Life-Cycle Wage Growth and the Dynamics of the Wage Distribution

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Abstract

Human capital accumulation and searching for better jobs are often regarded as two major engines of wage growth over worker's career. This paper introduces a third channel through which wages change - the depreciation of human capital, or loss of skills, during unemployment. I embed the three channels of wage dynamics into a structural equilibrium model of labor market, of the type of Burdett, Mortensen (1998) and Burdett et al. (2011). While analytically solving for endogenous equilibrium distribution of wage offers, the joint distribution of potential, actual experience, tenure and wage is obtained. Calibrating the model to the U.S. data, I analyze the life-cycle profiles of wages, identify the sources of wage growth at different ages, over the life-cycle, and for different percentiles of wage distribution. I identify the sources of disparities between the richest and the poorest groups at different ages, and explore how the spells of initial (youth) unemployment of different lengths impact future careers.

1 Introduction

The accumulation of experience and searching for better job opportunities are widely recognized as two important sources of wage growth over workers' careers. The predominant majority of up-to-date work in the field of explaining life-cycle wage outcomes includes both these channels.[1] These models are usually too complex to be solved analytically and to encompass equilibrium in a sense that they do not include strategic eqiulibrium wage-setting by the firms. I add to the existing literature along two lines: first, I introduce an additional channel of wage dynamics - the depreciation of human capital in unemployment, and second, I do it in an equilibrium framework, where the distribution of wages

[1] See Rubinstein, Weiss (2007) for a survey of the implications these two sources have on wage profiles. The models that encompass both of them unclude Yamaguchi (2010), Bowlus, Liu (2012), Bagger et al. (2013), Altonji et al. (2013)

offered by the firms takes into account the optimal search strategies and the quality of human capital of the workers and the strategies of other firms.

Two features of the model make it both tractable and instrumental for the analysis of life-cycle wage profiles: (i) The optimal behavior of the workers is independent of their employment history, and (2) At any point in time, wages reflect the previous employment history. These two features follow from the fact that both wages and unemployment income are modelled as proportional to worker's productivity. The main novelty of the paper is that I solve for the joint equilibrium steady-state distribution of wages, ages, and experiences, which allows to explore the dynamics of wage distribution within various groups (age groups, experience groups, income groups).

The main findings are as follows:

1. Life-cycle average wage growth

(i) Wages grow with age, at a slightly decreasing rate, but the implied average wage profile is insufficiently concave compared to existing empirical evidence. (ii) for the lower percentiles the wage profile is more concave than for the higher percentiles; (ii) on-the-job search is an important source of wage growth only at the beginning of the career, explaining about 1/3 of wage increase over the first 10 years. Over a longer horizon, its share is much lower, and 10 years into the career human capital accumulation is the only source of wage growth; (iii) the presence of human capital depreciation has little impact on the average wage profile, making it only slightly more concave; (iv) Compared to an economy without skills depreciation, it is very young and very old relatively poor workers who become much poorer when there is loss of productivity in unemployment. Relatively rich do not lose much in terms of their wages, and within their group, the old are impacted more than the young.

2. Life-cycle cross-sectional wage dispersion

(i) The cross-sectional wage dispersion follows a U-shape pattern over potential experience; (ii) the presence of loss of skills more than doubles the cross-sectional total variance of wages at all ages, and makes the U-shape pattern of variance more pronounced; (iii) The presence of loss of skills significantly improves model performance in terms of the frictional wage dispersion; (iv) Good sampled piece rates make workers relatively rich when young, and stable employment history makes them relatively rich in the second part of the career.

3. Expected lifetime value

(i) Expected lifetime value grows over the lifecycle both for employed and unemployed workers , and so does its variance; (ii) for the employed workers, 2/3 of the expected lifetime value at all ages comes from the current wage, and 1/3 - from learning by doing. For the unemployed, the lion's share of expected value comes from the search option, with loss of value due to skills depreciation being roughly offset by replacement income.

4. The impact of youth unemployment on expected career path

(i) Prolonged spells of initial unemployment have a permanent negative impact on expected average career profile; (ii) The gap relative to the benchmark profile shrinks towards the end of the career, but is never closed completely; (iii) The actual delay in wage profile relative to the benchmark case is higher than the duration of initial unemployment; (iv) workers who spend long time waiting for their first job are downgraded to lower percentiles of within-cohort wage distribution, and find it especially difficult to reach its upper tail.

2 Literature overview

On the theoretical side, recent structural models that combine on-the-job search with human capital accumulation are usually too complex to be solved analytically and to encompass general equilibrium, therefore these models are usually partial equilibrium ones, they are estimated by indirect inference, and the relative importance of various sources of wage growth is measured by simulating an artificial sample. Along these lines, Yamaguchi (2010) embeds deterministic human capital accumulation in the model of sequential bargaining in the spirit of Robin and Postel-Vinay (2002). The model is a partial equilibrium (the distribution of offers is given), and is estimated on NLSY data by indirect inference. When simulating counterfactual careers Yamaguchi (2010) decomposes wage growth into the effects of general human capital accumulation, on-the-job search, and wage bargaining. Bagger et al. (2013) also pursue the question of relative importance of various sources of wage growth, within the sequential auctions framework [2]. They differ from Yamaguchi (2010) in that they have a richer pattern of heterogeneity and shocks to ability, rather than match quality. The equilibrium is partial (the distribution of offers is exogenous), and is estimated by indirect inference using Danish matched employer-employee data. Bagger et al. (2013) then simulate the model, construct counterfactual wage paths over actual experience and measure the relative importance of general human capital, tenure effect (due to renegotiation with the current employer), and job switches for wage growth over the life-cycle. Bowlus and Liu (2012) relax the assumption of deterministic learning-by-doing and exogenous arrival of offers: they build on Christensen et al. (2005) and Ben Porath (1967) to construct a search model in the style of Burdett-Mortensen (1998), in which investment in general human capital and search intensity are optimally determined at each age. The equilibrium is partial (the distribution of offers is given), the model is solved numerically, and estimated on U.S. NLSY and SIPP panel data using indirect inference. The authors simulate counterfactual wage paths as a function of potential experience when one of the channels - human capital or job shopping - is switched off, thus measuring the relative inputs of each mechanism into total wage growth.

On the empirical reduced-form side, a number of recent papers use longitudinal data sets to characterize the career paths of individuals. This is usually done within a multinominal econometric model, that includes not only a Mincer-type wage equation, but also equations characterizing the transition probabilities between labor market states for different types of workers, experience levels, or stages of business cycles. Schonberg (2007) exploits the differences in wage

[2] proposed by Postel-Vinay and Robin (2002), extended by Dey and Flinn (2005), and Cahuc et al. (2006)

changes for workers depending on what kind of transition they make (stayers, movers, etc.) in order to recover the parameters of Mincer wage equation including returns to general human capital and tenure. She does it for the U.S. using NLSY data, and for Germany as well, to compare between two institutionally different labor markets. Buchinsky et al. (2010) use PSID panel to estimate returns to experience and seniority in the U.S., in a model including separate equations for employment, job-to-job mobility and wage. Adda et al. (2013) estimate a wage equation including experience and tenure effects, where they allow transition rates to differ by business cycle state, experience and level of skills. They use German panel data to compare the magnitude of the two sources of wage growth for skilled and unskilled workers, and compare the impact of economic downturns on the careers of these two worker types. Altonji et al. (2013) use the PSID data to estimate a rich model of earnings dynamics, that includes the characterization of wage rates, work hours, employment, and job changes over the life-cycle. They explore the potential experience profiles of wages, hours and transition rates, and measure the relative contributions of general human capital (a function of potential experience), tenure, job shopping and unemployment shocks on life-cycle wages. In addition, they are able to measure the relative inputs of various shocks (to employment status, to match quality, to wages) in the variance of lifetime earnings, and build impulse responses of the main variables to these shocks.

The literature above analyzes life-cycle wage profiles, where the main focus is on the evolution of wages over the entire career and on the relative importance of on-the-job search, human capital accumulation and, in some cases, tenure[3], in explaining wage growth at different stages in life. The authors mentioned above are not unanimous in their assessment of the relative importance of human capital accumulation and search for life-cycle wage growth. Some find that job search dominates human capital accumulation only in the beginning of career (Yamaguchi (2010) , Bowlus and Liu (2012), Schonberg (2007)). Some find an opposite pattern (Bagger et al. (2013)). Finally, Altonji et al. (2013) estimate that human capital is the main driver of growth over the entire career.

In a parallel strand of literature, researchers explored the effects of layoffs and youth unemployment on wage profiles

As far as youth unemployment is concerned, Kahn (2010) uses the NLSY79 data to estimate the effect of unemployment rate at the time of graduation from college, on wages over the first two decades of career. She finds that both national and state unemployment rate at the time of graduation have *a significant and persistent negative* impact on wage profiles. Oreoupulos et al. (2012) uses a reduced-form estimation on Canadian data, and finds that the impact of graduating in a recession, and the mechanics of the following catch-up process is different for different groups of workers, depending on their predicted

[3] Altonji et al. (2013) find that returns to tenure explain no more than 15% of wage growth over the first 30 years. Bagger et al. (2013) find that across experience groups the tenure component of wages is almost constant. Schonberg (2007) finds that wage growth due to firm-specific human capital is negligible for all groups in the U.S. except highly educated workers.

wage prospects (as a function of college background).

In addition to the impact of youth unemployment on wages, the impact of layoffs on wages (so-called "wage-scarring") has also been studied extensively in the empirical literature. Addison and Portugal (1989) use the US Displaced Workers Survey to measure the effect of past job duration and unemployment duration on post-displacement wages, and find that a 10% increase in unemployment duration lowers accepted wages by about 1%. Gregory and Jukes (2001) estimate the effects of unemployment on the subsequent earnings of British men and find that wage penalty after a six-month unemployment spell is 13% for the young and almost twice as high for the old. Buchinsky et al. (2010) include previous employment history in their econometric model, and find that this yields much lower estimates of the returns to experience. Altonji et al. (2013) find a statistically significant effect of cumulative unemployment shocks on the general human capital component of wages. More recent papers include Jung, Kuhn (2013), Davis, vonWachter (2011), who also find significant earnings losses. The general result of the aforementioned empirical literature is that unemployment is harmful for life-cycle wage growth. This suggests the relevancy of including the history of unemployment as an explicit factor in wage process. However, structural theoretical literature has avoided doing so, probably because of the complications that history-dependence brings into the model. One of the recent exceptions is the paper by Ortego-Marti (2012), which explores the impact of skills depreciation in unemployment on frictional wage dispersion.

3 The model

3.1 The implications of a simple stochastic model of individual careers

Section 3.1 presents a method for calculating the joint distribution of experience and unemployment durations. These are crucial for workers' productivity when there is learning-by-doing on the one hand, and loss of skills in unemployment, on the other. The distribution of experiences and unemployment spells will define the distribution of workers' productivities, and as such, will be taken into account by the firms when they decide on the offers to post, as presented in the next section 3.2.

