Time-Differentiated Monopolies

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Abstract

We consider sequential competition among sellers, with different consumers desiring the good at different times. Each consumer could buy from a later seller. Each seller recognizes that future sellers are potential competitors, and therefore does not necessarily set his monopoly price, which is the price he would set if consumers could only buy from the first seller they encounter. We show, however, that when sellers do not know the history of sales, an equilibrium price can be the same for all sellers only if it coincides with their monopoly prices. Moreover, the monopoly prices may be equilibrium prices also if sellers do know the history. Indeed, they are necessarily so unless two sellers may arrive in very quick succession. However, if sellers are perfectly informed about the history of sales, additional equilibria may exist with prices below the monopoly level. In this sense, sellers may be harmed by their own knowledge.

Keywords Sequential competition, Imperfect markets, Intertemporal consumer choice

JEL Classifications C73, D91, L13

1 Introduction

Firms or sellers that are alone in a particular market at a particular time have greater market power than they would in the presence of close competitors. This market power, however, may be constrained when consumers believe that future competitors will offer the good at a lower price and so defer buying the good.[1] In this respect, competition between present and future sellers resembles that between producers of partially substitutable products. However, temporal separation between physically identical goods differs

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[1] Zeithammer (2006) gives evidence that consumers' decisions are affected by expectations of future sales of a good. He finds that consumers on eBay bidding in a current auction and expecting a future auction for a similar good reduce their bids.

from product differentiation along other dimensions in several important respects. For example, if the current seller is later replaced by another seller, then a consumer who chooses not to buy from the current seller cannot later reverse himself, even if the price set by a later seller turns out to be higher than he expected. In addition, that price may be affected by the demand that the later seller faces, which in turn depends on the number of consumers who deferred buying the good. Moreover, sequential competition often involves uncertainty—both the current seller and the current consumers may be uncertain about when the good will again be on the market. Thus sequential competition creates unique strategic interdependencies between sellers and consumers, and also involves issues of information and beliefs that do not arise for other forms of competition.

Constructing a tractable model of sequential competition requires making some assumptions. In our model, a consumer buys one or no unit of a particular good, and sellers can supply any quantity. This rules out, for example, auctions in which sellers only own a single unit. Both sides of the market are assumed to stay there for a short time: a mere instant for the sellers, who therefore do not overlap, and a fixed, finite period for the buyers. This assumption rules out competition of a long-lived monopolist with itself at a later time as an explanation for non-monopolistic pricing (Coase, 1972). It also presents an alternative to time discounting as the cause of consumers' impatience. A consumer's utility from buying the good either continuously declines and hits zero at some point, or it abruptly vanishes after some deadline has been passed. An example is a consumer who wants to have a certain good by a certain date. If the delivery time is uncertain and an early shipping date increases the probability of on-time delivery, the consumer's willingness to pay for the good may be lower close to the deadline than earlier. An additional assumption in our model is that the time that different consumers have already spent in the market is the only factor differentiating them; there are no inherent differences in preferences.

The appearance of sellers at discrete times can apply when supply depends on the vagaries of nature. Consider some examples. An historically important one relates to sailing ships, which were subject to the whim of the winds, making the length of a voyage over a given route uncertain, and so creating much uncertainty about when the next ship will be in port and be available for another sea voyage.[2] A person contemplating sailing from Charleston to Liverpool would be unsure when the next ship would depart. In waiting for the next ship he might find a lower price, but at the risk of arriving much too late in Liverpool. For another example, given the risks of rockets failing or of the space shuttle needing repair, a firm wishing to launch a satellite is unsure about the times at which it would be able to launch. Indeed, the supplier of the launch services is also unsure about

[2]An exception was service by packet ships, in which an owner kept ships in reserve so that he could offer scheduled service even if several ships on their way were delayed. Such service was introduced in 1817 between New York and Liverpool, with great success. But packet service was available only on heavily traveled routes. And even in the second half of the nineteenth century, the fast passage provided by clipper ships was subject to uncertain departure times, with owners advertising a forthcoming sailing to a specific destination after the ship arrived at the departure port. See, for example, Marvin (1902), Chapter IX.

when the next launch will be.

With sequential competition, seller's and buyers' possible strategies depend on the information he has. To simplify the analysis, this paper considers only the two extreme cases of sellers with either perfect information about the history of prior sales or no information at all. Information or lack thereof turns out to greatly affect the sellers' market power, so allowing only one of these possibilities would be overly restrictive. In both cases, we suppose buyers are perfectly informed about the past. Our model allows for greater generality about the sellers' arrival times: deterministic or random arrival times, with dependent or independent inter-arrival intervals.

1.1 Overview of results

If consumers' utility from buying the good decreases over time, and if sellers are ignorant about the past, then a necessary condition for all to set the same price in equilibrium is that the price equals the price each seller would set as a monopolist. By 'monopolist' we mean a seller who behaves as if consumers can only buy from the first seller they encounter. The basic logic here is similar to Diamond's (1971) demonstration that, if each consumer bears an arbitrarily small search cost in switching from one seller to another, each of the identical firms enjoys monopoly power. The switching cost, which in our model is the loss of utility due to the waiting time to the next seller, means that a consumer will buy from the current seller at a price that leaves him positive utility even if it is slightly higher than the competitors' price. Hence, each seller can raise his price slightly above what other sellers charge, and will do so as long as others charge less than the monopoly price.

This analysis hints at the fundamental difference between the benchmark case of uninformed sellers and the more realistic one in which sellers know something about the history of sales. A seller who learns that his predecessor raised the price, and consequently lost some of his customers, may find it profitable to lower the price in order to attract these consumers. Since consumers can anticipate this outcome, it may affect their responses to a price increase by the current seller, and hence the profitability of such a move.

These considerations underline the limitations of applying search cost arguments to sequential competition. However, they do not necessarily invalidate the results, and in particular, do not imply that information necessarily eliminates monopoly prices. Indeed, we show that if each seller knows the time since the last seller visited the market, and if this time is never very short, then an equilibrium always exists in which each seller sets his monopoly price. However, a possible result of having perfectly informed sellers is multiplicity of equilibria, with some equilibria having prices below the monopoly ones. In this sense, sellers may suffer from being better informed.

Perfect information about the history of sales may harm sellers because it adds credibility to a threat by consumers not to buy at a price higher than some specified maximum. The threat is implicit in the following strategy. If a seller sets a high price, then some of the consumers who would get positive utility from buying the good choose not to buy it.

Their number, which is determined by the strategy, induces the next seller to lower the price precisely to the point which equalizes the utility of these consumers and those who did buy from the first seller. Hence, no consumer suffers from following the strategy. The credibility of the consumers' threat thus depends on the next seller's ability to react optimally to his predecessor's price, and in particular to set a lower price if old consumers, with relatively low valuations of the good, are sufficiently numerous.

2 The model

Consumers arrive at the market in a steady flow. They are all identical, demand one unit of a particular good, and stay in the market only for a limited time, which we take as the unit of time. Thus, each consumer arrives, or is "born", at a certain time c and "dies" at $c+1$. The total mass, or "number", of consumers that arrive in a unit of time is taken as the unit of mass. The time axis extends from $-\infty$ to ∞. The origin, $t = 0$, has no special meaning.

Sellers stay in the market only for an instant. Each has an unlimited supply of the good, produced at zero cost, and seeks to maximize the profit (which equals the revenue) from selling it. An arriving seller sets and announces a price for a unit of the good, sells the demanded quantity, and then leaves the market. We model the sellers' arrival at the market as a simple point process $\boldsymbol{T}$ on the time axis (Daley and Vere-Jones, 1988). A realization of the process is a finite or infinite collection of distinct points, which represent the sellers' arrival times. This excludes simultaneous arrivals of sellers but includes deterministic arrival times, for example, arrival at regular intervals. Other special case are predetermined number of sellers with random arrival times, and arrival only on positive t's. In general, however, both the sellers' arriving times and their number may be uncertain, and a first and a last seller may or may not exist. To exclude certain artificial, unreasonable cases, we assume that the total number of sellers arriving in any finite time interval has a finite expectation (which defines the so-called *mean measure* of the process $\boldsymbol{T}$) and that the probability of very short inter-arrival times is small. The latter assumption means that for every $\delta < 1$ there is $\epsilon > 0$ such that, at every time t at which a seller arrives and for all possible histories of previous arrivals, the (conditional) probability that the waiting time to the next seller is greater than ϵ (or that no more sellers arrive) is greater than δ.

