}$$

For stability of the supposed solution either all eigenvalues of the system above must lie within the unit circle (local sink) or at least one eigenvalue should be within the unit circle (local saddle path). In the latter case there are additional stability conditions required, which are not yet determined, but secure that the system does not leave the stable manifold.

In order to integrate the FOC for the optimal price chosen today into the solution, we log-linearize the interdependence between the marginal cost of a distinct domestic firm and the aggregate marginal cost of its local marketplace, i.e. $\dot{s}^k_{t+q} \equiv \int_{m_{int}\in M(k)} \dot{s}^{m_{int}}_{t+q} dm_{int}$, which can be derived from (40), and obtain

$$\dot{s}^{m_{int}}_{t+q} \;=\; \dot{s}^k_{t+q} - \dot{a}^{m_{int}}_{t+q} - \frac{\alpha_1}{1-\alpha_1}\dot{\tilde{k}}^{m_{int}}_{t+q} - \frac{\alpha_1 y^c}{(1-\alpha_1)(y^c+\phi_1)}\dot{\tilde{p}}^{m_{int}}_{t+q}. \tag{50}$$

Moreover we note that

$$\dot{\lambda}_t = \left(\dot{\lambda}_{t+q} - \sum_{s=1}^{q}\left(\ln(\tfrac{\lambda_{t+s}}{\lambda_{t+s-1}}) - \ln(1+\pi)\right)\right) = (\dot{\lambda}_{t+q} - \sum_{s=1}^{q}\dot{\pi}_{t+s}), \tag{51}$$

holds up to the first order.[27] Next substitute the firm-specific marginal costs obtained in (50) into the log-linearized version of (41), whereby $\phi^{m_{int}}/x^{m_{int}}_{t+q}$ can be neglected as an expansion variable due to its first order size in steady state. Subtract the discounted steady state deviation of each period's price level from the first term and add the same to the second while using the last result obtained above. Then isolate the summand containing the deviation of the current relative price of firm m_{int} from steady state, i.e. $\dot{\tilde{p}}^{*m_{int}}_t$, on the left side in order to yield

$$\frac{\dot{\tilde{p}}^{*m_{int}}_t}{1-\frac{\varrho\gamma(1+\pi)}{1+r}} \;=\; \hat{E}_t\left(\sum_{q=0}^{\infty}\left(\frac{\varrho\gamma(1+\pi)}{1+r}\right)^q \mathbf{\Lambda}\left(\dot{\tilde{k}}^{m_{int}}_{t+q}, \dot{\tilde{p}}^{m_{int}}_{t+q}, \dot{a}_{t+q}, \dot{s}^k_{t+q} + \sum_{s=1}^{q}\dot{\pi}_{t+s}\right)^T\right), \tag{52}$$

[26] The use of the pseudoinverse is necessary due to the singularity of the relevant matrices. While the singularity of $\mathbf{\Upsilon}_3$ has been detected within the calibrated model version, the singularity of $\mathbf{L}$ is obvious, since it is a vector. To solve for the parameters the pseudoinverse has been extensively used to solve linear systems of the form

$$\mathbf{AB} = \mathbf{C} \quad \Rightarrow \quad \mathbf{B} = \mathbf{A}^+\mathbf{C} + \big(\mathbf{I} + \mathbf{A}^+\mathbf{A}\big)\,\mathbf{Z} \quad \forall\, \mathbf{Z}_i \quad \text{with} \quad \dim(\mathbf{Z}) = \dim(\mathbf{B}).$$

Cf. Caspary et al. (1994), p.93.

[27] In the literature on New Keynesian models the last summand $\ln(1+\pi)$ is frequently ignored. This due to the assumption of either a zero or a negligible steady state inflation. Since in our model positive inflation plays a substantial role, as it is used to finance the factor inputs of the central bank, we deviate from this practice and account for the nonzero steady state inflation.

where matrix $\mathbf{\Lambda}$ is defined again in the appendix 12.2. Now decompose $\dot{\tilde{p}}_{t+q}^{m_{int}}$ into the two components $\dot{p}_{t+q}^{m_{int}}$ and $\dot{p}_{t+q}^{k}$, substitute $\dot{p}_t^{*m_{int}}(1+\pi)^q$ for the first one, which is allowed due to taking the conditional expectation value, i.e. $\hat{E}_t$,[28] and use $\hat{E}_t(\dot{\hat{p}}_{t+q}^{*m_{int}}) = \dot{\hat{p}}_t^{*m_{int}} + \sum_{s=1}^{q} \dot{\pi}_{t+s}$ to receive

$$\frac{(1-\Lambda_2)\dot{\hat{p}}_t^{*m_{int}}}{1-\frac{\varrho\gamma(1+\pi)}{1+r}} = \hat{E}_t\left(\sum_{q=0}^{\infty}(\tfrac{\varrho\gamma(1+\pi)}{1+r})^q \mathbf{\Lambda}\left(\dot{\tilde{k}}_{t+q}^{m_{int}}, \sum_{s=1}^{q} -\dot{\pi}_{t+s} - \dot{\hat{p}}_{t+q}^{k}, \dot{a}_{t+q}, \dot{\hat{s}}_{t+q}^{k} + \sum_{s=1}^{q}\dot{\pi}_{t+s}\right)^T\right). \quad (53)$$

The optimal price deviation for period t is therefore a function of the endogenously variable relative capital stock and some, from the firm's point of view, exogenous variables.
In order to replace the remaining endogenous variable (49) is integrated forward to receive

$$\sum_{q=0}^{\infty}(\varrho\gamma\beta)^q \begin{pmatrix} (\dot{\tilde{k}}_{t+q}^{m_{int}}, \dot{\tilde{N}}_{t+q-1}^{m_{int}}, \dot{\tilde{\psi}}_{1(t+q-1)}^{m_{int}})^T \\ \dot{\tilde{p}}_{t+q}^{m_{int}} \end{pmatrix} = \mathbf{\Theta} \begin{pmatrix} (\dot{\tilde{k}}_{t}^{m_{int}}, \dot{\tilde{N}}_{t-1}^{m_{int}}, \dot{\tilde{\psi}}_{1(t-1)}^{m_{int}})^T \\ \dot{\tilde{p}}_{t}^{m_{int}} \end{pmatrix}, \quad (54)$$

where the new matrices are defined as

$$\begin{aligned} \mathbf{\Sigma} &\equiv \begin{pmatrix} \mathbf{\Phi} & \psi_1 \\ (1-\varrho)\mathbf{\Omega} & \varrho \end{pmatrix} \\ \mathbf{\Theta} &\equiv \mathbf{\Sigma}\left(\mathbf{I}-\varrho\gamma\beta\mathbf{\Sigma}\right)^{-1} + \mathbf{I}. \end{aligned}$$

Replacing the relative capital stock in (53) with the help of the last equation finally yields, after some tedious algebra,

$$\begin{aligned} \tau\dot{\hat{p}}_t^{*m_{int}} &= (\Lambda_1(\Theta_{11},\Theta_{12},\Theta_{13}))\,(\dot{\tilde{k}}_{t}^{m_{int}}, \dot{\tilde{N}}_{t-1}^{m_{int}}, \dot{\tilde{\psi}}_{1(t-1)}^{m_{int}})^T - \Lambda_1\Theta_{14}\dot{\hat{p}}_t^{kc} \quad (55) \\ &+\hat{E}_t\left(\sum_{q=0}^{\infty}(\varrho\gamma\beta)^q(-\Lambda_2,\Lambda_3,\Lambda_4)\left(\dot{\hat{p}}_{t+q}^{kc} + \sum_{s=1}^{q}\dot{\pi}_{t+s}, \dot{a}_{t+q}, \dot{\hat{s}}_{t+q}^{k} - \sum_{s=1}^{q}\dot{\pi}_{t+s}\right)^T\right), \end{aligned}$$

with

$$\tau \equiv \frac{(1-\Lambda_2)}{1-\varrho\gamma\beta} - \Lambda_1\Theta_{14}.$$

For coincidence of (44) and (55) the following conditions must hold.

$$\begin{aligned} \tau\mathbf{\Omega} &= \Lambda_1(\Theta_{11},\Theta_{12},\Theta_{13}) \quad (56) \\ \dot{\hat{p}}_t^{kc*} &= \tau^{-1}\hat{E}_t\left(\sum_{q=0}^{\infty}(\varrho\gamma\beta)^q(-\Lambda_2,\Lambda_3,\Lambda_4,)\left(\dot{\hat{p}}_{t+q}^{kc} + \sum_{s=1}^{q}\dot{\pi}_{t+s}, a_{t+q}, \dot{\hat{s}}_{t+q}^{k} - \sum_{s=1}^{q}\dot{\pi}_{t+s}\right)^T\right) \\ &-\tau^{-1}\Lambda_1\Theta_{14}\dot{\hat{p}}_t^{kc}. \quad (57) \end{aligned}$$

At this point of the analysis it becomes obvious that, similarly to Woodford (2004), in our model the fully flexible price equilibrium, in which each firm is allowed to reoptimize

[28] The conditional expectation over all future states, in which prices will not be reoptimized, reflects the discussion of footnote 20.

in all periods (i.e. $\varrho = 0$), exhibits zero dispersion of prices across firms. Hence from (44) it follows that in flexible price equilibrium there is no dispersion in capital, equity nor in shadow prices of the equity constraint across firms.
Together (47), (48) and (56) determine the solution space for the remaining unknown coefficient matrix $\mathbf{\Phi}$. Substituting the former two into the latter yields the implicit function

$$\begin{aligned}\mathbf{\Phi} \;=\; & \left(\mathbf{L}\left[\frac{1-\varrho}{\tau}\Lambda_1(\Theta_{11},\Theta_{12},\Theta_{13}) + \mathbf{L}^{+}\mathbf{\Upsilon}_2 + \mathbf{L}^{+}\mathbf{\Upsilon}_1\mathbf{\Phi}\right] + \left(\mathbf{I}-\mathbf{L}^{+}\mathbf{L}\right)\mathbf{Z}_3\right)^{-1}\mathbf{\Upsilon}_3 \\ & -\mathbf{Z}_4^T\left(\mathbf{I}-\mathbf{\Upsilon}_3^{+}\mathbf{\Upsilon}_3\right).\end{aligned} \tag{58}$$

The fixed points of this function form the solution for $\mathbf{\Phi}$. Hence, given the conditions (47), (48), (56)-(58) and the requirement that the matrix $\mathbf{\Phi}$ has at least one eigenvalue with an absolute value smaller than one, equations (43), (44) comprise a stable solution for the dynamics of the endogenous variables of the firm. This solution depends on an array of variables, which are determined exogenously from the firm's point of view.
With respect to the information asymmetries within the economy the reaction of the capital to variation in the previous equity level within the local solution is governed by the matrix $\mathbf{\Phi}$. For a credit channel to be active, this reaction is expected to be positive.[29] In the same case the reaction of the interest spread between external and internal financing with respect to a variation in the riskless interest rate should also be positive, which is clearly guaranteed by (26).

4.3 The remaining suppliers: the central bank and financial intermediaries

A already discussed earlier the issuer of the payment medium, the central bank, is not a profit-maximizing entity. Due to its character as a public institution only wants to cover its production cots fully. Therefore the appropriate optimization problem is that of minimizing production costs. In order to do this the central bank chooses her optimal mix of input factors in order to minimize the costs for a given plan of current and future production of payment services. Doing this she has to pay attention to the intertemporal influence of her investment decisions, which is of exactly the same form as for a producer of consumption goods, i.e. (29) and (32), except for the size of the depreciation rate δ_2, which is slightly lower for the payment sector,[30] and her technological constraint given in (22). The cost function of the central bank can be written as the discounted sum of expenses for labor and capital, i.e.

$$E_t\left(\int_0^{\frac{1}{2}\iota}\sum_{q=0}^{\infty}\prod_{s=0}^{q}\frac{1}{1+r_{t+s-1}}\left(W_{t+q}^{k}l_{t+q}^{km} + (1+r_{t+q})Q_{t+q}I_{t+q}^{m} - (1-\delta_2)k_{t+q}^{m}\right)dk\right). \tag{59}$$

[29] Cf. Bernanke et al. (1998), p.15.

[30] Compared to other sectors the banking sector tends to be characterized by a slightly higher average capital lifetime. Cf. Costa et al. (1995), p.444, Table 1. This transforms into a relatively low depreciation rate. Cf. Hahn et al. (1984), p.295.

Note that procurement of input factors does not induce transaction costs for the firms, because input factors are traded locally.
Calculating the optimum of the central bank for given demand structure for money across marketplaces and given technology shows that the optimal ratio of labor and capital input is dependent on location and equivalent to

$$h^{1km}_{t+q+1} \equiv \frac{1-\alpha_2}{\alpha_2}\frac{Q_{t+q+1}}{W^k_{t+q+1}}\left(\frac{(1+r_{t+q})I^{m\prime}\left(\frac{k^m_{t+q+1}}{k^m_{t+q}}\right)Q_{t+q}}{Q_{t+q+1}} - I^{m\prime}\left(\frac{k^m_{t+q+2}}{k^m_{t+q+1}}\right)\frac{k^m_{t+q+2}}{k^m_{t+q+1}} + I\left(\frac{k^m_{t+q+2}}{k^m_{t+q+1}}\right)\right). \quad (60)$$

Since if the central bank operates in a given marketplace, it needs to hire the local labor input in the same marketplace, some heterogenity across marketplaces remains. Using this definition the current value of local nominal marginal costs of transaction services can be expressed as

$$s^k_{(t+q)m} = E_t\left(\frac{W^k_{t+q}}{a_{2(t+q)}(1-\alpha_2)}\left(h^{1m}_{t+q}\right)^{\alpha_2}\right). \quad (61)$$

As usual marginal costs depend on factor prices. Linearizing around the steady state one is left with

$$E_t(\dot{s}^k_{(t+q)m}) = E_t\left(\dot{W}^k_{t+q} + \alpha_2\dot{h}^{1km}_{(t+q)} - \dot{a}_{2(t+q)}\right), \quad (62)$$

whereby

$$\begin{aligned} E_t(\dot{h}^{1km}_{t+q+1}) = \frac{1-\alpha_2}{\alpha_2\hat{w}h^{1m}}\Bigg(&\frac{1+r}{1+\pi}(\dot{r}_{t+q} + \dot{Q}_{t+q} + I^{m\prime\prime}(1)(\dot{k}^{km}_{t+q+1} - \dot{k}^{km}_{t+q})) \quad (63)\\ &-I^{m\prime\prime}(1)(\dot{k}^{km}_{t+q+2} - \dot{k}^{km}_{t+q+1}) - (1-\delta_2)\dot{Q}_{t+q+1}\Bigg) - \dot{W}^k_{t+q+1}. \end{aligned}$$

The steady state level for the period's marginal nominal costs is

$$s^k_{t+q} = \frac{1}{a_2(1-\alpha_2)}\bar{W}^k_{t+q}\left(h^1_m\right)^{\alpha_2}, \quad (64)$$

whereby only those variables which are non-stationary in steady state keep their time indices.
For the business field of financial intermediation, i.e. credit extension, expected nominal marginal costs per unit of credit can be directly derived from the assumed market structure as

$$s^k_{(t+q)cred} = \frac{(1+r_{t-1})\chi}{\chi-1} - \frac{(1+r_{t-1})\lambda_t\bar{\chi}}{(\chi-1)(Q_{t-1}k^{m_{int}}_t - N^{m_{int}}_{t-1})} > 1. \quad (65)$$

5 Market Clearing Conditions

In any equilibrium all markets must clear, in order to coordinate demand and supply of the economic individuals. Therefore in all current and future periods, i.e. $q \in \mathbb{R}_+$, the following conditions must hold. The supply for each consumption good must meet aggregate demand consisting of private and public consumption of households and the demand of producers of investment goods, providers of monitoring services and the resources used up in the bequeathing of capital. Thus

$$y_{t+q}^{m_{int}} = \left(\frac{p_{t+q}^{m_{int}}(1+\nu_{t+q}^{ik})}{\lambda_{t+q}^{i}}\right)^{-\theta} \left(\int_0^{\varsigma} \left(c_{(t+q)}^{i} + y_{(t+q)}^{PC} + y_{(t+q)}^{MC} + y_{(t+q)}^{tc} + g_{t+q}^{d}\right) di \right. \quad (66)$$

$$\left. + \int_{\varsigma}^{1} \left(c_{(t+q)}^{i} + y_{(t+q)}^{PCf} + y_{(t+q)}^{MCf} + y_{(t+q)}^{tcf} + g_{t+q}^{f}\right) e_{t+q}^{\theta} di \right) \qquad \forall\, m_{int} \in [0,\varsigma].$$

must hold. The aggregation of this expression across all domestically produced varieties is denoted by y_{t+q}^{dc}. In the bond market, the demand for bonds by financial intermediaries must equal the riskless investment volume of households minus the central bank's bond demand.

$$Q_{t+q}k_{t+q}^{d} = \int_0^{\varsigma} b_{(t+q)}^{id} di + \int_{\varsigma}^{1} b_{(t+q)}^{if} di - b_{t+q}^{cb} \quad (67)$$

In the credit market the supply of loans by the financial intermediaries balances the financial needs of consumption good providers, which comprise the value of their capital assets, diminished by the equity position, and the capital stock of the central bank. Thus loans are not held directly by households, but are intermediated by the financial industry.

$$Q_{t+q}k_{t+q}^{d} = \int_0^{\varsigma} \left(Q_{t+q}k_{t+q+1}^{m_{int}} - N_{t+q}^{m_{int}}\right) dm_{int} + Q_{t+q}k_{t+q+1}^{m} \quad (68)$$

Within the locally segmented labor markets, the supply of labor by firms must be met by the demand of households in each marketplace k separately.

$$\int_0^{\varsigma} l_{t+q}^{ik} di = \int_{k\frac{2\varsigma}{\iota}}^{\min[(k+1)\frac{2\varsigma}{\iota},\varsigma]} l_{t+q}^{m_{int}} dm_{int} + l_{t+q}^{km} \qquad \forall\, k \in [0, \frac{1}{2}\iota] \quad (69)$$

With respect to payment services, the demand for gross transaction volume by households and others must be met locally by the central bank.

$$y_{t+q}^{mkB} = \int_0^{1} (x_{(t+q)m}^{ikB} - x_{(t+q)m}^{ikS}) di \qquad \forall\, k \in [0, \frac{1}{2}\iota] \quad (70)$$

In the markets for both investment goods and monitoring services supply must meet demand, whereby the competitive structure of both markets ensures that in equilibrium demand equals the resources consumed in the production process.

$$y_{t+q}^{PCd} = \phi^{-1}\left(\int_0^{\varsigma} I\left(\frac{k_{t+q+1}^{m_{int}}}{k_{t+q}^{m_{int}}}\right) k_{t+q}^{m_{int}} dm_{int} + I\left(\frac{k_{t+q+1}^{m}}{k_{t+q}^{m}}\right) k_{t+q}^{m}\right) \quad (71)$$

$$y_{t+q}^{MCd} = \frac{(1+r_{t+q-1})(Q_{t+q-1}k_{t+q}^{c} - N_{t+q-1}^{c})}{(\chi-1)\lambda_{t+q}} - \frac{\bar{\chi}}{\chi-1} \quad (72)$$

Finally, the resources consumed by the transition of capital from old firms to new ones are financed by the capital share of unsucessful firms, which perish during the shut down. Using (31) simplifies the right side of this relation.

$$y^{tc}_{(t+q)} = (1-\gamma)tc((1+r_{t+q+1})N_{t+q-1} + \Pi_{t+q}) = \frac{(1-\gamma)tc}{1-(1-\gamma)tc} N_{t+q} \tag{73}$$

With respect to (67)-(73) similar equations hold for the foreign economy. Moreover the aggregate version of (70) equals the supply of all domestically issued money across all marketplaces, to the aggregate demand for transaction services by all domestic economic individuals.

$$\begin{aligned} \int_0^{\frac{1}{2}\iota} y^{mkB}_{t+q} dk &= \left(\int_0^{\varsigma} p_{m_{int}(t+q)} di + \int_{\varsigma}^{1} p_{m_{int}(t+q)} e_{t+q} di \right) \cdot \\ &\quad (c_{t+q} + y^{PCd}_{t+q} + y^{MCd}_{t+q} + y^{tc}_{t+q} + g_{t+q}) + |b_{t+q} - (1+r_{t+q-1}) b_{t+q-1}| \\ &\quad + e_{t+q} |b^f_{t+q} - (1+r^f_{t+q-1}) b^f_{t+q-1}| + |x_{(t+q)m} - x_{(t+q-1)m}| \end{aligned} \tag{74}$$

6 The Aggregate Price Level, Inflation Dynamics and the Aggregate Dynamics of the Consumption Goods Sector

The construction of the domestic aggregate price level is complicated by two distinct features of the economy: transaction costs and consumer heterogeneity. If there are no transaction costs and consumers are homogenous due to identical income levels, the individual marginal costs of a unit of the aggregate consumption good, λ^i_t, are equal to the general price level. Introducing transaction costs alone does not alter this. But heterogeneity in income levels due to the confinement of labor forces to distinct marketplaces distorts the uniformity of individual cost-of-consumption indexes, since transaction costs and income levels can differ across households. Still the individual aggregate cost-of-consumption indexes of all subunits belonging to the same household family i are identical across marketplaces, but different marginal utilities drive a wedge between those of different households.[31]

In order to incorporate the idea of staggered nominal price setting as the Keynesian characteristic of price rigidity formulated in Fischer (1977) and Taylor (1980), we follow the modelling in Calvo (1983) and express the domestic aggregate price level λ_t as a convex combination of the theoretical optimal domestic price level and the domestic price level of

[31] Among others Hornstein et al. (2004) specifies non-zero steady state inflation as a further obstacle for aggregation. But this does not hold for rational anticipatory individuals, since any steady state inflation will be build into prices resulting in indexation. Hence heterogeneity between firms occurs only outside of steady state and there is no aggregation problem left, if the heterogeneity converges back to steady state homogeneity, which should hold for any stable model. Cf. Hornstein et al. (2004), p.58ff.

the last period, which both contain consumption expenditures as well as transaction costs. This is possible, because every firm has the same probability of being able to reoptimize prices, and hence the probability of price adjustment can be used as the fraction of firms altering their prices. But the price for one unit of the aggregated products of the whole population of firms is nothing but the average costs of such a unit across households, which of course also comprise some transaction costs.
Hence we use the microeconomic model of Calvo (1983) on the macroeconomic level for the modelled economy, and we express the actual price level as the combination of current and all former price levels $(\lambda^*_{t-q})_{q\in\mathbb{Z}}$, which are generated by the optimal price choice of firms, times their probabilities to survive until period t.[32] Therefore

$$\lambda_t = \sum_{q=0}^{\infty} (1-\varrho)\varrho^q \lambda^*_{t-q} \tag{75}$$

from which the expression[33]

$$\lambda_t = \frac{\int_0^{\varsigma} c_t^i \lambda_t^i di}{\int_0^{\varsigma} c_t^i di} = (1-\varrho)\lambda_t^* + \varrho(1+\pi)\lambda_{t-1} = (1-\varrho)\frac{\int_0^{\varsigma} c_t^i \lambda_t^{i*} di}{\int_0^{\varsigma} c_t^i di} + \varrho(1+\pi)\lambda_{t-1} \tag{76}$$

follows, if it is assumed that consumption goods are not used as payment media, because the technologies of the providers of utility-neutral goods are superior to those of the producers of consumption goods.[34] In this situation the living expenses of an individual consist of her consumption expenditures and her expenditures for the use of payment services, but do not comprise the expense of saving instruments.
Thus, the domestic general price level depends on the individual cost-of-living indexes, the related expenditures of the domestic households, if there is no price rigidity, and the weighted average of the past individual cost-of-living indexes. Equivalently, it can be interpreted as a combination of the optimal price level from those firms, which do adjust theirs prices and those which set the prices from period $t-1$. This form of the price level can be microfounded by assuming for example quadratic costs of price adjustments due to the existence of menu costs and costs generated by an aversion of consumers to price variability.[35]
Linearizing (76), applying some algebra and using $\int_0^{\varsigma} \dot{\lambda}_t^{i*} di = \int_0^1 \dot{p}_t^{m_{int}*} dm_{int}$, which holds due the fact that in flexible price equilibrium there is no change in the relative production structure nor in the relative structure of income or transaction costs across marketplaces,[36] yields the inflation rate deviation

$$\dot{\pi}_t = \frac{1-\varrho}{\varrho}\left(\int_0^{\varsigma} \dot{\lambda}_t^{i*} di - \dot{\lambda}_t\right) = \frac{1-\varrho}{\varrho}\left(\int_0^{\varsigma} \dot{\lambda}_t^{i*} di - \int_0^{\varsigma} \dot{\lambda}_t^{i} di\right) = \frac{1-\varrho}{\varrho}(\dot{p}_t^{c*} - \dot{\lambda}_t). \tag{77}$$

[32] A detailed and formal discussion can be found in Roberts (1995) and Calvo (1983).

[33] Cf. Christiano et al. (2001), p.11 or Gali (2001), p.8. Note that, equivalently to Obstfeld et al. (1995), the general price level is the average minimal cost of a unit of consumption in terms of the domestic central bank money.

[34] Formally this assumption imposes that the right part of (37) from Hespeler (2003) holds for every payment medium used.

[35] Cf. Benassi et al. (1994), p.286ff.

[36] Cf. Woodford (2003), p.150ff.