Consider the following simple stochastic process, set in continuous time. There is a measure one of workers, all of them are ex ante homogenous, they enter the market at Poisson rate ϕ, and this is also the rate at which they exit, so that the population remains constant. Assume that all workers enter the market unemployed, and when unemployed, they switch to employment at Poisson rate λ_0 . Finally, when they are employed, they may be laid off any time at Poisson rate δ. In this setting, the age (or potential experience) is the time that has passed since the moment of entry, and actual experience is the sum of all spells between λ_0 and δ events.

Note that the above simple structure is shared by a vast majority of search models, starting from Burdett, Mortensen (1998)[4]: a career is a stochastic process governed by Poisson rates of job offers arrival, job destruction, and worker's permanent exit. In conjunction with wage formation mechanism within the uninterrupted employment spell, i.e. the spell between λ_0 and δ events, the realization of these random shocks defines the wage profile of each particular worker.

In this setting, any employed worker's wage at a point in time can be pinned down by the following general steps:

- What is the link potential and actual experience?
- What is the link between actual experience and the length of the recent uninterrupted employment spell?
- What is the link between the length of uninterrupted employment spell and wage?

In what follows I build distributions describing the three pair-wise links above analytically.

Figure 1 below provides and example of how experience evolves with age, under some realization of random shocks, in a life of a worker:

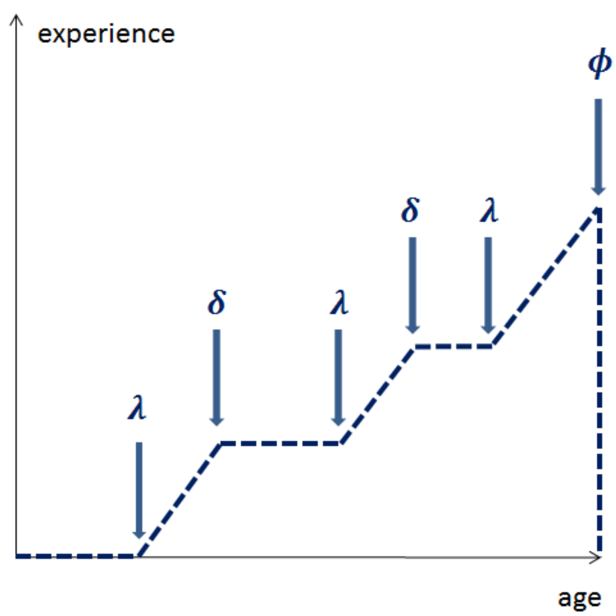

Figure 1: Actual career

[4]See Rogerson, Shimer, Wright (2005) for a survey of theory, and for empirical applications - see Ridder, van den Berg (2003), Jolivet et al. (2006)

I am interested in the distributions describing a continuum of actual careers, like in Figure 1. To make the problem tractable, I use the following analytical trick: given that any worker exiting the market at rate ϕ is immediately replaced by a new entrant, I replace the population of workers with each one having a career like in Figure 1, by a population of infinitely lived *synthetic* workers, where each one lives like any regular worker from Figure 1 above, but instead of exiting the labor market at rate ϕ , he is thrown to unemployment and his experience is reset to zero. Figure 2 provides an example of such a (synthetic) career, and shows how experience x evolves over time. Henceforth, for convenience, the periods between the two ϕ-events in a synthetic life are called cycles.

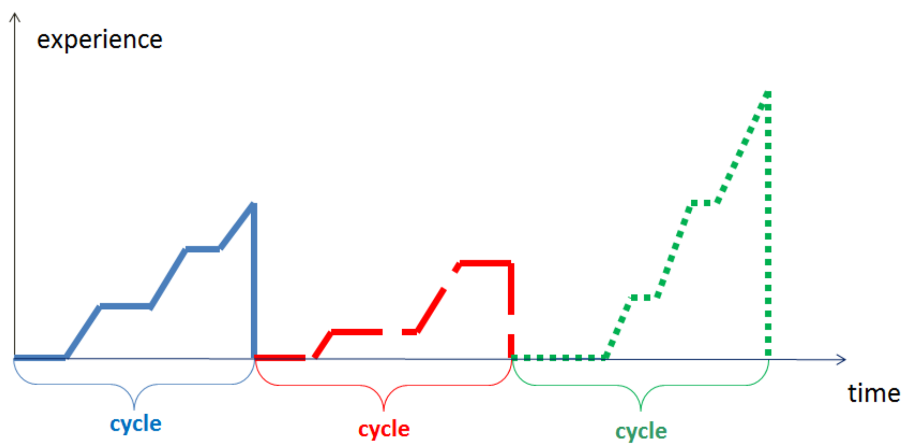

Figure 2: Synthetic career

Taking a snap-shot of such synthetic lives at a point in time is equivalent to taking a cross-section of the real workers who die at rate ϕ. Furthermore, in an ergodic environment, *a cross-section* of synthetic workers is equivalent to a career of *a single* synthetic worker. Therefore, I conclude that analyzing a single synthetic career is similar to analyzing a cross-section of "mortal" workers. As I will show below, operating with a single infinite synthetic life is rather straightforward, which allows me to obtain the distributions describing the life-cycle properties of individual careers, as laid out in the Introduction.

In the following subsections I derive the links between age, experience and uninterrupted tenure in the life of one synthetic worker, meaning, by ergodicity, in a cross-section of real workers.

3.1.1 General approach

Looking at a synthetic life, rather than at a cross-section, allows for a tractable calculation of various life-cycle statistics. Namely, to find the distribution of some variable z in a cross-section (z might represent age, experience, piece rate, etc.), one should answer the following question, in terms of synthetic life: what is the overall share of cycles, in which the variable of interest has reached a particular value? To illustrate the approach, below I present the detailed examples of the derivations of an unconditional distribution of actual experiences, and of the conditional distribution of actual experience, given potential experience. Other derivations are relegated to Appendix A.

3.1.2 Employed workers: actual experience

Denote by $S^E(x)$ a share of cycles in an infinite life of a hypothetical worker (a cycle is a spell between two $\phi-$events) in which she reached experience x.

$S^E(x)$ and $S^E(x+\Delta x)$ are linked as:

$$S^E(x+\Delta x) = S^E(x)\cdot(1-(\phi+\delta)\Delta x) + S^E(x)\cdot\left(\Delta x\delta\cdot\int_0^\infty \lambda_0 e^{-\lambda_0 u}e^{-\phi u}du\right)$$

The expression above states that the cycles in which one reaches experience $x+\Delta x$ are the cycles in which one had experience x and continued to be employed and did not die during Δx (the probability $1-(\phi+\delta)\Delta x$), or if one had x, got unemployed during Δx (probability $\delta\Delta x$) but managed to get an offer and come back to job before the cycle is terminated (probability $\int_0^\infty \lambda_0 e^{-\lambda_0 u}e^{-\phi u}du$). This gives us:

$$\frac{S^E(x+\Delta x)-S^E(x)}{\Delta x} = S(x)\cdot\left(-\underbrace{(\phi+\delta)}_{A1}+\underbrace{\frac{\delta\lambda_0}{\phi+\lambda_0}}_{A2}\right)$$

meaning that only the share $(1-A1\cdot\Delta x+A2\cdot\Delta x)$ of cycles where experience x was reached, are the cycles in which experience $x+\Delta x$ was reached as well.

Solving the differential equation yields:

$$S^E(x) = e^{-\dfrac{\phi\,(\phi+\delta+\lambda_0)\cdot x}{(\phi+\lambda_0)}}$$

The above is the share of cycles, in which experience x has been reached, in an infinite representative life. Since only employment adds to experience, this is also the share of cycles in which *an employed* worker with experience x has ever been observed. In terms of cross-section of "real" workers, $S^E(x)$ is the probability that a given employed worker has experience higher than x. For the ease of exposition, hereafter I denote by $S^E(x)$ the complement of the probability found about, that is, $S^E(x) = 1-e^{-\frac{\phi(\phi+\delta+\lambda_0)\cdot x}{(\phi+\lambda_0)}}$ will be the probability that a given employed worker has experience *below* x. Parameter values corresponding to more favorable market conditions for the workers (lower separation rate δ, higher job-finding rate λ_0) imply lower share of workers with relatively low experiences ($\frac{\partial S^E(x)}{\partial \delta} > 0$, $\frac{\partial S^E(x)}{\partial \lambda_0} < 0$). When workers' lives are on average shorter (ϕ is higher), they have less time to accumulate experience, and a higher share of them will have relatively low x, $\frac{\partial S^E(x)}{\partial \phi} > 0$.

3.1.3 Employed workers: Actual and potential experience

Over the cycle, a worker alternates between two states: employment and unemployment. Total time spent in employment (experience, x) together with the total time spent in unemployment (unemployment history q), constitutes one's potential experience, or age, on the labor market, denoted a.

Denote by $P^E(q|x)$ - the probability that an individual of experience x, who is employed now, has spent in the state of unemployment less than q over his life. In terms of synthetic life, one should be looking for the cycles in which one ever observes a worker in the state of employment with experience x, and with the time spent in unemployment over his life below q. It can be found by the following differential equation:

$$P^E(q|x+\Delta x) = \frac{1 - A1\Delta x}{1 - A1\Delta x + A2\Delta x} \cdot P^E(q|x) + \frac{\delta\Delta x \int_0^q P^E(v|x)e^{-\phi(q-v)}\lambda_0 e^{-\lambda_0(q-v)}dv}{1 - A1\Delta x + A2\Delta x}$$

The cycles where the unemployment accumulated over life is less than q, given that the experience $x + \Delta x$ has been reached, are all cycles where the experience x was reached, the accumulated unemployment was less than q, and the transition to $x+\Delta x$ went without the period of unemployment in the middle (probability $1 - A1\Delta x$), like in Case 1 on Figure 3; in addition, there are cycles where experience x was reached and the accumulated unemployment was below some $v < q$, so that the transition to $x + \Delta x$ can go through a period of unemployment, but so that the sum of v and this period of unemployment is still below q, see Case 2 on Figure 3.