Whether a consumer buys the good from a particular seller at a given price depends primarily on the value he assigns to the good. The value depends continuously on the consumers' "age", or the time he has been in the market, and does not increase with age. We take the maximum valuation, which is that of a newly born consumer, as the unit of wealth. Thus the value of the good for a consumer is described by a *valuation function* $v : [0,1] \to [0,1]$, which is continuous and nonincreasing with $v(0) = 1$.[3] The (quasi-

[3] It is sometimes convenient to view the valuation function as defined on the whole nonnegative ray, with $v(x) = 0$ for $x > 1$. This means that it may be discontinuous at 1.

linear) *utility* of a consumer of age x from buying the good at price p is $v(x) - p$. For $0 \leq p \leq 1$, the highest $0 \leq x \leq 1$ for which the utility is nonnegative, which is the oldest age of a consumer who may be willing to buy the good at that price, is denoted by $a(p)$. This defines a nonincreasing function $a : [0,1] \to [0,1]$ with $a(0) = 1$. It also sets an upper bound $\pi_0(p)$ to the revenue of a seller selling the good at price p, which is given by

$$\pi_0(p) = p\, a(p). \tag{1}$$

We assume that the function $\pi_0 : [0,1] \to [0,1]$ is concave, has a unique maximum point $0 < p^0 \leq 1$, and is strictly concave in $[p^0, 1]$ (trivially so if $p^0 = 1$).[4] This assumption implies that π_0 is continuous, strictly increasing in $[0, p^0]$ and (if $p^0 < 1$) strictly decreasing in $[p^0, 1]$. Examples of functions that satisfy the assumption are the constant function $v(x) = 1$, the linear function $v(x) = 1 - x$ (and, more generally, any valuation function that is piece-wise linear), the normalized power functions $v(x) = (1-x)^\alpha$ ($\alpha > 0$), and the normalized exponential functions $v(x) = \frac{\beta^x - \beta}{1-\beta}$ ($0 < \beta \neq 1$). Note that the linear and constant functions are essentially limit cases of exponential functions, obtained for β tending to 1 and ∞, respectively.

2.1 Strategies

The (possible) randomness of the sellers' arrivals makes our model a variant of a random-player game (Milchtaich, 2004). In such a game, strategies are ascribed not to individual agents but to agent types. A *seller's type* is his arrival time t. A *strategy* for type t is a rule that assigns an asking price $0 \leq p \leq 1$ to each possible *history* at time t. Such a history H_t is a complete description of all relevant past events: the arrival times of the previous sellers, the prices they set, and the total mass and age distribution of the consumers who bought the good from them. A strategy is *feasible* for a seller if it depends on information about the history that the seller actually has. Hence, the better informed are sellers about the past, the larger are their sets of feasible strategies. One extreme case has all the sellers (perfectly) *informed* about the past, and the other has all the sellers (completely) *uninformed* about the past. With informed sellers, the feasible strategies are all the strategies of the sellers' respective types. With uninformed sellers, the feasible strategies are simply specifications of an asking price, $0 \leq p \leq 1$. Other possibilities are that some sellers are informed and some are uninformed, or that sellers are only partially informed, for example, they are informed about the previous sellers' asking prices but not about the consumers' reaction to them. For tractability, however, we consider only the two extreme cases described above.

[4]For this assumption to hold, a sufficient, but not necessary, condition is that π_0 is twice differentiable in the open interval $(0,1)$ and satisfies $\pi_0'' < 0$. If v is strictly decreasing and $v(1) = 0$, an equivalent condition is that $\frac{1}{v}$ is twice differentiable in $(0,1)$ and $\left(\frac{1}{v}\right)'' > 0$. A sufficient condition for the last inequality is $v'' \leq 0$, which means that v is concave. However, as the examples below show, concavity is not a necessary condition. Indeed, v may be strictly convex and still satisfy $\left(\frac{1}{v}\right)'' > 0$.

Each seller has *posterior beliefs* about the other sellers' arrival times. Even for an uninformed seller, these beliefs are not necessarily identical to those derived from the *common prior*, which is the distribution of the point process $\boldsymbol{T}$. The difference arises because an uninformed seller who arrives at time t knows something that was not necessarily known in advance, namely, that a seller arrived at time t. This information may give an indication about the other arrival times. If the seller is uninformed, no additional information is available to him, so that his posterior about the arrival times is the conditional distribution of $\boldsymbol{T}$, given that a seller arrived at time t. This conditional distribution is called the *Palm distribution* (Kallenberg, 1986). For an informed seller, the posterior is obtained by further conditioning the Palm distribution on the actual arrival times of the previous sellers, that is, taking into consideration both the seller's arrival time and the history. For both kinds of sellers, the posterior induces a distribution for each variable that can be expressed as a function of (some or all of) the arrival times, such as the total number of sellers, the time σ from the last seller's appearance, and the waiting time τ to the next seller. If the (posterior) distribution is degenerate, that is, if it assigns probability 1 to a particular value, then we will say that the seller *knows* the variable. For example, an informed seller by definition knows σ, whereas an uninformed seller may or may not know it.

A *consumer's type* is his time of birth c. A strategy for a consumer of type c is a rule that assigns either the decision 'buy' or 'wait' to each *buying opportunity* the consumer may encounter. A buying opportunity is specified by the arrival time t of the seller (with $c \leq t \leq c + 1$), the posted price $0 \leq p \leq 1$ and the history H_t. To simplify the analysis, we assume that all consumers are informed, that is, they know the history, so that all the strategies of their respective types are feasible. This assumption entails that whenever a seller arrives at the market, all the consumers have identical posterior beliefs about the future arrival times, which coincide with those of the arriving seller if he is informed.

The sellers' and consumers' information, beliefs and strategies together determine each agent's expectation regarding his gain from unilaterally switching to any feasible strategy different from that specified for his type. For an uninformed agent, "unilateral" means that all the other agents' actions accord with their strategies. For an informed agent, the meaning is similar, but only concerning the actions from time t onward; the history may or may not be consistent with the strategy profile. If the expected gain is positive, the deviation is *profitable* for the agent. A strategy profile is a (Bayesian perfect) *equilibrium* if no profitable deviations exist. The requirement applies also to histories that are not consistent with the strategy profile, which means that it excludes irrational off-equilibrium behavior (by informed agents).

3 Monopoly prices

Our main concern in this paper is the effect of competition from future sellers on the sellers' market power. Competition has no effect if the prices sellers set and the profits they

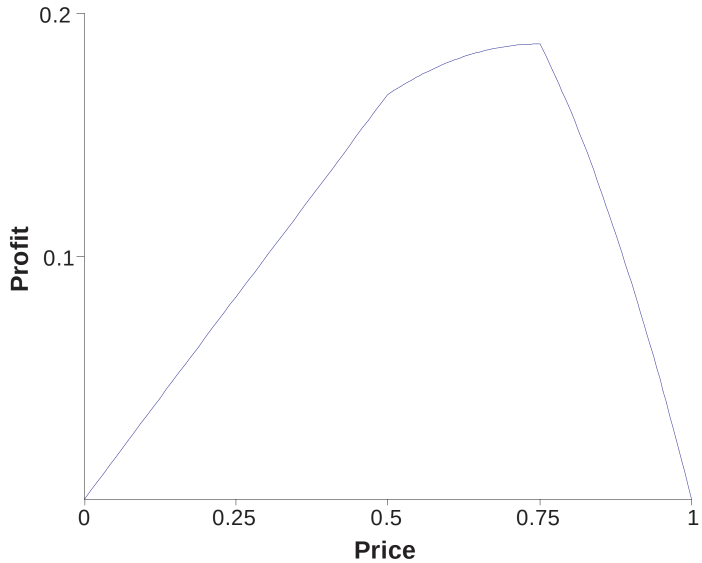

Figure 1: The monopoly profit function π_M for a seller who is uncertain about the time σ since the last seller appeared: with probability $\frac{2}{3}$, $\sigma = \frac{1}{4}$, and with probability $\frac{1}{3}$, $\sigma = \frac{1}{2}$. The consumers' valuation function is $v(x) = 1 - x$. The monopoly profit function peaks at the monopoly price $p^M = 0.75$.

earn are the same as they were if each consumer could only buy from the first seller he encounters, in other words, if sellers had complete monopoly power.

More specifically, a seller may face both young consumers, who were born after the previous seller appeared and so did not yet have a chance to buy the good, and old consumers, who could buy from the previous seller but did not. For $0 \leq p \leq 1$, denote by $\pi_M(p)$ the seller's expected profit from selling the good at that price to young consumers only, assuming that each such consumer who values the good at more than p buys it. This defines the seller's *monopoly profit function* $\pi_M : [0,1] \to [0,1]$ (see Figure 1). Its unique (see the Appendix) maximum point p^M is the *monopoly price* for the seller, and $\pi_M(p^M)$ is his *monopoly profit*.

A monopoly profit function as in Figure 1 applies only to sellers with specific beliefs about the length of time σ since the previous seller appeared. In other words, the distribution of σ is a parameter. On the other hand, the monopoly profit function is also relevant to sellers who have incomplete monopoly power. Specifically, for any seller with these beliefs and any price $0 \leq p \leq 1$, $\pi_M(p)$ is an upper bound on the expected revenue from selling the good at that price to young consumers. Additional revenue may come from selling to old consumers.

The counterpart on the consumers' side of monopolistic pricing is the *monopoly strategy*. It specifies that the consumer buys the good when he gets positive utility from doing so and does not buy when the utility is negative. This strategy would be optimal if the possibility of buying from a later seller were absent. Note that the action of the consumer whose valuation of the good equals the price is left unspecified. This omission is inconsequential, since any choice of the action would give the same utility to both the consumer and the other agents.

4 Uninformed sellers

By definition, uninformed sellers do not know the arrival times of previous sellers, the prices they set and the consumers who bought the good from them. The price an uninformed seller sets can therefore depend only on his own arrival time. If in addition the time of arrival does not provide any useful information (see Example 1 below), it is reasonable to expect that all prices will be identical. The intuition laid out in the Introduction suggests that this common price must be the sellers' monopoly price. The following theorem, whose proof is given in the Appendix, verifies this.