As in the model without any heterogeneity the current deviation of the inflation rate from steady state is determined by the percentage deviation of the average relative price of consumption from steady state in the case of fully flexible prices.[37]
Log-linearizing (9), summing across all domestic households, solving for the aggegate of all current consumption good prices $\dot{p}_t^c = \int_0^1 \dot{p}_t^{m_{int}} dm_{int}$ and using the inflation equation to substitute for the deviation of the price level yields

$$\dot{p}_t^c \;=\; \dot{p}_t^{c*} - \frac{\varrho}{1-\varrho}\dot{\pi}_t - \varsigma \frac{\nu^{dd}}{(1+\nu^{dd})}\dot{\nu}_t^{dd} - (1-\varsigma)\frac{\nu^{df}}{(1+\nu^{df})}\dot{\nu}_t^{df} - (1-\varsigma)\dot{e}_t. \tag{78}$$

We now return to the average deviation of optimal consumption good prices from steady state derived within the firms' optimization and recall (57). Aggregating across all marketplaces and quasi-differencing obtains

$$\begin{aligned}\dot{\hat{p}}_t^{c*} - (\varrho\beta\gamma)^{-1}\dot{\hat{p}}_{t-1}^{c*} \;=\; & \left(-\left(\Lambda_4\dot{\hat{s}}_{t-1} + \Lambda_3\dot{a}_{t-1} - \Lambda_2\dot{\hat{p}}_{t-1} + \frac{(\Lambda_4-\Lambda_2)\varrho\gamma\beta}{1-\varrho\gamma\beta}\dot{\pi}_t\right)(\varrho\gamma\beta)^{-1}\right. \\ & \left. -\Lambda_1\Theta_{14}(\dot{\hat{p}}_t^c - (\varrho\gamma\beta)^{-1}\dot{\hat{p}}_{t-1}^c)\right)\tau^{-1}.\end{aligned} \tag{79}$$

Finally we use (78) to substitute for average relative prices of consumption goods and (77) to substitute for average relative optimal prices, update for one period and receive, after some algebra, the inflation dynamics:

$$\begin{aligned}\dot{\pi}_t \;=\; & \frac{1-\varrho}{\tau\varrho}\Lambda_4\dot{\hat{s}}_t + \left(\frac{(\Lambda_4-\Lambda_2)\varrho\gamma\beta(1-\varrho)}{(1-\varrho\gamma\beta)\tau\varrho} + \varrho\gamma\beta\right)E_t(\dot{\pi}_{t+1}) + \frac{1-\varrho}{\tau\varrho}\Lambda_3\dot{a}_t \\ & -\frac{(1-\varrho)}{\tau\varrho}\Lambda_1\Theta_{14}\left(\frac{\varsigma\nu^{dd}}{1+\nu^{dd}}E_t(\dot{\nu}_{t+1}^{dd}) + \frac{(1-\varsigma)\nu^{df}}{1+\nu^{df}}E_t(\dot{\nu}_{t+1}^{df})\right) \\ & +\frac{1-\varrho}{\tau\varrho}\left(\Lambda_2 + \varrho\gamma\beta\Lambda_1\Theta_{14}\right)\left(\frac{\varsigma\nu^{dd}}{1+\nu^{dd}}\dot{\nu}_t^{dd} + \frac{(1-\varsigma)\nu^{df}}{1+\nu^{df}}\dot{\nu}_t^{df}\right) \\ & +(1-\varsigma)\frac{1-\varrho}{\tau\varrho}\left(\Lambda_2 + \varrho\gamma\beta\Lambda_1\Theta_{14}\right)\dot{e}_t - (1-\varsigma)\frac{(1-\varrho)}{\tau\varrho}\Lambda_1\Theta_{14}E_t(\dot{e}_{t+1}).\end{aligned} \tag{80}$$

Equation (80) resembles the New Keynesian Phillips Curve in the literature, in so far as it determines current inflation deviation as a linear function of the steady state deviation in current real costs and future expected inflation. Since the inclusion of endogenous capital does not alter the functional form, but only the coefficient of marginal costs, which is influenced in a non-trivial manner,[38] the remaining arguments are subject to the features of transaction costs, financial contracts, international relationships and the stochastic shock productivity with respect to consumption goods.[39] As one can easily perceive, financial

[37] Cf. Walsh (2003), p.263.

[38] This influence is discussed in detail in Woodford (2005). For the model presented the elasticity of inflation with respect to marginal costs is influenced by the function $\mathbf{\Theta}$, which depends amongst others on the matrix $\mathbf{\Upsilon}$ and thus on the adjustment cost parameter ϵ_I. Woodford (2005) shows that the case of an exogenous capital stock is restored when adjustment costs converge to infinity, and hence the model coincides with economic intuition.

[39] An extensive discussion of the last feature, which is referred to using the term cost-push stock, can be found in Clarida et al. (1999). It will be taken up again when the policy function of the central bank is introduced.

frictions do not influence the inflation dynamics directly except for the higher discount rate of consumption good producers. The reasons are that on the aggregate level financial contracts do not add additional information to the system, because equity is an exclusively function of capital and prices. For most calibrations the coefficients on current shadow prices of local income and exchange rates are unambiguously negative, while the sign of the coefficients for the current pendants remain ambiguous. Hence future increases in transaction costs or exchange rates tend to curb current inflation, while current variations have positive direct effects and negative indirect effects via aggregate demand. For symmetric calibrations of both economies, the coefficients of shadow prices on foreign and domestic marketplaces are identical and the equation collapses further. Note that the result in Clarida et al. (2002), which states that noncooperative monetary policy should resemble the policy for a closed economy, does not hold, because due to transaction costs neither the dynamic pattern nor the level of consumption need to coincide across both economies.
Eliminating the remaining mentioned non-standard features would restore the standard form of the New Keynesian Phillips Curve. As the parameter Λ_4 is equal to 1, Λ_1 would converge to 0, τ would converge to $1/(1-\varrho\beta)$ and the coefficient for future expected inflation would converge to β.[40] The fact that in contrast to most standard models of the New Keynesian literature the model presented includes positive inflation in steady state, does not alter the New Keynesian Phillips Curve conceptually, since merely the deviation of the inflation rate from steady state can not be replaced by the current inflation rate.

To receive equations for the development of the economy-wide capital, critical productivity, equity and shadow prices of consumption good producers we average (42) across all firms, use (77) and (78) to eliminate redundant terms, consolidate the deviations of the demand components into the deviation of aggregate output and obtain the reduced system

$$E_t\left(\boldsymbol{\Upsilon}\cdot L^3\left((\dot{k}^c_{t+q+2},\dot{N}^c_{t+q+1},\dot{\psi}^c_{1(t+q+1)})^T\right)\right)=E_t\left(\left(\varsigma\tilde{\tilde{\boldsymbol{\Xi}}}_e-(1-\varsigma)\tilde{\tilde{\boldsymbol{\Xi}}}_e\right)L^3(\dot{e}_{t+q+2})+\right. \tag{81}$$
$$\left.\tilde{\tilde{\boldsymbol{\Xi}}}\cdot L^3\left(\dot{p}^c_{t+q+2},\dot{r}_{t+q+1},\dot{y}^c_{t+q+2},\dot{\nu}_{t+q+2},\dot{\lambda}_{t+q+2},\dot{W}^k_{t+q+2},\dot{Q}_{t+q+1},\dot{a}^c_{(t+q+2)}\right)^T\right).$$

Herein both $\dot{r}_{t+q}$ and $\dot{Q}_{t+q}$ are the weighted average of the related economy-specific variables and the superindex c denotes the average across relevant firms. The elements of the matrices $\tilde{\tilde{\boldsymbol{\Xi}}}$ and $\tilde{\tilde{\boldsymbol{\Xi}}}_e$ are given explicitly in Appendix 12.2.

[40] A discussion of the standard form of this curve can be found in Walsh (2002). Cf. Walsh (2002), p.237.

7 Linearizing the Household Side and Market Equilibria

In order to evaluate the dynamics of the household side, the equations representing the dynamics of the household's optimal decision plan need to be linearized around the steady state. Doing this we should mention one theoretical obstacle for the linearization process. On a general mathematical level we need to pay attention to the fact that due to the discontinuity of the demand for transaction services, which is the difference between the amounts of a payment medium bought and sold on the same marketplace, no derivative is defined for a transaction volume of zero. However, we know from the demand analysis that individuals specialize with respect to the use of payment media between two marketplaces. Thus the local demand for these is zero and does not change at all, or it is significantly below or above zero. For this reason we do not need to bother ourselves with the problem of discontinuity at all, as the derivative should be well defined locally.
Log-linearizing the derived Euler conditions (16) and (12) after substituting for l_t by (17), the uncovered interest parity (19), the dynamic equation (96) of Appendix 12.1, which constitutes an additional Euler equation, the aggregate of the optimality conditions for the local payment media demand across markets, i.e. across (7) or rather (**??**), and the constraint (5) around the specific steady state described above and averaging across all households within the referring economy yields the system of difference equations

$$\mathbf{0} = \sqsupset \cdot L^3(\dot{c}^d_{t+1}, \dot{z}_{t+1}, \dot{\lambda}_{t+1}, \dot{\mu}_{t+1}, \dot{s}^m_{t+1}, \dot{r}_{t+1}, \dot{r}^f_{t+1}, \dot{\pi}_{t+1}, \dot{e}_{t+1}, \dot{x}_{(t+1)m}, \dot{b}^d_{t+1}, \dot{y}^d_{t+1}, \dot{\nu}_{t+1}, \dot{\hat{W}}_{t+1}). \quad (82)$$

While the first five equations of this system do not need any further explanation, the sixth is the linearized version of the zero-profit constraint of the central bank, which is taken into account by the individual households, i.e.

$$\frac{1}{\beta}(\frac{\hat{x}}{y}+\daleth_1\hat{s}^m\frac{\hat{b}}{y})\dot{r}_{t-1} = -\hat{s}^m\frac{\daleth_1}{\beta}\frac{\hat{b}}{y}(\dot{b}^d_{t-1}+\dot{r}^f_{t-1}+\dot{e}_{t-1}) - \frac{\hat{x}}{y}\frac{r+\daleth_2\hat{s}^m}{1+\pi}\dot{x}^m_{t-1} + \daleth_2\hat{s}^m\frac{\hat{x}^m}{y}\dot{x}^m_t \quad (83)$$
$$+\daleth_1\hat{s}^m\frac{\hat{b}}{y}(\dot{b}^d_t+\dot{e}_t) + \hat{s}^m\hat{p}^c\left(\theta(\dot{\lambda}_t - \frac{\nu}{1+\nu}\dot{\nu}_t - (1-\varsigma)\dot{e}_t) + \dot{y}^{dD}_t\right) + \frac{r\hat{x}}{y(1+\pi)}(\dot{s}^m_t - \dot{\lambda}_t).$$

Herein $\dot{b}^d_t$ is defined as $\dot{b}^{dd}_t + \dot{b}^{df}_t$, i.e. as the average of the domestic households demand for domestic and foreign bonds (domestic consumption $\dot{c}^d_t$ and domestic aggregate demand $\dot{y}^{dD}_t$ are defined similarly), and the two functions $\daleth_1$ and $\daleth_2$ are defined as

$$\daleth_1 \equiv \begin{cases} 1 & b_t \geq (1+r_{t-1})b_{t-1} \\ -1 & b_t < (1+r_{t-1})b_{t-1} \end{cases} \qquad \daleth_2 \equiv \begin{cases} 1 & x_t \geq x_{t-1} \\ -1 & x_t < x_{t-1} \end{cases}, \quad (84)$$

and h_2 denotes

$$h_2 \equiv c^{-\sigma} - \frac{(1-l)^{-\eta}}{\hat{w}}. \quad (85)$$

By displaying this version of the equation, we deviate from the strict principle of linear approximation around steady state, in order to indicate that generally asset holdings influence the transaction volume by altering the parameters $\daleth_1$ and $\daleth_2$. For example, if c.p. the current nominal demand declines less percentage points than twice the steady state rate of net inflation, the nominal transaction volume deviates negatively from steady state, because either the issue of new money is not accepted or nominal money savings are slightly reduced. If the decrease in current nominal money even exceeds the threshold mentioned, the nominal transaction value starts to deviate positively from steady state, since the initial reduction induced by the rejection of newly issued money is now overcompensated by massive sales of money by the private sector. Nevertheless, in our simulations we return to linear approximation around steady state and hence the values for $\daleth_1$ and $\daleth_2$ are determined by the steady state values.[41]

For a closed economy, aggregation of equilibria in the consumption good markets would be straightforward. We would simply aggregate (66) across all markets and linearize the result. For the open economy presented the equivalent would be an aggregation across all goods produced in an economy, in order to construct its aggregate output. But the CPI based inflation measure prevents an easy aggregation. Hence the method of Obstfeld et al. (1995) is employed, and all consumption good markets are aggregated to total world output and the resulting equation is linearized. Substitution of the linearized version of (71)-(73) into this equation and elimination of $\dot{p}^c_{t+q}$ by (78) delivers the final linearized condition for the balance between aggregate demand and supply (for the whole world). Instead of an equation balancing aggregate output and demand for a specific economy, the (linearized) law of one price

$$\dot{\lambda}^d_t - \frac{\nu}{1+\nu}\dot{\nu}^d_t \;=\; \dot{\lambda}^f_t - \frac{\nu}{1+\nu}\dot{\nu}^f_t + \dot{e}_t, \tag{86}$$

which holds only after adjustment for transaction costs, is used to close the linearized market equilibria block. With respect to equilibria in the remaining markets, (67)-(69) and (74) are linearized. Moreover substituting the obtained expression for (68) into (67) we obtain a condition for an equilibrium in the domestic bond market. The result for the labor market is obtained by replacing the labor demand of households by (95) and the labor supply of firms by using the products of optimal labor to capital ratios and capital stocks. Afterwards the labor to capital ratios are eliminated by using (39) and (60). Finally the result for the linearized version of (74), i.e. the aggregate condition for balanced money markets, is derived by using a second time (60) in order to substitute for $\dot{h}^m_{1t}$.
Since the optimality condition for the producers of capital goods determines an aggregate demand component and has been already used in the last equation, the linearization of (33)

$$\dot{Q}_t = \dot{\lambda}_t \tag{87}$$

is provided as a part of the market equilibrium conditions. Nevertheless the equation describes the optimal production plan of competitive providers of capital goods.

[41] If this assumption is not used, the coefficients for bonds and money would be endogenous functions in the model. This would complicate the analysis significantly and impose the risk of bifurcations.

Gathering the obtained equations into a system of the form

$$\mathbf{0} = \daleth \cdot L^3(\dot{y}^{jc}, \dot{c}^j, \dot{k}^{cj}, \dot{k}^{mj}, \dot{x}^j, \dot{N}^j, \dot{Q}^j, \dot{\lambda}^j, \dot{r}^j, \dot{\pi}^j, \dot{g}^j, \dot{\nu}^j, \dot{e}, \dot{\mu}^j, \dot{\bar{W}}^j_, \dot{s}^j, \dot{s}^{mj}, \dot{z}^j, \dot{b}^{dj}, \dot{b}^{fj}, \dot{a}^j_2)^{j\in\{d,f\}}_{t+1} \quad (88)$$

yields finally a condensed condition for domestic market equilibria. For the foreign economy a symmetric system is obtained.

8 The Central Bank

The central bank fulfills at least two functions in the model. Firstly it is the provider for the payment medium money, which serves as the nominal anchor of the economy. Therefore it supplies and accepts money, the good m, in all marketplaces in the domestic economy, at the same bid and ask prices, which are normalized to one without loss of generality and additionally acts as a clearing instance for transactions with abroad.[42] As in Mundell (1963), the resulting net position in foreign central bank money plus the private sector's net purchases in foreign bonds is the economy's accumulation of foreign assets.

Secondly, the central bank is an agency empowered by the government to conduct monetary policy, which minimizes the social loss resulting from at least five sources of distortions in the model: inefficiently high transaction costs, imperfect competition, suboptimal mark-ups due to price-stickiness, distortions in relative prices due to the lack of synchronization in price adjustments and finally wage dispersion and, hence, inefficient labor usage patterns.[43] Its objective function, the social loss function, which is derived in detail in appendix 12.4, is given by

$$E_t\left[\sum_{q=0}^{\infty} \beta^q \tilde{\mathbf{t}}^T_{t+q} \mathbf{P} \tilde{\mathbf{t}}_{t+q}\right] + t.i.p, \quad (89)$$

which is similar to many versions of the standard New Keynesian model, e.g. Clarida et al. (1999), Woodford (1999) amongst many others.[44] It involves quadratic expressions of the spreads between current and target values of several variables,[45] which for our economy are

[42] This model feature is motivated by the broad field of possible interactions of central banks, e.g. via the Bank for International Settlement (BIS). Transaction costs for settlement are ignored.

[43] A more detailed discussion of the first four sources can be found in Gali (2001). Cf. Gali (2001), p. 19f. In the model presented transaction costs indicate by no means a distortion, since even in the efficient steady state there remain positive transaction costs. Nonetheless, deviations of transaction costs from their optimal level results in an additional distortion. The distortions related to wage deviations are discussed in Erceg et al. (2000).

[44] In Woodford (2003), chapter 6, such objective functions has shown to serve as very close approximations of the social loss due to various kinds of distortions and, according to Svensson (1999), are widely accepted among academics and policymakers to be appropriate objectives of monetary policy.

[45] As emphasized in Woodford (2002) the loss function above essentially focuses on the stabilization goal with respect to the variability of endogenous variables, while the deterministic part in the equilibrium path of the endogenous variables is not of any concern from the perspective of monetary policy. Cf. Woodford (2002), p. 508f.

gathered in the vector $\tilde{\mathbf{t}}^T = (\dot{y}_{t+q}, \dot{x}_{t+q}, \dot{k}^c_{t+q}, \dot{k}^m_{t+q}, \dot{b}^d_{t+q}, \dot{r}_{t+q}, \dot{\tilde{W}}_{t+q}, \dot{e}_{t+q}, \dot{\pi}_{t+q}, \dot{x}_{t+q-1}, \dot{b}^d_{t+q-1}, \dot{e}_{t+q-1})$, which comprises the variables inflation, output and capital of both sectors, bonds, wages, interests and the change in exchange rates. The matrix $\mathbf{P}$ is defined such that the objective function equals (122) in appendix 12.4, while the term $t.i.p.$ contains all welfare determinants that can not be influenced by monetary policy.

Among the mentioned target variables, the inflation rate includes the third and fourth distortions discussed above into the policy function. In paerticular, in an economy with explicitly modelled transaction costs it is not only a measure of the change in the value of the central bank's asset, but also of the value change of all other active issuers' assets. The output gaps of both sectors reflect the traditional argument, first raised by Barro and Gordon (1983), that a time-consistent, discretionary monetary policy will try to dispose of the spread between the inefficient steady state of the model above, and an efficient steady state without any distortions from imperfect competition. Hence, no inflationary bias in the model remains.[46] But the uncertain productivities of consumption good producers induce a trade-off between inflation and output stabilization. This trade-off consists of the fact that a lower volatility in inflation rates can only be achieved by accepting a higher volatility in current output, and is represented by the efficiency frontier in Taylor (1993). It is integrated into the model by the appearance of stochastic shocks in the inflation dynamics.[47] Furthermore the output gap in the payment media sector clearly indicates that inefficiencies in the payment systems should be combated. The appearance of the capital gaps for both sectors reveals that investment and thus capital influences utility directly through production and indirectly by the extent, to which an output gap is balanced by an investment rather than a consumption gap.[48] Potential distortions caused by high interest rates, via the economizing effect on cash holdings and the incurred restrictions on transactions, call for a monetary policy, which cares about these.[49] An even more convincing argument for a direct role for interest rates within the policy objective is the influence of a credible commitment of the central bank policy. Low variances in interest rates display a policy using the rational expectations of the private sector by imposing an inertial series of small interest steps. Since the private sector therefore expects this inertial behavior, market forces will push the long term interest rates in the intended direction due to the expectations theory of the term structure of interest rates. Hence, there is no need for a policy of brute changes in the interest rate in order to influence

[46] Cf. Walsh (2002), p.521.

[47] This trade-off is discussed more extensively in Clarida et al. (1999). Cf. Clarida (1999), p.22f. It would be strengthened by the optimal inflation of Friedman (1969) designed to neutralize the distorting effects of potentially positive interests on real balances in payment media. According to Gali (2001) the trade-off can be incorporated into the model endogenously, if the assumption of staggered prices is extended to the labor markets, such as for example in Erceg et al. (2000). Because in our model spatial segmentation leads to a similar dispersion in mark-ups and wages, it is included endogenously even without stochastic shocks and staggered wages.

[48] Cf. Edge (2003), p.28

[49] Moreover, the criticism in Clarida et al. (1999) that standard loss functions do not capture the uncertainty generated from fluctuations in the inflation rate for intertemporal financial planning, seems to be overcome using a objective function containing, even indirectly, the variability of future interest rates as an explicit argument. Cf. Clarida (1999), p.15.

the markets.[50] Without the danger of discretionary policy changes, the probability of expectation errors by the public and the subsequent welfare costs shrink.[51] This becomes even clearer, if one follows the arguments in Woodford (1998) that a optimal policy with respect to a given objective function can be reproduced nearly perfectly by a discretionary policy with the same objective, but also including an explicit motivation for interest smoothing[52], or a interest rate policy with a high degree of inertial behavior with respect to prior interest rates. Considering real wages, monetary policy should try to reduce distortions from deviations of wages, and thus also real wages, from steady state, since these indicate deviations from the optimal marginal rate of substitution between leisure and consumption, and thus reduce welfare. Due to uncovered interest parity, exchange rate appreciation is equal to the spread between the domestic and the foreign inflation rate, which according to Leitemo et al. (2002) adds another mechanism to induce a trade-off between output and inflation stabilization. Moreover, both domestic and foreign inflation contribute to the welfare loss due to the arguments already given above.

When minimizing this objective function, the central bank has to respect the restrictions given by the structural model of the economy. These can be classified into the equations forming the supply block of the economy, which comprises (80), (81), the linearization of (21) for the consumption good sector and the conditions for an efficient production by the central bank (63) after replacing $\dot{h}^{1m}_{t+q+1}$ by (62) and for a nonnegative central bank profit. The last condition can be formalized by inserting (22) in (5) and linearizing the result, which yields

$$\frac{1+r}{r}\dot{r}_{t+q-1} + \dot{x}_{(t+q-1)m} = \frac{1}{\alpha_2}\dot{s}^m_{t+q} - \frac{1-\alpha_2}{\alpha_2}\dot{\bar{W}}^m_{t+q} + \dot{k}^m_{t+q} + \frac{1}{\alpha_2}\dot{a}_{2(t+q)}. \tag{90}$$

In order to incorporate a monetary shock component, (63) must be modified by adding a summand within the brackets of the first line, which can be interpreted as an exogenous component influencing the policy instrument, i.e. the riskless interest rate. This summand is of the form

$$\dot{\zeta}_{t+q+1} = \omega_1\dot{\zeta}_{t+q} + \dot{\epsilon}^{cb}_{t+q+1}, \tag{91}$$

whereby $\dot{\epsilon}^{cb}_{t+q+1}$ is a stochastic shock term, which is i.i.d. according to the standard normal distribution.[53] At first glance this modelling of monetary shocks deviates from the literature, of which most either assume some form of interest rate rules, e.g. Clarida et al. (2000), or a money growth rule, e.g. Casares (2001). Both ideas serve to pin down an exogenous liquidity supply. But in the presented model, liquidity is an endogenous concept. Therefore an exogenous disturbance is added to an endogenous equation in order to

[50]Cf. Goodfriend (1991).

[51]This offers the possibility to integrate an expectation channel of monetary transmission into the model.

[52]Clarida et al. (1999) argue that interest smoothing takes place due to the central bank's imperfect knowledge about the parameters of the economy. Cf. Clarida et al. (1999), p.55.

[53]Technically this is equivalent to multiplying the gross nominal interest rate in the central bank's version of (60) with the factor $\varpi_{t+q+1} = (\varpi_{t+q})^{\omega_1}\epsilon^{cb}_{t+q}$, where ϵ^{cb}_{t+q} is i.i.d across time according to the standard log-normal distribution.

allow for monetary policy shocks. Nevertheless, a closer inspection shows that combining (63) with the shock term and the previous equation, and repetitively substituting current and future money holdings with the linearized version of (12) and price levels with the next equation below yields some kind of augmented forward-looking Taylor rule,[54] which determines current nominal interest rates as a function of expectations for future interest and inflation rates and an array of further variables.
On the demand side, or as an IS block, the structural model includes (82) as the representation of the household, while the market equilibrium conditions (88) complete the model. Finally there is one last equation missing, which is the definitional equation for the inflation rate deviation, which is given by

$$\pi_{t+q+1} \equiv \dot{\lambda}_{t+q+1} - \dot{\lambda}_{t+q}. \tag{92}$$

This equation can be assigned to the supply block of the economy. For the foreign economy symmetric equations hold. Hence the structural model consists of 40 linear difference equations in the endogenous domestic variables ($\dot{k}_t^{cd}, \dot{k}_t^{mf}, \dot{x}_t^{m}, \dot{N}_t^{d}, \dot{\psi}_{1t}^{d}, \dot{r}_t, \dot{y}_t^{cd}, \dot{c}_t^{d}, \dot{b}_t^{dd}, \dot{b}_t^{df}, \dot{\bar{W}}_t^{d}$, $\dot{\pi}_t^{d}, \dot{e}_t, \dot{s}_t^{d}, \dot{s}_t^{md}, \dot{\nu}_t^{d}, \dot{z}_t^{d}, \dot{\lambda}_t^{d}, \dot{\mu}_t^{d}, \dot{Q}_t^{d}$) and the foreign variables ($\dot{k}_t^{cf}, \dot{k}_t^{mf}, \dot{x}_t^{mf}, \dot{N}_t^{f}, \dot{\psi}_{1t}^{f}, \dot{r}_t^{f}, \dot{y}_t^{cf}, \dot{c}_t^{f}$, $\dot{b}_t^{ff}, \dot{b}_t^{fd}, \dot{\bar{W}}_t^{f}, \dot{\pi}_t^{f}, \dot{e}_t, \dot{s}_t^{f}, \dot{s}_t^{mf}, \dot{\nu}_t^{f}, \dot{z}_t^{f}, \dot{\lambda}_t^{f}, \dot{\mu}_t^{f}, \dot{Q}_t^{f}$). In addition, these equations are influenced by the exogenous stochastic shocks ($\dot{a}_t, \dot{\epsilon}_t^{cb}, \dot{\epsilon}_t^{a2}$).
By assumption central banks conduct the noncooperative monetary policy proposed by Clarida et al. (2002), i.e. there are no efforts to coordinate policy action. Instead, the domestic central bank behaves according to the Nash-assumption and minimizes its objective function, while assuming that foreign variables, especially the foreign interest rate as the foreign policy variable, are independent of its policy and therefore are exogenous. Thus for the domestic central bank, a system results of 120 equations in 240 variables, since the four subsequent periods of the structural model can be represented as a first-order difference equation of a vector with dimension 3, and comprises 20 equations affected by 20 domestic optimization variables. Since the foreign central bank solves the symmetric problem, the FOCs can be combined to a system of size 240×240. The redundancies are eliminated by dropping 3 double entries for the exchange rate and the repetitions of three equations, i.e. the equilibrium in the global market for aggregate consumption goods, the interest parity and the law of one price, within the system for the foreign economy. Hence a system of 237 equations in 237 variables is obtained.[55]
Reconsidering this optimization problem and recalling the transmission theories presented in Chapter 5, apparently money does not play a preeminent role in the transmission of monetary policy. Instead, as emphasized by Svensson (1999), it is the short term nominal interest rate, which serves as central variable and as the policy instrument of the central bank. For a given level of this variable the money supply adjusts endogenously to the pre-

[54] The Taylor rule is a linear connection between the nominal interest rate and the inflation and output gap, i.e. the deviation of those variables from their target or rather potential level, of the same period. It is first proposed in Taylor (1993) and suggests that the interest rate should be adjusted with an elasticity of more than 1 to a deviation in the inflation rate. This elasticity is also known as the Taylor principle. Cf. Woodford (2003), p.90ff.