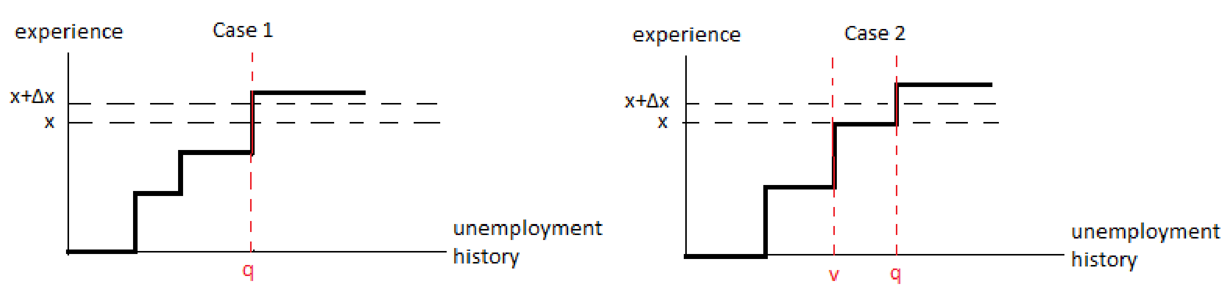

Figure 3: Experience and unemployment history

That is, for any incremental unemployment period $q-v$, if unemployment accumulated before that was below v ($prob = P^E(v|x)$), then the total unemployment will be below q. The incremental unemployment will last $q-v$ if a worker does not die and finds a job exactly at $q-v$ ($prob = e^{-\phi(q-v)}\lambda_0 e^{-\lambda_0(q-v)}$). After some work (see Appendix A for details), the equation above becomes a partial differential equation (denote $P^E(q|x) = F^E(q,x)$):

$$\frac{\partial^2 P^E(q|x)}{\partial q \partial x} = -\frac{\delta\lambda_0}{\phi+\lambda_0} \cdot \frac{\partial P^E(q|x)}{\partial q} - (\phi+\lambda_0) \cdot \frac{\partial P^E(q|x)}{\partial x} \tag{1}$$

Boundary conditions: $P^E(q|0)$ is the probability that a worker (who is employed) spent in unemployment (since he was born) less than q and then found his first work. This is the case when a worker did not die till some $u < q$, and waited till the first offer exactly u periods. As it is a conditional probability, one should normalize it by the cumulative probability of all cases in which a worker manages to enter employment before he dies:

$$P^E(q|0) = \frac{\int_0^q \lambda_0 e^{-\lambda_0 u} e^{-\phi u} du}{\int_0^\infty \lambda_0 e^{-\lambda_0 u} e^{-\phi u} du} = 1 - e^{-(\phi+\lambda_0)q}$$

Now, for a second boundary condition, let us find $P^E(0|x)$ - which is the probability that an employed worker of experience x has spent in unemployment less or exactly 0. Obviously, zero unemployment is impossible as all the workers are born unemployed. Therefore, $P^E(0|x) = 0$. Thus we have the following PDE with boundary conditions:

$$\begin{aligned}
\frac{\partial^2 P^E(q|x)}{\partial q \partial x} &= a^E \cdot \frac{\partial P^E(q|x)}{\partial q} + b^E \cdot \frac{\partial P^E(q|x)}{\partial x} \\
P^E(0|x) &= 0 \\
P^E(q|0) &= 1 - e^{bq}
\end{aligned}$$

where

$$\begin{aligned}
a^E &= -\frac{\delta\lambda_0}{\phi+\lambda_0} < 0 \\
b^E &= -(\phi+\lambda_0) < 0
\end{aligned}$$

Its solution is given by (see Appendix A for details):

$$P^E(q|x) = e^{a^E x} e^{b^E q} \cdot \sum_{n=0}^{\infty} \sum_{k=n+1}^{\infty} \frac{\left(-a^E x\right)^n}{n!} \frac{(-b^E q)^k}{k!} \tag{2}$$

It is important to note that higher experiences are associated with longer unemployment histories, because in this framework of alternating Poisson events, the way to high x lies through many periods of unemployment. Mathematically it can be shown that $\frac{\partial P^E(q|x)}{\partial x} < 0$ (see Appendix A). The higher is the separation rate, the more likely it is that accumulating a given level of experience takes more time, as the periods of unemployment happen more frequently, and the accumulated unemployment will be higher for a given level of experience, $\frac{\partial P^E(q|x)}{\partial \delta} < 0$. The opposite holds for the job-finding rate λ_0 - in case it is high, the accumulation of a given level of experience will be accompanied by shorter unemployment spells, that is, $\frac{\partial P^E(q|x)}{\partial \lambda_0} > 0$. Finally, the more frequent are the exits (higher ϕ), the harder it is to accumulate any given experience level x, meaning that if it has been nonetheless reached, the process must have taken on average less time, that is, the accumulated unemployment should be on average lower ($\frac{\partial P^E(q|x)}{\partial \phi} > 0$).

In this model one's age on the labor market, a, or potential experience, is a sum of actual experience and unemployment history: $a = x + q$. Once $P^E(q|x)$ is found, the probability that one's potential experience, or age, is less than a, when actual experience is x, is simply $P^E(a - x|x)$:

$$G^E(a|x) = P^E(q = a - x|x)$$

Having the distribution of ages, conditional on experiences, and having the overall distribution of experiences, I derive the *joint* distribution of ages *and* experiences among the employed, $G^E(x, a)$ (see Appendix A). Thereafter, I can derive the distribution of experience, conditional on age:

$$G^E(x|a) = \frac{\frac{\partial G^E(x,a)}{\partial a}}{\frac{\partial S^E(a)}{\partial a}} \tag{3}$$

where $S^E(a)$ is the probability that an employed worker is younger than a. The distribution of ages differs by employment status, because by assumption everyone is born unemployed, and therefore, on average, the unemployed tend to be younger than the employed (see Appendix C for details).

3.1.4 Employed workers: actual experience and recent uninterrupted employment

Denote by t the length of the current employment spell. It can be shown (see Appendix A), that the conditional distribution of t, given x is:

$$\begin{aligned} P^E(t|x) &= 1 - e^{-\frac{\delta\lambda_0}{\phi+\lambda_0}t} \\ P^E(t &= x|x) = 1 \text{ (there is a mass at } t = x) \end{aligned} \tag{4}$$

It turns out that the conditional distribution of employment spells does not depend explicitly on the level of experience that we take, it is only that experience serves as an upper bound of the range of possible employment spells. The more frequently unemployed workers leave to employment (higher λ_0) , and the more often matches are destroyed (higher δ), the more likely it will be that an employed person of experience x has been uninterruptedly employed for less than t : $\frac{\partial P^E(t|x)}{\partial \lambda_0} > 0, \frac{\partial P^E(t|x)}{\partial \delta} > 0$. In other words, the more churning there is in the labor market, the more new-hires from unemployment will be there at a given point in time, therefore, the more likely it is that an individual is new-hired. When workers exit the labor market often (high ϕ), they have on average less time to accumulate a given level of experience. In other words, any x is on average reached through less periods of unemployment, and, accordingly, there is a lower probability that someone who managed to accumulate x, has currently been uninterruptedly employed for less than t, $\frac{\partial P^E(t|x)}{\partial \phi} < 0$.

3.1.5 Employed workers: uninterrupted employment and wage, with wage posting

Assume that when employed, the workers get job offers at Poisson rate λ_1. Assume that these offers come from some given distribution $F(w), w \in [\underline{w}, \overline{w}]$. Assume also that employed workers accept any offer w' that is above their current wage w (a standard optimal strategy in wage posting model).

Now take all employed worker whose current employment spell lasts t and ask a question: "How are their wages distributed, given t?" Denote this distribution by $P(w|t)$. Obviously, if $t = 0$, the wages are distributed as wage offers:

$$P^E(w|0) = F(w)$$

In the extreme case of an infinite employment spell all workers will eventually arrive at the highest possible wage:

$$P^E(w|\infty) = \begin{Bmatrix} 0 \text{ if } w < \overline{w} \\ 1 \text{ if } w = \overline{w} \end{Bmatrix}$$

In between, for positive employment spells, I find $P(w|t)$ recursively. Suppose a worker is employed at a wage below w at tenure t. Then, at tenure $t + dt$ his wage will still be below w only in case he did not get an offer higher than w during Δt. That is:

$$\begin{aligned} P^E(w|t+dt) &= P^E(w|t) \cdot [1 - \lambda_1 dt \cdot (1 - F(w))] \\ &\implies P^E(w|t) = F(w) \cdot e^{-\lambda_1 \cdot (1-F(w)) \cdot t} \end{aligned} \quad (5)$$

Here, a higher rate of job-to-job transitions (higher λ_1) means a lower probability to earn a wage below w : $\frac{\partial P^E(w|t)}{\partial \lambda_1} < 0$. Higher λ_1 means that given t, workers manage to change works more frequently, and thus attain higher wages. The expression (5) is the "employment premium" of Christensen et al. (2005), that is, the stochastic dominance of the distribution of earned wages over the distribution of offers, measured at a particular employment spell length t.

This completes the presentation of the method for calculating the life-cycle distributions. Note that combining the steps (3), (4) and (5), I obtain joint distribution of wage, potential, actual experience and tenure (see Appendix A), and I return to it in the analysis in the second part of the paper.

To sum up, given a basic stochastic structure of individual careers, I analytically derived distributions linking age, experience, uninterrupted employment and wages for the employed workers, as well as the corresponding characteristics for the unemployed (see Appendix A for details). I used three main assumptions: (1) the distributions are stationary, (2) unemployed workers accept the first offer that they get, and (3) employed workers accept any offer above their current wage.

In what follows I use the above distributions to characterize a stationary equilibrium in case when workers accumulate and lose their human capital over the course of career, and firms take it into account when forming the distribution of wage offers to post. The structure of the model builds on the paper by Burdett et al. (2011), which is one of the first attempts to unite two sources of wage growth (learning-by-doing and on-the-job search) in a single tractable equilibrium framework.

3.2 Human capital accumulation and loss, on-the-job search and equilibrium

3.2.1 The setting

The basic stochastic structure of the career is the same as laid out in the previous section. For the ease of exposition, here I assume that $\lambda_0 = \lambda_1 = \lambda$ - this assumption has no impact on the qualitative properties of the model, and is relaxed in the simulations section. All the derivations for the full model with two different rates can be found in Appendix B.

Workers accumulate human capital (HC) when employed and lose it when unemployed. In particular, human capital grows in employment at a constant rate ρ, and depreciates in unemployment at a constant rate η, so that one's current productivity is $y = y_0 e^{\rho x} e^{-\eta q}$ where x is actual experience and q is the total unemployment tenure. While unemployed, any worker of productivity y enjoys flow income $b \cdot y$. When employed, one's wage is a piece rate θ of productivity, $\theta \cdot y$. Piece rate offers are posted by firms, and are distributed according to $F(\theta)$, $\theta \in [\underline{\theta}, \overline{\theta}]$ which will be derived endogenously. All agents are risk-neutral, the continuous time interest rate is r. The economy is at a steady-state.

Note that while wages are defined as a share (piece rate) of flow productivity, it is important not only how much experience one has, but also how much unemployment one has lived through while accumulating this actual experience.

3.2.2 Worker behavior

Here I take the distribution of piece rate offers, $F(\theta)$, as given and characterize optimal worker behavior. For convenience, I set the initial productivity $y_0 = 1$ for all the workers, this simplification has no impact on the derivations. Denote $y_1 = e^{\rho x}$ and $y_2 = e^{-\eta q}$, so that $y = y_1 \cdot y_2$. Let $W^U(y_1, y_2)$ be the expected lifetime payoff of an unemployed worker with experience x and total unemployment history q, using an optimal search strategy. Let $W^E(y_1, y_2, \theta)$ denote the expected lifetime payoff of an employed worker with experience x and total unemployment q, who is currently employed at a piece rate θ.