Theorem 1 *Suppose that the sellers are uninformed and that the consumers' valuation function v is strictly decreasing. If there is an equilibrium in which all the sellers set the same price, then this is necessarily the monopoly price for each of the sellers.*

The sellers' monopoly power is not reduced by competition with future sellers not because competition does not affect *demand*. In fact, the anticipated arrival of future sellers may well affect the consumers' willingness to buy the good at any given price. The reason the equilibrium price is nevertheless the monopoly one is more subtle. As the detailed analysis in the Appendix shows, for prices that are close to the competitors' price the effect of competition on demand is of a second order. It therefore does not affect the equilibrium price, which is determined by the first-order condition of zero marginal profit.

The assumption in Theorem 1 of a decreasing valuation function cannot be dropped. This is demonstrated by the simple example in which this function is constant, $v = 1$, and the sellers arrive at regular intervals: the time from one seller to the next is always $\frac{1}{2}$. Here, any price $0 \leq p^* \leq 1$ is an equilibrium price. The consumers' equilibrium strategy is to buy the good if and only if its price is at most p^*. Hence, a seller cannot increase sales by setting a lower price, and cannot sale any amount at all at a price higher than p^*.

Theorem 1 is illustrated by the following example. In the example, a seller's arrival time gives no indication about the time since the last seller. Hence, the sellers' monopoly profit functions are identical. The assertions concerning this example and the other ones in this paper are proved in the Appendix.

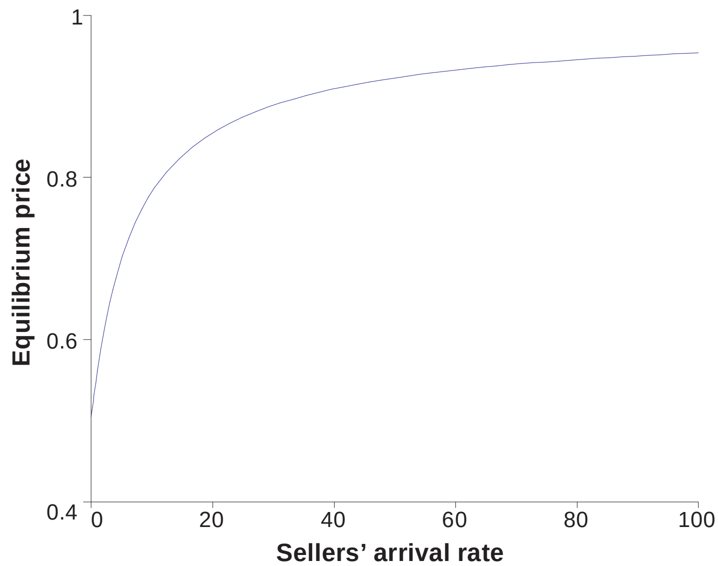

Figure 2: The unique equilibrium price in Example 1 as a function of the sellers' arrival rate η.

Example 1 *Uninformed sellers arrive according to a Poisson process. The time τ from one seller to the next is independent of past arrivals and has an exponential distribution with parameter η, which is hence equal to the sellers' arrival rate. The consumers' valuation function is linear, $v(x) = 1 - x$. If all the sellers set the same price, then this is an equilibrium price if and only if it is equal to the sellers' (common) monopoly price p^M. The consumers' equilibrium strategy is to buy at any price that gives them positive utility, if a higher expected utility cannot be obtained by waiting to buy from the next seller at price p^M.*

As Figure 2 shows, the equilibrium price in Example 1, which coincides with the sellers' monopoly price, is determined by the arrival rate of the sellers as an increasing function. This finding contrasts with the normal effect of increased supply, which is to lower the price.[5] The price rises because when sellers arrive soon after each other, a seller faces relatively young potential customers, who are willing to pay more.[6]

[5] Chen and Frank (2004) observe a related phenomenon in a queuing system. In their model, a monopolistic server charges a profit-maximizing service fee. Because an increase in the number of customers admitted increases the expected queuing time, this fee generally declines with demand.

[6] It is shown in the Appendix that, of the consumers for whom buying the good at the requested price would give positive utility, those who are actually willing to buy it are the younger ones. To understand why, note that consumers who wait enjoy an *option value* that reflects their option to buy nothing if the next seller arrives only after a long time. A young consumer is unlikely to use this option, and therefore cares mostly about the price. An older consumer is more affected by the option value, and therefore gains more from waiting.

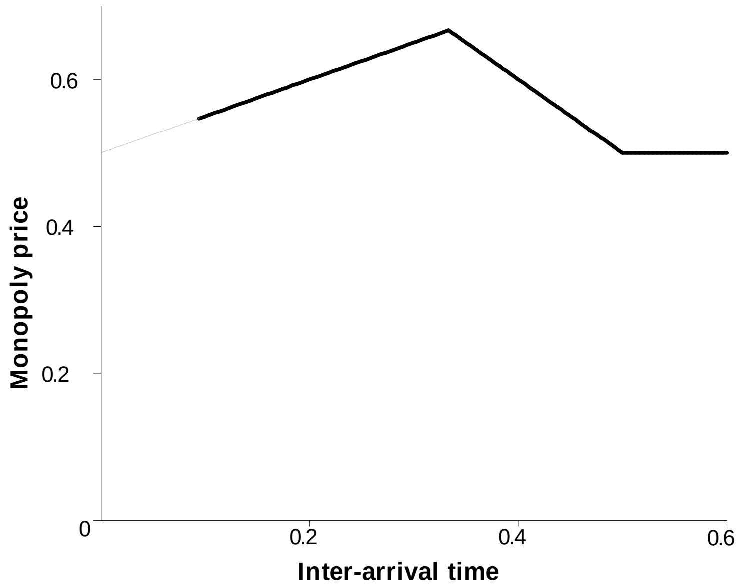

Figure 3: The monolopy price of the sellers in Example 2 as a function of the time s between their arrivals. A thick line designates an equilibrium price.

It should be pointed out, that even with the assumptions in Theorem 1, an identical monopoly price for all the sellers is not a sufficient condition for this to also be an equilibrium price. As the following example shows, it is possible that no equilibrium exists.

Example 2 *There are two uninformed sellers, one arriving s units of time after the other. The consumers know whether a seller is first or second but the sellers themselves do not know that. The consumers' valuation function is $v(x) = 1 - x$. Then, an equilibrium in which both sellers set the same price exists if and only if $s \geq \frac{1}{5+\sqrt{32}}$ (≈ 0.094).*

The sellers' monopoly price in Example 2 is shown in Figure 3. By Theorem 1, it coincides with the equibrium price whenever the latter exists.

5 Different monopoly prices

The identity between the monopoly and the equilibrium prices does not generally extend to the case in which different sellers have different monopoly prices. As Example 3 below shows, the identity may not hold even if the equilibrium prices are unique. However, this can happen only if sellers may appear in very short succession (where the exact meaning of "short" depends on the consumers' valuation function) or if they do not know the time since the appearance of the previous seller. Otherwise, the monopoly prices are equilibrium prices, although not necessarily the unique ones.

Theorem 2 *Suppose each seller knows the time σ since the appearance of the previous seller, and that time is never shorter than the smallest solution s_1 of the equation*[7]

$$s \cdot v(s) = \max_{0 \leq p \leq 1} \left[p \left(a(p) - a(p^0) \right) \right]. \tag{2}$$

Then an equilibrium exists in which each seller sells at his monopoly price and the consumers use the monopoly strategy.

The following example illustrates the theorem. Although the sellers in the example are uninformed, the second seller knows how long ago the first seller arrived. Depending on the length of this time interval, the equilibrium prices may be lower than the monopoly prices, may coincide with them, or may not exist.

Example 3 *There are at most two, uninformed sellers. The first seller arrives for sure, at time t_1, and the second arrives with probability $\frac{1}{2}$ at time $t_2 = t_1 + s$, with known $s > 0$. The consumers' valuation function is $v(x) = 1 - x$. If $s \leq \frac{1}{26}$ or $s \geq \frac{1}{8+\sqrt{48}}$, then there is for each seller a unique equilibrium price, which is respectively lower than or equal to the seller's monopoly price (see Figure 4). Otherwise, an equilibrium does not exist.*

The reason the second seller in Example 3 sets a price lower than his monopoly price (and than the price set by the first seller) if he comes shortly after the first one is that a short ($\leq \frac{1}{26}$) inter-arrival time means that few consumers are young, and willing to pay a high price for the good. The seller can therefore profit from setting a low price, which will attract some of the old consumers born before the first seller appeared. Anticipating this price reduction, some consumers who would gain little by buying from the first seller wait for the second seller. Therefore, the first seller cannot assume that all the consumers with positive utility will buy from him, and so he is forced to lower his price and receive a profit lower than his monopoly profit. The profit for the second seller exceeds his monopoly profit, which he can always get regardless of what the first seller does.

[7] Equation (2) always has has at least one solution in the interval $[0, a(p^0)]$, since the valuation function v is continuous and $0 \cdot v(0) \leq \max_p \left[p \left(a(p) - a(p^0) \right) \right] \leq \max_p \left[pa(p) \right] = p^0 a(p^0) \leq a(p^0) v(a(p^0))$. For example, if $v(x) = 1 - x$, then $a(p) = 1 - p$ and $p^0 = \arg\max_p \left[p\, a(p) \right] = \frac{1}{2}$, which means that (2) is the quadratic equation $s\,(1 - s) = \frac{1}{16}$. The smallest solution is then $s_1 = \frac{1}{8+\sqrt{48}}$ (≈ 0.067). Another, in a sense more general, example is the one-parameter family of exponentially-decreasing valuation functions $v(x) = \frac{\beta^x - \beta}{1-\beta}$, with $0 < \beta \neq 1$. It can be shown numerically that, for any such β, s_1 is less than about 0.082.