[55] The optimization problem of the central bank is not discussed formally, since it is a mere extension of Söderlind (1999) already presented in Hespeler (2006).

ferred level of money holdings.[56] Of course, using the interest rate as a policy instrument does not imply that a central bank determines interest rates. Uncertainty in the economic environment, imperfect information of policy-makers as well as economic individuals and multifaceted intertemporal interdependencies restrict not only the degree of control over final targets, but also over instruments. Even if the central bank employs strategies such as intermediate targets and indicator variables as well as forecasts for economic developments in order to mitigate this restriction,[57] it is still confined to giving positive impulses to economic dynamics rather than to determining the future path of the economy.
With respect to the implementation of the optimal monetary policy, it is emphasized by Clarida et al. (1999) that apart from any discretionary bias resulting from trying to increase the output level, binding commitment to a policy without an inflationary bias can improve the future policy possibilities via expectations. The central bank signals that it reacts not only today, but also in the future more aggressively with respect to inflationary pressure by accepting a greater output gap. It is the threat of a continued future contraction in output that reduces inflation today. Hence the current contraction may be lower and the short run trade off between inflation and output is improved.[58] The problem with this kind of policy, of course, is its time-inconsistency.[59] Given the commitment of the last period and the resulting expectations of the private sector, a central banker facing her decision in the current period has the incentive to deviate from her commitment and to act in a discretionary maner in order to stabilize the output gap and to exploit the improved tradeoff between output and inflation. But this behavior will be anticipated by rational households. Hence, commitment is neither credible nor exploitable. As shown by Woodford (2003) the incentive to break the commitment can be eliminated, if the central bank commits itself to conduct monetary policy by respecting the private sector's past expectations concerning monetary policy. If the central bank incorporates the formation of past expectations into its policy framework, it is optimal to honor the past commitment, since this again induces an incentive for the private sector to rely on the central bank's future commitment. This timeless perspective of monetary policy results in policy rules which should be constant over time.[60] Amongst others, Walsh

[56]These arguments are due to the missing special role of money in the transmission process and to the fact that, even under money growth targeting, the relevant instrument variable is the short term interest rate, which in the case of an efficient policy, does not depend on the target variable money, but on the determinants of this variable and hence on the structure of the economy. Again money only plays an endogenous role. Cf. Svensson (1999), p.611,636ff.

[57]Svensson (1999) introduces intermediate targets as variables which are highly correlated with the goal, easier to control and to observe as well as more transparent. Variables which forecast or determine intermediate targets, are called indicators. This terminology matches roughly with that of the ECB.

[58]Cf. Clarida et al. (1999), p.35ff.

[59]This expression traces back to Kydland et al. (1977). It is equivalently used by all authors working on the New-Keynesian model.

[60]From a technical perspective this means that the first order conditions of the policy problem are time invariant, while in the case of the traditional commitment, they are different for the initial and all subsequent periods. This result is driven by the inclusion of an additional constraint into the policy problem, which consists of a time invariant difference equation, whose coefficients will be determined by the solution, for all non-state variables assuring constant laws of motion across time for all variables. Cf. Woodford (2003), p.536ff.

(2003) argues that the same effect can be reached by altering the objective function in the appropriate manner. Explicitly he mentions price level stabilization and discretionary speed-limit policy, in which the output gap is substituted within the policy function by the change in this gap. In particular the latter induces a dependence on past variables similar to the time-consistent policy proposal.[61]

9 Exploring and Calibrating Steady States

Following Woodford (2003) we assume a steady state that is characterized by such a low degree of distortion due to monopolistic power, that the gap between the model's steady state and a theoretically efficient steady state implies only a third-order deviation within the welfare approximation and a second-order deviation within the structural equations of the model.[62] This inefficiency does not affect technological efficiency, but reduces the amount of labor used as an input in the economy and thus distorts the price system away from the Pareto optimum. With respect to inflation it is quite obvious that our model can not be characterized by zero steady state inflation, because the central bank must cover its production costs by inducing inflation. This steady state inflation rate is denoted by π and drives a wedge between real and nominal interest rates. Hence only the real interest rate in steady state coincides with the time preference rate of households. The wedge between shadow prices for local income in domestic and foreign marketplaces is neglected in steady state, because for each domestic marketplace the low probability of being the one in which the household is hired, is too low to drive a first order difference between domestic and foreign marketplaces.

Within the calibration of the model we choose the strategy to begin with standard parameters from the literature, for the model's conventional parts and to pin down the remaining non-standard parameters in order to comply with first order moments for simple empirical time series.[63] This method is inspired by the two stage character of calibration, in which in the first stage parameters are set in order to match at least a part of the available dependent data and afterwards, in a verification stage, the results are compared to the remaining, yet unused, data for dependent variables.[64] We start by setting the quarterly nominal interest rate to $r = 0.015$ and the quarterly inflation rate to $\pi = 0.005$. The

[61] Cf. Walsh (2003), p.268ff.

[62] Regarding this somewhat unfamiliar concept of the efficient steady state one should firstly recognize that transaction costs do not imply an additional inefficiency, and secondly that indeed a sufficient weak market-power implies a deviation from efficiency of first-order size. Turning to the first argument, transaction costs are an essential feature of the model's structure and therefore do not imply distortions per se, even if they imply some frictions, which can also be seen in the famous money-in-utility approach first presented in Sidrauski (1967). With respect to the second argument, second order-terms in the approximation of the structural equations do not influence utility, because the first-order-size of marginal utility reduces those to third-order size. Cf. Woodford (2003), p.385ff.

[63] Appendix 12.5 provides details for the computation of steady state values from the exogenously determined parameters.

[64] The origin of this method, its relation to formal econometric testing and connected problems are discussed extensively in Hannsen et al. (1996). Cf. Hannsen et al. (1996), p.6ff.

discount factor arises as $\beta \approx 0.9901$, which is approximately the standard value used in the New Keynesian literature.[65] We fix the capital coefficient within the production of consumption goods to $\alpha_1 = 0.36$, which is at the upper end of the span used in the literature, but in line with some very closely related models.[66] The quarterly depreciation rate of capital is set to $\delta = 0.02$ for the consumption good sector and $\delta_2 = 0.012$ for the payment services sector.[67] Setting the elasticity of substitution between consumption goods to $\theta = 11$, which matches the mark-up of roughly 10 percent used for example in Gali et al. (2001), and the marginal and fixed costs within the monitoring technology to $\chi = 100$ and $\bar{\chi}/k^c = 0.08$, while the share of capital bequeathed to other firms in case of a firm's demise takes on the value $tc = 0.6$ and the leverage ratio equals $\hat{N}/k = 0.5$, yields a risk spread of around 86 basis points per quarter.[68] With a quarterly steady state ratio of net profits to equity fixed at 0.005 we obtain $\gamma = 0.9917$ for the quarterly survival probability of a producer of consumption goods and thus are close to the value in Carlstrom et al. (1997) and the literature on the financial accelerator. Due to identical steady state real wages the marginal products of labor coincide across marketplaces. This enables the consistent calculation of the parameter α_2, i.e. the capital coefficient within the technology of the payment service sector. Employing the values above and setting the total factor productivity of the payment service industry to $a_2 = 14$ we obtain approximately $\alpha_2 = 0.65$ along with the steady state ratios of several endogenous variables to output of consumption goods.[69] The exact method for the computation of α_2 is described in Appendix 12.5.

A Frisch elasticity of labor supply of unity,[70] and a steady state ratio of leisure to labor of 2 determine the labor coefficient within the utility function as $\eta \approx 0.56$. The risk aversion parameter within the utility function is set to the standard value of $\sigma = 5$.

[65] This discount factor is used in models with zero steady state inflation as well as with positive steady state inflation. Cf. Bernanke et al. (1998), p.32 and Erceg et al. (2000), p.299.

[66] Cf. Casares (2003), p.23. Edge et al. (2003) report a markedly lower parameter as representative for the US non-farm industries. Cf. Edge et al. (2003), p.15.

[67] Cf. Casares (2003), p.23f. Hahn et al. (1984) show that for most services the average life span of capital equipment is higher than for the rest of the economy. Cf. Hahn et al. (1984), p.299. Costa et al. (1995) report the same for capital in the form of buildings in the banking sector. Cf. Costa et al. (1995), p444. This evidence spills over to lower depreciation in the payment industry compared to the aggregate economy.

[68] Bernanke et al. (1998) calibrate the risk spread to 50 basis points per quarter and use the same leverage ratio. Cf. Bernanke et al. (1998), p.33.

[69] In an estimation of an Cobb-Douglas production function for the banking industry, which includes both loan issuance and payment services, Wang (2003) reports a strong bias to labor as a factor input (1.014 vs. 0.015) and a total factor productivity of 3.115. Cf. Wang (2003), p.21. Our settings conflict with this empirical evidence. But reflecting that payment services require much more physical capital and less labor than credit extensions, and recalling the low contribution of payment services to bank returns due to pricing behavior (cross subsidizing) the findings of Wang (2003) seem quite unrepresentative for a provider of payment services

[70] The Frisch elasticity is defined in Frisch (1959) as the elasticity of labor with respect to real wages assuming a constant marginal utility of consumption. Christiano et al. (2001) emphasize the reference of the reported value to empirical studies. Nevertheless the value seems to be controversial, since Casares (2002) proposes a value of 0.25, which is also drawn from empirical studies.

Outside of steady state capital dynamics are governed by an adjustment cost parameter equal to $\epsilon_I = 2$, which is lower than that proposed in Woodford (2003). The value for price stickyness, $\varrho = 0.75$, is taken from Bernanke et al. (1998).[71] For the open economy characteristics we assume economies of identical sizes. Hence the share of consumption goods produced in the domestic economy is set to $\varsigma = 0.5$. Following the assumption of zero net external assets in Obstfeld et al. (1995), foreign bond holdings are set to the level of domestic bonds in steady state, i.e. $b^d = b^f$. However, as discussed below, open economy features have failed to produce usable results and hence have been dismissed from the model. Consequently there are no parameters left to calibrate.

Table 1: Selected Macroeconomic Data for Industrial Countries (Long-run Averages)

	AU	CA	CH	Euro12	GER	JP	SE	UK	US
ratio of consumption to GDP for									
total economy	0.755	0.764	0.726	-	0.744	0.679	0.774	0.826	0.806
private sector	0.596	0.587	0.617	-	0.570	0.565	0.541	0.632	0.642
data	1950-2004, except for: CH 1950-2003, GER 1950-1998, JP 1955-2003, UK 1950-2003								
interest rate spread in basis points (per year)									
	-	148.3	194.0	307.6	-	289.4	-	147.8	169.9
data	CA 1975-2004, CH 1981-2004, Euro12 1996-2002, JP 1966-2004, UK 1969-2004, US 1955-2004								
monetary aggregates to GDP (per quarter)									
M1	0.139*	-	1.356	1.342	0.684	0.330*	1.410	-	0.169*
M2	-	-	3.361	2.718	1.178	0.903*	1.804	-	0.565*
data	AU Q1 1075 - Q1 2006, CH Q4 1974 - Q3 2005, Euro12 Q1 1998 - Q3 2005, GER Q1 1969 - Q4 1998, JP Q1 1957 - Q3 2005, SE Q1 1998 - Q1 2006, US Q1 1959 - Q4 2005								

*annual data in quarterly frequency
source: IFS

Comparing the steady state values of several endogenous variables in our base line model to real-world benchmarks, reveals that the model reproduces roughly the macroeconomic long-run characteristics of most industrialized countries. Thus the model's steady state ratios of private and total consumption to the production of consumption goods of 0.513 and 0.713 respectively are well within the domain of the empirical data for both measures.[72] The calibrated value for the interest spread within the model of 324 basis points

[71] This value departs significantly from the estimates reported in Eichenbaum et al. (2004). Hence below we use a lower value while checking for robustness.

[72] In order to compare the model's steady state ratios of private and total consumption to output, which are 0.476 and 0.676, to empirical data, these have been adjusted for the production of the payment services sector and the resources used up for bequests, which are included in the empirical data, because they constitute services demanded by the private and public sector. The adjusted values are reported in

per year is slightly above the upper bound of the empirical data. But choosing the lending rate (or rather the prime bank loan rate) as the risky rate is quite conservative. Hence the sample should support the relatively high number of the model. Finally the data for the ratio of monetary aggregates to GDP shows a large dispersion even in the small sample presented.[73] Without going into the reasons for the dispersion we simply note that the model's steady state value of money over production of consumption goods is with 1.00 well within the domain of the empirical data for the ratio between M1 and GDP. Finally, the model implies transaction costs of 1.4 percent of real output, which are comparable to the transaction costs derived in the shopping-time model in Casares (2002).

10 Simulation Results

Before presenting the results of the simulation experiments a few comments have to be made with respect to the solution method, numerical precision, robustness and open economy features.

The model is solved with the solution algorithm proposed in Hespeler (2006), which generalizes the method in Sims (2001) and applies it to the policy problem in Söderlind (1999). The generalizations mainly refer to a broader set of solutions, the use of the pseudoinverse in the solution algorithm and a correct decomposition for sparse systems.

Numerical precision is limited by the complexity of the problem, which impairs two numerical methods: the ordered Generalized Schur Decomposition and the Singular Value Decomposition.[74] For the former the negative effects manifest themselves in the generalized eigenvalues close to infinity, which deviate, if computed on different systems, due to varying computer arithmetic settings.[75] The second problem affects the pseudoinverse by choosing the rank of the associated matrix according to a tolerance level, below which all singular values are set to zero and thus potential additional ranks are ignored. By setting this tolerance level rather than employing the default value of the software this problem

the main text.

[73]Adjusting the time series to the same length does not change this phenomenon.

[74]Both decompositions are standard operations in modern linear algebra, which decompose a matrix into the product of three matrices. Details can be found for the Generalized Schur Decomposition in Van Dooren (1981) and for the Singular Value Decomposition in Meyer (2000). Cf. Meyer (2000), p.411ff.

[75]This problem is also acknowledged by the provider of Mathematica, Wolfram Inc., who refers to it as follows. "Small numerical residuals may be different because of the different machine arithmetic. ... you can usually confirm that you are still getting a correct result by testing some property of the resulting system (e.g., eigenvalues of the closed loop system after a pole placement). Finally, the concrete values after random distortions will, of course, be different in your experiments." Cf. Wolfram (2003), 1.8. In order to check for the responsibility of the software we constructed a random matrix of dimension 100×100 and computed the Ordered Generalized Schur decompositions and the associated eigenvalues for the same matrix on both systems. The norm of the differences between matching matrices of the decompositions is below 10^{-7}, while the norm of differences between the associated eigenvalues is below 10^{-12}, where the strongest contribution comes from the highest eigenvalues. For our model these differences are with 1700 or rather up to 10^{11} much higher and thus the related problems are much more serious. This analysis supports the arguments above.

can be at least partially controlled. Although numerical precision is still impeded, computations on two different computers reveal that qualitatively the results yield roughly the same conclusions. Hence in general our results are quite reliable, while all obtained numerical values should never be interpreted in a quantitative, but only in a qualitative manner.

With respect to open economy features we find that these either prevent a numerical analysis completely, or add substantial instability to the system, whereas the closed economy version is characterized by stable and smooth reactions to the same shocks. In the first case the reordering of the Generalized Schur decomposition destroys the theoretically guaranteed non-singularity of certain diagonal blockmatrices. In the second case both the number of stable generalized eigenvalues as well as the specific values of the unstable eigenvalues diverge more between the two simulation scenarios for the open economy than for the closed economy version. There are no apparent economic reasons, but from a technical perspective some natural explanations are close at hand. Firstly almost doubling the system's dimensions manifolds the potential for interdependencies and increases the probability of persistent fluctuations. Secondly the numerical problems discussed above are severely aggravated by the rise in dimension. And finally, more complex eigenvalues, which tend to induce fluctuations, appear on the unstable manifold of the system. Because these difficulties could not be overcome, open economy features have been skipped and the remainder of this study is confined to the analysis of a closed economy version of the presented model.

10.1 Transmission Mechanism of Monetary Shocks

As already presented, monetary policy shocks are implemented into the model as stochastic shocks to (63), which have exactly the same coefficient within this equation as the nominal interest rate. Since (63) can be easily transformed to an interest rate rule similar to augmented forward-looking Taylor rules, the shock component can be interpreted as a policy shock on nominal interest rates, whereby these do not fully absorb the shock, because the other endogenous variables react simultaneously. In the following we analyze a shock which would force nominal interest rates 1 percent of their steady state value downwards, if no other variable would react. This translates roughly to a reduction of the policy rate by 1 basis point.

As in many models of the New Keynesian type, which assume some limiting factor in access to financial markets such as e.g. Christiano et al. (1997),[76] in the short run the interest rate channel is dominated by the liquidity effect driven by a strong initial reaction of the bond markets to the shock on interest rates. The liquidity driven out of the bond markets expands the demand for money as a saving instrument, because the cost of inflation falls. The fall in nominal interest rates together with the immediately rising inflation,

[76] The issue of existence or nonexistence of a liquidity effect depends not only on this limitation, but also on the degree of risk aversion within the utility function, which needs to exceed some critical value. Cf. Gali (2001), p.16.

which is due to future inflation expectations, pushes real interest rates downwards. This shift in real interest rates induces on the one hand investment in the private sector, and on the other hand increases consumption through the intertemporal substitution channel. Both components increase aggregate demand and support the rise in inflation, because aggregate output reacts relatively sluggishly. But it should be noted that both demand components react slowly or even with some delay indicating that adjustment costs, as well as risk aversion influence the transmission mechanism.

Both the strong positive reaction of equity as well as the persistent decrease in the interest rate spread, after potential initial fluctuations due to a short-time contraction in the funds available for financial intermediation, document that the standard credit channel is at work. Together with the negative reaction of real interest rates this suggests that the credit channel plays an important role within the transmission mechanism. Since the effect on the interest rate spread seems to be comparatively weak, while equity reacts quite considerably, we can conclude that strong effects on the profits of monopolists and a strong demand for bonds apparently determine the effects on the market for risky loans.[77]

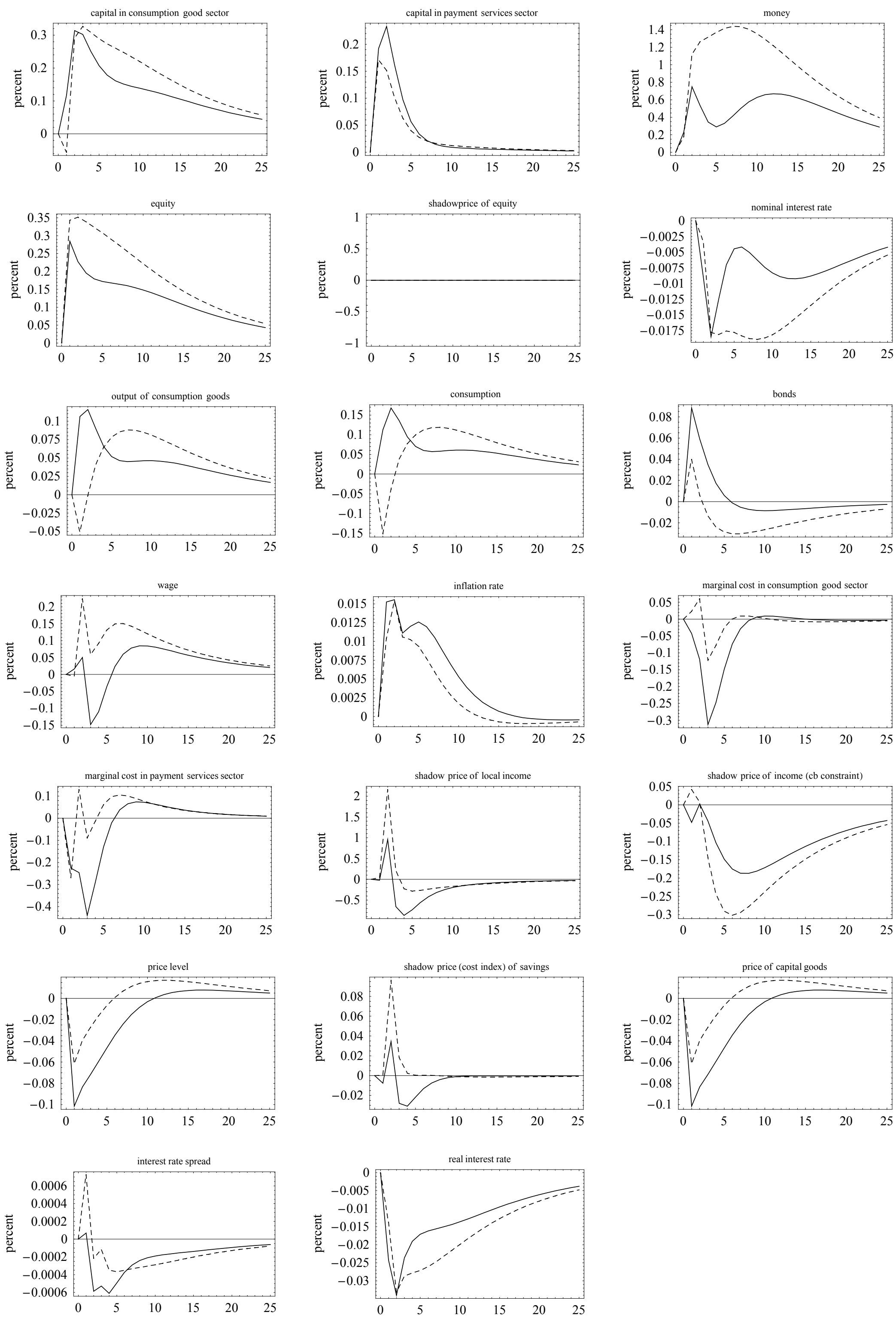
The model includes a transmission through the activities and budget of the central bank, which from now on will be denoted as the central bank effect. Despite the prevalence of the liquidity effect and the related ambiguous reaction of central bank returns from seignorage, the investment demand of the public sector, i.e. the central bank, increases significantly in the short run. As simultaneously the shadow prices for local income soar, this reaction seemed to be driven by a substantial increase in the demand for actual and/or future transaction services. The resulting immediate increase in aggregate demand explains the strong initial reaction of the inflation rate, which tends to diverge from the monotonic hump-shaped reaction of the more standard New Keynesian models, e.g. Bernanke et al. (1998) or Casares (2001), due to its faster reaction. Since shadow prices for local income decline sharply after the initial peak, this effect is limited to the very short run. But in this period the related higher supply of transaction services tends to push wages and marginal costs in the consumption good sector upwards, and contributes to the delay in the reaction of private investment demand and production. Nevertheless marginal costs tend to fall for several quarters, because the effect on real interest rates outweighs potential positive fluctuations in nominal as well as real wages.

The persistent fall of the shadow prices of global income demonstrates that the increase in consumption is driven by household income, which is affirmed further by the intermediate positive reaction of the shadow prices of savings. This indicates the contribution of wealth or income channels to the transmission within the model.

The transmission through income channels does not prevent the strong rise in equity nor a tendency for decreases in the marginal costs of consumption good producers in the short run. Together these reactions demonstrate a significant increase in monopolistic profits, which can be interpreted as transmission through the supply-side channels of Lucas (1973, 1975). The effect of these channels can also be seen in the trend to a deviation of output from the strict hump-shaped form in standard New Keynesian models.

[77]Nevertheless, it should be noted that the risk premium is expressed as a gross rate, which is very close to one. For this reason also minor percentage changes can be related to substantial changes in the net rate. To a lesser extent this holds also for real and nominal interests.

Figure 1: Simulation Responses to Monetary Shock



Calibration Values					$\frac{(1-l)}{\eta l}$	1	ω_1	0.3	χ	100	$\frac{\bar{X}}{k^c}$	0.08	$\frac{\hat{N}}{k^c}$	0.5
α_1	0.36	δ_1	0.02	ϵ_I	2	$\frac{\Pi^c}{N}$	0.005	ϱ	0.75	$\delta_1 - \delta_2$	0.008	a_2	14	θ	11
μ	1	σ	5	$\frac{1-l}{l}$	2	r	0.015	π	0.005	$\frac{g}{y^c}$	0.2	tc	0.2	P	1

Comparing the impulse reactions of the two scenarios with low and high productivity within the payment industry in general reveals that, with respect to non-price variables, the effects for the model with higher productivity are somewhat higher in the short and/or intermediate run but display a similar persistence as the model with lower productivity. A notable exception is nominal money, for which the reaction in the second scenario remains above that of the first throughout the whole adjustment period. The reactions of most price variables are very similar in both scenarios, whereby interest rates react slightly less in the intermediate run in the scenario with lower productivity.