Bellman equation for the unemployed:

$$\begin{aligned}(\phi + r)\, W^U(y_1, y_2) \quad &= \quad b \cdot y_1 \cdot y_2 + \frac{\partial W^U(y_1, y_2)}{\partial t} + \\ &+\lambda \int_{\underline{\theta}}^{\overline{\theta}} \max\left[W^E(y_1, y_2, \theta') - W^U(y_1, y_2), 0\right] dF(\theta')\end{aligned} \tag{6}$$

Unemployed workers enjoy flow benefits, their lifetime value changes due to the depreciation of human capital, and they might also get a job offer, which they accept if the welfare gain is positive. Bellman equation for the employed at piece rate θ:

$$\begin{aligned}(\phi + r)\, W^E(y_1, y_2, \theta) \quad &= \quad \theta \cdot y_1 \cdot y_2 + \frac{\partial W^E(y_1, y_2, \theta)}{\partial t} + \\ &+\lambda \int_{\theta}^{\overline{\theta}} \left[W^E(y_1, y_2, \theta') - W^E(y_1, y_2, \theta)\right] dF(\theta') + \\ &+\delta \cdot \left(W^U(y_1, y_2) - W^E(y_1, y_2, \theta)\right)\end{aligned} \tag{7}$$

Employed worker gets a wage, his lifetime value grows due to learning-by-doing on the job, he might quit to an outside offer (as it is always better to be employed at a higher piece rate, $W^E(.)$ is increasing in θ and it is, therefore, optimal, to accept any offer higher than the current piece rate), or the match might dissolve exogenously.

The solution to these Bellman equations is standard, all unemployed workers use the reservation piece rate strategy. Proposition 1 below shows that all unemployed workers have the same reservation piece rate θ^R, and provides the conditions that solve for it. For convenience, define:

$$q(\theta) = \phi + \delta + \lambda \cdot (1 - F(\theta))$$

which is the rate at which any employee leaves the firm paying θ.

Proposition 1 *Optimal job search implies:*

(i) all unemployed workers have the same reservation piece rate θ^R*;*
(ii) the reservation piece rate θ^R *is determined by the following system of two equations in two unknowns* (θ^R, α^U)*:*

$$\theta^R = b - (\rho + \eta) \cdot \alpha^U \tag{8}$$

$$(\ \phi + \eta + r) \cdot \alpha^U = b + \lambda_0 \cdot \left[\int_{\theta^R}^{\overline{\theta}} \frac{1 - F(\theta')}{q(\theta') + r - \rho} d\theta' \right] \tag{9}$$

Further, for any F*, the solution exists, is unique, implies* $\theta^R < b$ *and* θ^R *is strictly decreasing in* ρ *and* η*.*

The proof is presented in the Appendix B, it goes along the same lines as in Burdett et al. (2011), and it hinges on the very useful property of the value functions above, namely, that they are proportional to one's productivity, $y_1 \cdot y_2$. If there is no learning-by-doing and no human capital depreciation, $\theta^R = b$.

However, for a given F, higher ρ makes experience (and employment) relatively more valuable, and higher η makes unemployment relatively more harmful, and both these effects drive the reservation piece rate below b. When the interest rate is high, and the discounting stronger, the normalized present value of being unemployed , α^U, goes down, as equation (9) shows, and the reservation rate of the unemployed goes up. The latter follows from the fact that the value of being employed includes the value of unemployment, in addition to the search option, and therefore falls twice as a result of high r.

In the next section I take the workers' behavior as given and characterize profits.

3.2.3 Profits

First note that offering $\theta < \theta^R$ implies that the firm attracts no workers and thus makes zero profit. Offering a θ above θ^R , for example $\theta = b$, generates strictly positive profit ($b < 1$), and thus strictly dominates offering $\theta < \theta^R$. Thus, in any market equilibrium it must be that $\underline{\theta} \geq \theta^R$, and any unemployed worker accepts the first offer received.

Given the distributions calculated in Section 1, and given an offer $\theta \geq \theta^R$, steady-state flow profit is:

$$\begin{aligned} \pi(\theta) \quad = \quad & \lambda U \cdot \left[\int_{x=0}^{\infty} \int_{q=0}^{\infty} \int_{\tau=0}^{\infty} e^{-r\tau} e^{-q(\theta)\tau} \cdot (1-\theta)\, e^{\rho(x+\tau)} e^{-\eta q} d\tau d \left(1 - S^U(q)\right) dP^U(x|q) \right] + \\ & + \lambda(1-U) \cdot \left[\int_{x=0}^{\infty} \int_{q=0}^{\infty} \int_{\underline{\theta}}^{\theta} \int_{\tau=0}^{\infty} e^{-r\tau} e^{-q(\theta)\tau} \cdot (1-\theta)\, e^{\rho(x+\tau)} e^{-\eta q} d\tau \frac{\partial^2 P^E(\theta, x)}{\partial x \partial \theta} \frac{\partial P^E(q|x)}{\partial q} dx dq d\theta \right] \end{aligned}$$

where $S^U(q)$ is the distribution of unemployment histories among the unemployed, $P^U(x|q)$ is the distribution of experiences, conditional on unemployment history, among the unemployed, $P^E(\theta, x)$ is the joint distribution of experiences and piece rates among the employed (see Appendix A), and $P^E(q|x)$ is the distribution of unemployment histories, conditional on experience, for the employed.

Simplifying:

$$\begin{aligned}\pi(\theta) &= \frac{\lambda U\cdot(1-\theta)}{q(\theta)+r-\rho}\cdot\left[\int_{x=0}^{\infty}\int_{q=0}^{\infty}e^{\rho x}e^{-\eta q}\cdot dS^U(q)dP^U(x|q)\right]+\\ &\quad+\frac{\lambda(1-U)\cdot(1-\theta)}{q(\theta)+r-\rho}\cdot\left[\int_{x=0}^{\infty}\int_{q=0}^{\infty}\int_{\underline{\theta}}^{\theta}e^{\rho x}e^{-\eta q}\cdot d^2P^E(x,\theta)dP^E(q|x)\right]\end{aligned}$$

The first element in the profit expression is the expected profit from hiring an unemployed worker with experience x and accumulated employment q. The second element is the expected profit from poaching a worker who was previously employed at a piece rate below θ, with experience x and accumulated unemployment q. The distributions $S^U(q), P^U(x|q), P^E(x,\theta)$ and $P^E(q|x)$ are all found in Part 1.

Now I define the market equilibrium:

A Steady-State Market Equilibrium is a triple $\left\{\theta^R, F(\cdot), U\right\}$ [5]such that:

(i) θ^R is the optimal reservation piece rate of any unemployed worker

(ii) $F(\cdot)$ satisfies the constant profit condition:

$$\begin{aligned}\pi(\theta) &= \overline{\pi}>0 \text{ for all } \theta \text{ where } dF(\theta)>0\\ \pi(\theta) &\leq \overline{\pi} \qquad \text{for all } \theta \text{ where } dF(\theta)=0\end{aligned}$$

(iii) U is consistent with steady-state turnover

The following useful result from Burdett et al. (2011) applies here as well:

Lemma 1. In Equilibrium defined above, (i) $F(\cdot)$ contains no mass points, (ii) $F(\cdot)$ has a connected support, and (iii) $\underline{\theta}$=θ^R. The proof is relegated to the Appendix B.

Now I turn to characterizing the equilibrium $F(\cdot)$, using the constant profit condition. First, the steady-state inflows and outflows from the pool of unemployed must be equal, meaning that newborns and those separated from employers must be exactly offset by those unemployed who die or find a job:

[5] As we are looking for the steady-state equilibria, we automatically assume that the links between x, q and θ hold, as described in Part 1

$$\begin{aligned} \phi + \delta(1-U) &= \phi U + \lambda U \\ U &= \frac{\phi+\delta}{\phi+\delta+\lambda} \end{aligned}$$

Substituting for U and using the distributions found in Part 1, one can show after some work (see Appendix B) that [6]:

$$\pi(\theta) = \frac{1-\theta}{\phi+\delta+\lambda\cdot(1-F(\theta))+r-\rho}\cdot\Omega(F(\theta)) = \overline{\pi} \tag{10}$$

The constant profit condition implies that the profit is the same for all offers, including the bounds of the support of $F(.)$:

$$\pi(\theta) = \pi(\underline{\theta}) = \pi(\overline{\theta}) = \overline{\pi}$$

Inserting $\pi(\underline{\theta})$ instead of $\overline{\pi}$ into (10) above, one gets the following equation:

$$\begin{aligned} &\frac{1-\theta}{\phi+\delta+\lambda\cdot(1-F(\theta))+r-\rho}\cdot\Omega(F(\theta)) \\ = \quad &\frac{1-\underline{\theta}}{\phi+\delta+\lambda+r-\rho}\cdot\frac{\lambda\phi\left(\phi+\delta-\rho\right)}{\left(\phi+\delta-\rho\right)\left(\phi+\lambda+\eta\right)-\delta\lambda} \end{aligned} \tag{11}$$

In addition, $\pi(\underline{\theta}) = \pi(\overline{\theta})$ gives the following link between the lowest and the highest offers:

$$\frac{(\phi+\delta-\rho)\cdot(\phi+\delta+r-\rho)}{(\phi+\delta+\lambda-\rho)\cdot(\phi+\delta+\lambda+r-\rho)} = \frac{1-\overline{\theta}}{1-\underline{\theta}} \tag{12}$$

Now the characterization of $F(.)$ goes as follows:

- (8) is substituted into the LHS of (9) to get rid of α^U.
- The upper bound $\overline{\theta}$ is found through (12)
- (9) is solved for $\underline{\theta} = \theta^R$ (see Lemma 1), where the integral on the RHS is calculated using $F(.)$ that solves 11, which is a cubic equation in $F(.)$ for any θ, given a particular $\underline{\theta}$, and the bounds of integration are the functions of $\underline{\theta}$ as well

[6]
$$\Omega(F(\theta)) = \frac{\lambda_0\phi(\phi+\delta-\rho)}{(\phi+\delta-\rho)(\phi+\lambda_0+\eta)-\delta\lambda_0} +$$
$$\frac{F(\theta)\phi\lambda_1\lambda_0}{\delta\lambda_0+(\phi+\lambda_0)\lambda_1\cdot(1-F(\theta))}\cdot\left[\frac{\delta\lambda_0}{(\phi+\delta-\rho)(\eta+\phi+\lambda_0)-\delta\lambda_0}+\frac{\lambda_1(1-F(\theta))\cdot(\phi+\lambda_0)}{(\eta+\phi+\lambda_0)\left(\frac{\delta\lambda_0}{\phi+\lambda_0}+q(\theta)-\rho\right)-\delta\lambda_0}\right],$$
$$\lambda_0 = \lambda_1$$

This completes the characterization of the equilibrium. The equilibrium unemployment rate is a constant, like in a classic Burdett-Mortensen (1998) model, and the equilibrium distribution of offers is a continuous distribution, with the support $\left[\underline{\theta}, \overline{\theta}\right]$. When the rate of human capital accumulation ρ goes up, (12) implies that the range of offers expands: the unemployed are ready to accept much lower wages ($\underline{\theta}$ goes down), because the value of employment is higher. Higher profits emerging from the offers at the bottom of the piece rates range, make the offers at the top of the range go down as well, to maintain constant profit (from (12) it follows that the lower and the upper bound move together). An increase in the rate of loss of skills η has a similar effect on the distribution of offers.