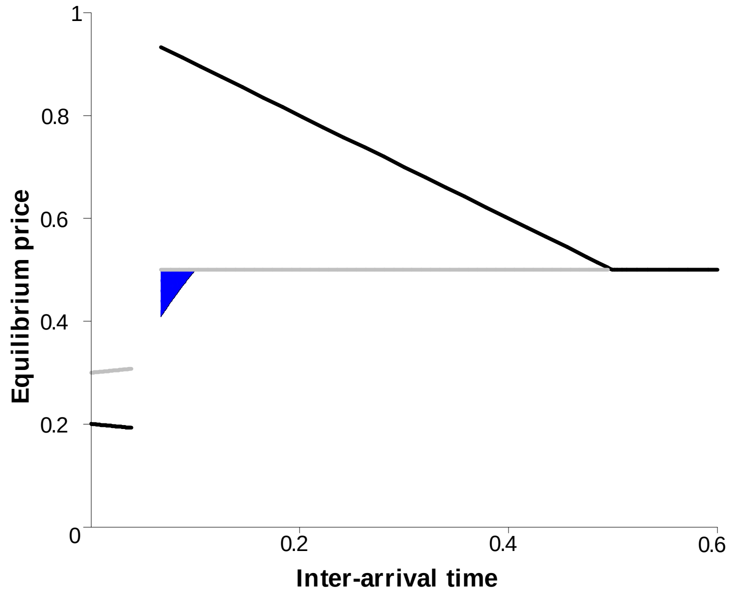

Figure 4: The equilbrium prices for the first and second seller (gray and black line, respectively) in Examples 3 and 4 as a function of the time s between their arrivals. For Example 4 only, the shaded area shows additional equilbrium prices for the first seller, which are lower than his monoploy price.

6 Informed sellers

The consumers' role in the determination of the equilibrium prices in Example 3 is, in a sense, a passive one. They strive to predict the price the second seller will set, not to affect it. Consumers, however, can sometimes force lower prices in situations where higher equilibrium prices also exist. This reduction of equilibrium prices requires that sellers know the prices that previous sellers set and the times of their visits, or at least know the size and the age composition of the set of consumers who did not yet buy the good (which entails also knowing their valuations).

The following example demonstrates these ideas. It differs from Example 3 in that the second seller is informed (and also in only considering inter-arrival times that fall within a narrow range). This difference results in additional equilibria in which the first seller sets lower prices than his monopoly price of $\frac{1}{2}$.

Example 4 *One seller arrives for sure at time t_1. Another, informed seller arrives with probability $\frac{1}{2}$ at time $t_2 = t_1 + s$, with $\frac{1}{8+\sqrt{48}} \leq s \leq \frac{1}{10}$. The consumers' valuation function is*

Figure 5: The consumers' equilibrium strategy in Example 4. A consumer buys the good from the first seller if and only if his age and the price fall within the shaded area. The depicted strategy corresponds to an equilibrium price of $p^* = \frac{23}{53}$ when the second seller comes $\frac{4}{53}$ units of time after the first one.

$v(x) = 1 - x$. *Then, for any given price* $\frac{1}{2}s + \frac{3}{2}\sqrt{s(1-s)} \le p^* \le \frac{1}{2}$, *an equilibrium exists with the first seller setting the price* p^* *and the second seller setting the price* $1 - s$ *(see Figure 4).*

The equilibrium strategy of the consumers in this example is to buy the good from the second seller at any price that leaves them with positive utility. From the first seller, however, they buy only if the price also does not exceed some threshold, which depends on their age. In particular, only consumers who receive high utility from buying the good at the equilibrium price p^* would buy it at any price that is even just slightly above p^* (see Figure 5). This strategy entails that the punishment of a seller who sets a higher price than p^* reflects the size of the deviation: it is not an all-or-nothing punishment. An all-or-nothing punishment would fail here, since the threat lacks credibility. If all the consumers refrained from buying at the higher price, the next seller would have no incentive to set a low price, so that it would be against a consumer's own interests to join the others in severely punishing a seller who sets a price that is not much higher than p^*.

Credibility is an important issue. Even with a single seller, equilibria exist in which a price lower than the monopoly price is supported by a consumers' threat not to buy at any higher price. However, these equilibria are not subgame perfect. If the seller sets a moderately high price, it may be optimal for individual consumers to buy the good after all. A credible threat is possible only with sequential competition, and only if the sellers are sufficiently informed about the previous sellers' behavior or about the consumers. Since all the equilibria identified in Example 4 satisfy the credibility requirement, and information is symmetric, none of them is eliminated by any obvious notion of equilibrium refinement. Multiplicity of equilibrium prices in that example thus appears to be a robust inherent property.

A Appendix: proofs

The appendix gives the proofs of the various results in this paper and of the assertions made in the examples. The proofs use the following lemma. Recall that the monopoly profit function $\pi_M : [0,1] \to [0,1]$ is defined by

$$\pi_M(p) = p\mathbb{E}\left[\min(\sigma, a(p))\right] = \mathbb{E}\left[\min(\sigma p, a(p))\right], \tag{3}$$

where the function π_0 is defined in (1), σ is the time from the previous seller's appearance, and the expectation is with respect to the seller's beliefs about that time (which are degenerate if the seller knows σ).

Lemma 1 *The monopoly profit function π_M is continuous and concave, has a unique maximum point $0 < p^M \leq 1$, and is strictly concave on $[p^M, 1]$. If the seller knows the time σ since the last seller's appearance, then $p^M = \max(v(\sigma), p^0)$.*

Proof. The assumed continuity and concavity of π_0 imply that the functions $\{\min(ps, \pi_0(p))\}_{s>0}$ are equicontinuous and concave in $[0,1]$, and therefore π_M is also continuous and concave. Let p^M be a maximum point of π_M. Since $\min(ps, \pi_0(p)) < \min(p^0 s, \pi_0(p^0))$ for all $s > 0$ and $p < p^0$, necessarily $p^0 \leq p^M \leq 1$. To prove that the maximum point is unique, it suffices to show that π_M has no other maximum point in the interval $[p^M, 1]$. For this, it suffices to show that π_M is strictly concave there. Strict concavity follows from the Claim below. Since the function a is nonincreasing, it follows from the Claim that, with positive probability, $\min(\sigma p, \pi_0(p)) = \pi_0(p)$ for all $p^M \leq p \leq 1$. Since π_0 is strictly concave on the interval $[p^M, 1]$, this implies that π_M is also strictly concave (rather than just concave) there.

Claim Suppose that $p^M < 1$. With positive probability, $\sigma \geq a(p^M)$.
Suppose otherwise, that $\sigma < a(p^M)$ almost surely, so that

$$\pi_M(p^M) = p^M \mathbb{E}\left[\sigma\right]. \tag{4}$$

The concavity of π_0 implies that this function, and hence also a, have one-sided derivatives at p^M. Therefore, there exists some $\delta > 0$ such that $\frac{a(p^M) - a(p^M + \epsilon)}{\epsilon} < \frac{\mathbb{E}[\sigma]}{\delta}$ for sufficiently small $\epsilon > 0$. The continuity of a implies that $\Pr(\sigma > a(p^M + \epsilon)) < \delta$ for sufficiently small $0 < \epsilon < 1 - p^M$. However, this inequality and the previous one lead to a contradiction:

$$\begin{aligned}
0 &> \delta\left(a(p^M) - a(p^M + \epsilon)\right) - \epsilon \mathbb{E}[\sigma] \\
&\geq \Pr(\sigma > a(p^M + \epsilon))\left(a(p^M) - a(p^M + \epsilon)\right) - \epsilon \mathbb{E}[\sigma] \\
&\geq \left(p^M + \epsilon\right)\mathbb{E}\left[\sigma - \min(\sigma, a(p^M + \epsilon))\right] - \epsilon \mathbb{E}[\sigma] \\
&= p^M \mathbb{E}[\sigma] - \left(p^M + \epsilon\right)\mathbb{E}\left[\min(\sigma, a(p^M + \epsilon))\right] \\
&= \pi_M(p^M) - \pi_M(p^M + \epsilon) \\
&\geq 0,
\end{aligned}$$

where the third inequality holds since $a(p^M) > \sigma$ almost surely and $p^M + \epsilon < 1$, the second equality holds by (4) and (3), and the last inequality holds since p^M is a maximum point of π_M. This contradiction proves the Claim.