In detail, the reaction of nominal interest rates is somewhat more pronounced after an initial period in the second scenario. This results in the corresponding behavior of the monetary aggregate and in a slightly stronger reaction of investment demand by the private sector and equity in the intermediate run. The relative difference for the central bank's investment demand reflects the relative reactions of real interest rates and money demand, because it is financed out of central bank revenues. The total effect is ambiguous, since the baseline calibration shows a weaker reaction of public investment for the second scenario, while this result is reversed for stronger shock sizes. Consumption and aggregate output behave ambiguously in the first quarters, but afterwards the reactions of the second scenario clearly supersede. In the long run the reactions are nearly identical and differences converge slowly to zero. The impulse reaction of the inflation rate is very similar for both scenarios. Thus the real transmission via the liquidity effect or rather the interest rate channel seems to be stronger for more efficient transaction technologies, a conclusion which is reconfirmed by the significantly stronger reaction of nominal interest rates. Moreover the shadow prices for local income are more clearly inclined to a positive reaction suggesting a stronger sensitivity with respect to transaction costs and volume. This suggests an even stronger contribution of the central bank's budget to the transmission to aggregate demand, a conclusion, which at least for some parametrizations is reinforced by the stronger reaction of public investments. For the credit channel, our results are more ambiguous, because on the one hand equity reacts more strongly at least in the short run, but on the other hand the interest rate spread gives ambiguous signals, since for higher productivity it actually rises in the short run and falls below the level of the first scenario only slightly after a delay of five quarters. This indicates that direct effects of the profitability of firms on their production plans rather than credit supply restrictions contribute to a stronger reaction by output as well as demand. Hence the credit channel seems to be weakened, while supply side channels are reinforced. Overall the grip of the central bank on the economy seems to be strengthened, because real reactions with respect to monetary shocks are stronger for the scenario with a higher productivity of the payment industry, while the fact that the short-term reactions of output and consumption are somewhat out of line with our arguments, may well be interpreted as an additional delay within the monetary transmission mechanism.

It is striking that the traditional question of the difference in money demand between the two scenarios is not a driving factor within the model. Of course in a steady state with higher productivity in the payment service industry, real money holdings are most likely to be higher, because the real interest rate remains unchanged, while the substitution of resources from the payment industry to the consumption sector increases output as well

as transaction volume. As long as the efficiency gain does not overcompensate the increase in transaction volume, real money holdings need to rise. Likewise after a monetary shock, the endogenous money demand adjusts to the money supply, which is influenced strongly by the effects of monetary policy on seignorage. The important question here is whether the effects on the demand for gross transaction volume support or oppose, or even outweigh, price effects. The higher the total factor productivity of the payment industry the higher the probability for the second alternative is. Hence monetary aggregates might even fall after an expansive monetary shock and the probability for such behavior rises with productivity. Since this would require a faster transmission of the effects to aggregate demand, we can conclude that in this case actually higher productivity speeds up transmission through the interest rate channel. The contradiction to the conclusion derived above elucidates that the change in the speed of the transmission depends on the parameter configuration of the baseline scenario as well as on the size of the productivity difference between the scenarios compared.
Finally the welfare differences between the two scenarios reconfirm the supposition that a higher efficiency in the payment industry reduces transaction costs and thus increases welfare.

10.2 Persistent Productivity Shocks on the Payment Services Sector

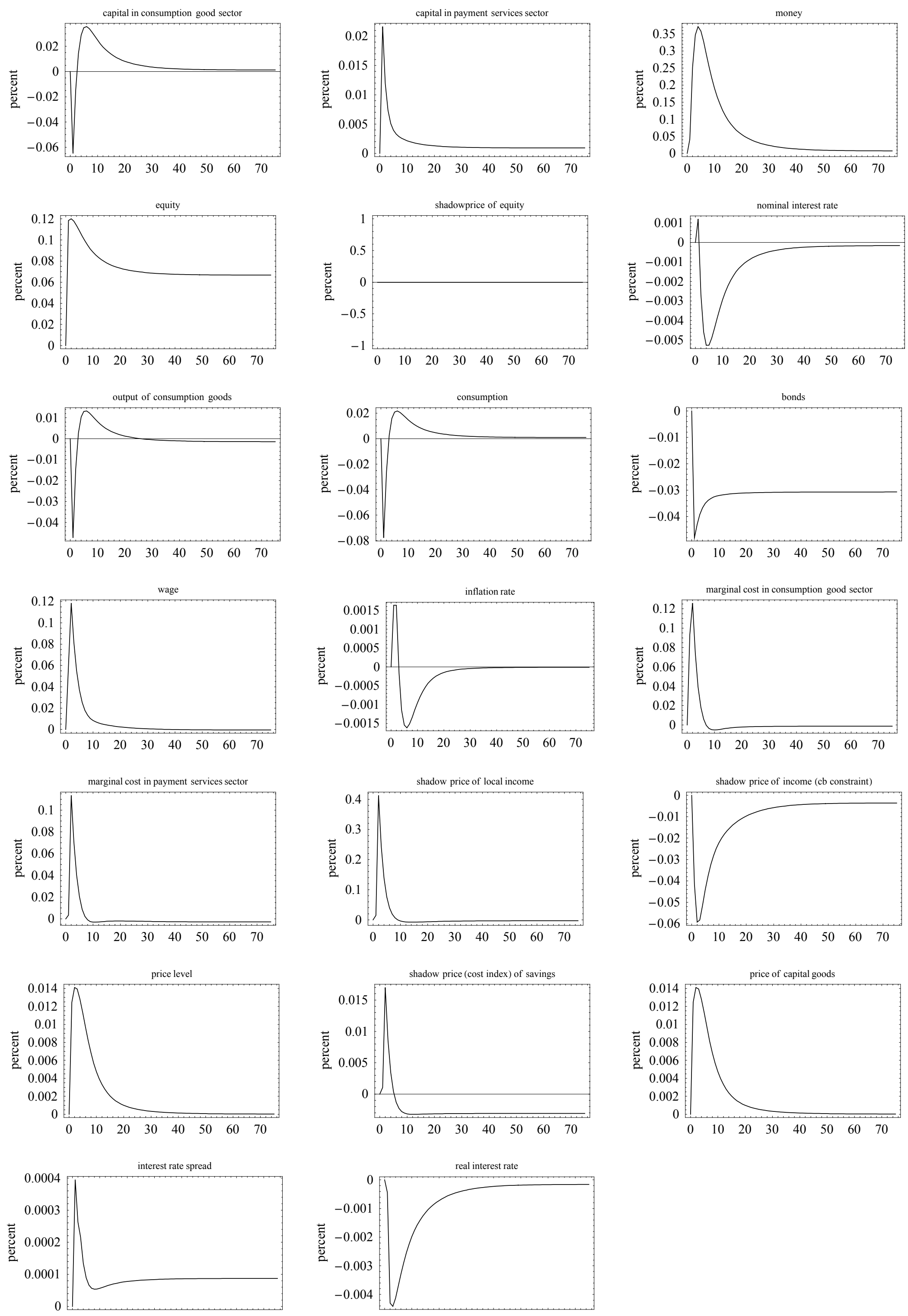
The transitory consequences of a completely persistent productivity shift in the payment service sector can be analyzed by means of an impulse response function, as long as the complete future shock series is convergent.[78] If this condition holds, the consequences of a technological innovation in the payment service sector are analyzed by the impulse responses of a positive shock to the total factor productivity of the payment service industry which is of size one per mil of its steady state value.[79]

As Fig. 2 documents, the positive shock generates in the very short run a recessionary reaction, which is supplanted after the initial periods by an intermediate expansion and afterwards the model economy tends to the new steady state characterized by a slightly lower aggregate output. The initial response is driven by a strong positive reaction of the

[78]According to Suzuki (1976) a sufficient condition for convergence of a series is that the last summand of the infinite series converges to zero. For the model presented this implies that the kth-power of the product of the two matrices defining the unstable generalized eigenvalues should converge to zero, if k converges to ∞.

[79]The surprising tiny size of the shock is due to the fact that is has been designed to resemble the transition path between the two scenarios analyzed for monetary shocks, which must be very close to each other, since otherwise steady state inflation in the second scenario would be very close to zero or eventually even negative. Of course the transition path can not end in the second steady state of the monetary policy experiment, because it is calibrated by normalizing the price value to 1, which does not hold for the new steady state resulting from our technology shock due to inflation in the meantime. Nevertheless, restricting the size of the shock to the same level may allow some comparisons.

Figure 2: Simulation Responses to Technology Shock



Calibration Values						$\frac{(1-l)}{\eta l}$	1	ω_1	0.3	χ	100	$\frac{\bar{X}}{k^c}$	0.08	$\frac{\hat{N}}{k^c}$	0.5
α_1	0.36	δ_1	0.02	ϵ_I	2	$\frac{\Pi^c}{N}$	0.005	ϱ	0.75	$\delta_1 - \delta_2$	0.008	a_2	14	θ	11
μ	1	σ	5	$\frac{1-l}{l}$	2	r	0.015	π	0.005	$\frac{g}{y^c}$	0.2	tc	0.2	P	1

demand for transaction services, which pushes up the demand for money as well as the central bank's investment demand. The latter drives up wages and marginal costs in both sectors and in the short run even nominal interest rates, while the former is financed by a substitution of bonds for money. Consequently, private demand components as well as output of consumption goods react negatively on impact, and liquidity increases. Lower private demand as well as the sharp initial rise in the inflation rate dampen the need for transaction services, while the increased liquidity starts to push down nominal and real interest rates and revives private sector demand. Output increases less than private sector demand, but nevertheless inflation falls below its steady state level. This indicates that despite the rise in consumption and private investment demand, aggregate demand still lags behind output and the resulting deflationary pressure restricts the price-sensitive output. The explanation can be found in decreasing monitoring costs due to the strong increase in equity and the related decrease in marginal costs of financial intermediation. Since the reduction in the resources demanded for monitoring services remains in the long run, the new steady state value for output is slightly below the old steady state, while demand components except for demand for monitoring services tend to a higher level. But considerable long-run movements are only found for equity, the risk premium and the shadow prices for savings and bonds. While the first two are related to the discussed decrease in monitoring costs, whereby the increase in the interest spread is due to slightly lower nominal interest rates, the decrease in bonds and the shadow prices of savings can be explained by the fact that due to higher equity less external funding is needed in order to finance capital. Again the results imply that the credit channel loses weight within the monetary transmission mechanism, because the shift to equity reduces the role for financial intermediation, while the marginal influence of risky loans on the firms' pricing decisions remains unchanged. In the long run the combined reaction of shadow prices for global income and savings is slightly negative, while wages return to their steady state level. Hence labor is reduced and leisure increased. But surprisingly, and in contradiction to the result of the comparative analysis within the last section, welfare is not changed in the long run, while in the short and intermediate run the fluctuations are accompanied by welfare losses compared to steady state. One possible interpretation is that compared to a stationary path of the economy, in which all real growth rates are constant over time, the analyzed transition path does not lead back to the old steady state, but instead reaches another.

10.3 Robustness

As summarized in Hannsen et al. (1996) the use of calibrated general equilibrium models in order to analyze macroeconomic policy impacts, or even to generate policy proposals may lead to spurious and doubtful results due to methodological flaws in the selection of the data used to value the numerous elasticities and parameters within macroeconomic models. As a common way to check for such failures, informal robustness tests, i.e. tests for non-sensitivity to parameter changes, are used in many, but not all contributions to

the standard literature on New Keynesian models.[80]
In order to assess the robustness of our model we conduct a series of parameter shifts. Due to the high need for computation time for computing steady state values, we do not claim that these experiments constitute an extensive test of robustness. Moreover the analysis of parameter shifts is inhibited in many cases by the fact that those shifts produce differences in either the number of stable eigenvalues, or similar characteristics, which imply that the two scenarios are no longer comparable.[81] Nevertheless, the results should suffice to examine at least some robustness patterns with respect to parameter changes. As can be expected for a relatively detailed structured dynamic general equilibrium model with a considerable degree of autoregression, robustness can be found for a certain manifold of the parameter space, but is in no sense a global phenomenon.
A higher intertemporal autocorrelation in the monetary shock series does not influence the general results, except for more persistent reactions, which adapt to the more hump-shaped pattern typical for many New Keynesian models. Only when the persistence parameter ω_1 converges to 0.9, the credit channel becomes more important after the productivity increase. Similarly a stronger divergence in the productivity of both scenarios merely emphasizes the differences more strongly without altering the results presented above. On the other hand a mere increase in the level of the total factor productivity for both scenarios by roughly 14 percent does not alter the general transmission mechanism, but reverses the results of the comparison between the first and the second scenario. Since both capital as well as equity are nearly 10 percent higher after the parameter change, while marginal transaction costs are around 14 percent lower, the reaction of the economy to monetary shocks seems to be connected to its contemporary endowments.
To substantiate the claim that there exists a manifold, on which the model's responses to our shocks are fairly robust, we return to the observation that our baseline calibration is characterized by a relatively low private consumption and a high risk spread. Adjusting parameters in order to generate a configuration more in line with the empirical evidence, i.e. a quarterly risk rate spread of 0.51 percent and adjusted ratios of private and public consumption to output of 0.533 and 0.733 respectively,[82] obtains results, which are, apart from a short run fluctuation in output and consumption for the second scenario, very similar to those of the baseline calibration. This experiment, for which the impulse responses are depicted in Fig. 3, and Fig. 4 for the technology shock, of Appendix 12.7, supports the claim of limited robustness strongly. Moreover, the results obtained from the new calibration are fairly robust with respect to changes in the persistence of the shock. A lower degree of risk aversion, i.e. $\sigma = 3$, does not change the results for the persistence level of shocks within the baseline calibration, but for more persistent shocks the results

[80] As Novales (2000) points out, these tests would ideally not only include parameter changes, but also changes in the structure of the model. Cf. Novales (2000), p.10. Nevertheless bowing to such high standards encumbers research projects heavily. Probably this explains why in many publications these standards are not fully met. An alternative approach might be to comprehend the related literature as a pool of control models in order to assess the scope of one's own model.

[81] Again this points to the complexity of the system, since in systems with lower dimension these changes are less likely.

[82] The adjusted ratios include besides private and public consumption also the resources for bequeathing capital and transaction costs.

of the comparison between the two scenarios are again reversed. On the other hand a slight reduction in fixed costs for the evading technology again supports the results, which now also hold in the short run, except for the fact that the credit channel becomes more important, when the productivity of the payment industry increases.
For technology shocks the results are even more robust, since the expansive reaction in the intermediate run as well as the long run effects remain quite similar across all simulations conducted. Parameter shifts do not seem to have any impact in this respect at all. Hence, in general, robustness experiments support the results of the model insofar, as the steady state characteristics are not changed too much by parameter shifts.

11 Discussion of Results

Within this paper we presented a computable general equilibrium model, which encloses a rationale for the existence of money and other payment media. The model has been constructed in order to analyze the impact of a variation in transaction costs on monetary transmission. Herein transaction costs are operationalized as the costs of producing transaction services by issuing the associated payment media. While the model presented is a two-country model, the simulations are restricted to a simpler closed economy version without private issuers of payment media. These simplifications are unavoidable due to numerical problems connected to dimensionality. Some general vagueness in the results remains, stemming from the fact that the available numerical procedures have difficulties in dealing with eigenvalues which are relatively close to infinity. But fortunately this only concerns the quantitative character of the results, while the broad qualitative characteristics are scarcely impeded.
Turning to the results of the analysis, we find that the impulse responses to monetary shocks are comparable to those in the literature on New Keynesian models including monetary frictions. In particular, the transmission through the interest rate channel is dominated by the liquidity effect, while additionally the risky credit contract integrates a credit channel into the model. Moreover the central bank's consumption of real resources in order to produce payment services generates an additional transmission mechanism, in which an increase in expected transaction volume raises public investment demand and therefore temporarily delays private reactions by raising factor prices. Thus it contributes to the persistence of the overall effects. With respect to transaction cost variations, a comparative analysis of two scenarios with diverging efficiency parameters in the production function for payment services reveals that generally lower transaction costs go together with a higher degree of transmission through the interest rate channel and a weakened credit channel. Moreover the supply side becomes more important within the transmission process, if efficiency in the payment service industry rises. Finally the central bank channel is reinforced by an improvement in its technology. Altogether, the influence of the central bank on the real economy increases, because real reactions are stronger. The model's responses to permanent productivity shocks on the transaction technology show that in the new long run equilibrium a higher equity stock decreases aggregate bond

holdings and the scarcity of money, which is also revealed by the higher stock of nominal money. Moreover, despite the shift of resources out of the payment service sector into the consumption good industry, aggregate output of consumption goods still decreases, because aggregate demand shrinks due to lower monitoring costs. The intermediate effects show the intuitively expansive impact of the technological improvement, as well as the deflationary pressure which finally forces aggrega te output downwards. The higher equity position goes well with the lower importance of the credit channel in the scenario with lower transaction costs, because now less funds depend on the external risk premium and therefore the restrictive role of the banking system is diminished.

Our results seem to be more or less in line with a substantial part of the literature. Thus the responses presented to both monetary and technology shocks are broadly comparable to those in Casares (2001), who incorporates transaction costs into a money-in-utility model by means of a shopping-time function. For shocks on the transaction technology the only important difference between the two models' responses is the reaction of the money demand, which remains markedly lower in Casares (2001), since in his model it is directly determined by and positively correlated with shopping-time costs. For monetary shocks the responses are also quite similar and diverge only with respect to the more hump-shaped reactions of consumption and output in our model. A very similar model is presented by Kim et al. (2006). But unlike Casares (2001) they derive an explicit New Keynesian Phillips curve including an implicit interest rate term within marginal costs, and a central bank objective which firstly does not care about interest rates explicitly, and secondly implies a less aggressive policy with respect to inflation relative to output stabilization than the standard model due to the presence of transaction costs. A comparison to our model reveals that the dependence of marginal costs on interest rates becomes quite insignificant, if capital dynamics are present,[83] while the absence of an explicit interest term in their policy function is due to the missing financial intermediation and the zero debt of the public sector. The last feature changes, if financial intermediation is integrated, leaving one with a policy objective which contains an interest rate term similar to that in Woodford (2003), whereby the interest term is due to monetary frictions.[84] However, parameter shifts reducing transaction costs for given levels of consumption and real money tend to reduce the role of interest rates in the New Keynesian Phillips curve and thus also mitigate the relative laxity of the central bank towards inflation. This implies that the model is shifted back towards the standard New Keynesian model and hence the transmission to output is strengthened. Hence despite smaller differences the model seems to support our results.
With respect to the financial accelerator part of our model the responses are quite similar to those of the original work of Bernanke et al. (1998). In particular their model version with two sectors, which are distinguished by different adjustment costs for investment, shows similar reactions to the model presented above, whereby the sector with lower adjustment costs in Bernanke et al. (1998) is comparable to the payment services sector

[83] Ravenna et al. (2002) derive a similar New Keynesian Phillips curve by assuming that the loan bill must be prefinanced by issuing debt.

[84] Cf. Woodford (2003), p.419ff.

in our model characterized by a lower depreciation. For both models the hump-shaped reaction of output, a persistent decrease in the risk premium and a slower reaction by investment in the sector with higher adjustment costs or rather depreciation rate hold. But since Bernanke et al. (1998) omit a significant dynamic role of money, an experiment similar to a transaction cost reduction can not be analyzed. However, the related work of Carlström et al. (2000) can be used for this purpose. Like our model they find a liquidity effect in a financial accelerator model with capital dynamics. But while the liquidity effect in their model is driven by private consumption,[85] in our model this role is played by the expansion in the seignorage base in order to refinance the higher expected transaction volume. Nevertheless, both models produce similar hump-shaped patterns. Although Carlström et al. (2000) integrate a cash-in-advance restriction into the model, its role for the liquidity effect and the monetary transmission mechanism is not discussed. But because investments, which are linked via monitoring costs to the endogenous costs of capital corresponding to the risk spread of our model, directly influence this condition, a positive monetary shock reinforces its binding character and would probably raise transaction costs in the economy. This should dampen private demand and hamper the interest rate channel, resulting in a stronger interest channel for lower transaction costs. Hence even this feature seems to be quite similar in both models.

In Edge et al. (2003) a cash-less New-Keynesian economy is modelled and analyzed with respect to technology shocks. The reactions of the model to interest rate shocks coincide roughly with a VAR-model,[86] which explains the times series for the modelled variables for US data between the first quarter of 1960 and the last quarter of 2004 using four lag periods, as well as with the patterns in our model. Again the hump-shaped responses for output, consumption, private investment and inflation hold, while interest rates are dominated by a liquidity effect in the short run. More interestingly, besides obvious differences, their model show interesting parallels to ours for permanent technology shocks. Reactions in interest rates, output, consumption, investment of the affected sectors and wages are quite similar, even if in Edge et al. (2003) effects on many variables are permanent, while they are only transitory in ours. But this only reflects that in the model presented above the consumption sector is not shocked directly. Rather the payment service sector absorbs the permanent shocks, while the shocks used in Edge et al. (2003) directly affect the consumption good sector.

Belongia et al. (2002) offer a rather different supporting piece of evidence to our results. Starting from the hypothesis that neither interest rates nor monetary aggregates, but the costs of money holdings are a correct representation of monetary policy shocks, they show in both an empirical VAR model and a real business cycle model, which integrates money by a shopping-time function, that positive shocks to the costs of money produce a hump-shaped negative reaction in output growth. In their theoretical model the costs of money are represented by the costs of financial intermediation, while in the empirical part they

[85] Cf. Carlström et al. (2000), p.23.

[86] Vector autoregressive (VAR) models can be interpreted as unrestricted reduced form multivariate time series models, in which all variables are a priori endogenous. In these models all variables depend on lags of all variables. Thus there is no clear distinction between exogenous and endogenous variables. An introduction and discussion can be found in Lütkepohl et al. (1996).

are interpreted as the foregone interest returns. These results coincide with our findings for a positive shock on the efficiency of the central bank. Furthermore, Belongia et al. (2002) interpret their findings as evidence for a new model of monetary transmission, in which monetary policy is neither transmitted through interest rates nor through monetary aggregates, but in accordance to real business cycle theory by real price changes. While this interpretation may be somewhat rash, their new transmission channel displays similarities to our central bank channel, in which monetary policy is also transmitted by the cost covering of the central bank.
Finally the analysis of the welfare costs of inflation with respect to price adjustments in Casares (2002) supports the welfare implications of our model. Under the assumption of constant real interest rates the steady state inflation in our model is inversely related to efficiency within the transaction technology. Thus higher transaction costs imply higher inflation rates. Casares (2002) show that for a fully indexed economy, an increase from zero inflation to 4 percent inflation per year induce a welfare loss of 0.5 percent.[87] In the baseline calibration of our model a shift from the steady state inflation rate from 2 percent per year to roughly 1.6 percent per year yields a welfare gain of 0.4 percent. Thus welfare effects are even stronger than in Casares (2002).

Altogether most of the results of the model presented can be supported by evidence drawn from the related literature. In particular, the reinforcement of the real effects of monetary policy shocks by a reduction in transaction costs as well as a strengthening of the interest rate channel and a dampening of the credit channel are reaffirmed. Welfare increases and a stronger role for supply side channels associated with the monopolistic pricing behavior of consumption good producers are also reconfirmed. Interestingly, there is also some support for a new finding of the model: the central bank channel, which proposes monetary transmission through the expenses of the central bank for inputs into the production of payment services and the financing of these costs. Similarly the literature also bolsters the model's responses to technology shocks, which again reconfirms the decline in the importance of the credit channel, if transaction costs decrease permanently. On the other hand the puzzle between the results of the comparative static and the dynamic welfare analysis are neither resolved nor discussed by the literature.

[87] Cf. Casares (2000), p.17.