Returning to the life-cycle distributions emerging from the stochastic properties of careers as laid out in Section 2.1, and using the equilibrium distribution $F(\theta)$ as described above, I obtain the full characterization of career in a stationary labor market equilibrium: I have the link between potential and actual experience according to (3), the link between actual experience and recent employment spell (4), and the link between employment spell and current piece rate (5). Combining the three, I can analyze the total distribution of wages, its cross-sections by potential or actual experience levels, and by percentiles.

4 Calibration

In this section I calibrate the model in order to illustrate its implications for the life-cycle wage distributions. When computing the distributions, I release the assumption of a unified offers arrival rates, and distinguish between λ_0 and λ_1. The reference period is one quarter. I set the value of $\phi = 0.0063$, to ensure that on average, the working life lasts 160 periods, or 40 years. I set $r = 0.0099$ per quarter, that amounts to 4% per annum.

4.1 Separation rate δ and the offers arrival rate in unemployment λ_0

Shimer (2012) highlights the shortcomings of time aggregation when deriving the probabilities of transitions between labor market states from CPS flows data. In the survey, the respondents are observed at discrete time intervals, while the underlying process of transitions between the pools is continuous. Therefore, one fails to register completed unemployment/employment spells that take place between the interview dates, and thus underestimates the underlying transition rates. In the model, workers alternate between two states, U and E, and the probability if being in state $X \in \{U, E\}$ at t_1 , conditional on being state $Y \in \{U, E\}$ at time t_0, can be found from the mathematics of the Poisson processes. For example, consider the case when someone reports being unemployed when interviewed at t_0. At some point in time $t \in [0, t_1 - t_0]$ between the two interviews, the probability of being in state "U" can be found from the following differential equation:

$$S(U|t + \Delta t) = S(U|t) \cdot [1 - \lambda_0 \cdot \Delta t] + (1 - S(U|t)) \cdot \delta \Delta t$$

where we have the initial condition $S(U|0) = 1$, as we look only at those who reported "U" at t_0. Having found $S(U|t)$, and using t = time between interviews, one finds the probability of reporting U at t_1, conditional on reporting U at t_0 - in other words, the UU transition rate. In a similar way one can find all possible transition rates, when the time interval between the interviews is t:

$$\begin{aligned} P_{EE} &= \frac{\lambda_0}{\delta + \lambda_0} + \frac{\delta}{\delta + \lambda_0} \cdot e^{-(\delta + \lambda_0)t} \\ P_{UU} &= \frac{\delta}{\delta + \lambda_0} + \frac{\lambda_0}{\delta + \lambda_0} \cdot e^{-(\delta + \lambda_0)t} \\ P_{UE} &= \frac{\lambda_0}{\delta + \lambda_0} - \frac{\lambda_0}{\delta + \lambda_0} \cdot e^{-(\delta + \lambda_0)t} \\ P_{EU} &= \frac{\delta}{\delta + \lambda_0} - \frac{\delta}{\delta + \lambda_0} \cdot e^{-(\delta + \lambda_0)t} \end{aligned}$$

The values for P_{EE}, P_{UU}, P_{UE} and P_{EU} can be computed directly from the CPS data, by dividing relevant flows by corresponding stocks. I use seasonally adjusted monthly CPS data for men, for the period February 1990 till August 2013. Note that as Poisson rate is an instantaneous arrival rate, or a measure of the process intensity in an infinitely small time interval, it is inessential what data frequency I use.

In the model, there are only two possible labor market states: employed and unemployed, therefore, in the model the above equations imply the following restrictions: $P_{EE} + P_{EU} = 1, P_{UU} + P_{UE} = 1$. In the data, there are also flows into and from the state "not in the labor force", therefore, these restrictions do not hold. This leaves 4 possible alternatives for calibrating δ and λ_0, depending on which two equations we chose in order to solve for δ and $\lambda_0 :\ 1 : (P_{EE}, P_{UU})$, $2 : (P_{EE}, P_{UE})$, $3 : (P_{EU}, P_{UU})$, $4 : (P_{EU}, P_{UE})$. All these options give different estimates for δ and λ_0. First, I reject the pairs (P_{EE}, P_{UU}), (P_{EE}, P_{UE}) for the reason that the job-finding and employment-exit rates implied by these equations are strongly positively correlated ($corr = 0.5 - 0.7$) over the sample period, which stands in stark contrast with the data (see, for example, Table 1 in Shimer (2005)). Second, of the two remaining pairs I prefer (P_{EU}, P_{UU}) , because the other remaining pair (P_{EU}, P_{UE}) implies an implausibly long average unemployment spell of around 4 months, while the implied average employment duration is approximately the same for both remaining pairs.

Thus, we have a system of two equations in two unknowns, δ and λ_0 :

$$\left(\begin{array}{l} P_{UU} = \frac{\delta}{\delta+\lambda_0} + \frac{\lambda_0}{\delta+\lambda_0} \cdot e^{-(\delta+\lambda_0)} \\ P_{EU} = \frac{\delta}{\delta+\lambda_0} - \frac{\delta}{\delta+\lambda_0} \cdot e^{-(\delta+\lambda_0)} \end{array} \right)$$

which I solve separately for each month in the sample period.

The obtained Poisson rates λ_0 and δ for the period Feb1990-Aug2013 are presented in the following Figure 4:

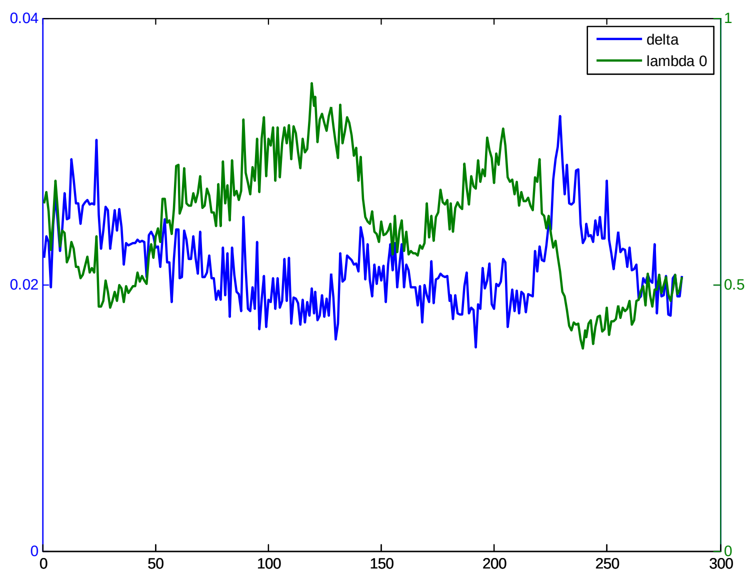

Figure 4: Poisson arrival rates

The average job finding rate λ_0 for the sample period is 0.61, and the average separation rate δ is 0.022. Following Shimer (2012), I compute the corresponding monthly job finding and employment exit *probabilities* (where the *probability* to leave unemployment in less time than t, F_t is linked to the instantaneous Poisson *arrival rate* f_t in period t according to the formula: $f_t = -\log(1 - F_t)$). I get that the average *monthly* job-finding *probability* is 0.46, and the average *monthly* employment exit *probability* is 0.021. These numbers are in line with the values computed for US male workers by Hornstein et al. (2011) - 0.43 and 0.03, and by Shimer (2012) - 0.44 and 0.034. The implied average unemployment duration $(1/\lambda_0)$ is just over two months, and average uninterrupted employment duration $(1/\delta)$ is just below 4 years.

4.2 On-the-job offers arrival rate λ_1

To find the value for the Poisson arrival rate of offers on-the-job, I use the methodology of Nagypal (2008), Hornstein et al. (2011), and Ortego-Marti (2012). Namely, per-period job-to-job flow in the model is:

$$\chi = \lambda_1 \cdot \int_{\underline{\theta}}^{\overline{\theta}} (1 - F(\theta)) \cdot dG(\theta) \tag{13}$$

where $G(\theta)$ is the earned piece rates distribution. The equation above simply reflects the fact that employed workers move to any job that offers a higher piece rate than their current one.

Integrating by parts yields:

$$\begin{aligned} \int_{\underline{\theta}}^{\overline{\theta}} (1 - F(\theta)) \cdot dG(\theta) &= \int_{\underline{\theta}}^{\overline{\theta}} G(\theta) \cdot dF(\theta) \\ &\Rightarrow \chi = \lambda_1 \cdot \int_{\underline{\theta}}^{\overline{\theta}} G(\theta) \cdot dF(\theta) \end{aligned} \tag{14}$$

The distribution of earned piece rates $G(\theta)$ can be found in SS from the inflow-outflow condition:

$$U \cdot \lambda_0 \cdot F(\theta) = (1 - U) \cdot (\delta \cdot G(\theta) + \lambda_1 \cdot G(\theta) \cdot (1 - F(\theta)))$$

Given the steady-state unemployment rate $U = \frac{\phi+\delta}{\phi+\delta+\lambda_0}$:

$$G(\theta) = \frac{(\phi + \delta) \cdot F(\theta)}{(\delta + \lambda_1 \cdot (1 - F(\theta)))}$$

Which can be used to replace $G(\theta)$ in (14):

$$\chi = \lambda_1 \cdot (\phi + \delta) \cdot \int_{\underline{\theta}}^{\overline{\theta}} \frac{F(\theta)}{(\delta + \lambda_1 \cdot (1 - F(\theta)))} \cdot dF(\theta)$$

Substituting $F(\theta) = z$, and integrating by parts yields a closed-form solution (see Appendix G for details):

$$\chi = (\phi + \delta) \cdot \left[\frac{(\delta + \lambda_1)}{\lambda_1} \cdot \ln \left(\frac{\delta + \lambda_1}{\delta} \right) - 1 \right]$$

Hornstein et al. (2011) point out that "the most recent empirical evidence sets monthly job-to-job flows χ between 2.2 percent and 3.2 percent of employment" (see p. 2889 in Hornstein et al. (2011)). Using the middle of the above interval, I set the target χ at 2.7 percent. Solving the last equation above for λ_1, given the monthly values $\phi = 0.0021, \delta = 0.022$ I obtain the value of $\lambda_1 = 0.11$.

4.3 Human capital accumulation rate ρ and depreciation rate η

I start with the mean-min ratio derivations, along the lines of Hornstein et al. (2011). Note that as the mean-min ratio is a measure of frictional wage dispersion, that is, wage differences between workers with the same observable characteristics. Therefore, a counterpart in the model will be the mean-min ratio of the distribution of earned piece rates, $G(\theta)$.