Consider now the case where the seller knows σ. It follows from the definition of a that $a(p) \geq \sigma$ for every $p \in [0, v(\sigma)]$, and $a(p) < \sigma$ for every $p \in (v(\sigma), 1]$. Hence, by (3), in the first interval, $\pi_M(p) = p\sigma$, so that π_M is strictly increasing there, and in the second interval, $\pi_M(p) = \pi_0(p)$. If $v(\sigma) \geq p^0$, then π_0, and hence also π_M, are strictly decreasing in the closed interval $[v(\sigma), 1]$, so that they attain their maximum there at the point $v(\sigma)$, at which $\pi_M = \sigma v(\sigma)$. If $v(\sigma) < p^0$, then π_0 and π_M attain their maximum at the point p^0, at which $\pi_M = \pi_0(p^0)$. This proves that $p^M = \max(v(\sigma), p^0)$. ■

A.1 Theorem 1

Consider an equilibrium in which all the sellers set the same price p^E. By definition of equilibrium, for any $0 \leq p \leq 1$, the expected profit $\pi(p)$ for any single seller from setting the price p is not greater than the profit $\pi(p^E)$ from setting p^E. Therefore, to prove the theorem it suffices to show that the latter condition does not hold if p^E is not equal to the sellers' monopoly price p^M.

If $p^E > p^M$, then

$$\pi(p^M) \geq \pi_M(p^M) > \pi_M(p^E) = \pi(p^E). \tag{5}$$

The strict inequality holds because, by Lemma 1, p^M is the unique maximizer of the seller's monopoly profit function π_M. The equality holds because the assumption of decreasing valuations implies that at equilibrium consumers never wait to the next seller, and therefore any consumer who was born after the arrival of the previous seller will buy the good at price p^E if this gives him positive utility, and any consumer born before the arrival of the previous seller but did not buy from him will also not buy now. By the definition of the monopoly profit function, this means that the seller's expected profit from setting price p^E is $\pi_M(p^E)$. The weak inequality in (5) holds because the lower price p^M *may* also attract consumers who where born before the arrival of the previous seller but did not buy from him (at price p^E). It follows from (5) that, if $p^E > p^M$, the seller would gain from reducing the price to p^M.

Consider now the case $p^E < p^M$. Suppose that this inequality holds, and consider a price $p^E < p \leq 1$ and a consumer for whom buying the good is an optimal decision if the price is p^E and waiting is an optimal decision if the price is p. This means that the consumer's age x is such that (i) $v(x) - p^E \geq 0$ and (ii) $v(x) - p$ is less than or equal to the consumer's expected utility if he defers buying the good. Condition (ii) holds for two kinds of consumers: consumers with negative utility from buying at price p (i.e., $v(x) - p < 0$), and consumers for whom the utility is nonnegative (i.e., $v(x) - p \geq 0$) but is not greater than the expected utility from waiting for the next seller. In the rest of the proof, the main idea is to show that, for p sufficiently close to p^E, consumers of the first kind greatly outnumber those of the second kind, so that the anticipated arrival of future sellers has a vanishingly small effect on consumers' decisions.

For a consumer of age x who defers buying, the utility is $v(x+\tau)-p^E$ if the waiting time τ to the next seller makes this expression positive; otherwise the utility is zero. Condition (ii) above is therefore equivalent to $v(x) - p \leq \mathbb{E}[\max(v(x+\tau) - p^E, 0)]$, or

$$\mathbb{E}\left[\min\left(v(x) - p^E, v(x) - v(x+\tau)\right)\right] \leq p - p^E, \tag{6}$$

where the expectation is with respect to the consumer' beliefs about τ, that is, about when the next seller will arrive. (Since, by assumption, all the consumers are informed, their beliefs about τ are identical.) If the consumers know when the next seller will arrive, the distribution of (the random variable) τ is degenerate.

Fix $\delta > 0$. By the assumption in Section 2, there is some (small) $0 < \epsilon < 1$ that makes $\Pr(\tau > \epsilon) > \frac{1}{1+\delta}$. Since by assumption v is continuous and strictly decreasing in the unit interval, there is some price $p^E < p < \frac{p^M + \delta p^E}{1+\delta}$ that is sufficiently close to p^E to make

$$v(x) - v(x+\epsilon) \geq (1+\delta)\left(p - p^E\right) \tag{7}$$

for all $0 \leq x \leq 1 - \epsilon$. It follows from (7) that for any consumer whose age y satisfies $v(y) \geq p^E + (1+\delta)\left(p - p^E\right)$, if the waiting time τ satisfies $\tau > \epsilon$ (which means that either $\epsilon < \tau \leq 1 - y$ or $1 - y < \tau$, and in the latter case, trivially $v(y+\tau) = 0$), then

$$\min\left(v(y) - p^E, v(y) - v(y+\tau)\right) \geq (1+\delta)\left(p - p^E\right).$$

Therefore, for a consumer of such an age y,

$$\mathbb{E}\left[\min\left(v(y) - p^E, v(y) - v(y+\tau)\right)\right] \geq \Pr(\tau > \epsilon)\,(1+\delta)\left(p - p^E\right) > p - p^E.$$

Thus, (6) does not hold for $x = y$, and therefore a consumer of this age prefers buying the good at price p over waiting for the next seller. This conclusion shows that the age x of any consumer who *is* willing to wait satisfies $v(x) < p^E + (1+\delta)\left(p - p^E\right)$. If, in addition, the consumer would have nonnegative utility from buying at price p (that is, if he is of the second kind considered above), then $v(x) \geq p$, and hence

$$a(p^E + (1+\delta)\left(p - p^E\right)) < x \leq a(p). \tag{8}$$

If the seller charges p^E, he sells to all the consumers who were born after the previous seller appeared and have positive utility from buying at p^E. The expected profit is then $\pi_M(p^E)$. Raising the price to p changes the profit to some other value, $\pi(p)$. The consumers' response to the price increase may be thought of as having two stages. In the first stage, all the consumers with nonnegative utility from buying at price p still do so, giving the seller a profit of $\pi_M(p)$. In the second stage, the consumers who are better off waiting to the next seller drop out. The resulting reduction in the number of customers is constrained by (8), which gives the following upper bound on the second-stage loss of profit:

$$\pi_M(p) - \pi(p) \leq p\left(a(p) - a(p^E + (1+\delta)\left(p - p^E\right))\right). \tag{9}$$

Since, by Lemma 1, π_M is concave and has a maximum only at p^M, and $p^E < p < p^M$, the ratio $R = \frac{\pi_M(p^M) - \pi_M(p^E)}{p^M - p^E}$ is strictly positive and satisfies

$$\pi_M(p) - \pi_M(p^E) \geq R\ \left(p - p^E\right). \tag{10}$$

Inequalities (9) and (10) give

$$\frac{\pi(p) - \pi_M(p^E)}{p - p^E} \geq R + p\ \left(\frac{a(p^E + (1+\delta)\left(p - p^E\right)) - a(p^E)}{p - p^E} - \frac{a(p) - a(p^E)}{p - p^E}\right). \tag{11}$$

If $p^E = 0$, the right-hand side of (11) tends to R as p tends to p^E. If $p^E > 0$, it tends to $R + \delta p^E a'(p^E)$, where $a'(p^E)$ is the right derivative of a at p^E. (The existence of this one-sided derivative follows from its existence for the convex function π_0.) Therefore, choosing sufficiently small δ guarantees that in both cases the limit is positive, so that $\pi(p) > \pi_M(p^E)$ for p sufficiently close to (but greater than) p^E. Thus, raising the price by a small amount increases the seller's profit, contradicting the assumption that p^E is an equilibrium price.

A.2 Example 1

Suppose that all the sellers set the same price p^E. Any (perfect) equilibrium strategy for the consumers must specify that they buy the good at that price or lower if doing so gives them positive utility. Moreover, they will also buy at a higher price if the utility is higher than that expected from waiting and (optionally) buying from the next seller at price p^E, and not buy if the utility is lower than that. Therefore, p^E is an equilibrium price if and only if, with such a strategy for the consumers, no seller can gain from setting a price different than p^E. We show below that a necessary and sufficient condition for this is that $p = p^E$ is a solution of

$$p + \frac{1}{\eta} \ln (1 + \eta p) = 1. \tag{12}$$

Consider first a seller who deviates by setting a certain higher price, $p > p^E$. The utility of a consumer of age x from buying at price p is $1 - x - p$. The utility from waiting to the next seller (who will sell in price p^E) is $\max(1 - (x + \tau) - p^E, 0)$, where τ is the (random) waiting time to the next seller. Hence, the expected utility from waiting is the maximum between 0 and

$$\int_0^{1-x-p^E} \left(1 - (x + \tau) - p^E\right) \eta e^{-\eta\tau} d\tau = 1 - x - (p^E + \frac{1}{\eta}) + \frac{1}{\eta} e^{-\eta\left(1-x-p^E\right)}.$$

Therefore, a necessary condition for selling any units at all at price p is

$$p < p^E + \frac{1}{\eta}.$$

If this condition holds, then a consumer of age x is better off buying immediately (at price p) than waiting to the next seller or not buying at all if and only if $x < x_p$, where

$$x_p = 1 - p^E + \frac{1}{\eta} \ln \left(1 - \eta \left(p - p^E\right)\right). \tag{13}$$

The threshold value x_p is lower than $1 - p$. If positive, it is the age at which a consumer is indifferent between buying and waiting.