12 Appendix

12.1 First Order Conditions of Households

Using the concept of the current-value Hamiltonian,[88] we obtain the following Hamiltonian function.

$$u_t^i(c_t,l_t)+\frac{z_t^i}{\mu_{tm}^i}\left(\bar{W}_t l_t^i+\Pi_t^i-\lambda_t^i c_t^i-(1+\nu_t^{i(K+1)})\left(b_t^i-(1+r_{t-1})b_{t-1}^i\right)\right.$$
$$\left.-(1+\nu_t^{i(K+2)})e_t\left(b_t^{if}-(1+r_{t-1}^f)b_{t-1}^{if}\right)-t_t^i\right)$$
$$+\rho_t\left(\int_0^{\varsigma} r_{t-1}x_{(t-1)m'}^i di-\int_0^{\varsigma}\int_0^1 s_{tm}^k\left(\frac{x_{tm}^{ikB}-x_{tm}^{ikS}}{P_t^k}+\phi^m\right)dkdi\right)+\upsilon_{tm}x_{tm}^i, \tag{93}$$

where z_t^i, ρ_t and υ_{tm} act as shadow prices for the associated constraints.
According to the relevant maximum principle[89] we get the following conditions,

$$0 = \frac{\partial u_t^i}{\partial c_t^i}-\frac{z_t^i}{\mu_{tm}^i}\lambda_t^i-\rho_t\frac{\partial\left(\int_0^1 s_{tm}^k\frac{\bar{x}_{tm}^{ik}}{P_t^k}dk\right)}{\partial c_t^i} \tag{94}$$

$$0 = \frac{\partial u_t^i}{\partial l_t^i}+\frac{1}{\mu_{tm}^i}z_t^i\bar{W}_t \tag{95}$$

$$0 = \beta E_t\left(\frac{(1+\nu_{t+1}^{ib})(1+r_t)}{\mu_{(t+1)m}^i}z_{t+1}^i\right)-\frac{1+\nu_t^{ib}}{\mu_{tm}^i}z_t^i$$
$$-\rho_t\frac{\partial\left(\int_0^1 s_{tm}^k\frac{\bar{x}_{tm}^{ik}}{P_t^k}dk\right)}{\partial b_t^i}+E_t\left(\rho_{t+1}\beta\frac{\partial\left(\int_0^1 s_{(t+1)m}^k\frac{\bar{x}_{(t+1)m}^{ik}}{P_{t+1}^k}dk\right)}{\partial b_t^i}\right) \tag{96}$$

$$0 = \beta E_t\left(\frac{(1+\nu_{t+1}^{ibf})(1+r_t^f)}{\mu_{(t+1)m'}^i}z_{t+1}^i e_{t+1}\right)-\frac{1+\nu_t^{ibf}}{\mu_{tm}^i}z_t^i e_t$$
$$-\rho_t\frac{\partial\left(\int_0^1 s_{tm}^k\frac{\bar{x}_{tm}^{ik}}{P_t^k}dk\right)}{\partial b_t^{if}}+E_t\left(\rho_{t+1}\beta\frac{\partial\left(\int_0^1 s_{(t+1)m}^k\frac{\bar{x}_{(t+1)m}^{ik}}{P_{t+1}^k}dk\right)}{\partial b_t^{if}}\right) \tag{97}$$

[88] The basic idea of the current-value Hamiltonian is that the discounting factor is factored out by integrating its inverse into the shadow prices of the constraints. Cf. Kamien et al. (1991), p.127,165,198ff. and 230ff. The Hamiltonian approach simplifies a dynamic optimization problem by absorbing the law of motion into the objective function and giving explicit conditions as FOCs for the problem. Cf. Kamien et al. (1991), p.124ff. In fact, the Hamiltonian approach can be understood as a simplification of the Lagrangian formulation for the same problem.

[89] The maximum principle of dynamic control problems is presented in many textbooks. We give two references in order to offer different perspectives on the topic. Cf. Chiang (1992), p.299ff. or Kamien et al. (1991) p. 163,198f.,230,248ff.

$$x^i_{tm} - x^i_{(t-1)m} = \frac{1}{\mu^i_{tm}}\big(\bar{W}_t l^i_t + \Pi^i_t + (1+\nu^{ib}_t)((1+r_{t-1})b^i_{t-1} - b^i_t) \tag{98}$$
$$-\lambda^i_t c^i_t - t^i_t + (1+\nu^{ibf}_t)e_t((1+r^f_{t-1})b^{if}_{t-1} - b^{if}_t)\Big)$$

$$z^i_{t+1} - z^i_t = \left(\frac{1}{\beta} - 1\right) z^i_t + \frac{1}{\beta}\rho_t \frac{s^{km}_t}{\lambda_t} - \rho_{t+1}\frac{s^{km}_{t+1}}{\lambda_{t+1}} - r_t\rho_{t+1} - \frac{1}{\beta}\upsilon_{tm} \quad [90], \tag{99}$$

where $\bar{x}^{ik}_{(t+q)m} \equiv x^{ikB}_{(t+q)m} - x^{ikS}_{(t+q)m}$ denotes the local transaction value settled in central bank money. Additionally the transversality condition remains as in (14).

[90] Denoting the shadow price of the state equation in the regular Hamiltonian as κ_t, this is subject to

$$z^{ik}_{t+1} - z^{ik}_t = (\beta^{-(t+1)} - \beta^{-t})\kappa_t + \beta^{-t-1}(\kappa_{t+1} - \kappa_t) = (\beta^{-1} - 1)z^{ik}_t - \frac{\partial H_t}{\partial x_t},$$

which is the discrete version of equation (9) on page 65 in Kamien et al. (1991). The result is identical to that obtained from the Lagrangian approach or, alternatively, from the standard formulation of the discrete Hamiltonian approach. Cf. Lewis et al. (1995), p.32.

12.2 Coefficients in the First Order Conditions for Producers of Consumption Goods

matrix element	coefficient	variable
Υ_{11}	$+I''(1)(1+\pi)\gamma\left(\frac{\chi}{\chi-1}-\frac{1+\pi}{1+r}(1-\delta)\right)$	$\dot{k}^c_{t+q+2}$
Υ_{14}	$-\left(\frac{h_{1m_{int}}^{1-\alpha_1}(1+\pi)\hat{s}\alpha_1}{1-\alpha_1}+I''(1)(1+\pi)\gamma\left(\frac{\chi}{\chi-1}-\frac{1+\pi}{1+r}(1-\delta)\right)\right.$ $\left.+(1+I''(1))(1+\pi)\left(\frac{(1+r)\chi}{(1+\pi)(\chi-1)}-(1-\delta)\right)\right)$	$\dot{k}^c_{t+q+1}$
Υ_{17}	$+(1+I''(1))\left(\frac{(1+r)\chi}{\chi-1}-(1+\pi)(1-\delta)\right)$	$\dot{k}^c_{t+q}$
Υ_{22}	$+\left(\gamma+\gamma^2\left(1-\frac{\chi}{\chi-1}\right)\right)\varphi_1$	$\dot{\varphi}_{1(t+q+1)}$
Υ_{26}	$-\gamma^2\varphi_1$	$\dot{\varphi}_{1(t+q)}$
Υ_{35}	$-\hat{N}$	$\dot{N}_{t+q}$
Υ_{37}	$+k^c(1-\gamma)(1-tc)\left(\frac{h_{1m_{int}}\hat{w}\alpha_1}{(1-\alpha_1)}-\frac{(1+r)\chi}{(1+\pi)(\chi-1)}+\frac{\bar{\chi}}{k^c(\chi-1)}+(1-\delta)\right)$	$\dot{k^c}_{t+q}$
Υ_{38}	$+\frac{1+r}{1+\pi}\left((1-tc(1-\gamma))+(1+\pi)\frac{(1-tc)(1-\gamma)}{\chi-1}\right)\hat{N}$	$\dot{N}_{t+q-1}$
Λ_1	$-\frac{\alpha_1 h_{1m_{int}}^{\alpha_1}\hat{w}}{(1-\alpha_1)^2\hat{s}}$	$\dot{k^c}_{t+q}$
Λ_2	$-\frac{\alpha_1 h_{1m_{int}}^{\alpha_1}\hat{w}}{(1-\alpha_1)\hat{s}}\frac{y^c}{y^c+\phi_1}$	$\dot{p}_{t+q}$
Λ_3	$-\frac{h_{1m_{int}}^{\alpha_1}\hat{w}}{(1-\alpha_1)\hat{s}}$	$\dot{a}_{t+q}$
Λ_4	$+1$	$\dot{s}_{t+q}$

matrix element of			coefficient	variable
Ξ^a	$\tilde{\Xi}$	$\tilde{\tilde{\Xi}}^b$		
Ξ_{19}	-	$\tilde{\tilde{\Xi}}_{17}$	$-(1+\pi)(1-\delta)-\{\theta\frac{\hat{s}(1+\pi)\alpha_1 y^c}{(1-\alpha_1)k^c}\}$	$\dot{Q}_{t+q+1}$
Ξ_{112}	-	$\tilde{\tilde{\Xi}}_{19}$	$\frac{(1+r)\chi}{\chi-1}$	$\dot{r}_{t+q}$
Ξ_{113}	-	$\tilde{\tilde{\Xi}}_{110}$	$-\frac{\hat{s}(1+\pi)\alpha_1 y^c}{(1-\alpha_1)k^c}\frac{c}{y^c}$	$\dot{c}_{t+q+1}$
Ξ_{114}	-	$\tilde{\tilde{\Xi}}_{111}$	$-\frac{\hat{s}(1+\pi)\alpha_1 y^c}{(1-\alpha_1)k^c}\frac{y^{PC}}{y^c}$	$\dot{y}^{PC}_{t+q+1}$
Ξ_{115}	-	$\tilde{\tilde{\Xi}}_{112}$	$-\frac{\hat{s}(1+\pi)\alpha_1 y^c}{(1-\alpha_1)k^c}\frac{y^{MC}}{y^c}$	$\dot{y}^{MC}_{t+q+1}$
Ξ_{111}	$\tilde{\Xi}_{12}$	-	$\theta\frac{\hat{s}(1+\pi)\alpha_1 y^c}{(1-\alpha_1)k^c}$	$\dot{p}_{t+q+1}$
Ξ_{e1}	-	-	$\theta\frac{\hat{s}(1+\pi)\alpha_1 y^c}{(1-\alpha_1)k^c}(1-\xi)$	$\dot{e}_{t+q+1}$
Ξ_{116}	-	-	$\theta\frac{\hat{s}(1+\pi)\alpha_1 y^c}{(1-\alpha_1)k^c}\varsigma\frac{\nu}{1+\nu}$	$\dot{\nu}_{t+q+1}$
Ξ_{118}	-	$\tilde{\tilde{\Xi}}_{114}$	$-\frac{h_{1m_{int}}(1+\pi)\hat{w}\alpha_1}{1-\alpha_1}$	$\dot{\hat{w}}_{t+q+1}$
Ξ_{119}	-	$\tilde{\tilde{\Xi}}_{115}$	$\frac{(1+r)\chi}{\chi-1}$	$\dot{Q}_{t+q}$
Ξ_{312}	-	$\tilde{\tilde{\Xi}}_{39}$	$+\frac{(1+r)(1-\gamma)(1-tc)(k^c-\hat{n})\chi}{(1+\pi)(\chi-1)}$	$\dot{r}_{t+q}$
Ξ_{317}	-	$\tilde{\tilde{\Xi}}_{315}$	$-k^c(1-\gamma)(1-tc)\left(1-\delta+\frac{\bar{\chi}}{k^c(\chi-1)}\right)$ $\{+\theta(1-\gamma)(1-tc)\frac{y^c(h_{1m_{int}}^{\alpha_1}\hat{w}-\hat{p}^{m_{int}}(1-\alpha_1))}{1-\alpha_1}\}$ $[-\hat{p}^{m_{int}}(1-\gamma)(1-tc)y^c]$	$\dot{Q}_{t+q}$
Ξ_{321}	$\tilde{\Xi}_{33}$	-	$-\hat{p}^{m_{int}}(1-\gamma)(1-tc)y^c$ $-\theta(1-\gamma)(1-tc)\frac{y^c(h_{1m_{int}}^{\alpha_1}\hat{w}-\hat{p}^{m_{int}}(1-\alpha_1))}{1-\alpha_1}$	$\dot{p}^{m_{int}}_{t+q}$
Ξ_{322}	-	$\tilde{\tilde{\Xi}}_{317}$	$-\frac{1+r}{1+\pi}(1-tc(1-\gamma))\hat{N}$	$\dot{r}_{t+q-1}$
Ξ_{323}	-	$\tilde{\tilde{\Xi}}_{318}$	$+(1-\gamma)(1-tc)\frac{y^c(h_{1m_{int}}^{\alpha_1}\hat{w}-\hat{p}^{m_{int}}(1-\alpha_1))}{1-\alpha_1}\frac{c}{y^c}$	$\dot{c}_{t+q}$
Ξ_{324}	-	$\tilde{\tilde{\Xi}}_{319}$	$+(1-\gamma)(1-tc)\frac{y^c(h_{1m_{int}}^{\alpha_1}\hat{w}-\hat{p}^{m_{int}}(1-\alpha_1))}{1-\alpha_1}\frac{y^{PC}}{y^c}$	$\dot{y}^{PC}_{t+q}$
Ξ_{325}	-	$\tilde{\tilde{\Xi}}_{320}$	$+(1-\gamma)(1-tc)\frac{y^c(h_{1m_{int}}^{\alpha_1}\hat{w}-\hat{p}^{m_{int}}(1-\alpha_1))}{1-\alpha_1}\frac{y^{MC}}{y^c}$	$\dot{y}^{MC}_{t+q}$
Ξ_{e3}	-		$-\theta(1-\gamma)(1-tc)\frac{y^c(h_{1m_{int}}^{\alpha_1}\hat{w}-\hat{p}^{m_{int}}(1-\alpha_1))}{1-\alpha_1}(1-\xi)$ $[+\hat{p}^{m_{int}}(1-\gamma)(1-tc)y^c(1-\xi)]$	$\dot{e}_{t+q}$
Ξ_{326}	-	$\tilde{\tilde{\Xi}}_{321}$	$-\theta(1-\gamma)(1-tc)\frac{y^c(h_{1m_{int}}^{\alpha_1}\hat{w}-\hat{p}^{m_{int}}(1-\alpha_1))}{1-\alpha_1}\frac{\nu}{1+\nu}\xi$ $[+\hat{p}^{m_{int}}(1-\gamma)(1-tc)y^{m_{int}}\frac{\nu}{1+\nu}\xi]$	$\dot{\nu}_{t+q}$
Ξ_{328}	-	$\tilde{\tilde{\Xi}}_{322}$	$+k^m_{int}h_{1m_{int}}\hat{w}(1-\gamma)(1-tc)$	$\dot{W}_{t+q}$
Ξ_{329}	-	$\tilde{\tilde{\Xi}}_{323}$	$+\frac{(1+r)(1-\gamma)(1-tc)k^m_{int}\chi}{(1+\pi)(\chi-1)}$	$\dot{Q}_{t+q-1}$
Ξ_{330}	-	$\tilde{\tilde{\Xi}}_{324}$	$-\frac{h_{1m_{int}}k^m_{int}\hat{w}\alpha_1(1-\gamma)(1-tc)}{(1-\alpha_1)}$	$\dot{a}_{t+q}$

[a] without parts in square brackets, i.e. [...]
[b] without parts in curly brackets, i.e. {...}

12.3 Coefficients in First Order Conditions of consumers and the market equilibria conditions

matrix element of	coefficient	variable
$\beth_{11}$	$\sigma c^{-\sigma}\left(1+\frac{r}{\hat{s}^m}\right)$	$E_t(\dot{c}^d_{t+1})$
$\beth_{12}$	$\frac{(1-l)^{-\eta}}{\hat{w}}\left(1+\frac{r}{\hat{s}^m}\right)-\frac{1+\hat{s}^m}{\hat{w}(1-l)^\eta}$	$E_t(\dot{z}_{t+1})$
$\beth_{13}$	$\frac{(1-l)^{-\eta}}{\hat{w}}\left(1+\frac{r}{\hat{s}^m}\right)-\frac{1+\hat{s}^m}{\hat{w}(1-l)^\eta}\theta\left(1-\frac{1+\pi}{\beta}\right)-h_2\frac{r}{\hat{s}^m}$	$E_t(\dot{\lambda}_{t+1})$
$\beth_{14}$	$-\frac{(1-l)^{-\eta}}{\hat{w}}\left(1+\frac{r}{\hat{s}^m}\right)$	$E_t(\dot{\mu}_{t+1})$
$\beth_{15}$	$h_2\frac{r}{\hat{s}^m}$	$E_t(\dot{s}^m_{t+1})$
$\beth_{18}$	$h_2\theta\frac{1+\pi}{\beta}$	$E_t(\dot{\pi}_{t+1})$
$\beth_{19}$	$-h_2\theta\frac{1+\pi}{\beta}(1-\zeta)+\frac{1+\hat{s}^m}{\hat{w}(1-l)^\eta}\theta\left(1-\frac{1+\pi}{\beta}\right)(1-\varsigma)$	$E_t(\dot{e}_{t+1})$
$\beth_{113}$	$-h_2\theta\frac{1+\pi}{\beta}\frac{\nu}{1+\nu}-\frac{1+\hat{s}^m}{\hat{w}(1-l)^\eta}\theta\left(1-\frac{1+\pi}{\beta}\right)\frac{\nu}{1+\nu}$	$\dot{\nu}_{t+1}$
$\beth_{115}$	$-\sigma c^{-\sigma}\frac{1+\pi}{\beta}$	$\dot{c}^d_t$
$\beth_{116}$	$-\frac{(1-l)^{-\eta}}{\hat{w}}\frac{1+\pi}{\beta}+\frac{1+\hat{s}^m}{\hat{w}(1-l)^\eta}\frac{1+\pi}{\beta}\dot{z}_{t\frac{1+\hat{s}^m}{\hat{w}(1-l)^\eta}\frac{1+\pi}{\beta}}$	$\dot{z}^d_t$
$\beth_{117}$	$-\frac{(1-l)^{-\eta}}{\hat{w}}\frac{1+\pi}{\beta}$	$\dot{\lambda}^d_t$
$\beth_{118}$	$\frac{(1-l)^{-\eta}}{\hat{w}}\frac{1+\pi}{\beta}$	$\dot{\mu}_t$
$\beth_{120}$	$-h_2\frac{1+r}{s^m}$	$\dot{r}_t$
$\beth_{123}$	$h_2\theta\frac{1+\pi}{\beta}(1-\zeta)$	t
$\beth_{127}$	$+h_2\theta\frac{1+\pi}{\beta}\cdot\frac{\nu}{1+\nu}$	$\dot{\nu}_t$
$\beth_{21}$	$-\sigma c^{-\sigma}\frac{\hat{w}}{\hat{p}^c}\daleth_1$	$E_t(\dot{c}_{t+1})$
$\beth_{22}$	$\frac{1+\nu}{(1-l)^\eta}-\daleth_1\frac{\hat{w}}{\hat{p}^c}\frac{(1-l)^{-\eta}}{\hat{w}}$	$E_t(\dot{z}_{t+1})$
$\beth_{23}$	$-\daleth_1\frac{\hat{w}}{\hat{p}^c}\left(\frac{(1-l)^{-\eta}}{\hat{w}}+h_2\theta\right)$	$E_t(\dot{\lambda}_{t+1})$
$\beth_{24}$	$-\frac{1+\nu}{(1-l)^\eta}+\daleth_1\frac{\hat{w}}{\hat{p}^c}\frac{(1-l)^{-\eta}}{\hat{w}}$	$E_t(\dot{\mu}_{t+1})$
$\beth_{29}$	$\daleth_1\frac{\hat{w}}{\hat{p}^c}h_2\theta(1-\zeta)$	$E_t(\dot{e}_{t+1})$
$\beth_{213}$	$\frac{\nu}{(1-l)^\eta}+\daleth_1\frac{\hat{w}}{\hat{p}^c}h_2\theta\frac{\nu}{1+\nu}$	$E_t(\dot{\nu}_{t+1})$
$\beth_{215}$	$\daleth_1\frac{\hat{w}}{\hat{p}^c}\sigma c^{-\sigma}$	$\dot{c}_t$
$\beth_{216}$	$-\frac{1+\nu}{(1-l)^\eta}+\daleth_1\frac{(1-l)^{-\eta}}{\hat{p}^c}$	$\dot{z}_t$
$\beth_{217}$	$\daleth_1\frac{(1-l)^{-\eta}}{\hat{p}^c}+\daleth_1\frac{\hat{w}}{\hat{p}^c}h_2\theta$	$\dot{\lambda}_t$
$\beth_{218}$	$\frac{1+\nu}{(1-l)^\eta}-\daleth_1\frac{(1-l)^{-\eta}}{\hat{p}^c}$	$\dot{\mu}_t$
$\beth_{220}$	$\frac{1+\nu}{(1-l)^\eta}+\daleth_1\frac{\hat{w}}{\hat{p}^c}h_2$	$\dot{r}_t$
$\beth_{223}$	$-\daleth_1\frac{\hat{w}}{\hat{p}^c}h_2\theta(1-\zeta)$	$\dot{e}_t$
$\beth_{227}$	$-\frac{1+\nu}{(1-l)^\eta}\frac{\nu}{1+\nu}+\daleth_1\frac{\hat{w}}{\hat{p}^c}h_2\frac{\nu}{1+\nu}\theta$	$\dot{\nu}_t$
$\beth_{39}$	$\beta\frac{1+r}{1+\pi}$	$E_t(\dot{e}_{t+1})$
$\beth_{320}$	$-\beta\frac{1+r}{1+\pi}$	$\dot{r}_t$
$\beth_{321}$	$\beta\frac{1+r}{1+\pi}$	$\dot{r}^f_t$
$\beth_{323}$	-1	$\dot{e}_t$
$\beth_{415}$	$-\frac{c\hat{p}^c}{1+\hat{s}^m}$	$\dot{c}_t$

$\beth_{416}$	$\frac{\hat{w}\hat{p}^c}{1+\hat{s}^m}\frac{1-l}{\eta}$	$\dot{z}_t$
$\beth_{417}$	$-\frac{c\hat{p}^c}{1+\hat{s}^m}-\frac{ty\hat{p}^c}{1+\hat{s}^m}$	$\dot{\lambda}_t$
$\beth_{418}$	$-\frac{\hat{w}l\hat{p}^c}{1+\hat{s}^m}\left(1+\frac{1-l}{l\eta}\right)+\frac{\hat{b}}{1+\hat{s}^m}(1-\frac{1}{\beta})+\frac{c\hat{p}^c}{1+\hat{s}^m}+\frac{ty\hat{p}^c}{1+\hat{s}^m}$	$\dot{\mu}_t$
$\beth_{423}$	$-\frac{\hat{b}}{1+\hat{s}^m}$	$\dot{e}_t$
$\beth_{424}$	-1	$\dot{x}_{tm}$
$\beth_{425}$	$-\frac{\hat{b}}{1+\hat{s}^m}$	$\dot{b}_t^d$
$\beth_{426}$	$-\frac{ty\hat{p}^c}{1+\hat{s}^m}$	$\dot{y}_t$
$\beth_{427}$	$-\frac{\hat{b}}{1+\hat{s}^m}\frac{\nu}{1+\nu}(1-\frac{1}{\beta})$	$\dot{\nu}_t$
$\beth_{428}$	$\frac{\hat{w}l\hat{p}^c}{1+\hat{s}^m}\left(1+\frac{1-l}{l\eta}\right)$	$\dot{\bar{W}}_t$
$\beth_{434}$	$\frac{\hat{b}}{1+\hat{s}^m}\frac{1}{\beta}$	$\dot{r}_{t-1}$
$\beth_{435}$	$\frac{\hat{b}}{1+\hat{s}^m}\frac{1}{\beta}$	$\dot{r}_{t-1}^f$
$\beth_{437}$	$\frac{\hat{b}}{1+\hat{s}^m}\frac{1}{\beta}$	$\dot{e}_{t-1}$
$\beth_{438}$	$\frac{\hat{x}^m}{1+\pi}$	$\dot{x}_{(t-1)m}$
$\beth_{439}$	$\frac{\hat{b}}{1+\hat{s}^m}\frac{1}{\beta}$	$\dot{b}_{t-1}^d$
$\beth_{54}$	$-\frac{1}{1+r}$	$E_t(\dot{\mu}_{t+1})$
$\beth_{513}$	$\frac{1}{(1+r)(1+\hat{s}^m)}\frac{\nu}{1+\nu}$	$E_t(\dot{\nu}_{t+1})$
$\beth_{517}$	$\frac{\hat{s}^m}{1+\hat{s}^m}$	$\dot{\lambda}_t$
$\beth_{518}$	1	$\dot{\mu}_t$
$\beth_{519}$	$-\frac{\hat{s}^m}{1+\hat{s}^m}$	$\dot{s}_t^m$
$\beth_{520}$	$\frac{\frac{\hat{s}^m}{1+\hat{s}^m}}{1+r}$	$\dot{r}_t$
$\beth_{527}$	$-\frac{\nu}{1+\nu}$	$\dot{\nu}_t$
$\beth_{617}$	$\hat{s}^m\hat{p}^c\theta-\frac{r\hat{x}}{y(1+\pi)}$	$\dot{\lambda}_t$
$\beth_{619}$	$\frac{r\hat{x}}{y(1+\pi)}$	$\dot{s}_t^m$
$\beth_{623}$	$\daleth_1\hat{s}^m\frac{\hat{b}}{y}-\hat{s}^m\hat{p}^c\theta(1-\varsigma)$	$\dot{e}_t$
$\beth_{624}$	$\daleth_2\hat{s}^m\frac{\hat{x}^m}{y}$	$\dot{x}_t^m$
$\beth_{625}$	$\daleth_1\hat{s}^m\frac{\hat{b}}{y}$	$\dot{b}_t^d$
$\beth_{626}$	$\hat{s}^m\hat{p}^c$	$\dot{y}_t$
$\beth_{627}$	$-\hat{s}^m\hat{p}^c\theta\frac{\nu}{1+\nu}$	$\dot{\nu}_t$
$\beth_{634}$	$-\frac{1}{\beta}(\frac{\hat{x}}{y}+\daleth_1\hat{s}^m\frac{\hat{b}}{y})$	$\dot{r}_{t-1}$
$\beth_{635}$	$-\hat{s}^m\frac{\daleth_1}{\beta}\frac{\hat{b}}{y}$	$\dot{r}_{t-1}^f$
$\beth_{637}$	$-\hat{s}^m\frac{\daleth_1}{\beta}\frac{\hat{b}}{y}$	$\dot{e}_{t-1}$
$\beth_{638}$	$-\frac{\hat{x}}{y}\frac{r+\daleth_2\hat{s}^m}{1+\pi}$	$\dot{x}_{t-1}^m$
$\beth_{639}$	$-\hat{s}^m\frac{\daleth_1}{\beta}\frac{\hat{b}}{y}$	$\dot{b}_{t-1}^d$