First, recall from the above analysis the link between the distribution of offered piece rates $F(\theta)$, and the distribution of earned piece rates $G(\theta)$:

$$\begin{aligned} G(\theta) &= \frac{(\phi+\delta)\cdot F(\theta)}{(\delta+\lambda_1\cdot(1-F(\theta)))} \\ 1-G(\theta) &= \frac{(\delta+\lambda_1+\phi)\,(1-F(\theta))-\phi}{\delta+\lambda_1\cdot(1-F(\theta))} \end{aligned}$$

Using approximations like in Hornstein et al. (2011), as ϕ, r and ρ are of the second order of magnitude relative to λ_1 and δ:

$$\begin{aligned} 1-G(\theta) &= \frac{(\delta+\lambda_1+\phi)\,(1-F(\theta))-\phi}{\delta+\lambda_1\cdot(1-F(\theta))} \simeq \\ &\simeq \frac{(\delta+\lambda_1+\phi+r-\rho)}{\phi+\delta+r-\rho+\lambda_1\cdot(1-F(\theta))}\cdot(1-F(\theta)) \end{aligned} \tag{15}$$

Second, from the value functions of the worker there are (see Appendix B) two equations that can be solved for θ^R :

$$\begin{aligned} (1) &: \quad \lambda_0\theta^R = \lambda_1\cdot b+\alpha^U\cdot[\lambda_0\,(\phi+r-\rho)-\lambda_1\cdot(\,\phi+r+\eta)] \\ (2) &: \quad (\,\phi+\eta+r)\cdot\alpha^U = b+\lambda_0\cdot\left[\int_{\theta^R}^{\overline{\theta}}\frac{1-F(\theta')}{q(\theta')+r-\rho}d\theta'\right] \end{aligned}$$

Re-arranging gives the following expression:

$$\theta^R = b\frac{(\phi+r-\rho)}{(\,\phi+r+\eta)}+\frac{\lambda_0\,(\phi+r-\rho)-\lambda_1\cdot(\,\phi+r+\eta)}{(\,\phi+r+\eta)}\cdot\left[\int_{\theta^R}^{\overline{\theta}}\frac{1-F(\theta')}{\phi+\delta+\lambda_1\left(1-F(\theta')\right)+r-\rho}d\theta'\right] \tag{16}$$

Third, the mean piece rate in the economy is given by (using integration by parts):

$$\theta^{mean} = \int_{\theta^R}^{\overline{\theta}} \theta dG(\theta) = \int_{\theta^R}^{\overline{\theta}} (1 - G(\theta))\, d\theta + \theta^R$$

$$\theta^{mean} - \theta^R = \int_{\theta^R}^{\overline{\theta}} (1 - G(\theta))\, d\theta$$

Fourth, use (15) to re-write the integral in (16):

$$\theta^R = b\frac{(\phi + r - \rho)}{(\ \phi + r + \eta)} + \frac{\lambda_0\,(\phi + r - \rho) - \lambda_1 \cdot (\ \phi + r + \eta)}{(\ \phi + r + \eta)\,(\delta + \lambda_1 + \phi + r - \rho)} \cdot \left[\int_{\theta^R}^{\overline{\theta}} \left(1 - G(\theta')\right) d\theta'\right]$$

From the last equation above one can isolate $\int_{\theta^R}^{\overline{\theta}} \left(1 - G(\theta')\right) d\theta'$:

$$\int_{\theta^R}^{\overline{\theta}} \left(1 - G(\theta')\right) d\theta' = \left(\theta^R - b\frac{(\phi + r - \rho)}{(\ \phi + r + \eta)}\right) \cdot \frac{(\ \phi + r + \eta)\,(\delta + \lambda_1 + \phi + r - \rho)}{\lambda_0\,(\phi + r - \rho) - \lambda_1 \cdot (\ \phi + r + \eta)}$$

Inserting into the expression for $\theta^{mean} - \theta^R$ one gets:

$$\theta^{mean} - \theta^R = \left(\theta^R - b\frac{(\phi + r - \rho)}{(\ \phi + r + \eta)}\right) \cdot \underbrace{\frac{(\ \phi + r + \eta)\,(\delta + \lambda_1 + \phi + r - \rho)}{\lambda_0\,(\phi + r - \rho) - \lambda_1 \cdot (\ \phi + r + \eta)}}_{A}$$

$$\frac{\theta^{mean}}{\theta^R} = A + 1 - \frac{b \cdot A}{\theta^R} \cdot \frac{(\phi + r - \rho)}{(\ \phi + r + \eta)} \tag{17}$$

I set the replacement rate in unemployment at $b = 0.4$ (as in the basic calibration of Hornstein et al. (2011), Shimer (2005), and Shimer (2012)). For any pair (ρ, η), given the rest of parameters, the equation (17) defines the mean-min ratio, given θ^R on the RHS solves the system (8-9).

One of the latest benchmarks for returns to experience in the US comes from Altonji and Williams (2005), that estimate the return to 30 years of experience, for the period 1988-2001, to be between 0.4 and 0.6 (see Table 6 in their paper). In monthly terms, this implies a rate of 0.0011-0.0017 per month, and this is also consistent with the value 0.0017 assumed by Hornstein et al. (2011). This implies, in quarterly terms, the rate ρ of human capital accumulation of 0.0033-0.0050 per quarter. At the same time, Hornstein et al. (2011) report empirical counterparts of the mean-min ratio (17) [7], to be between 1.7 and 1.9.

[7] The 50-10 percentile ratio of the residual in a Mincerian wage regression for men

Given ρ, the mean-min ratio (17) grows with η (both directly, and through θ^R , that goes down when workers lose skills fast), and vice-versa - so that higher mean-min ratios require both higher ρ and higher η. When using the highest value for ρ that is compatible with Altonji and Williams (2005) $\rho = 0.005$, one needs skills depreciation rate $\eta = 0.02$ in order to obtain the mean-min ratio of 1.7 - a minimum needed to be consistent with Hornstein et al. (2011). However, it seems implausible that workers lose skills four times faster than they accumulate them. This illustrates a tension between fitting returns to experience from Altonji and Williams (2005) and fitting mean-min ratio from Hornstein et al. (2011). I chose a compromise, which implies tolerable deviations from these two benchmarks. In particular, I set the quarterly rate of human capital accumulation to be $\rho = 0.006$ (implying the return to 30 years of experience to be 0.72, somewhat higher than in Altonji and Williams (2005)), and I set the quarterly rate of skill depreciation to be the highest reasonable $\eta = 0.006$ (implying the mean-min ratio (17) of 1.55 . The corresponding monthly rate of human capital accumulation 0.002 is also completely in line with what Carrillo-Tudela (2012) finds basing on the British BHPS survey.

To sum up, the final calibration that I use is the following:

$$
\begin{aligned}
\lambda_0 &= 1.83 \\
\lambda_1 &= 0.33 \\
\phi &= 0.0063 \\
\delta &= 0.066 \\
r &= 0.0099 \\
\rho &= 0.006 \\
\eta &= 0.006
\end{aligned}
$$

5 The life-cycle profiles of careers

In this section I combine the results from the two previous sections in order to analyze the evolution of wage over workers' age.

In a stationary environment, the overall distribution of wages does not change from period to period. However, the overall distribution at each instant is composed of wages of different age groups. As workers age, the distribution of earned wages changes; in other words, workers travel within the overall distribution of wages, which is stationary. The results obtained in Sections 1 and 2 allow me to look both at the overall distribution of wages, as well as the distribution of wages by age groups. Denote the overall cumulative distribution of wages by $P^E(w)$, and the distribution of wages, given age a, by $P^E(w|a)$. The mathematical expression for the cumulative distribution of total wages is the following (see Appendix C for details):

$$P^E(w) = \int_{\underline{\theta}}^{\overline{\theta}} \int_{y=0}^{w/\theta} \frac{\partial^2 P^E(y,\theta)}{\partial y \partial \theta} dy d\theta$$

where y is productivity of the employed and θ is the earned piece rate, and $P^E(y,\theta)$ is the joint cumulative distribution of productivity and earned piece rates among the employed. The expression above states that in order for the wage to be below a certain level of w, for each possible θ the value of productivity should be below w/θ. The distribution of wages, conditional on age, is (see Appendix C for details):

$$P^E(w|a) = \Pr(\theta e^{(\rho+\eta)x} < w \cdot e^{\eta a}|a) = \int_{\underline{\theta}}^{\overline{\theta}} \int_{0}^{\min(a, \frac{1}{\rho+\eta} \ln \frac{w \cdot e^{\eta a}}{\theta})} \frac{\partial P^E(x|a)}{\partial x} \cdot \frac{\partial P^E(\theta|x)}{\partial \theta} dx d\theta$$

,

where $P^E(x|a)$ is the conditional distribution of experience, given age, and $P^E(\theta|x)$ is the conditional distribution of earned piece rate, given experience. The expression above states that for a given age a, and for all possible piece rates θ, the wage will be below w, if experience x is below a certain threshold which takes into account that for some pairs of (a, w), the wage can not be higher than w, for all levels of experience, feasible at age a.

I calculate the above distributions numerically, using the formulas derived in Sections 2 and 3. The details of the calculations and the Matlab code are available upon request.

Figure 5 below presents the overall cumulative distribution of wages (black dashed line), and the distributions of wages within different age groups.

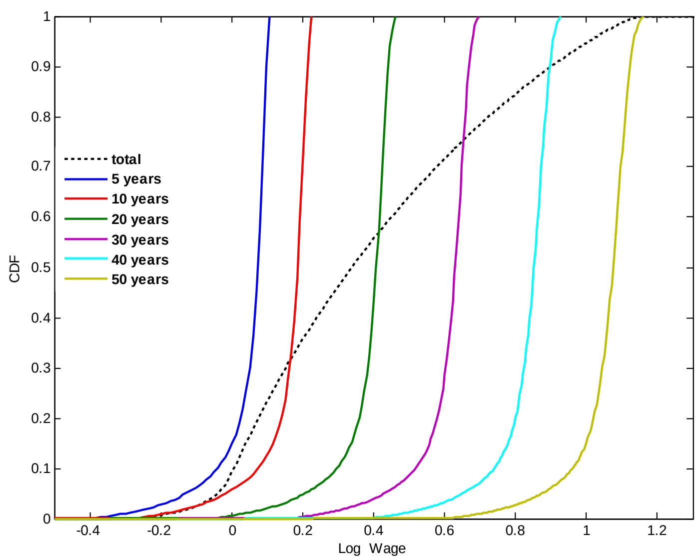

Figure 5: Life Cycle Wage Distributions

As they age, people gradually move within the total distribution of wages. For each age, the cumulative distribution of wages first order stochastically dominates that of any younger cohort. Conversely, the mean wage of an average employed worker grows with his age, and the oldest cohort, though the lest populous, is the richest one in this economy. A more detailed analysis of the life-cycle profiles follows.

5.1 Life-cycle wage growth

In this subsection, I use the model to explore the relative magnitude of two channels - human capital evolution and on-the-job search - in shaping wages at different stages of a career. To do so, I take different age groups, and look at what their wage would be if it equalled their actual current productivity (I denote this counterfactual profile by "HC only"), or if it equalled their current actual piece rate (I call it "OTJ only"). The mathematical derivations are relegated to Appendix F.