From the current seller's perspective, the time σ since the previous seller appeared is exponentially distributed with parameter η. Denote by $\pi(p)$ the expected profit for the seller if he sets a price $p^E < p < p^E + \frac{1}{\eta}$. A consumer who did not buy from the previous seller at price p^E will certainly not buy at the higher price p. Therefore, if $x_p > 0$ (which is a necessary condition for $\pi(p) > 0$), then

$$\begin{aligned}
\pi(p) &= p\, \mathbb{E}\left[\min(\sigma, x_p)\right] \\
&= p \left[\int_0^{x_p} s\eta e^{-\eta s} ds + \int_{x_p}^{\infty} x_p \eta e^{-\eta s} ds\right] \\
&= \frac{p}{\eta} \left(1 - e^{-\eta x_p}\right) \\
&= \frac{p}{\eta} \left(1 - \frac{e^{-\eta(1-p^E)}}{1 - \eta\left(p - p^E\right)}\right),
\end{aligned}$$

where the last equality follows from (13). Differentiation gives

$$\begin{aligned}\pi'(p) &= \frac{1}{\eta} - \frac{e^{-\eta(1-p^E)}(1+\eta p)}{\eta(1-\eta(p-p^E))^2} \\ \pi''(p) &= -\frac{e^{-\eta(1-p^E)}\left(3+\eta p+\eta p^E\right)}{(1-\eta(p-p^E))^3}.\end{aligned}$$

The second derivative is negative, and therefore $\pi(p) \leq \pi(p^E)$ for all $p^E < p < p^E + \frac{1}{\eta}$ if and only if $\pi'(p^E) \leq 0$, or

$$e^{\eta(1-p^E)} \leq 1 + \eta p^E. \tag{14}$$

Consider next a price $p \leq p_e$. Any consumer of age $x < 1 - p$ will buy at that price. However, such a consumer will still be in the market only if no previous seller arrived $0 < s < x$ units of time earlier, for every $s < x$ with $1 - (x - s) > p^E$. (The consumer would have bought the good from such a seller.) The probability of this is $e^{-\eta x}$ if $x < 1-p^E$, and $e^{-\eta(1-p^E)}$ if $x \geq 1 - p^E$. Therefore, the expected profit of a seller selling at price p is

$$\begin{aligned}\pi(p) &= p\left[\int_0^{1-p^E} e^{-\eta x}dx + \int_{1-p^E}^{1-p} e^{-\eta(1-p^E)}dx\right] \\ &= \frac{p}{\eta}\left(1 - \left(1+\eta\left(p-p^E\right)\right)e^{-\eta(1-p^E)}\right).\end{aligned}$$

Since this is a quadratic, concave function, the inequality $\pi(p) \leq \pi(p^E)$ holds for all $p \leq p^E$ if and only if $\pi'(p^E) \geq 0$, or

$$e^{\eta(1-p^E)} \geq 1 + \eta p^E.$$

The price p^E satisfies both the last inequality and the reverse one (14) if and only if it solves (12). This proves that the latter is indeed a necessary and sufficient condition for an equilibrium price.

Equation (12) has a unique solution, since the expression on its left-hand side is strictly increasing and is less or greater than 1 for $p = 0$ or $p = 1$, respectively. The solution is the sellers' monopoly price p^M. This is because the exponential distribution (with parameter η) of the time σ since the previous seller appeared implies that the monopoly profit function is

$$\begin{aligned}\pi_M(p) &= p\mathbb{E}\left[\min(\sigma, 1-p)\right] \\ &= p\left[\int_0^{1-p} s\eta e^{-\eta s}ds + \int_{1-p}^{\infty}(1-p)\,\eta e^{-\eta s}ds\right] \\ &= \frac{p}{\eta}\left(1 - e^{-\eta(1-p)}\right).\end{aligned}$$

The monopoly price p^M maximizes π_M, and hence satisfies the first-order condition

$$\frac{d\pi_M}{dp} = \frac{1}{\eta}\left(1 - (1+\eta p)\, e^{-\eta(1-p)}\right) = 0,$$

which is equivalent to (12).

Eq. (12) thus gives the equilibrium price as an implicit function of the sellers' arrival rate η (or of the expected time between successive arrivals, which equals $\frac{1}{\eta}$). As Figure 2 shows, this function is increasing. This can also be shown analytically. Implicitly differentiating (12) gives:

$$\left(1 + \frac{1}{1+\eta p}\right)\frac{dp}{d\eta} + \frac{1}{\eta^2}\left(1 - \frac{1}{1+\eta p} - \ln(1+\eta p)\right) = 0.$$

Since $1 - \frac{1}{x} - \ln x < 0$ for all $x > 1$, this proves that $\frac{dp}{d\eta} > 0$.

A.3 Example 2

The monopoly profit function in this example is given by

$$\pi_M(p) = \frac{1}{2}p\,(1-p) + \frac{1}{2}p\min(s, 1-p),$$

since a seller is equally likely to be first or second. The maximum point of this function, which is the monopoly price p^M (see Figure 3), is given by

$$p^M = \begin{cases} \frac{1+s}{2} & 0 < s \le \frac{1}{3} \\ 1-s & \frac{1}{3} \le s \le \frac{1}{2} \\ \frac{1}{2} & s \ge \frac{1}{2} \end{cases}. \tag{15}$$

By Theorem 1, in any equilibrium with a single equilibrium price, that price is p^M. To check whether such an equilibrium exists, suppose that both sellers set the price to p^M but one of them contemplates setting a different price p.

If $p > p^M - s$, then the seller's profit from setting this price will not be greater than $\pi_M(p)$, since consumers who had a chance to buy the good, at price p^M, s units of time earlier but chose not to will also not buy now at price p. It follows that the profit from setting price p will be strictly less than $\pi_M(p^M)$, the (monopoly) profit from selling at price p^M. If $0 < p \le p^M - s$, then the profit will be $\frac{1}{2}p\,(1-p) + \frac{1}{2}p(p^M - p)$. This is because, if the seller is the second one, some consumers who did not buy from the first seller at price p^M, namely, those older than $1 - p^M + s$ but younger than $1 - p$, *will* buy at price p. By (15), $0 < p \le p^M - s$ is possible only if $\frac{1}{3} \le s < \frac{1}{2}$ or $s < \frac{1}{3}$. If $\frac{1}{3} \le s < \frac{1}{2}$, then $p^M = 1 - s$, and therefore $\pi_M(p^M) = s\,(1-s)$, whereas $\max_p\left[\frac{1}{2}p\,(1-p) + \frac{1}{2}p(p^M - p)\right] = \frac{1}{16}\left(1 + p^M\right)^2 =$

$\frac{1}{16}(2-s)^2 \leq \frac{1}{16}\left(2-\frac{1}{3}\right)^2 < \frac{1}{3}\left(1-\frac{1}{3}\right) \leq s(1-s)$. This proves that p^M is indeed an equilibrium price if $s \geq \frac{1}{3}$. If $s < \frac{1}{3}$, then $p^M = \frac{1+s}{2}$, and therefore $\pi_M(p^M) = \frac{1}{8}(1+s)^2$ and $\max_p\left[\frac{1}{2}p(1-p) + \frac{1}{2}p(p^M - p)\right] = \frac{1}{16}\left(1+p^M\right)^2 = \frac{1}{64}(3+s)^2$. This maximum is greater than $\frac{1}{8}(1+s)^2$ if and only if $s < \frac{1}{5+\sqrt{32}}$. The maximum point is $p = \frac{3+s}{8}$, which is less than $p^M - s$ $(= \frac{1-s}{2})$ for such s. Therefore, an equilibrium does not exist for $s < \frac{1}{5+\sqrt{32}}$, but does exist for $s \geq \frac{1}{5+\sqrt{32}}$.

A.4 Theorem 2

We must show that if each seller charges his monopoly price, and each consumer buys the good if and only if its value for him is equal to or higher than the price, then no single seller or consumer can benefit from deviating from these strategies. We will consider three generic agents: a consumer born at time c and two sellers arriving at t_1 and t_2, with $c \leq t_1 < t_2$.

To prove that it is optimal for the consumer to buy the good at time t_1 if this gives him nonnegative utility, it suffices to show that buying at t_2 will give him nonpositive utility. That utility is the difference between the value of the good to the consumer at time t_2 and the monopoly price p^M for the seller at time t_2. Thus, it suffices to show that

$$p^M \geq v(t_2 - c). \tag{16}$$

By assumption, the seller at t_2 will know the time σ since the appearance of his immediate predecessor (who may be the seller at t_1 or a later one). Therefore, by Lemma 1, the seller's monopoly price p^M satisfies

$$p^M = \max(v(\sigma), p^0) \geq v(\sigma).$$

Since $\sigma \leq t_2 - c$ and v is nonincreasing, the inequality implies (16).

It remains to show that the monopoly price p^M maximizes the profit for the seller at t_2. According to the consumers' strategy, any consumer who was born after the appearance of the previous seller, and so had no earlier opportunity to buy the good, will buy at any price that gives him nonnegative utility. Therefore, by the definition of the monopoly profit function, the seller's profit from selling at price p^M is at least $\pi_M(p^M)$. Therefore, a price p can be more profitable than p^M only if the corresponding profit exceeds $\pi_M(p^M)$, and hence, by definition of the monopoly price, also exceeds $\pi_M(p)$. This is possible only if some consumers who could have bought the good from an earlier seller, but did not, will buy at time t_2 at price p.