matrix element of	coefficient	variable
$\daleth_{13}$	$\frac{\delta k^c}{y^{dD}}$	$\dot{k}^{cd}_{t+1}$
$\daleth_{14}$	$\frac{\delta_2 k^m}{y^{dD}}$	$\dot{k}^{md}_{t+1}$
$\daleth_{124}$	$\frac{\delta k^c}{y^{fD}}$	$\dot{k}^{cf}_{t+1}$
$\daleth_{125}$	$\frac{\delta_2 k^m}{y^{fD}}$	$\dot{k}^{mf}_{t+1}$
$\daleth_{143}$	-1	$\dot{y}^{dc}_t$
$\daleth_{144}$	$\frac{c}{y^{dD}}$	$\dot{c}^d_t$
$\daleth_{145}$	$-\left(\frac{\delta k^c}{y^{dD}}(1-\delta) - \frac{k^c(1+r)}{(\chi-1)(1+\pi)y^{dD}}\right)$	$\dot{k}^{cd}_t$
$\daleth_{146}$	$-\frac{\delta_2 k^m}{y^{jD}}(1-\delta_2)$	$\dot{k}^{md}_t$
$\daleth_{148}$	$+\frac{(1-\gamma)tc}{1-(1-\gamma)tc}\frac{\hat{N}}{y^{dD}}$	$\dot{N}^d_t$
$\daleth_{152}$	$-\frac{(k^c-\hat{N})(1+r)}{(1+\pi)(\chi-1)y^{dD}}$	$\dot{\pi}^d_t$
$\daleth_{153}$	$\frac{g}{y^{dD}}$	$\dot{g}^d_t$
$\daleth_{164}$	-1	$\dot{y}^{fc}_t$
$\daleth_{165}$	$\frac{c}{y^{fD}}$	$\dot{c}^f_t$
$\daleth_{166}$	$-\left(\frac{\delta k^c}{y^{fD}}(1-\delta) - \frac{k^c(1+r)}{(\chi-1)(1+\pi)y^{fD}}\right)$	$\dot{k}^{cf}_t$
$\daleth_{167}$	$-\frac{\delta_2 k^m}{y^{fD}}(1-\delta_2)$	$\dot{k}^{mf}_t$
$\daleth_{169}$	$+\frac{(1-\gamma)tc}{1-(1-\gamma)tc}\frac{\hat{N}}{y^{fD}}$	$\dot{N}^f_t$
$\daleth_{173}$	$-\frac{(k^c-\hat{N})(1+r)}{(1+\pi)(\chi-1)y^{fD}}$	$\dot{\pi}^f_t$
$\daleth_{174}$	$\frac{g}{y^{fD}}$	$\dot{g}^f_t$
$\daleth_{190}$	$-\frac{\hat{N}(1+r)}{(1+\pi)(\chi-1)y^{dD}}$	$\dot{N}^d_{t-1}$
$\daleth_{191}$	$\frac{k^c(1+r)}{(\chi-1)(1+\pi)y^{dD}}$	$\dot{Q}^d_{t-1}$
$\daleth_{192}$	$-\frac{k^c(1+r)}{(\chi-1)(1+\pi)y^{dD}}$	$\dot{\lambda}^d_{t-1}$
$\daleth_{193}$	$+\frac{(k^c-\hat{N})(1+r)}{(1+\pi)(\chi-1)y^{dD}}$	$\dot{r}^d_{t-1}$
$\daleth_{1111}$	$-\frac{\hat{N}(1+r)}{(1+\pi)(\chi-1)y^{fD}}$	$\dot{N}^f_{t-1}$
$\daleth_{1112}$	$\frac{k^c(1+r)}{(\chi-1)(1+\pi)y^{fD}}$	$\dot{Q}^f_{t-1}$
$\daleth_{1113}$	$-\frac{k^c(1+r)}{(\chi-1)(1+\pi)y^{fD}}$	$\dot{\lambda}^f_{t-1}$
$\daleth_{1114}$	$+\frac{(k^c-\hat{N})(1+r)}{(1+\pi)(\chi-1)y^{fD}}$	$\dot{r}^f_{t-1}$
$\daleth_{250}$	-1	$\dot{\lambda}^d_t$
$\daleth_{254}$	$\frac{\nu}{1+\nu}$	$\dot{\nu}^d_t$
$\daleth_{255}$	1	$\dot{e}_t$
$\daleth_{271}$	1	$\dot{\lambda}^f_t$
$\daleth_{275}$	$-\frac{\nu}{1+\nu}$	$\dot{\nu}^f_t$
$\daleth_{33}$	$-k$	$\dot{k}^{cd}_{t+1}$
$\daleth_{34}$	$-k^m$	$\dot{k}^{md}_{t+1}$
$\daleth_{347}$	$\hat{x}_m$	$\dot{x}^d_{(t)m}$
$\daleth_{348}$	$+\hat{N}$	$\dot{N}^d_t$
$\daleth_{349}$	$-(k^c+k^m)$	$\dot{Q}^d_t$

$\gimel_{355}$	$-\hat{b}$	$\dot{e}_t$
$\gimel_{361}$	$\hat{b}$	$\dot{b}_t^{dd}$
$\gimel_{362}$	$\hat{b}$	$\dot{b}_t^{fd}$
$\gimel_{445}$	$-k^c h_1^c$	$\dot{k}_t^d$
$\gimel_{446}$	$-k^m h_1^m$	$\dot{k}_t^{md}$
$\gimel_{456}$	$-\frac{1-l}{\eta}$	$\dot{\mu}_t^d$
$\gimel_{457}$	$\left(\frac{h_1^c}{\alpha_1}k + \frac{h_1^m}{\alpha_2}k^m\right) + \frac{1-l}{\eta}$	$\dot{W}_t^d$
$\gimel_{458}$	$-\frac{h_1^c}{\alpha_1}k^c$	$\dot{s}_t^d$
$\gimel_{459}$	$-\frac{h_1^m}{\alpha_2}\frac{k^m}{y}$	$\dot{s}_t^{md}$
$\gimel_{460}$	$\frac{1-l}{\eta}$	$\dot{z}_t^d$
$\gimel_{543}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{\hat{p}^m int\, y^{dD}}{a_2(h_1^m)^{1-\alpha_2}k^m}$	$\dot{y}_t^{dc}$
$\gimel_{544}$	$\frac{\alpha_2}{1-\alpha_2}\frac{\hat{p}^m int\, y^{dD}}{a_2(h_1^m)^{1-\alpha_2}k^m}\frac{c}{y^{dD}}$	$\dot{c}_t^d$
$\gimel_{545}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{\hat{p}^m int\, y^{dD}}{a_2(h_1^m)^{1-\alpha_2}k^m}\left(\frac{\delta k^c}{y^{dD}}(1-\delta) - \frac{k^c(1+r)}{(\chi-1)(1+\pi)y^{dD}}\right)$	$\dot{k}_t^{cd}$
$\gimel_{546}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{\hat{p}^m int\, y^{dD}}{a_2(h_1^m)^{1-\alpha_2}k^m}\frac{\delta_2 k^m}{y^{jD}}(1-\delta_2) - \frac{\alpha_2}{1-\alpha_2}$	$\dot{k}_t^{md}$
$\gimel_{547}$	$\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_2\hat{x}_m$	$\dot{x}_{mt}^d$
$\gimel_{548}$	$\frac{\alpha_2}{1-\alpha_2}\frac{\hat{p}^m int\, y^{dD}}{a_2(h_1^m)^{1-\alpha_2}k^m}\frac{(1-\gamma)tc}{1-(1-\gamma)tc}\frac{\hat{N}}{y^{dD}}$	$\dot{N}_t^d$
$\gimel_{550}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\left(\daleth_1(1-\frac{1+r}{1+\pi})\hat{b} + \daleth_2(\hat{x}_m - \frac{\hat{x}_m}{1+\pi})\right)$	$\dot{\lambda}_t^d$
$\gimel_{552}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{\hat{p}^m int\, y^{dD}}{a_2(h_1^m)^{1-\alpha_2}k^m}\frac{(k^c-\hat{N})(1+r)}{(1+\pi)(\chi-1)y^{dD}}$	$\dot{\pi}_t^d$
$\gimel_{553}$	$\frac{\alpha_2}{1-\alpha_2}\frac{\hat{p}^m int\, y^{dD}}{a_2(h_1^m)^{1-\alpha_2}k^m}\frac{g}{y^{dD}}$	$\dot{g}_t^d$
$\gimel_{554}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{\hat{p}^m int\, y^{dD}}{a_2(h_1^m)^{1-\alpha_2}k^m}\frac{\nu}{1+\nu}$	$\dot{\nu}_t^d$
$\gimel_{555}$	$\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_1\hat{b}$	$\dot{e}_t$
$\gimel_{557}$	1	$\dot{W}_t^d$
$\gimel_{559}$	-1	$\dot{s}_t^{md}$
$\gimel_{561}$	$\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_1\hat{b}$	$\dot{b}_t^{dd}$
$\gimel_{562}$	$\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_1\hat{b}$	$\dot{b}_t^{df}$
$\gimel_{563}$	$-\frac{2}{1-\alpha_2}$	$\dot{a}_{2t}$
$\gimel_{589}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_2\frac{\hat{x}_m}{1+\pi}$	$\dot{x}_{m(t-1)}^d$
$\gimel_{593}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_1\frac{1+r}{1+\pi}\hat{b}$	$\dot{r}_{t-1}^d$
$\gimel_{597}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_1\frac{1+r}{1+\pi}\hat{b}$	$\dot{e}_{t-1}$
$\gimel_{5103}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_1\frac{1+r}{1+\pi}\hat{b}$	$\dot{b}^d d_{t-1}$
$\gimel_{5104}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_1\frac{1+r}{1+\pi}\hat{b}$	$\dot{b}^{dft-1}$
$\gimel_{5114}$	$-\frac{\alpha_2}{1-\alpha_2}\frac{1}{a_2(h_1^m)^{1-\alpha_2}k^m}\daleth_1\frac{1+r}{1+\pi}\hat{b}$	$\dot{r}_{t-1}^f$
$\gimel_{649}$	-1	$\dot{Q}_t^d$
$\gimel_{650}$	1	$\dot{\lambda}_t^d$

12.4 Derivation of the Welfare Loss Approximation

In order to derive the sum of all domestic households' utility from current and future consumption we begin with the approximation of the utility of an arbitrary household within the single period t.[91] Using a second order Taylor approximation the first of the two summands in (15) can be expressed as

$$\frac{1}{1-\sigma}c^{1-\sigma}+c^{-\sigma}dc_t-\frac{1}{2}c^{-\sigma-1}(dc_t)^2. \tag{100}$$

Using (21) and its predecessor, the equilibrium labor demand of an arbitrary consumption good producer is derived as

$$l_{tm_{int}}=\left(\frac{1}{k_t^{m_{int}}}\right)^{\frac{\alpha_1}{1-\alpha_1}}\left(\frac{y_{tm_{int}}^{kB}+\phi^{m_{int}}}{a_t}\right)^{\frac{1}{1-\alpha_1}} \tag{101}$$

and, similarly,

$$l_{tm}=\left(\frac{1}{k_t^{m}}\right)^{\frac{\alpha_2}{1-\alpha_2}}\left(\frac{y_{tm}^{kB}+\phi^{m}}{a_2}\right)^{\frac{1}{1-\alpha_2}} \tag{102}$$

for an arbitrary payment issuer. Remembering the assumption of perfectly competitive labor markets in each marketplace k, the expected labor supply of each household can be derived from (69) as

$$l_t^i=\frac{1}{\varsigma}\left(\int_0^{\varsigma}l_{tm_{int}}dm_{int}+\int_0^{\frac{1}{2}\iota}\sum_{m\in M(k)}l_{tm}dk\right). \tag{103}$$

Nested substitution of the last three equations into the second summand of (15), linearizing with respect to all the arguments of the labor demand and using the following approximation

$$dz_t=z(\frac{z_t}{z}-1)\approx z\left(\ln(\frac{z_t}{z})-\frac{1}{2}\left(\ln(\frac{z_t}{z})\right)^2\right) \tag{104}$$

yields

$$\frac{1}{1-\eta}(1-l)^{1-\eta}+(1-l)^{-\eta}\left(\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T}-\frac{1}{2}\left(\eta\frac{\hat{\mathbf{z}}_t^{\mathbf{i}}(\nabla_l)^T\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T}}{(1-l)}+\hat{\mathbf{z}}_t^{\mathbf{i}}\mathbf{h}_{\mathbf{1m_{int}}}\hat{\mathbf{z}}_t^{\mathbf{i}T}\right)\right). \tag{105}$$

Herein notation is simplified by $\hat{z}_t\equiv z_t\dot{z}_t$ and denominating the vector of variables, on which l^i depends, as $\mathbf{z^i}$ and the Hessian matrix of l^i as $\mathbf{H}$. Adding the two components of utility, adjusting the notation in the first term to the convention presented, isolating the

[91] To simplify notation the index i for a distinct household is omitted. Nevertheless the reader should keep in mind that we are for the moment concerned with only one household.

two constant terms, which can not be influenced by policy, in a term denoted as $t.i.p$ and finally summing up across all domestic households one is left with

$$\int_0^{\varsigma} c^{1-\sigma}\dot{c}_t^i - \frac{1}{2}c^{1-\sigma}(\dot{c}_t^i)^2 + (1-l)^{-\eta}\left(\nabla_l \hat{\mathbf{z}}_t^{\mathbf{i}T} - \frac{1}{2}\left(\eta\frac{\hat{\mathbf{z}}_t^{\mathbf{i}}\nabla_l^T\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T}}{(1-l)} + \hat{\mathbf{z}}_t^{\mathbf{i}}\mathbf{H}_l\hat{\mathbf{z}}_t^{\mathbf{i}T}\right)\right)di + t.i.p. \tag{106}$$

Following Edge (2003) we can substitute $\lim_0^{\varsigma}\dot{c}_t^i di$ by a linearized and rearranged version of the aggregate of (66) across all goods in the economy. Using the assumption of the constancy of all foreign variables, which characterizes the noncooperative policy case in Clarida et al. (2002), the utility approximation changes to

$$c^{-\sigma}\left(\int_0^{\varsigma}\hat{y}_{tm_{int}} - \hat{k}_{t+1}^{m_{int}} + (1-\delta)\hat{k}_t^{m_{int}} - \hat{y}_{tm_{int}}^{MC}dm_{int} + \int_0^{\frac{1}{2}\iota}\sum_{m\in M(k)}(1-\delta)\hat{k}_t^m - \hat{k}_{t+1}^m dk\right) \tag{107}$$
$$+\int_0^{\varsigma}\frac{1}{2}c^{1-\sigma}(\dot{c}_t^i)^2 + (1-l)^{-\eta}\left(\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T} - \frac{1}{2}\left(\eta\frac{\hat{\mathbf{z}}_t^{\mathbf{i}}(\nabla_l)^T\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T}}{(1-l)} + \hat{\mathbf{z}}_t^{\mathbf{i}}\mathbf{H}_l\hat{\mathbf{z}}_t^{\mathbf{i}T}\right)\right)di + t.i.p.$$

Collecting the first order terms with respect to output deviations of consumption good producers and payment issuers within this expression and ignoring all other terms yields

$$c^{-\sigma}\int_0^{\varsigma}\hat{y}_{tm_{int}}dm_{int} + (1-l^i)^{-\eta}\nabla_{l^i}\left(\int_0^{\varsigma}\hat{y}_{tm_{int}}dm_{int}, 0, 0, \sum_{m\in M([0,0.5\iota])}\hat{x}_{tm}, 0\right)^T. \tag{108}$$

Now we partially differentiate the supply of labor (103) with respect to production for the various consumption goods and aggregate across the obtained partial differentials for all consumption goods. The result is nothing but the differential of labor supply with respect to consumption, which is equal to the negative marginal rate of substitution between consumption and leisure displayed in (18). Returning to (108) we can divide the expression by $(1-l^i)^{-\eta}$. Taking a closer look at the result shows that the coefficient of the first term is again the LHS of (18), while the second term can be identified as the differential of labor supply with respect to consumption just discussed, if we realize that $\hat{x}_{tm}$ can be substituted by $\frac{\partial x_m}{\partial y_{m_{int}}}\int_0^{\varsigma}\hat{y}_{tm_{int}}dm_{int}$. Hence the three terms in (108) add up to zero and thus (107) simplifies to

$$c^{-\sigma}\left(\int_0^{\varsigma} -\hat{k}_{t+1}^{m_{int}} + (1-\delta)\hat{k}_t^{m_{int}} - \hat{y}_{tm_{int}}^{MC}dm_{int} + \int_0^{\frac{1}{2}\iota}\sum_{m\in M(k)}(1-\delta)\hat{k}_t^m - \hat{k}_{t+1}^m dk\right) \tag{109}$$
$$+\int_0^{\varsigma}\frac{1}{2}c^{1-\sigma}(\dot{c}_t^i)^2 + (1-l)^{-\eta}\left(\nabla_l\tilde{\mathbf{z}}_t^{\mathbf{i}T} - \frac{1}{2}\left(\eta\frac{\hat{\mathbf{z}}_t^{\mathbf{i}}(\nabla_l)^T\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T}}{(1-l)} + \hat{\mathbf{z}}_t^{\mathbf{i}}\mathbf{H}_l\hat{\mathbf{z}}_t^{\mathbf{i}T}\right)\right)di + t.i.p.,$$

where $\tilde{\mathbf{z}}_t$ denotes $\hat{\mathbf{z}}_t$ after substituting all elements referring to output deviations by 0. In order to supersede $\int_0^{\varsigma}(\dot{c}_t^i)^2 di$ we again follow Edge (2003) and linearize (18) around the steady state up to the first order, solve for the deviation of consumption, square the result and sum across all domestic households to obtain

$$\int_0^{\varsigma}(\dot{c}_t^i)^2 di = \frac{\int_0^{\varsigma}\left(\lambda_t^i - \int_0^{\frac{1}{2}\iota}\dot{W}_t^k dk + \frac{\eta}{1-l}\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T} + \left(1 - \frac{(1-l)^{-\eta}}{\hat{w}c^{-\sigma}}\right)(\dot{\mu}_t^i - \dot{z}_t^i)\right)^2 di}{\sigma^2\left(\left(c^{-\sigma} - \frac{(1-l)^{-\eta}}{\hat{w}}\right) - 1\right)^2}. \tag{110}$$

Substituting into (109) yields

$$
\begin{aligned}
& c^{-\sigma}\left(\int_0^{\varsigma} -\hat{k}_{t+1}^{m_{int}} + (1-\delta)\hat{k}_t^{m_{int}} - \hat{y}_{tm_{int}}^{MC} dm_{int} + \int_0^{\frac{1}{2}\iota} \sum_{m\in M(k)} (1-\delta)\hat{k}_t^m - \hat{k}_{t+1}^m dk\right) \qquad (111)\\
& \quad + \frac{1}{2}c^{1-\sigma}\frac{\int_0^{\varsigma}\left(\dot{\lambda}_t^i - \int_0^{\frac{1}{2}\iota}\dot{W}_t^k dk + \frac{\eta}{1-l}\nabla_l \hat{\mathbf{z}}_t^{\mathbf{i}aT} + \left(1 - \frac{(1-l)^{-\eta}}{\hat{w}c^{-\sigma}}\right)(\dot{\mu}_t^i - \dot{z}_t^i)\right)^2 di}{\sigma^2\left(\left(c^{-\sigma} - \frac{(1-l)^{-\eta}}{\hat{w}}\right) - 1\right)^2}\\
& \quad + (1-l)^{-\eta}\left(\int_0^{\varsigma} \nabla_l \hat{\mathbf{z}}_t^{\mathbf{i}aT} - \frac{1}{2}\left(\eta\frac{\hat{\mathbf{z}}_t^{\mathbf{i}a}(\nabla_l)^T\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}aT}}{(1-l)} + \hat{\mathbf{z}}_t^{\mathbf{i}}\mathbf{H}_l\hat{\mathbf{z}}_t^{\mathbf{i}T}\right) di\right) + t.i.p.,
\end{aligned}
$$

after recognizing that all terms exclusively containing deviation rates of the productivity shocks to consumption good producers, $a_{m_{int}}$, are independent of any policy measures and can therefore be integrated into the term $t.i.p.$. The associated elements in the first and second derivatives of the labor supply are substituted by one and this is indicated in the vector of deviation rates by the superindex a. Note that without being able to indicate this within the above formula purely quadratic forms in $\dot{a}_{tm_{int}}$ are also shifted to the policy-independent term.

We proceed by dividing the last equation by $(1-l^i)^{-\eta}$, gathering all remaining first order terms, using (21) to (22), (38), (60), (129) and (130) to express the steady state derivatives of the labor supply of the various firms in terms of model parameters and obtain

$$
\begin{aligned}
& \left(\left(\frac{1}{1+\hat{s}^m}\left(\frac{\chi}{\chi-1}\frac{1}{\beta} - (1-\delta)\right) + (1-\delta)\right)\int_0^{\varsigma}\hat{k}_t^{m_{int}} dm_{int} - \int_0^{\varsigma}\hat{k}_{t+1}^{m_{int}} + \hat{y}_{tm_{int}}^{MC} dm_{int}\right. \qquad (112)\\
& \quad \left. + \int_0^{\frac{1}{2}\iota}\left(\frac{1}{1+\hat{s}^m}\left(\frac{1}{\beta} - (1-\delta)\right) + (1-\delta)\right)\sum_{m\in M(k)}\hat{k}_t^m - \hat{k}_{t+1}^m dk\right)\frac{c^{-\sigma}}{(1-l)^{-\eta}} + \\
& \quad \frac{1}{2}\frac{c^{1-\sigma}}{(1-l)^{-\eta}}\frac{\int_0^{\varsigma}\left(\dot{\lambda}_t^i - \int_0^{\frac{1}{2}\iota}\dot{W}_t^k dk + \frac{\eta}{1-l}\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T} + \left(1 - \frac{(1-l)^{-\eta}}{\hat{w}c^{-\sigma}}\right)(\dot{\mu}_t^i - \dot{z}_t^i)\right)^2 di}{\sigma^2\left(\left(c^{-\sigma} - \frac{(1-l)^{-\eta}}{\hat{w}}\right) - 1\right)^2}\\
& \quad -\frac{1}{2}\int_0^{\varsigma}\left(\eta\frac{\hat{\mathbf{z}}_t^{\mathbf{i}a}(\nabla_l)^T\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}aT}}{(1-l)} + \hat{\mathbf{z}}_t^{\mathbf{i}}\mathbf{H}_l\hat{\mathbf{z}}_t^{\mathbf{i}T}\right) di + t.i.p..
\end{aligned}
$$

Due to the calibration of our model we are now able to assume that the coefficients for the current deviations of the capital stocks in both industries reduce to $1/\beta$, because the deviation of expected marginal monitoring costs $\frac{\chi}{\chi-1}$ from 1 and real marginal costs within the payment industry are all terms of order two. Moreover we follow the arguments of Bernanke et al. (1998) and ignore the influence of deviations in monitoring costs on welfare, since under sensible parameterizations their influence is nearly of the third order.[92]

[92] Cf. Bernanke et al. (1998), p.26.