Figure 6 below illustrates the actual average log wage, as well as two counterfactual paths - one when only on-the-job search is in place, and the other when workers do not sample better offers on-the-job, but simply alternate between the states of employment and unemployment, accumulating human capital and getting the entire flow product in each period:

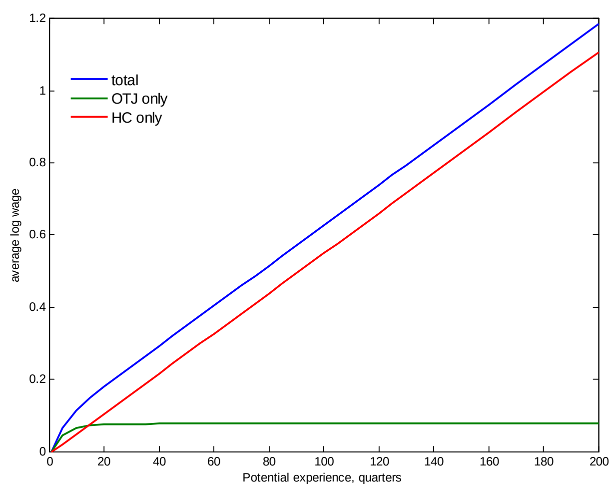

Figure 6: Mean Log Wage Profile

Over the first ten years of career, the cumulative wage increase implied by the model is 0.29 log points. This is substantially lower than found in the literature: Altonji et al. (2013) estimate the total wage growth of 0.51 over the first 10 years in the PSID data, Bowlus and Liu (2012), and Yamaguchi (2010) also obtain a roughly 0.5 increase using NLSY and SIPP. Over a longer horizon of 30 years, the model predicts total wage growth of 0.74 - somewhat lower than 0.83 estimated by Altonji et al. (2013), but consistent with the value of around 0.75 provided in Bowlus and Liu (2012). Consistent with existing evidence, the first ten years contribute the highest share to this lifetime total wage growth, however, due to the lack of concavity, the differences between the inputs of the first and the following decades are not sufficiently stark (0.29, 0.22, 0.22 and 0.22, respectively). Summing up, the average wage profile implied by the model is not sufficiently concave at the beginning of career, but the implied cumulative growth over a longer horizon is reasonable. The insufficient concavity follows from the fact that I did not assume decreasing rate of human capital accumulation, for the sake of tractability of the model. The only source of concavity in my model is in the decreasing returns to on-the-job search.

The sources of wage growth differ over the life-cycle. Job shopping is the main driver of growth at the very beginning of career (first year-two), but its share declines monotonically, as workers climb up the piece rate ladder and attractive offers become scarce, and after first 7.5 years, the entire cumulative wage growth is due to human capital. In this sense, the predictions of the model are close to what Schonberg (2007), Yamaguchi (2010) and Bowlus and Liu (2012) find, but the reversal of the relative weights of the two channels happens here relatively early.

Over the first decade the input of search into cumulative wage growth is 0.26, and the rest is due to human capital accumulation. This result is close to the estimates of Topel and Ward (1992), but contradicts the findings of Altonji et al. (2013), who assign a much lower share (0.13) to search over the same period. Notably, the shares attributed to human capital accumulation over the first decade are almost the same in my model and in Altonji et al. (2013) - 0.74 and 0.74, respectively[8]. However, in their paper the share of human capital declines slightly with age, whereas in my model it grows rapidly, and over a longer horizon of 30 years I obtain a relatively high weight of human capital accumulation in total wage growth: 0.90 versus 0.71 in Altonji et al. (2013).

As human capital can not be measured directly, and its life-cycle profile can not be observed, economists have to make ad hoc assumptions about its dynamics. Obviously, these assumptions have direct implications for the estimates of the relative weight assigned to learning in wage growth. Altonji et al. (2013) assume that the general-experience profile is a deterministic polynominal function of potential experience, and the estimates imply a significant concavity of this profile, meaning that the returns for each additional year of potential experience are highest at the beginning of career, therefore, the impact of learning on wage growth is highest at the beginning of career. In my model, the return to each additional year of potential experience is a result of the stochastic division

[8] Altonji et al. (2013) have two more channels of wage dynamics: returns to tenure and the impact of employment history

of this year into the periods of employment and non-employment. While the transition rates are Poisson rates without memory, this division (and thus the return to additional year) is roughly constant over career (it is a bit lower at the very beginning, because the first year contains on average more unemployment periods as everyone is borne unemployed). Therefore, the relative importance of the two channels of wage growth is driven mainly by the decline in the input of job search.

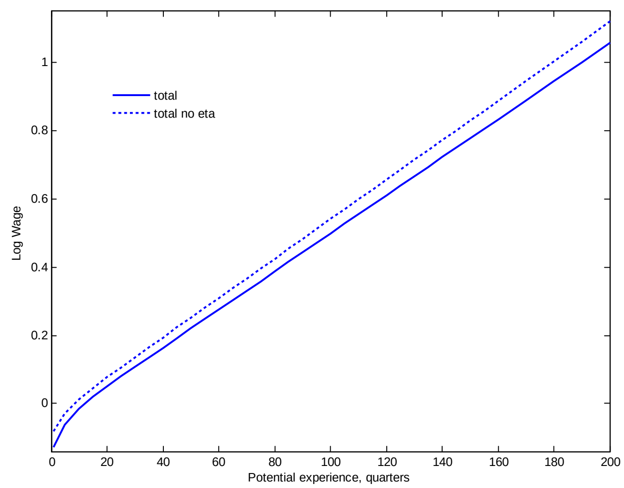

Figure 7: Log Wage Profile - With and Without η

As can be seen from Figure 7 above, the introduction of human capital depreciation makes the log-wage profile a little more concave at the beginning, but overall has moderate impact on the life-cycle path of average log wage. This is a result of two effects. First, when workers face loss of skills in unemployment, they lower their reservation wage, so that the "on-the-job-search-only" profile of wages lies lower when there is human capital depreciation. At the same time, the returns to potential experience decline, as workers live through unemployment and lose skills. The total minor impact of loss of skills on average log wage is the sum of the direct negative impact, through a decrease in productivity, and an indirect negative equilibrium impact, through the drop in the lower bound of the range of offers. The increased concavity at the beginning, in the phase when on-the-job search is the main driver of growth, follows from the fact that when the lowest wage declines substantially, and the highest wage declines only slightly, the range of offers expands. The higher the variety of offers, the higher the gains from job-shopping, and the steeper will be the increase in wages due to on-the-job search.

The difference between the two human capital accumulation profiles in the presence and in the absence of loss of skills represents the direct impact of human capital loss on wages (see Figure 1H in the Appendix H). The drag on wages is higher for older workers, it reaches 0.05 log points for workers who have been 50 years in the labor market, meaning that these workers would have been around 5% better off towards the end of their careers, had they not lost skills during unemployment periods that they have lived through. Older workers are affected more, because they accumulate longer histories of unemployment.

5.2 Life-Cycle Cross-Sectional Wage Inequality

The variance of log wages follows a clear U-shaped pattern, where the variance is highest within the youngest group, then it falls rapidly, is the lowest among the workers with about 5 years of potential experience, and grows slowly thereafter. Figure 7 illustrates this, where solid lines represent the full model, and dashed lines represent the model without loss of skills.

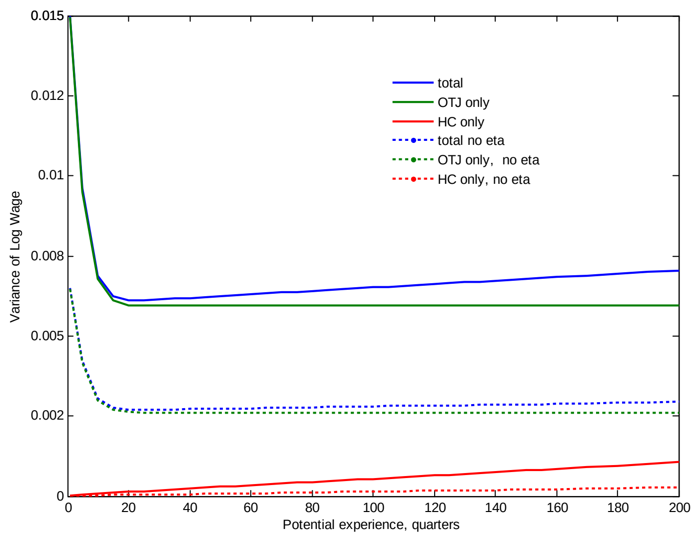

Figure 8: Variance Profile of Log Wage

A U-shaped pattern arises because the total variance is a sum of two components: the variance of productivities and the variance of piece rates (there is also an interaction term, but within an age group, the covariance between productivity and piece rate is negligible):

$$Var(\ln w) = Var(\ln \theta) + Var(\ln y) + 2 \cdot cov(\ln \theta, \ln y)$$

The variance of piece rates goes down with potential experience, and more and more workers on average move up the piece rate ladder. The variance of productivities, on the contrary, goes up. As time goes by, workers accumulate more diverse histories of shocks, and their productivities become more dispersed.

Such a U-shaped pattern arises also in the Mincer (1974) model of investment in human capital, however, for a different reason: those who chose to invest into human capital at the beginning of the career will have relatively low initial earnings, but due to their higher productivity, at some stage they will later overtake those who chose not to go to school.

The inequality of piece rates is the main source of overall cross-sectional wage inequality over career, but its share declines steadily from 0.99 after a year in the market, to 0.85 at the highest levels of potential experience.

The empirical evidence on the life-cycle patterns of wage inequality in the U.S. is mixed. Rubinstein and Weiss (2007) use PSID (1968-1997) and NLSY (1979-2000) samples to show that the variance of the residuals of the Mincer equation (in this model, the corresponding object is the variance of piece rates) follows a U-shape pattern over potential experience, with a minimum at around 11 years. In this model, frictional wage dispersion goes down monotonically, due to on-the-job search. Heathcote et al. (2005) use PSID (1980-1997) and find that the total cross-sectional wage inequality rises over the life-cycle. Again, as the blue solid curve in Figure 7 shows, this is inconsistent with what my model implies. Finally, Heckman (2006) uses the U.S. 1960-1990 Censuses data, and finds a clear U-shaped pattern in the variance of log earnings, with the minimum between 5 and 20 years of potential experience, depending on education group. The pattern implied by my model is especially close to the variance profile of low-educated workers, according to Heckman's (2006) results, when the variance is higher at the beginning of the career, than at the end. For medium- and high educated workers the empirical variance profile is also U-shaped, but the right side of U is higher than the left one (see Heckman (2006), Figure 3).