Consider such a consumer, who was born at time c and had his first chance to buy the good at time t_1, with $c \leq t_1 < t_2$. Since the seller at t_1, like all the others, chose his monopoly price, but the consumer did not buy from him, Lemma 1 implies that the price was p^0 and the consumer was older than $a(p^0)$. Hence, at time t_2 the consumer is older than $a(p^0) + t_2 - t_1$. This shows that any potential customer of the seller at time t_2 is either younger than σ or older than $a(p^0) + \sigma$, where σ is the time since the appearance of the seller's immediate predecessor. The older consumers may buy the good only at a price p that satisfies $p \le v(a(p^0) + \sigma)$, and so gives the seller a profit that is no more than $p\left(a(p) - a(p^0)\right)$ (since the age gap between the older and younger consumers is at least $a(p^0)$). Therefore, any price p more profitable than p^M must give a profit that is bounded from above by the expression on the right-hand side of (2). To prove that such price p does not exist, it suffices to show that the same expression is a *lower* bound on the profit from setting the price p^M. For this, it suffices to show that $\pi_M(p^M) \ge \pi_M(v(s_1)) \ge s_1 v(s_1)$. The first inequality holds by definition of p^M. The second follows from the assumption $\sigma \ge s_1$, which implies that all the consumers younger than s_1 did not have a chance to buy from an earlier seller. This proves that a price more profitable than p^M for the seller at t_2 does not exist.

A.5 Example 3

Consider a generalized version of the example, in which the probability that the second seller arrives is $0 < \alpha < 1$. We prove below that there is a number $0 < s_0(\alpha) < \frac{1}{8+\sqrt{48}}$ (≈ 0.067) such that the following holds:

1. If $0 < s \le s_0(\alpha)$, the unique equilibrium prices are $p_1 = \frac{1-\alpha}{4-3\alpha}(2 - \alpha + 2\alpha s)$ for the first seller and $p_2 = \frac{1-\alpha(1+s)}{4-3\alpha}$, which is less than p_1, for the second seller.

2. If $s \ge \frac{1}{8+\sqrt{48}}$, the unique equilibrium prices are $p_1 = \frac{1}{2}$ and $p_2 = \max(1-s, \frac{1}{2})$ ($\ge p_1$).

3. If $s_0(\alpha) < s < \frac{1}{8+\sqrt{48}}$, no equilibrium exists.

In Case 1, the price p_1 is less than the first seller's monopoly price, which is $\frac{1}{2}$, and p_2 is less than the second seller's monopoly price, which is $\max(1-s, \frac{1}{2})$. In particular, for $\alpha = \frac{1}{2}$ and $s < s_0(\frac{1}{2}) = \frac{1}{26}$, the equilibrium prices satisfy $p_1 = \frac{3}{10} + \frac{1}{5}s < \frac{4}{13}$ and $p_2 = \frac{1}{5}(1 - s)$. The critical value $s_0(\alpha)$ is determined by the probability α that the second seller will arrive as a continuous and strictly decreasing function (see (26) below). It tends to 0 as α tends to 1, and to $\frac{1}{8+\sqrt{48}}$ as α tends to 0.

We must show that the conditions given in Cases 1–3 are indeed necessary and sufficient for the stated p_1 and p_2 to be equilibrium prices. Consider a consumer for whom at time t_1 the value of the good v exceeds p_1. If he buys the good, his utility is $v - p_1$. If he waits, his expected utility is $\max\left(\alpha\left(v - s - p_2\right), 0\right)$. It follows that buying is preferable to waiting if and only if v exceeds the critical value v^c given by

$$v^c = \max\left(p_1, \frac{p_1 - \alpha(p_2 + s)}{1 - \alpha}\right) = p_1 + \frac{\alpha}{1 - \alpha}\max\left(p_1 - p_2 - s, 0\right).$$

Therefore, if $p_1 \leq p_2 + s$, all the consumers who would get positive utility from buying at time t_1 do so. But if $p_1 > p_2 + s$, then the consumers with $v < v^c$, who are the ones older than $\frac{1-p_1-\alpha(1-p_2-s)}{1-\alpha}$, do not buy. We conclude that the first seller's profit π_1 depends on the price p_1 he sets as follows:

$$\pi_1(p_1) = \begin{cases} p_1(1-p_1) & p_1 \leq p_2 + s \\ p_1 \frac{1-p_1-\alpha(1-p_2-s)}{1-\alpha} & p_2 + s < p_1 \leq 1 - \alpha\,(1 - p_2 - s) \\ 0 & p_1 > 1 - \alpha\,(1 - p_2 - s) \end{cases} . \tag{17}$$

The maximal profit is attained at

$$p_1 = \begin{cases} \frac{1-\alpha}{2} + \frac{\alpha}{2}\,(p_2 + s) & p_2 + s \leq \frac{1-\alpha}{2-\alpha} \\ p_2 + s & \frac{1-\alpha}{2-\alpha} \leq p_2 + s \leq \frac{1}{2} \\ \frac{1}{2} & p_2 + s \geq \frac{1}{2} \end{cases} . \tag{18}$$

Consider now the profit of the second seller, specifically, the effect on the profit of setting a price p different from p_2. A sufficiently low p may attract old consumers, who were born before t_1 but did not buy from the first seller. As explained above, at time t_1 these consumers valued the good at less than v^c. Since the value of the good decreases linearly with time, and it is at most 1, selling to such consumers requires setting a price $0 \leq p \leq \min(v^c, 1) - s$. The profit from setting such a price, which comes from selling to both young and old consumers, is

$$p\,(\min(v^c, 1) - p)\,. \tag{19}$$

The alternative is to sell only to young consumers, who were born after t_1, by setting a price $p \geq 1 - s$. This alternative gives the profit $p\,(1-p)$, which has the unique maximum point $p = 1-s$ if $0 < s < \frac{1}{2}$ and $p = \frac{1}{2}$ if $s \geq \frac{1}{2}$. If $s \geq \frac{1}{2}$, then by (18) the profit-maximizing price for the first seller is also $\frac{1}{2}$. Thus, $\frac{1}{2}$ is the unique equilibrium price for both sellers. In the rest of the proof, we assume that $0 < s < \frac{1}{2}$, so that the second seller's maximum profit for prices $p \geq \min(v^c, 1) - s$ is attained at $p = 1 - s$, where it is equal to $s(1-s)$.

Consider the maximum of the second seller's profit (19) for prices $0 < p \leq \min(v_c, 1) - s$. It is attained at a unique point $\bar{p}$. Depending on whether the maximum is greater or less than $s(1-s)$, a price p_2 is the seller's profit-maximizing price if and only if $p_2 = \bar{p}$ or $p_2 = 1 - s$, respectively. In the following, we examine these conditions more closely.

Suppose first that $p_2 + s > \frac{1}{2}$. By (18), $p_1 = \frac{1}{2}$, and hence $v^c = \frac{1}{2}$. Therefore, (19) is equal to $p\left(\frac{1}{2} - p\right)$, and its maximum for prices $p \leq \min(v_c, 1) - s$ is attained at $p = \min\left(\frac{1}{4}, \frac{1}{2} - s\right)$, which is a lower price than p_2. The maximum is equal to $\frac{1}{16}$ if $s \leq \frac{1}{4}$ and to $s\left(\frac{1}{2} - s\right)$ if $\frac{1}{4} < s < \frac{1}{2}$, to that is it less than or equal to $s(1-s)$ if and only if $s \geq \frac{1}{8+\sqrt{48}}$ (≈ 0.067). This proves that, for $\frac{1}{8+\sqrt{48}} \leq s < \frac{1}{2}$, $p_2 = 1 - s$ and $p_1 = \frac{1}{2}$ are equilibrium prices, and for any $0 < s < \frac{1}{2}$, there are no other equilibrium prices with $p_2 > \frac{1}{2} - s$.

Suppose now that $p_2 + s \leq \frac{1}{2}$. For such p_2, (18) can be written as

$$p_1 - p_2 - s = \frac{1-\alpha}{2} - \frac{2-\alpha}{2} \min\left(p_2 + s, \frac{1-\alpha}{2-\alpha}\right). \tag{20}$$

The right-hand side is clearly nonnegative, and therefore $v^c = p_1 + \frac{\alpha}{1-\alpha}(p_1 - p_2 - s)$, and $p_2 \leq p_1 - s \leq v^c - s$. Since also $p_2 \leq \frac{1}{2} - s < 1 - s$ (hence, $p_2 \leq \min(v_c, 1) - s$), p_2 is a profit-maximizing price if and only if (i) it maximizes (19) and (ii) the maximum (which is the second seller's equilibrium profit) is greater than or equal to $s(1-s)$. Now, (19) is equal to

$$p\left(\min\left(\frac{p_1 - \alpha p_2 - \alpha s}{1-\alpha}, 1\right) - p\right), \tag{21}$$

which is maximal at $p = \frac{1}{2}\min\left(\frac{p_1 - \alpha p_2 - \alpha s}{1-\alpha}, 1\right)$, where it equals p^2. This proves that $p_2 \leq \frac{1}{2} - s$ is a profit-maximizing price if and only if

$$p_2 = \frac{p_1 - \alpha s}{2-\alpha} \tag{22}$$

and

$$p_2^2 \geq s(1-s). \tag{23}$$

If $\frac{1-\alpha}{2-\alpha} \leq p_2 + s \leq \frac{1}{2}$, the solution of (20) and (22) is $p_1 = 2s$ and $p_2 = s$ ($< \frac{1}{2}$), which does not satisfy (23). If $p_2 + s < \frac{1-\alpha}{2-\alpha}$, the solution is

$$\begin{aligned} p_1 &= \frac{1-\alpha}{4-3\alpha}(2 - \alpha + 2\alpha s) && (24) \\ p_2 &= \frac{1-\alpha(1+s)}{4-3\alpha}. && (25) \end{aligned}$$

These prices satisfy (23) and the inequality $p_2 + s < \frac{1-\alpha}{2-\alpha}$ (which also guarantees that they are positive) if and only if $s \leq s_0(\alpha)$, where

$$s_0(\alpha) = \frac{2(1-\alpha)^2}{(8-7\alpha)(2-\alpha) + (4-3\alpha)\sqrt{\alpha^2 + 12(1-\alpha)}}. \tag{26}$$

For such s, the unique equilibrium prices are (24) and (25). By (17), the first seller's equilibrium profit is $p_1 \frac{1-p_1-\alpha(1-p_2-s)}{1-\alpha}$, which equals $(1-\alpha)\left(\frac{2-\alpha(1-2s)}{4-3\alpha}\right)^2$. For $s > s_0(\alpha)$,there are no equilibrium prices with $p_2 + s \leq \frac{1}{2}$.