Therefore we are left with

$$\frac{c^{-\sigma}}{(1-l)^{-\eta}}\left(\frac{1}{\beta}\int_0^{\varsigma}\hat{k}_t^{m_{int}}dm_{int}-\int_0^{\varsigma}\hat{k}_{t+1}^{m_{int}}dm_{int}+\int_0^{\frac{1}{2}\iota}\frac{1}{\beta}\sum_{m\in M(k)}\hat{k}_t^m-\sum_{m\in M(k)}\hat{k}_{t+1}^m dk\right) \quad (113)$$
$$+\frac{1}{2}\frac{c^{1-\sigma}}{(1-l)^{-\eta}}\frac{\int_0^{\varsigma}\left(\dot{\lambda}_t^i-\int_0^{\frac{1}{2}\iota}\dot{W}_t^k dk+\frac{\eta}{1-l}\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}T}+\left(1-\frac{(1-l)^{-\eta}}{\hat{w}c^{-\sigma}}\right)(\dot{\mu}_t^i-\dot{z}_t^i)\right)^2 di}{\sigma^2\left(\left(c^{-\sigma}-\frac{(1-l)^{-\eta}}{\hat{w}}\right)-1\right)^2}$$
$$-\frac{1}{2}\int_0^{\varsigma}\left(\eta\frac{\hat{\mathbf{z}}_t^{\mathbf{i}a}(\nabla_l)^T\nabla_l\hat{\mathbf{z}}_t^{\mathbf{i}aT}}{(1-l)}+\hat{\mathbf{z}}_t^{\mathbf{i}}\mathbf{H}_l\hat{\mathbf{z}}_t^{\mathbf{i}T}\right)di+t.i.p..$$

To proceed, we use a set of assumptions already presented in the main text. Firstly remember, that all consumption good producers are subject to the assumption of Nash behavior. Hence no covariations remain between the variables of two arbitrary consumption good producers and thus the relevant covariation submatrix reduces to a matrix solely containing variations and covariations between the firm-specific variables of a distinct firm. Secondly, we recognize that the stochastic location of households implies identical expected demands for transaction services in all marketplaces. Hence any non-zero covariance or variance between variables including one associated with an arbitrary payment issuer is precluded. Imposing these restrictions on (113) and again shifting all terms containing exogenous productivity shocks to the policy independent term yields

$$\frac{c^{-\sigma}}{(1-l)^{-\eta}}\left(\frac{1}{\beta}\int_0^{\varsigma}\hat{k}_t^{m_{int}}dm_{int}-\int_0^{\varsigma}\hat{k}_{t+1}^{m_{int}}dm_{int}+\int_0^{\frac{1}{2}\iota}\frac{1}{\beta}\sum_{m\in M(k)}\hat{k}_t^m-\sum_{m\in M(k)}\hat{k}_{t+1}^m dk\right) \quad (114)$$
$$+\left(\varsigma(\int_0^{\frac{1}{2}\iota}\dot{W}_t^k dk)^2+\int_0^{\varsigma}\left(\dot{\lambda}_t^i\right)^2 di-2\left(\int_0^{\varsigma}\dot{\lambda}_t^i di+\frac{h_2}{c^{-\sigma}}(\dot{\mu}_t^i-\dot{z}_t^i)\right)\varsigma\int_0^{\frac{1}{2}\iota}\dot{W}_t^k dk\right.$$
$$+2\frac{\eta}{1-l}\nabla_l\hat{\mathbf{z}}_t^a\left(\int_0^{\varsigma}\dot{\lambda}_t^i di-\varsigma\int_0^{\frac{1}{2}\iota}\dot{W}_t^k dk+\frac{h_2}{c^{-\sigma}}\int_0^{\varsigma}\dot{\mu}_t^i-\dot{z}_t^i di\right)+2\frac{h_2}{c^{-\sigma}}\int_0^{\varsigma}\dot{\lambda}_t^i\left(\dot{\mu}_t^i-\dot{z}_t^i\right)di$$
$$\left.+\left(1-\frac{(1-l)^{-\eta}}{\hat{w}c^{-\sigma}}\right)^2\left(\int_0^{\varsigma}(\dot{\mu}_t^i)^2+(\dot{z}_t^i)^2 di-2\int_0^{\varsigma}\dot{z}_t^i\dot{\mu}_t^i di\right)\right)\frac{1}{2}\frac{\frac{c^{1-\sigma}}{(1-l)^{-\eta}}}{\sigma^2\left(h_2-1\right)^2}$$
$$-\frac{1}{2}(1-l)^{-\eta}\varsigma\left(\hat{\mathbf{z}}_t^a\mathbf{H}_l\hat{\mathbf{z}}_t^{aT}+\eta\frac{\hat{\mathbf{z}}_t^a(\nabla_l)^T\nabla_l\hat{\mathbf{z}}_t^{aT}}{(1-l)}-\frac{\frac{c^{1-\sigma}\eta^2}{\sigma^2(1-l)^{2-2\eta}}}{(h_2-1)^2}\hat{\mathbf{z}}_t^a\nabla_l^T\nabla_l\hat{\mathbf{z}}_t^{aT}\right)+t.i.p.,$$

where $\hat{\mathbf{z}}_t^a$ denotes the vector $\hat{\mathbf{z}}_t^{\mathbf{i}a}$ after recognizing that the latter is identical for all households in the economy.[93] The linearized version of (95) can be used in order to eliminate the terms $\dot{\mu}_t^i-\dot{z}_t^i$ and $\int_0^{\varsigma}(\dot{\mu}_t^i)^2+(\dot{z}_t^i)^2-2\dot{z}_t^i\dot{\mu}_t^i di$. Discounting and adding across current and future periods while noticing that the result of this operation for the first summand

[93]The different notations are used to indicate the different levels of aggregation in the respective contexts.

is a further policy independent summand,[94] we can express the welfare approximation for the economy as

$$\sum_{q=0}^{\infty}\beta^{q}\left[\frac{1}{2}\frac{\frac{c^{1-\sigma}}{(1-l)^{-\eta}}}{\sigma^{2}\left(h_{2}-1\right)^{2}}\left(\left(2-\frac{h_{2}}{c^{-\sigma}}\right)^{2}\varsigma\left(\int_{0}^{\frac{1}{2}\iota}\dot{W}_{t}^{k}dk\right)^{2}\right.\right. \tag{115}$$

$$+2\frac{\eta}{1-l}\nabla_{l}\hat{\mathbf{z}}_{t+q}^{a}\left(\left(2-\frac{h_{2}}{c^{-\sigma}}\right)\int_{0}^{\varsigma}\dot{\lambda}_{t+q}^{i}di-\left(2-\frac{h_{2}}{c^{-\sigma}}\right)^{2}\varsigma\int_{0}^{\frac{1}{2}\iota}\dot{W}_{t}^{k}dk\right)$$

$$\left.-2\left(2-\frac{h_{2}}{c^{-\sigma}}\right)\int_{0}^{\varsigma}\dot{\lambda}_{t+q}^{i}\int_{0}^{\frac{1}{2}\iota}\dot{W}_{t}^{k}dkdi+\int_{0}^{\varsigma}\left(\dot{\lambda}_{t+q}^{i}\right)^{2}di\right)-\frac{1}{2}(1-l)^{-\eta}\varsigma\cdot$$

$$\left.\left(\hat{\mathbf{z}}_{t+q}^{a}\mathbf{H}_{l}\hat{\mathbf{z}}_{t+q}^{aT}+\left(\frac{\eta}{(1-l)}-\frac{\frac{c^{1-\sigma}\eta^{2}}{\sigma^{2}(1-l)^{2-2\eta}}\left(2-\frac{h_{2}}{c^{-\sigma}}\right)^{2}}{\left(h_{2}-1\right)^{2}}\right)\hat{\mathbf{z}}_{t+q}^{a}\nabla_{l}^{T}\nabla_{l}\hat{\mathbf{z}}^{aT}_{t+q}\right)\right]+t.i.p..$$

Using (66) and (78) to calculate the variance of the production in consumption goods across all different types of goods, we obtain

$$Var_{m_{int}}[\dot{y}_{tm_{int}}]=\theta^{2}Var_{m_{int}}[\dot{p}_{tm_{int}}+\left(\frac{\nu^{dd}}{1+\nu^{dd}}\int_{0}^{\varsigma}\dot{\nu}_{t}^{ik}di+\frac{\nu^{df}}{1+\nu^{df}}\int_{\varsigma}^{1}\dot{\nu}_{t}^{ik}di\right)]. \tag{116}$$

Due to the assumed Nash behavior, price changes of consumption good producers do not influence the aggregate demand for other consumption goods. Hence neither foreign income levels nor foreign households' shadow prices of domestic local income are affected by domestic price variation. This is due to the fact that the probability of synchronous uniform price movements of a substantial fraction of prices within a marketplace is approximately zero. Hence we have

$$\begin{aligned} Var_{m_{int}}[\dot{y}_{tm_{int}}] &= \theta^{2}Var_{m_{int}}[\dot{p}_{tm_{int}}+\frac{\nu^{dd}}{1+\nu^{dd}}\int_{0}^{\varsigma}\dot{\nu}_{t}^{ik}di] \\ &= \theta^{2}\left(\Delta_{t-1}+\frac{\varrho}{1-\varrho}\left(\varsigma\pi_{t}+(1-\varsigma)ln\left(\frac{e_{t}}{e_{t-1}}\right)\right)^{2}\right), \end{aligned} \tag{117}$$

in which the second equality is established by following Woodford (2003) and assuming, that the distortion of the model's steady state relative to an efficient steady state without transaction costs is sufficiently small,[95] whereby the temporary variable Δ_t is a measure of domestic price dispersion. Integrating this expression forward to yield similar versions

[94] For details of the elimination of the first-order terms cf. Edge (2003), p. 16.

[95] The exact procedure is not described, as it is rather tedious, but scarcely differs from the original (Cf. Woodford (2003), p.295f). The only difference is to define the temporary variables involved not as the expectation value, respectively variance, of the current log prices of consumption goods but as those of the log current absolute values of the variables in (117). The rest of the method proceeds as in the original.

for all subsequent periods $t+q$ and adding the discounted results across the entire time horizon yields

$$\sum_{q=0}^{\infty} \beta^q Var_{m_{int}}[\dot{y}_{(t+q)m_{int}}] = \theta^2 \sum_{q=0}^{\infty} \beta^q \frac{\varrho}{1-\varrho} \left(\varsigma \pi_{t+q} + (1-\varsigma) \ln \left(\frac{e_{t+q}}{e_{t+q-1}} \right) \right)^2 + t.i.p., \tag{118}$$

where the obviously policy independent price dispersion of period $t-1$ determines the last summand.
In the final steps we split the Hessian matrix and the gradient of labor into its remaining components, writing the first and the second derivative of labor with respect to argument x as l_x and l_{xx}, use the mathematical definitions of variances and covariances to eliminate all sums of squared deviations, and substitute the RHS of (118) into (115). Furthermore we use the linearized version of (76) to substitute the price level deviation for the mean value of individual price level deviations, and aggregate price level and wage deviations to derive the deviation of the real wage from its steady state value. Performing these operations we obtain

$$\begin{aligned}
\sum_{q=0}^{\infty} \beta^q \Bigg[\Bigg(& 2 \left(\dot{\lambda}_{t+q} - \left(2 - \frac{h_2}{c^{-\sigma}}\right) \int_0^{\frac{1}{2}\iota} \dot{W}_t^k dk \right) (l_{k^{m_{int}}} \hat{k}^c_{t+q} + l_y \hat{y}_{t+q} + l_x \hat{x}_{t+q} + l_{k^m} \hat{k}^m_{t+q}) \qquad (119) \\
& \frac{\eta(2 - \frac{h_2}{c^{-\sigma}})}{1-l} - 2\left(2 - \frac{h_2}{c^{-\sigma}}\right) \dot{\lambda}_{t+q} \int_0^{\frac{1}{2}\iota} \dot{W}_t^k dk + \left(2 - \frac{h_2}{c^{-\sigma}}\right)^2 \left(\int_0^{\frac{1}{2}\iota} \dot{W}_t^k dk \right)^2 \\
& + Var_i[\dot{\lambda}^i_{t+q}] + \dot{\lambda}^2_{t+q} \Bigg) \frac{1}{2} \frac{\varsigma \frac{c^{1-\sigma}}{(1-l)^{-\eta}}}{\sigma^2 (h_2 - 1)^2} - \frac{1}{2} \frac{\varsigma}{(1-l)^\eta} \left(l_{yy} + h_3 {l_y}^2 \right) \cdot \\
& \left(\frac{\varsigma \theta^2 \varrho y^2}{1-\varrho} \left(\varsigma^2 \pi^2_{t+q} + (1-\varsigma)^2 \left(\ln \left(\frac{e_{t+q}}{e_{t+q-1}} \right) \right)^2 + 2\varsigma(1-\varsigma)\pi_{t+q} \ln \left(\frac{e_{t+q}}{e_{t+q-1}} \right) \right) \right. \\
& + 2 \left(l_{yk} + h_3 l_y l_k \right) \varsigma Cov_{m_{int}}[\hat{y}^{m_{int}}_{t+q}, \hat{k}^{m_{int}}_{t+q}] + \left(l_{k^{m_{int}} k^{m_{int}}} + h_3 (l_{k^{m_{int}}})^2 \right) \varsigma Var_{m_{int}}[\hat{k}^{m_{int}}_{(t+q)}] \\
& \left. + \mathbf{t}_{t+q} \mathrm{diag}(\varsigma^{-1}, \varsigma^{-1}, M^{-1}, M^{-1}) (\tilde{\mathbf{H}} + h_3 \tilde{\nabla}_l^T \tilde{\nabla}_l) \mathrm{diag}(\varsigma^{-1}, \varsigma^{-1}, M^{-1}, M^{-1}) \mathbf{t}^T_{t+q} \right) \Bigg] + t.i.p.,
\end{aligned}$$

where $\mathbf{t}_{t+q}$ denotes the vector of target variables $(\hat{y}^c_{t+q}, \hat{k}^c_{t+q}, \hat{x}^m_{t+q}, \hat{k}^m_{t+q})$, the $[4,4]$-matrix $\tilde{\mathbf{H}}$ and the $[1,4]$-vector $\tilde{\nabla}_l$, are defined as the steady state Hessian matrix and the steady state gradient of labor after contracting all submatrices, subtraces or subvectors with identical elements into a single element and eliminating all elements with respects to stochastic productivity shocks. Furthermore h_3 is defined as

$$h_3 \equiv \frac{\eta}{(1-l)} - \frac{\frac{c^{1-\sigma}\eta^2}{\sigma^2(1-l)^{2-2\eta}} \left(2 - \frac{(1-l)^{-\eta}}{\hat{w}c^{-\sigma}}\right)^2}{\left(\left(c^{-\sigma} - \frac{(1-l)^{-\eta}}{\hat{w}} \right) - 1 \right)^2}. \tag{120}$$

Now recognize, that under the prevailing assumptions the variances of individual price indexes and shadow prices of local income across all households are equal. Moreover (**??**)

implies a linear dependence of the flat rates on the demand for privately issued payment money in the steady state. Using this within the linearized version of the same equation, isolating the current shadow price of local income and integrating forward shows that the variance of the current deviations of this variables across all households is zero. In addition, when renaming the average nominal wage across all domestic marketplaces as $\bar{W}_{t+q}$ we are left with the expression

$$\sum_{q=0}^{\infty}\beta^q\Bigg[\left(2\frac{\eta}{1-l}\left(\left(2-\frac{h_2}{c^{-\sigma}}\right)\dot{\lambda}_{t+q}-\left(2-\frac{h_2}{c^{-\sigma}}\right)^2\dot{\bar{W}}_{t+q}\right)\tilde{\nabla}_l\mathbf{t}_{t+q}^T+\dot{\lambda}_{t+q}^2\right. \tag{121}$$
$$\left.-2\left(2-\frac{h_2}{c^{-\sigma}}\right)\dot{\lambda}_{t+q}\dot{\bar{W}}_{t+q}+\left(2-\frac{h_2}{c^{-\sigma}}\right)^2\dot{\bar{W}}_{t+q}^2\right)\frac{\frac{1}{2}\frac{c^{1-\sigma}}{\sigma^2(1-l)^{-\eta}}}{(h_2-1)^2}-\frac{1}{2}\frac{1}{(1-l)^{\eta}}\left(\left(l_{yy}+h_3{l_y}^2\right)\right.$$
$$\frac{\varsigma\theta^2\varrho y^2}{1-\varrho}\left(\varsigma^2\pi_{t+q}^2+(1-\varsigma)^2\left(\ln\left(\frac{e_{t+q}}{e_{t+q-1}}\right)\right)^2+2\varsigma(1-\varsigma)\pi_{t+q}\ln\left(\frac{e_{t+q}}{e_{t+q-1}}\right)\right)$$
$$+2\left(l_{yk}+h_3l_yl_k\right)\varsigma Cov_{m_{int}}[\hat{y}_{t+q}^{m_{int}},\hat{k}_{t+q}^{m_{int}}]+\left(l_{k^{m_{int}}k^{m_{int}}}+h_3(l_{k^{m_{int}}})^2\right)\varsigma Var_{m_{int}}[\hat{k}_{(t+q)}^{m_{int}}]$$
$$\left.+\mathbf{t}_{t+q}\mathrm{diag}(\varsigma^{-1},\varsigma^{-1},M^{-1},M^{-1})(\tilde{\mathbf{H}}+h_3\tilde{\nabla}_l^T\tilde{\nabla}_l)\mathrm{diag}(\varsigma^{-1},\varsigma^{-1},M^{-1},M^{-1})\mathbf{t}_{t+q}^T\right)\Bigg]+t.i.p.$$

Next we use the principle of certainty equivalence proven for stochastic, forward-looking linear-quadratic control problems in Svensson et al. (2003) to eliminate the remaining terms referring to second order moments.[96] According to this argument the solution of any stochastic optimal linear control problem with a purely quadratic objective function and linear constraints is independent of the second moments of the stochastic variables. Hence second moments do not influence optimal policy, and can thus be assigned to the independent policy term. Implementing this, we conclude that aggregate welfare within the presented economic model can be approximated up to the second order by the following measure

$$\sum_{q=0}^{\infty}\beta^q\Bigg[\left(2\frac{\eta}{1-l}\left(\left(2-\frac{h_2}{c^{-\sigma}}\right)\dot{\lambda}_{t+q}-\left(2-\frac{h_2}{c^{-\sigma}}\right)^2\dot{\bar{W}}_{t+q}\right)\tilde{\nabla}_l\mathbf{t}_{t+q}^T+\dot{\lambda}_{t+q}^2\right. \tag{122}$$
$$\left.-2\left(2-\frac{h_2}{c^{-\sigma}}\right)\dot{\lambda}_{t+q}\dot{\bar{W}}_{t+q}+\left(2-\frac{h_2}{c^{-\sigma}}\right)^2\dot{\bar{W}}_{t+q}^2\right)\frac{\frac{1}{2}\frac{c^{1-\sigma}}{\sigma^2(1-l)^{-\eta}}}{(h_2-1)^2}-\frac{1}{2}\frac{1}{(1-l)^{\eta}}\left(\left(l_{yy}+h_3{l_y}^2\right)\right.$$
$$\frac{\varsigma\theta^2\varrho y^2}{1-\varrho}\left(\varsigma^2\pi_{t+q}^2+(1-\varsigma)^2\left(\ln\left(\frac{e_{t+q}}{e_{t+q-1}}\right)\right)^2+2\varsigma(1-\varsigma)\pi_{t+q}\ln\left(\frac{e_{t+q}}{e_{t+q-1}}\right)\right)$$
$$\left.+\mathbf{t}_{t+q}\mathrm{diag}(\varsigma^{-1},\varsigma^{-1},M^{-1},M^{-1})(\tilde{\mathbf{H}}+h_3\tilde{\nabla}_l^T\tilde{\nabla}_l)\mathrm{diag}(\varsigma^{-1},\varsigma^{-1},M^{-1},M^{-1})\mathbf{t}_{t+q}^T\right)\Bigg]+t.i.p..$$

Using the simple fact that $\ln(e_{t+q}/e_{t+q-1})) = \dot{e}_{t+q} - \dot{e}_{t+q-1} - \ln(e_{t+q-1}^{ss}/e_{t+q}^{ss})$, in which the last summand vanishes, because the central bank takes the foreign interest rate as given

[96] The proof can be found in Appendix 1 of the paper mentioned for the case of discretionary policy and in an accompanying paper for the case of an optimal policy with commitment. Since the technique of the proof is essentially the same we restrict ourselves to the more easily available source.

and therefore the steady state gross appreciation rate is equal to the steady state gross inflation rate, which is relatively close to one, yields

$$\sum_{q=0}^{\infty}\beta^q\left[\left(2\frac{\eta}{1-l}\left(\left(2-\frac{h_2}{c^{-\sigma}}\right)\dot{\lambda}_{t+q}-\left(2-\frac{h_2}{c^{-\sigma}}\right)^2\dot{\bar{W}}_{t+q}\right)\tilde{\nabla}_l\mathbf{t}^T_{t+q}+\dot{\lambda}^2_{t+q}\right.\right. \tag{123}$$
$$-2\left(2-\frac{h_2}{c^{-\sigma}}\right)\dot{\lambda}_{t+q}\dot{\bar{W}}_{t+q}+\left(2-\frac{h_2}{c^{-\sigma}}\right)^2\left.\dot{\bar{W}}^2_{t+q}\right)\frac{\frac{1}{2}\frac{c^{1-\sigma}}{\sigma^2(1-l)^{-\eta}}}{(h_2-1)^2}-\frac{1}{2}\frac{1}{(1-l)^{\eta}}\left((l_{yy}+h_3{l_y}^2)\right.$$
$$\frac{\varsigma\theta^2\varrho y^2}{1-\varrho}\left(\varsigma^2\pi^2_{t+q}+(1-\varsigma)^2(\dot{e}_{t+q}-\dot{e}_{t+q-1})^2+2\varsigma(1-\varsigma)\pi_{t+q}(\dot{e}_{t+q}-\dot{e}_{t+q-1})\right)$$
$$\left.\left.+\mathbf{t}_{t+q}\mathrm{diag}(\varsigma^{-1},\varsigma^{-1},M^{-1},M^{-1})(\tilde{\mathbf{H}}+h_3\tilde{\nabla}_l^T\tilde{\nabla}_l)\mathrm{diag}(\varsigma^{-1},\varsigma^{-1},M^{-1},M^{-1})\mathbf{t}^T_{t+q}\right)\right]+t.i.p..$$

The steady state deviation of the output level of the payment service industry can be substituted by the term within the brackets on the right side of (??). Doing this and gathering deviation rates into the new vector $\tilde{\tilde{\mathbf{t}}}^T=(\hat{y}_{t+q},\hat{\hat{x}}_{t+q},\hat{\hat{x}}_{t+q-1},\hat{k}^c_{t+q},\hat{k}^m_{t+q},\hat{\hat{b}}_{t+q},\hat{\hat{b}}_{t+q-1},\hat{r}_{t+q},$ $\hat{\pi}_{t+q})$ and their coefficients together with the original Hessian matrix, gradients and partial derivatives of (123) in the Hessian $\tilde{\tilde{\mathbf{H}}}$ or the gradient $\tilde{\tilde{\nabla}}_l^T$, respectively, we are finally left with

$$\sum_{q=0}^{\infty}\beta^q\left[\left(2\frac{\eta}{1-l}\left(\left(2-\frac{h_2}{c^{-\sigma}}\right)\dot{\lambda}_{t+q}-\left(2-\frac{h_2}{c^{-\sigma}}\right)^2\dot{\bar{W}}_{t+q}\right)\tilde{\tilde{\nabla}}_l\tilde{\tilde{\mathbf{t}}}^T_{t+q}+\dot{\lambda}^2_{t+q}\right.\right. \tag{124}$$
$$-2\left(2-\frac{h_2}{c^{-\sigma}}\right)\dot{\lambda}_{t+q}\dot{\bar{W}}_{t+q}+\left(2-\frac{h_2}{c^{-\sigma}}\right)^2\left.\dot{\bar{W}}^2_{t+q}\right)\frac{\frac{1}{2}\frac{c^{1-\sigma}}{\sigma^2(1-l)^{-\eta}}}{(h_2-1)^2}-\frac{1}{2}\frac{1}{(1-l)^{\eta}}\left((l_{yy}+h_3{l_y}^2)\right.$$
$$\frac{\varsigma\theta^2\varrho y^2}{1-\varrho}\left(\varsigma^2\pi^2_{t+q}+(1-\varsigma)^2(\dot{e}_{t+q}-\dot{e}_{t+q-1})^2+2\varsigma(1-\varsigma)\pi_{t+q}(\dot{e}_{t+q}-\dot{e}_{t+q-1})\right)$$
$$\left.\left.+\tilde{\tilde{\mathbf{t}}}_{t+q}\sqsupset(\tilde{\tilde{\mathbf{H}}}+h_3\tilde{\tilde{\nabla}}_l^T\tilde{\tilde{\nabla}}_l)\sqsupset\tilde{\tilde{\mathbf{t}}}^T_{t+q}\right)\right]+t.i.p.,$$

where

$$\sqsupset=\mathrm{diag}(\varsigma^{-1},M^{-1},M^{-1},\varsigma^{-1},M^{-1},M^{-1},M^{-1},M^{-1},M^{-1}). \tag{125}$$

Concerning the adequacy of this approximation as a measure for social welfare, we must first recognize that in steady state the derivatives of utility with respect to the variables used for the Taylor-approximation above are zero, because in the state of minimal costs the derivatives of the production functions balance the output sensitivity. Hence there are no non-zero terms that make a contribution to a second-order utility approximation, while they do not influence the first-order approximation of the model's structural equations. Therefore the functioning of the utility approximation as a welfare measure is not disturbed. Secondly, the distortions in the labor decision of the consumption good producers are likely to be relatively small, since they tend to offset each other. Hence the difference between an efficient steady state and the steady state in question seems to be small. Finally, the disturbances in our analysis are assumed to be local.[97]

12.5 Derivation of Steady State Relationships

In order to derive the steady state in the economy, we first recall its characteristics. Due to transaction costs there is positive inflation in steady state, because otherwise the central bank would not be able to cover its operation costs. Hence steady state is characterized by constant real and constantly growing nominal variables. From (37) we can directly calculate the shadow price for the equity constraint of consumption good producers as

$$\psi_1 = \frac{\frac{\bar{\chi}}{\chi-1}}{\gamma\frac{\chi}{\chi-1} - 1}. \tag{126}$$

Substituting this into (36), the linear system in the variables $\hat{p}^{m_{int}}$, $\hat{s}^{m_{int}}$ and $\frac{\hat{N}^{m_{int}}}{y}$, consisting of the real versions of equations (35), (36) and (37) after dividing by y, can be used to solve for these variables as functions of the exogenously determined variables, or rather parameters, tc, α_1, r, θ, δ, χ, $\bar{\chi}$ and the variables $\hat{w}$, γ and k/y, which are as yet still undetermined. Using the definition of the interest rate spread (26) we can solve for the steady state interest rate spread as

$$r^c - r = \frac{1+r}{\chi - 1} - \frac{(1+\pi)\bar{\chi}}{(\chi-1)(1-\frac{\hat{N}}{k})}. \tag{127}$$

Next we use the steady state version of (38) to calculate the steady state ratio of $\frac{k}{y}$ as a function of the parameters and the yet undetermined variables $\hat{w}$ and γ. Afterwards γ is calculated as a function of the parameters by using (31) and the parameterized ratio $\frac{\hat{\Pi}}{\hat{N}}$. Leaving the financial contract part of the model, (64) determines the steady state value for the real marginal costs of the payment service industry $\hat{s}^m$ as a function of $\hat{w}$ and α_2 and the exogenous parameters. We substitute the solutions so far obtained into the steady state versions of equations (66), (67), (68) and (69) and divide these by y. Using the production function and (68) to eliminate capital from (67) and joining the remaining three equations with the real steady state version of (5)

$$r\frac{\hat{x}}{y} = (1+\pi)\left(\hat{p}^{m_{int}} + \left|1 - \frac{1+r}{1+\pi}\right|\frac{\hat{b}}{y} + \left|1 - \frac{1}{1+\pi}\right|\frac{\hat{x}}{y}\right)\hat{s}^m, \tag{128}$$

where the steady state volume of transactions from (70) is substituted into the right side, we are left with a system of four linear equations in the variables c/y, $\hat{b}/y$, l/y and $\hat{x}/y$. The solutions for these variables are functions of exogenously determined parameters and the yet undetermined variables α_2 and $\hat{w}$.

[97] Cf. Woodford (2003), p.383 ff.

