The Optimal Scoring Rule

Abstract

When a scoring rule is applied n voters rank m alternatives. Every voter assigns $v_1 = 1$ points to his most preferred alternative, $v_2 \leq v_1$ points to his second preferred one, and on down to $v_m = 0$ points for his least preferred alternative. The aggregate ranking is determined by the total number of points assigned to each alternative. Common scoring rules are the Borda rule, in which $V = (1, \frac{m-2}{m-1}, \dots, \frac{m-k}{m-1}, \dots, \frac{1}{m-1}, 0)$, and the Plurality rule, which selects as a winner the alternative with the greatest number of first place votes, and is thus defined by the scoring vector $V = (1, 0, \dots, 0)$, and there are many other scoring rules.

Many scoring rules differ in their scoring vector, and hence possess different properties. While each scoring rule has its advantages, it also has certain flaws. For instance, the Borda rule and Plurality rule do not meet the Condorcet criterion (if an alternative defeats all other alternatives on the basis of majority in pair-wise contests, this alternative should be the social outcome). The Borda rule does not satisfy the majority criterion as well (if an alternative is preferred by a majority of voters, then this alternative should win). As each scoring rule holds different properties, one may wonder whether a particular rule is optimal, given some definition of optimality. In this research we try to answer this question and present an optimal scoring method. The novelty of this study is expressed not only in the selected rule, which so far is unknown in the literature, but also in the optimization method.

While standard approaches in selecting scoring rules are all proposed prior to the election, we propose to select the scoring rule based on the votes received. By doing so, we can interpret the scores as a maximum likelihood estimator and thus may solve some of the flaws of standard scoring rules.