A.6 Example 4

Consider again a generalized version of the example, in which the second seller arrives with probably $\frac{1}{2} \leq \alpha \leq 1$. (Unlike in the generalized version of Example 3, the second seller may arrive for sure.) For any $\alpha s+(2-\alpha)\sqrt{s(1-s)} \leq p^* \leq \frac{1}{2}$, we show that the following is an equilibrium: The first seller sets the price p^*, the second seller responds to whatever price p the first seller chooses and to the consumers' actions by choosing a profit-maximizing price, a consumer buys from the second seller (at time t_2) if and only if doing so gives him positive utility, and he buys from the first seller at the requested price $0 \leq p \leq 1$ if and only if his age (at time t_1) is less than x_p, where

$$x_p = \begin{cases} 1-p & p \leq p^* \\ 1-\frac{2}{2-\alpha}(p-\alpha s) & p^* < p < 1-\alpha\left(\frac{1}{2}-s\right) \\ 0 & p \geq 1-\alpha\left(\frac{1}{2}-s\right) \end{cases}. \tag{27}$$

In other words, if $p \leq p^*$, then the consumer buys the good if and only if he values it at more than p, but if $p > p^*$, he buys if and only if the value to him exceeds $\frac{2}{2-\alpha}(p-\alpha s)$. The latter value is higher than p, since for $p > p^*$ ($\geq \alpha s+(2-\alpha)\sqrt{s(1-s)}$),

$$\begin{aligned} \frac{2}{2-\alpha}(p-\alpha s)-p &> \frac{2}{2-\alpha}(p^*-\alpha s)-p^* \\ &= \frac{\alpha}{2-\alpha}(p^*-2s) \\ &\geq \frac{\alpha}{2-\alpha}\left(\alpha s+(2-\alpha)\sqrt{s(1-s)}-2s\right) \\ &= \alpha\left(\sqrt{s(1-s)}-s\right) \\ &> 0. \end{aligned} \tag{28}$$

For example, if the second seller arrives with probability $\alpha=\frac{1}{2}$, and he does that $s=\frac{4}{53}$ (≈ 0.075) units of time after the first seller, then for every price between $\frac{23}{53}$ and $\frac{1}{2}$ there is an equilibrium in which the first seller sets this price. If the second seller arrives with certainty $\frac{1}{10}$ units of time after the first seller, then every price between $\frac{2}{5}$ and $\frac{1}{2}$ is an equilibrium price for the first seller.

Assuming that the consumers act according to the specified strategy, the dependence of the second seller's profit-maximizing price on the price p set by the first seller is as follows. If p is such that $1-x_p \leq 2\sqrt{s(1-s)}$, then the second seller chooses the price $p_2=1-s$, which gives him the profit $s(1-s)$. Only consumers who were not yet born at t_1 buy at that price. Selling also to consumers who were then older than x_p, and for this reason did not buy from the first seller, would require setting a price $p_2 \leq 1-x_p-s$, for which the profit would be $p_2(1-p_2-x_p)$. However, for any p_2, $p_2(1-p_2-x_p) \leq \frac{1}{4}(1-x_p)^2 \leq s(1-s)$. If $1-x_p > 2\sqrt{s(1-s)}$, then the second seller chooses the price $p_2=\frac{1-x_p}{2}$ ($< 1-x_p-s$, since $1-x_p>2s$) and the corresponding profit is $p_2(1-p_2-x_p)=\frac{1}{4}(1-x_p)^2$ ($> s(1-s)$). Therefore, by (27) and the inequalities $\frac{1}{4} \leq \sqrt{s(1-s)} \leq \frac{3}{10}$, which are an alternative presentation of the assumption $\frac{1}{8+\sqrt{48}} \leq s \leq \frac{1}{10}$: (i) if $p \leq p^*$ ($\leq \frac{1}{2}$), then $1-x_p=p\leq\frac{1}{2}\leq 2\sqrt{s(1-s)}$ and $p_2=1-s$, and (ii) if $p>p^*$ ($\geq \alpha s+(2-\alpha)\sqrt{s(1-s)}$), then $1 - x_p = \min\left(\frac{2}{2-\alpha}\left(p - \alpha s\right), 1\right) > 2\sqrt{s\left(1-s\right)}$ and

$$p_2 = \frac{1 - x_p}{2} = \min\left(\frac{p - \alpha s}{2 - \alpha}, \frac{1}{2}\right). \tag{29}$$

In case (i), the price p_2 the second seller will set satisfies $p \leq p^* < p_2$, and so it is optimal for consumers who value the good at more than p at time t_1 to buy it at that price, as strategy (27) instructs them to do. In case (ii), buying the good at time t_1 (at price p) is optimal for a consumer of age $x < 1 - p$ if and only if $1 - x - p \geq \alpha\left(1 - x - p_2 - s\right)$, or $\left(1 - \alpha\right)x \leq 1 - p - \alpha\left(1 - p_2 - s\right)$. Waiting for the second seller is optimal if and only if the reverse inequality holds, and if an equality holds, then both options are equally good. By (29),

$$1 - p - \alpha\left(1 - p_2 - s\right) \leq \left(1 - \alpha\right)\left(1 - \frac{2}{2 - \alpha}\left(p - \alpha s\right)\right), \tag{30}$$

with equality if $x_p > 0$. Thus, if $p > p^*$ and $x_p > 0$, the consumers' strategy (27) prescribes optimal actions for them. The same holds if $x_p = 0$, or $p \geq 1 - \alpha\left(\frac{1}{2} - s\right)$, since this inequality implies that the right-hand side of (30) is nonpositive, and hence not buying the good at time t_1 is an optimal action for all consumers.

It remains to show that the given p^* is the profit-maximizing price for the first seller. If the seller sets a price $0 < p \leq p^*$, his profit will be $p\left(1 - p\right)$, which is less than or equal to $p^*\left(1 - p^*\right)$, since $p^* \leq \frac{1}{2}$. By (27), the profit for any price $p > p^*$ is either zero or $p\left(1 - \frac{2}{2-\alpha}\left(p - \alpha s\right)\right)$. The latter depends on p as a quadratic, concave function, with a maximum at $\frac{1}{2} - \frac{\alpha}{2}\left(\frac{1}{2} - s\right)$. This maximum point lies to the left of p^*, since

$$\begin{aligned} p^* - \left(\frac{1}{2} - \frac{\alpha}{2}\left(\frac{1}{2} - s\right)\right) &\geq \alpha s + \left(2 - \alpha\right)\sqrt{s(1-s)} - \left(\frac{1}{2} - \frac{\alpha}{2}\left(\frac{1}{2} - s\right)\right) \\ &= \left(2 - \alpha\right)\left(\sqrt{s\left(1 - s\right)} - \frac{1}{4}\right) + \frac{1}{2}\alpha s \\ &> 0. \end{aligned}$$

Therefore, for any price $p > p^*$, $p\left(1 - \frac{2}{2-\alpha}\left(p - \alpha s\right)\right) \leq p^*\left(1 - \frac{2}{2-\alpha}\left(p^* - \alpha s\right)\right) \leq p^*\left(1 - p^*\right)$, where the last inequality follows from (28). Thus, no such price gives the first seller a higher profit than p^* does.

References

[1] Chen, H. and Frank, M. (2004) "Monopoly pricing when customers queue." *IIE Transactions*, 36:569–581.

[2] Coase, R. (1972) "Durability and Monopoly." *Journal of Law and Economics*, 15:143–149.

[3] Daley, D. J. and Vere-Jones, D. (1988) *An Introduction to the Theory of Point Processes.* Springer-Verlag, New York.

[4] Diamond, P. A. (1971) "A model of price adjustment." *Journal of Economic Theory*, 3: 156–168.

[5] Kallenberg, O. (1986) *Random Measures,* 4th Edition. Akademie-Verlag, Berlin, Academic Press, London.

[6] Marvin, W. L. (1902) *The American Merchant Marine: Its History and Romance From 1620 to 1902.* Scribner's, New York.

[7] Milchtaich, I. (2004) "Random-player games." *Games and Economic Behavior*, 47:353–388.

[8] Zeithammer, R. (2006) "Forward-looking bidding in online auctions." *Journal of Marketing Research*, 43:462–476.