From (9) follows $(1+\nu) = 1/\hat{p}^{m_{int}}$ in steady state. Hence in steady state the shadow price of local income on those markets, in which consumption goods are traded, is constant across time. Using this result in (**??**) and (7), it is proven easily that also the shadow price of saving in central bank money μ is constant over time. Moreover the ratio of the marginal transaction costs implied by savings of central bank money and consumption in steady state can be solved to

$$\frac{\mu}{1+\nu} = 1 + \hat{s}^m. \tag{129}$$

By substituting (99) and (129) into (96), applying the constancy of the shadow prices related to transaction costs and using the results derived thus far, we obtain a time-dependent solution for the shadow price of the global income restriction z_{t+q}, which of course depends on the same yet unknown variables as the solution derived above.

$$z_{t+q} = -\frac{1}{r}(\beta(1+r)\rho_{t+q+1} - \rho_{t+q})(\hat{s}^m(\alpha_2,\hat{w}))^2 + \beta(1+r)\rho_{t+q+1} - \hat{s}^m(\alpha_2,\hat{w})\rho_{t+q} \tag{130}$$

Substituting this solution back into the steady state version of (99) yields a second order difference equation in ρ

$$\begin{aligned} & \frac{1+r}{r}\beta(r - (\hat{s}^m(\alpha_2,\hat{w}))^2)\rho_{t+q+2} & (131) \\ + \frac{1+r}{r}\left(\left(\frac{1}{1+r}+1\right)(\hat{s}^m(\alpha_2,\hat{w}))^2 - \frac{r}{1+r}\right)\rho_{t+q+1} - \frac{1}{\beta r}(\hat{s}^m(\alpha_2,\hat{w}))^2\rho_{t+q} & = 0. \end{aligned}$$

Next we plug (129) into (94) and equate the results for two consecutive time periods in order to obtain

$$\frac{1}{1+\hat{s}^m(\alpha_2,\hat{w})}(z_{t+q} - (1+\pi)z_{t+q+1}) = \hat{s}^m(\alpha_2,\hat{w})(\rho_{t+q+1}(1+\pi) - \rho_{t+q}). \tag{132}$$

Using (130) to eliminate z for all periods the resulting equation can be solved for the growth rate of ρ between the periods $t+q$ and $t+q+1$, which turns out to be either $1/(1+\pi)$ or $\frac{(s^m(\alpha_2,\hat{w}))^2}{(-r+(s^m(\alpha_2,\hat{w}))^2)\beta}$, whereby the second solution can be rejected for all sensible calibrations. Using this result we can divide (131) by ρ_{t+q}/β and solve the resulting quadratic equation for the steady state value of the discount factor β. The solutions are real, if the condition

$$\frac{r}{2(r-2)} < (\hat{s}^m(\alpha_2,\hat{w}))^2 < \frac{r^2-2r-4}{2r(r-2)} \tag{133}$$

holds.[98] The multiplicity of solutions for the quadratic equation generates the potential for multiple steady states. But if one of the two solutions is outside the sensible range of β, i.e. above one or below zero, the potential solution is unique. In any case it can

[98]Remember that the interest rate should be above zero or and below one in order to be of economic interest. Hence the ordering of the interval bounds follows.

be shown that the simpler expression $\beta = \frac{1+r}{1+\pi}$ is one of the solutions to the quadratic equation. Thus far we have obtained solutions for all steady state values of variables and the free parameters as functions of the exogenous parameters and the yet undetermined variables α_2 and $\hat{w}$.

Because marginal costs in the consumption good sector deviate from the mark-up, the steady state value of the real wage can not be pinned down directly from the optimality conditions of firms. Instead, we can use the implicit definition of ν used in (11)

$$\left(\frac{r+\pi}{1+\pi} - \frac{(1+\hat{s}^m(\alpha_2,\hat{w}))\pi}{1+\pi}\right)\frac{\hat{x}}{y} = \nu\left((1+\hat{s}^m(\alpha_2,\hat{w})\frac{\pi}{1+\pi})\frac{\hat{x}}{y} + \left(1-\frac{1+r}{1+\pi}\right)\frac{\hat{b}}{y}\right) \quad (134)$$

and the steady state version of (98) in order to solve for the parameter α_2 and the variable $\hat{w}$. Plugging in all the solutions obtained thus far we obtain two equations of transcendental form in these variables, which can not be solved algebraically.[99] Therefore Newton's method of approximating the roots of an equation system is employed to search for solutions, which are within the possible range for the variables α_2 and $\hat{w}$. For α_2 this is the interval $[0,1]$ and for $\hat{w}$ set of positive real numbers. Because the software used, Mathematica 5.2, can not identify solutions according to its internal precision goals in all cases, we used the following identification procedure. The values of (98) and (134) are calculated for a grid of (α_2,$\hat{w}$) starting with (0,0) and a step-width of 0.01 for 100 steps in α_2 and a step-width of 0.1 and 30 steps for $\hat{w}$. Among the results the one with the smallest absolute norm across both equations is chosen. The associated arguments are used as starting values within Newton's method. If the result of the approximation lies within the domain for α_2 and $\hat{w}$ and the absolute value of both equations are below 10^{-12} at this point, the result is accepted as sufficiently precise. If not, a set of random starting values is drawn from the domain of α_2 and $\hat{w}$, for each one a Newton approximation is performed, and among the results the one with the smallest absolute norm is chosen. If it meets the presented criteria for acceptance, the result is accepted as sufficiently precise. Of course, even for an unique β this solution does not necessarily secure an unique steady state. But exploring the solution with various starting values does not change the solution for the base line calibrations. Hence the existence of a unique steady state at least seems to be likely.

Combining (96) and (97) we can eliminate all summands related to the central-bank constraint, insert (130) and the steady state value for the growth rate of the shadow price of the non-profit constraint for the central bank, and yield

$$-1+\beta\frac{1+r}{1+\pi} = \left(-1+\beta\frac{1+r^f}{1+\pi}\frac{e_{t+q+1}}{e_{t+q}}\right)e_{t+q}. \quad (135)$$

This equation holds for a unitary steady state exchange rate in the first period, if the growth rate of the exchange rate matches the ratio between domestic and foreign gross interest rates. If we assume a symmetric steady state without interest differentials the

[99] A function is of transcendental form if it can not be constructed by using a finite number of elementary operations, which include addition, subtraction, multiplication and division as well as rational root extraction. Cf. Weisstein (2006), "Elementary Functions" and "Transcendental Functions".

exchange rate is constant over time and unitary.
Finally, to compute the absolute level of the variables in steady state, we start with the definition of the elasticity of labor, which is given as

$$\frac{1}{\eta}\frac{1-l}{l} = \epsilon_w. \tag{136}$$

Since ϵ_w is given exogenously we can directly calculate η. We substitute the result into the steady state version of (18), assume a shadow price of the central bank's constraint of zero in the first period and solve approximately for the steady state value of the consumption c by applying once again Newton's method. Except for the parameters of the assumed linear solution of consumption good producers, for which the steady state solution will be presented below, all remaining steady state values follow directly from this value.
For the scenario with a higher productivity in the payment service sector, the solution procedure has to be changed, since now neither technological parameters nor discount factors can be determined endogenously, because these belong to the model's parameters and should remain unchanged in both scenarios. This is necessary, because the model analyzes the differences in the reactions of endogenous variables to a variation in total factor productivity in two otherwise identical models. Hence in the calibration the capital coefficient within the payment industry α_2, and β are taken from the first scenario and plugged as inputs into the second one. Since the resulting overdetermination must be avoided, the endogenous variables of inflation and the nominal interest rate are allowed to be free even in steady state, and the steady state is solved for these variables along with all the others discussed above.
With respect to the parameters of the solution for consumption good producers, the analytical solution could not be solved to obtain numerical values due to restrictions in computing capacities.[100] Hence, for the case that $\mathbf{Z}_4 = \mathbf{Z}_3 = \mathbf{Z}_2 = \mathbf{Z}_1 = \mathbf{0}$, we employ Newton's method to detect potential roots of the equations (46) and (56), which can be simplified by using the assumption $\mathbf{\Xi}_1 = 0$ justified by the calibration of the model. From the second equation of (46) and using a zero vector within the last row of $\mathbf{\Upsilon}_3$, which is due to the model calibration, it follows that $\mathbf{\Phi}_{13} = \mathbf{\Phi}_{23} = \mathbf{\Phi}_{33} = 0$. These values are substituted into the equations mentioned and the three equations within the second part of (46) concerning these parameters are dropped from the system. Since there is no further ex ante information about sensible starting values for the approximation, we employ real random numbers from the interval $[-100, 100]$ for all remaining parameters. Running 500 iterations with maximum 1000 iteration steps and an internal working precision of at least 30 digits, which implies an accuracy of around 27 digits, in each case the average of the function value's Euclidean norm across all cases is often still above 10^1. Hence we calculate the mean norm of the 5 per cent of cases with the lowest norms. Using this measure as a critical value, all cases with larger norms are eliminated in order to construct from those remaining intervals for each parameter by choosing the maximum (minimum)

[100] Neither a 2.4 GHz CPU with 240 MByte memory and a virtual memory of 8 GByte nor a 2.4 GHz server CPU with up to 1 Gbyte memory were sufficient for this computation. Since the use of a cluster computer would have inferred substantial financial costs and delays, these problems have not been pursued further.

across all these cases as an upper (lower) limit. These intervals are used as domains for the random starting values in a second stage of the iteration, which takes the same form as the first one. While the mean value of the Euclidian vector norms of the function value across all cases need not be substantially lower than the one obtained from the first stage, we can identify a relatively large number of cases with norms smaller than the new critical value 10^{-14}. Because there may be still a dispersion for nearly all parameters, we eliminate all values for each parameter which are outside the interval defined by the mean plus/minus variance. As the final data set for the calculation of the root approximation, we use the union of the resulting twelve sets of cases. From this the means are calculated for each parameter and used as an approximation for the root in question, if none of the left sides of the equations (46) and (56) except for the three eliminated has a value above 10^{-14}.
Since we have to do this exercise not only one but two times, because we compare two steady states, we can theoretically use the result for the first model as a priori knowledge for the second approximation, which should yield a solution very close to the first in order to compare similar models differing only in variation in transaction costs. Therefore we use within the second approximation in the first stage starting values which are drawn from an interval centered around the results of the first approximation and characterized by an interval length of 0.2. To ensure that the extremely low size of the random component does not distort the solution, a case has been tested by calculating the same procedure for interval lengths of 2. The result did not change, and hence we conclude that the method should be fairly stable. However, during the computations it turned out that the usage of the a priori knowledge did not obtain any advantages neither in terms of precision nor of computation time. Hence we ignored the apriori information and applied simply the procedure described in the last paragraph for a second time.
As a final remark it should be emphasized that we do not claim that the detected roots are unique, and thus there is an additional potential for multiple steady states. Nevertheless the algorithm picks the specific roots, because these can be calculated relatively precisely and are supposed to be close to each other for both scenarios.

12.6 Derivation of the Analytical Solution for the Parameters within the Firm Solution.

Starting with the equations in (46) and assuming $\mathbf{\Xi}_1 = 0$, one is left with simplified versions of both equations. Transposing the second yields an equation which can be solved for the transpose of the matrix, that is assumed to be non-singular, by usage of the concept of generalized inverses to yield

$$(\mathbf{\Upsilon}_1(\mathbf{\Phi} + (1-\varrho)\mathbf{\Psi\Omega}) + \mathbf{\Upsilon}_2 - \tilde{\mathbf{\Xi}}_2(1-\varrho)\mathbf{\Omega})^{-1} = -(\mathbf{\Upsilon}_3^T)^{+}\mathbf{\Phi}^T + (\mathbf{I} - (\mathbf{\Upsilon}_3^T)^{+}\mathbf{\Upsilon}_3^T)\mathbf{Z}_1.\text{[101]} \quad (137)$$

Within this solution $\mathbf{Z}_1$ denotes any arbitrary real matrix with the same dimensions as $\mathbf{\Phi}$. Substituting this expression into the first of the equations in (46) and solving for $\mathbf{\Psi}$

[101] Cf. Caspary et al. (1994), p.93f.

we obtain (47). Using this solution to substitute for $\mathbf{\Psi}$ in (137) and applying once more the concept of generalized inverses, the equation can be solved for the parameter $\mathbf{\Omega}$ as it is given in (48). Since with (56) we have a second equation for $\mathbf{\Omega}$, inserting into (48) and moving some summands from the right side to the left yields

$$\begin{aligned}\frac{1-\varrho}{\tau}&(\Lambda_1(\Theta_{11},\Theta_{12},\Theta_{13},\Theta_{14})+(\Lambda_2+\Lambda_9+\varrho\beta\gamma\Lambda_3)(\Theta_{21},\Theta_{22},\Theta_{23},\Theta_{24})+\\ &\Lambda_4(\Theta_{41},\Theta_{42},\Theta_{43},\Theta_{44})+(0,0,0,\Lambda_5))-(\mathbf{I}-\mathbf{L}^+\mathbf{L})\mathbf{Z}_2+\mathbf{L}^+\mathbf{\Upsilon}_2+\mathbf{L}^+\mathbf{\Upsilon}_1\mathbf{\Phi}=\\ &\qquad\mathbf{L}^+\left(-\mathbf{\Phi}\mathbf{\Upsilon}_3^++\mathbf{Z}_1^T\left(\mathbf{I}-\mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right)\right)^{-1}.\end{aligned}\tag{138}$$

Next solve this equation for the second factor on the right side, and invert the result to be left with

$$\begin{aligned}-\mathbf{\Phi}\mathbf{\Upsilon}_3^++\mathbf{Z}_1^T\left(\mathbf{I}-\mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right)&=\left(\left(\mathbf{L}^+\right)^+\left(\frac{1-\varrho}{\tau}(\Lambda_1(\Theta_{11},\Theta_{12},\Theta_{13},\Theta_{14})\right.\right.\\ +(\Lambda_2+\Lambda_9+\varrho\beta\gamma\Lambda_3)(\Theta_{21},\Theta_{22},\Theta_{23},\Theta_{24})&+\Lambda_4(\Theta_{41},\Theta_{42},\Theta_{43},\Theta_{44})+(0,0,0,\Lambda_5))\\ &-(\mathbf{I}-\mathbf{L}^+\mathbf{L})\mathbf{Z}_2+\mathbf{L}^+\mathbf{\Upsilon}_2+\mathbf{L}^+\mathbf{\Upsilon}_1\mathbf{\Phi}+(\mathbf{I}-\mathbf{L}^+\mathbf{L})\mathbf{Z}_3^{-1}.\end{aligned}\tag{139}$$

Finally subtracting the last summand on the left side, transposing, solving for the parameter $\mathbf{\Phi}^T$, and once again transposing yields

$$\begin{aligned}\mathbf{\Phi}=-&\left(\left(\left(\mathbf{L}^+\right)^+((\Lambda_1(\Theta_{11},\Theta_{12},\Theta_{13},\Theta_{14})+(\Lambda_2+\Lambda_9+\varrho\beta\gamma\Lambda_3)(\Theta_{21},\Theta_{22},\Theta_{23},\Theta_{24})+\right.\right.\\ &\left.\Lambda_4(\Theta_{41},\Theta_{42},\Theta_{43},\Theta_{44})+(0,0,0,\Lambda_5))\frac{1-\varrho}{\tau}-(\mathbf{I}-\mathbf{L}^+\mathbf{L})\mathbf{Z}_2+\mathbf{L}^+\mathbf{\Upsilon}_2+\mathbf{L}^+\mathbf{\Upsilon}_1\mathbf{\Phi}\right)\\ &+(\mathbf{I}-\mathbf{L}^+\mathbf{L})\mathbf{Z}_3\Big)^{-1}-\mathbf{Z}_1^T\left(\mathbf{I}-\mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right)\left(\left(\left(\mathbf{\Upsilon}_3^+\right)^T\right)^+\right)^T+\mathbf{Z}_4^T\left(\mathbf{I}-\left(\left(\mathbf{\Upsilon}_3^+\right)^T\right)^+\left(\mathbf{\Upsilon}_3^+\right)\right).\end{aligned}\tag{140}$$

Now we can use the fact that every generalized inverse is also a pseudoinverse, since it minimizes the Euclidian norm of the distance of all approximate solution vectors of a linear matrix equation within an arbitrary environment of the exact solution, which are, as already mentioned, defined by the generalized inverse. But for the pseudoinverse the following relationships hold: $(A^+)^+=A$, $(A^+)^T=(A^T)^+$, $AA^+A=A$.[102] Hence the solution for $\mathbf{\Phi}$ simplifies to

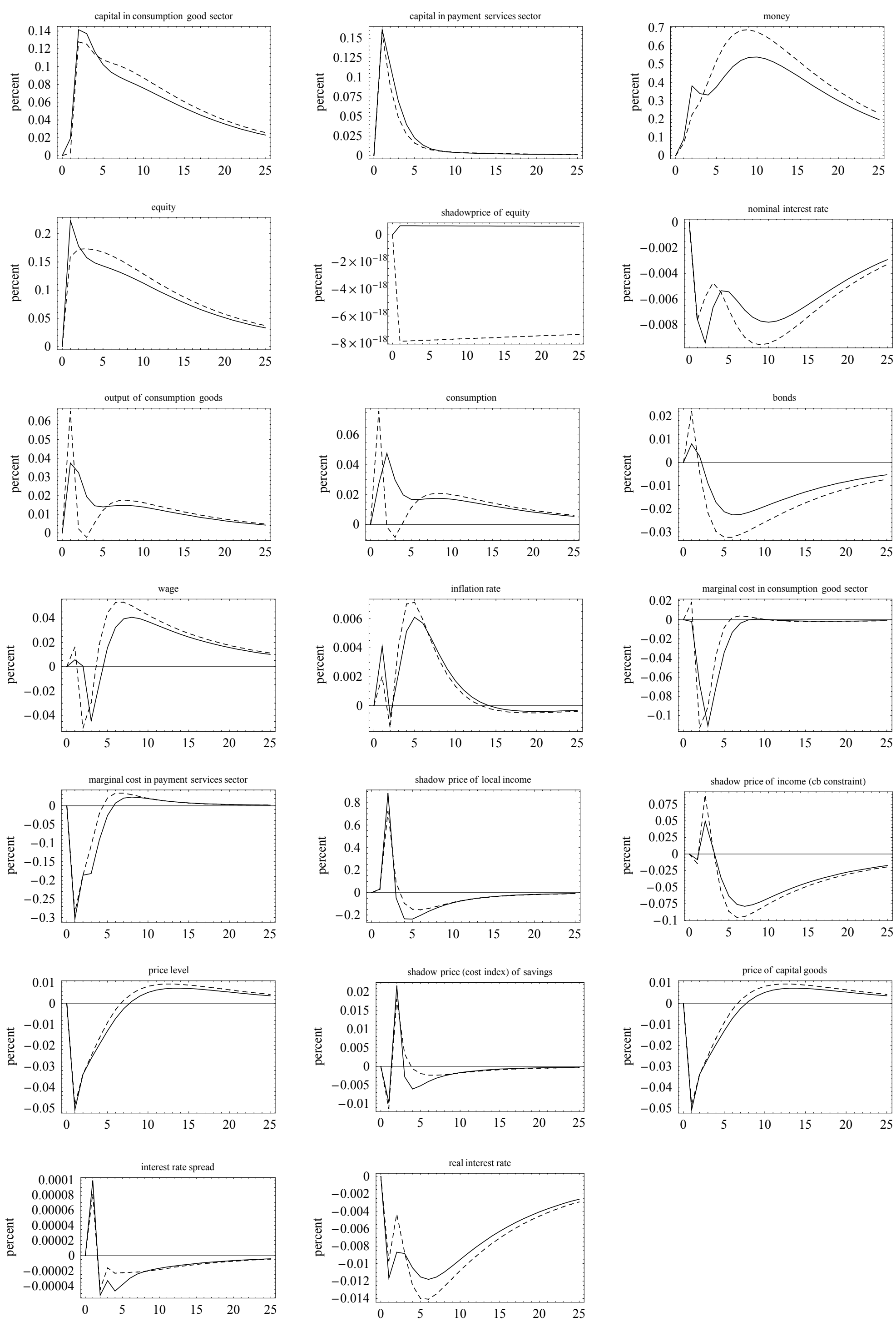
$$\begin{aligned}\mathbf{\Phi}=-&\left(\mathbf{L}\left(\frac{1-\varrho}{\tau}(\Lambda_1(\Theta_{11},\Theta_{12},\Theta_{13},\Theta_{14})+(\Lambda_2+\Lambda_9+\varrho\beta\gamma\Lambda_3)(\Theta_{21},\Theta_{22},\Theta_{23},\Theta_{24})+\right.\right.\\ &\left.\Lambda_4(\Theta_{41},\Theta_{42},\Theta_{43},\Theta_{44})+(0,0,0,\Lambda_5))+\mathbf{L}^+\mathbf{\Upsilon}_2+\mathbf{L}^+\mathbf{\Upsilon}_1\mathbf{\Phi}\right)+(\mathbf{I}-\mathbf{L}^+\mathbf{L})\mathbf{Z}_3\Big)^{-1}\mathbf{\Upsilon}_3\\ &+\mathbf{Z}_4^T\left(\mathbf{I}-\mathbf{\Upsilon}_3\mathbf{\Upsilon}_3^+\right).\end{aligned}\tag{141}$$

This is exactly the condition (58) given in chapter 4.2.

12.7 Simulation Results corresponding to Robustness Checks

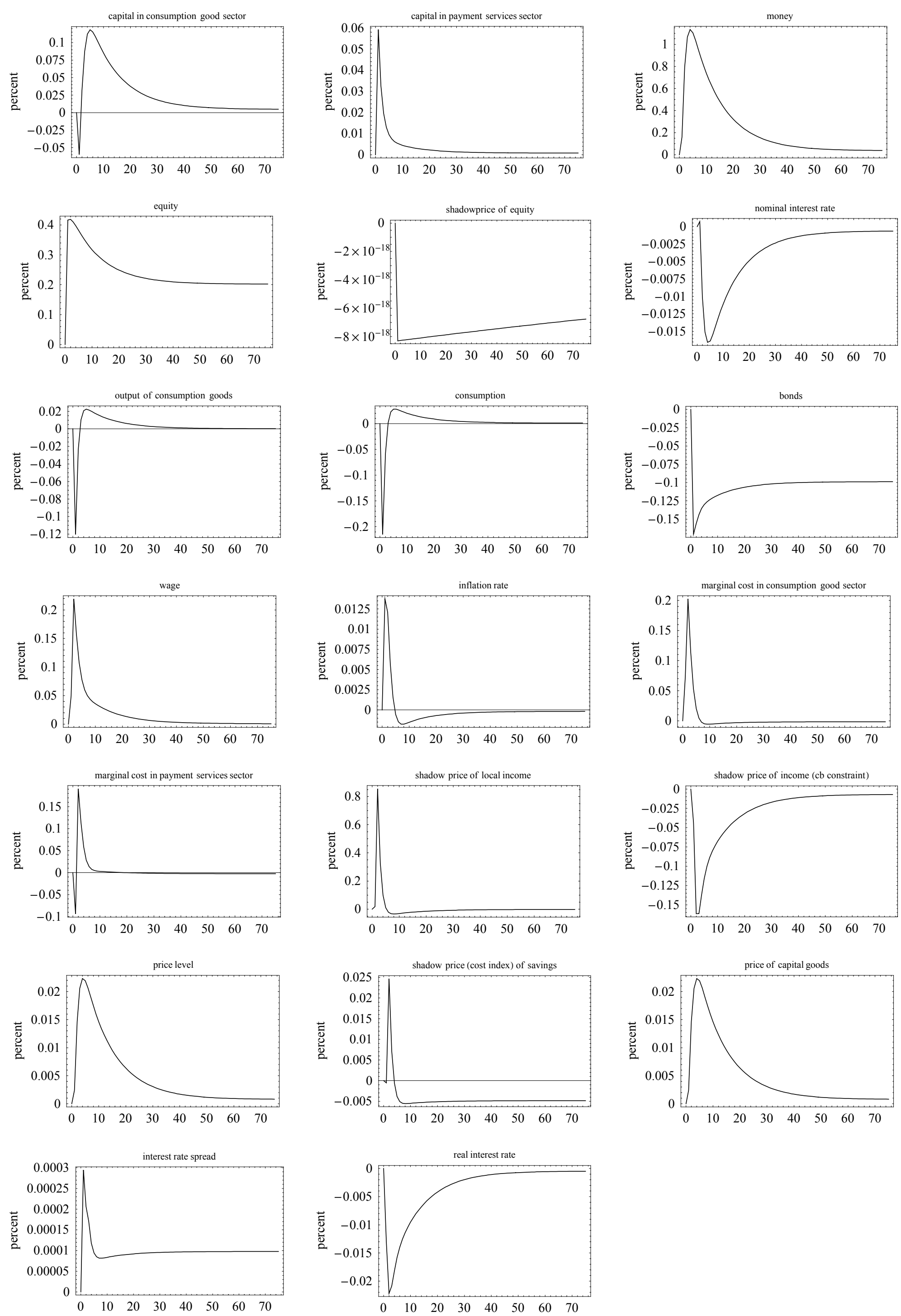
[102] Cf. Caspary (1994), p.123.

Figure 3: Simulation Responses to Monetary Shock



Calibration Values						$\frac{(1-l)}{\eta l}$	1	ω_1	0.3	χ	180	$\frac{\bar{X}}{k^c}$	0.05	$\frac{\hat{N}}{k^c}$	0.5
α_1	0.33	δ_1	0.02	ϵ_I	3	$\frac{\Pi^c}{N}$	0.005	ϱ	0.65	$\delta_1 - \delta_2$	0.008	a_2	15	θ	11
μ	1	σ	5	$\frac{1-l}{l}$	2	r	0.015	π	0.005	$\frac{t}{y^c}$	0.2	tc	0.6	P	1

Figure 4: Simulation Responses to Technology Shock



Calibration Values						$\frac{(1-l)}{\eta l}$	1	ω_1	0.3	χ	180	$\frac{\bar{X}}{k^c}$	0.05	$\frac{\hat{N}}{k^c}$	0.5
α_1	0.33	δ_1	0.02	ϵ_I	3	$\frac{\Pi^c}{N}$	0.005	ϱ	0.65	$\delta_1 - \delta_2$	0.008	a_2	15	θ	11
μ	1	σ	5	$\frac{1-l}{l}$	2	r	0.015	π	0.005	$\frac{t}{y^c}$	0.2	tc	0.6	P	1

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