The inclusion of loss of skills in unemployment into the model has a substantial impact both on the magnitude, and the pattern of life-cycle log wage variance. It more than doubles the wage dispersion at all ages, with the relative increase (140%) being most pronounced for most experienced workers. Both components of total inequality go up under the assumption of skills loss: the variance of earned piece rates increases at all ages, more than two-fold, due to the fact the the unemployed are ready to accept lower offers and the range of equilibrium piece rates expands. Also, the variance of productivity goes up dramatically, by almost 300%, at all ages, now that small levels of actual experience at each age imply much lower productvities than earlier. The presence of skills depreciation also makes a U-shaped pattern more pronounced, which brings it closer to empirical evidence (see Heckman (2006)).

Notably, the model predicts that the variance of the expected flow value of employment grows monotonically over the life-cycle. Presumably, an interplay between current wage and the value of the search option, and current wage and the value of the separation (see eq. (7) for details) are responsible for this implication. A simulation of an artificial sample of workers can shed more light on the variance-covariance structure of the components of the value function, as defined by (7), and this is left for future research.

5.3 Frictional Wage Dispersion

This section looks at the measure of total residual wage dispersion, the mean-min ratio. Hornstein et al. (2011) point at a difficulty of standard search model in generating sufficient frictional wage dispersion, measured as the mean-min ratio of the residuals of Mincer wage equation. The authors claim that the introduction of learning-by-doing or on-the-job search into the model makes the problem less acute, but does not resolve it completely, for reasonable values of non-market time relative to average wage. Burdett et al. (2011) combine on-the-job search and learning-by-doing and obtain, for their calibration, mean-min ratio of more than 2, where the empirical counterparts reported by Hornstein et al. (2011) range from 1.7 to 1.9. However, I find that this success on generating enough frictional wage dispersion hinges to a significant extent on the assumption that offers arrival rate to unemployed is as low as the rate for the employed. When I take the calibration of Burdett et al. (2011) and increase the offer-arrival rate for the unemployed λ_0 so that its ratio to λ_1 is the same as in my model[9], the mean-min ratio drops to 1.27, much less than claimed in Burdett et al. (2011). The reason is that when offers arrive more often to the unemployed, they are more picky regarding which piece rate to accept, their reservation rate (the denominator in the mean-min ratio) goes up. Ortego-Marti (2012) combines on-the-job search with human capital depreciation in unemployment and obtains the mean-min ratio of around 2. However, he calibrates the rate of skills loss to be more than 1% per month (that is, the loss in wage of 1% for each additional month of unemployment history), which seems quite high. I calibrate it to be 0.6% per quarter, and as showed in the calibration section, higher values of η imply higher mean-min ratios, because the reservation rate of the unemployed goes down. Indeed, when a value of 0.2% per month is used to compute the mean-min ratio according to Ortego-Marti (2012) (see eq. 3.13 on p. 95), the result is a mean min ratio of 1.26. I claim that both human capital accumulation *and* loss of skills are necessary in order to match the magnitude of frictional wage dispersion, in a reasonable calibration in the model with on-the-job search. Both of them are needed in order to push down the reservation piece rate of the unemployed. To illustrate this, I add the ingredients of the model one-by-one and compute the implied mean-min ratios, according to (17). I get that for the basic Burdett-Mortensen (1998) model, the mean-min ratio is 1.15. When human capital accumulation is added (the model by Burdett et al. (2011)), the ratio goes up to 1.32. Finally, when I introduce skills depreciation, the ratio goes up to 1.55, which is a plausible value given the results in Hornstein et al. (2011).

5.4 Sources of Growth in the Tails of Wage Distribution

Previous analysis suggests that there is a substantial dispersion of wages within all age groups. Here I analyze the factors that shape the differences between

[9] Now, instead of having $\lambda_1 = \lambda_0 = \lambda = 0.15$, I now use $\lambda_1 = 0.15, \lambda_0 = 0.83$

the richest and the poorest groups over the life-cycle. Namely, for each age I look at the properties of the group of workers whose earnings are in the highest and the lowest deciles of the wage distribution for a given age.

For both groups of workers, the total mean log wage, and its components are presented in Figure 8 below:

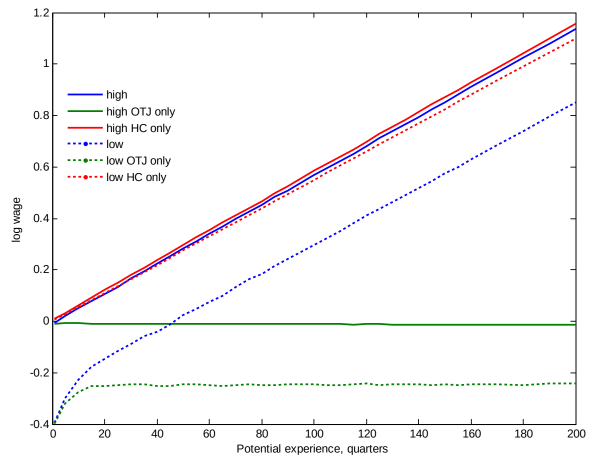

Figure 8: Mean Log Wage Profile in Top 10% and Bottom 10% of Wages

As Figure 8 illustrates, the profiles of the poorest and the richest group differ. At each age, the richest group are those who are lucky to receive back a high share of their flow product (the average piece rate within the richest group remains close to the highest possible over career), and those who have a fortunate history of few separation shocks and whose actual experience is therefore very close to their potential one. In the poorest group, the piece rate grows with age, closing the gap partly, but remains relatively low. The actual experience is significantly lower than the potential one. While, unlike the rich, job shopping is an important source of early wage growth for the poor, their life-cycle wage profile is more concave.

To highlight the differences between the groups, Figure 9 depicts the gap between the average piece rate earned by the richest and the poorest, the experience gap, as well as the wage gap:

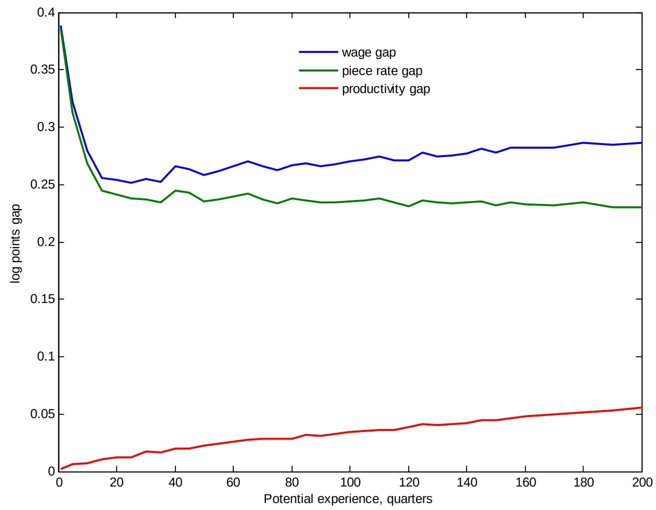

Figure 9: Components of 90/10 Log Wage Gap

The total wage gap follows a U-shape over the life-cycle. For the very young, the experience gap is negligible, and the total wage gap follows mostly from the fact that some were lucky to quickly sample high offers while others were not. Over the first five years of career, the piece rate of the poor increases (still it falls short of the average piece rate of the rich) and the piece rate of the top 10 percent stays stable, thus almost 1/3 of the gap is closed. Over the next 45 years, those who are in the poorest group spend more time in unemployment, their actual experience more and more lags behind that of the rich. For example, towards the end of an average career, the model predicts that the poor have overall spent 1.25 years more in unemployment, than the rich. Less actual experience and more unemployment means less productivity, and so the wage gap increases back,still staying short of the initial level.

To sum up, within the group of young workers the rich differ from the poor in that they were lucky to sample high piece rate offers from the start. Later in life, the rich differ from the poor mostly in that they spend less time in unemployment

5.5 The Impact of Loss of Skills on Quantiles of Wage Distribution

In this subsection I compare two economies - with and without loss of skills. I take three percentiles of the wage distribution - the 10th, the 50th (median), and the 90th, and look at their evolution with age. Figure 10 below presents the profiles:

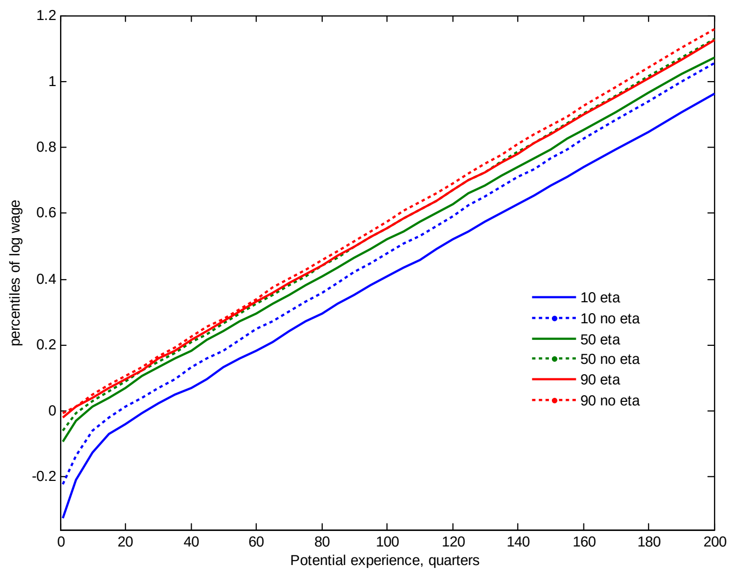

Figure 10: Log Wage Percentiles Profiles

Several points stand out: (i) The lower the percentile, the more concave is the life-cycle profile. As previous analysis suggests, wages at the bottom of the distribution are driven by job search to a much greater extent (especially at the beginning of career), than wages at the top, hence higher concavity. The concavity is enhanced in the presence of human capital depreciation, due to a wider span of piece rates and more rapid progression up the ladder, as described above. (ii) The negative impact of skills loss (expressed as the distance between solid and dotted lines of the same color) is weakest for the high percentiles. The reason is two-fold: first, under both regimes (with or without η) the richest workers are those who are lucky to sample high piece rates, where we know that the upper bound of $F(.)$ is almost insensitive to η. Second, the richest workers are those who live through few separations, and therefore do not spend much time in unemployment whatsoever, therefore their careers are less influenced by what happens during unemployment. (iii) Finally, when one looks at the magnitude of impact over a given percentile over periods, it turns out that for the median and for the 90th percentile the relative losses rise very moderately, if at all, over career. This moderate rise follows from the fact that for these relatively rich groups who do not spend much time in unemployment, the effect will be more pronounced towards the end of the career, when they accumulate enough unemployment (though much less than the poor), for η to have an impact. However, for the 10th percentile the negative effect is highest both at the end and at the beginning of career. The reason is that the young spend more time in unemployment in their first years, due to the fact that they start their careers unemployed, and the deterioration of skills lowers their wages. Also, as discussed extensively above, the lower bound of $F(.)$ falls when workers lose skills in unemployment, meaning that those young workers will start from lower wages. The losses become more moderate towards the middle of the career, due to the job search driving the poor up the distribution, and thereafter the losses grow again, because the relatively high accumulated unemployment and the corresponding deterioration of productivity become a substantial burden. Figure 11 below illustrates the log points losses - the differences between the solid and the dashed profiles depicted in Figure 10 above:

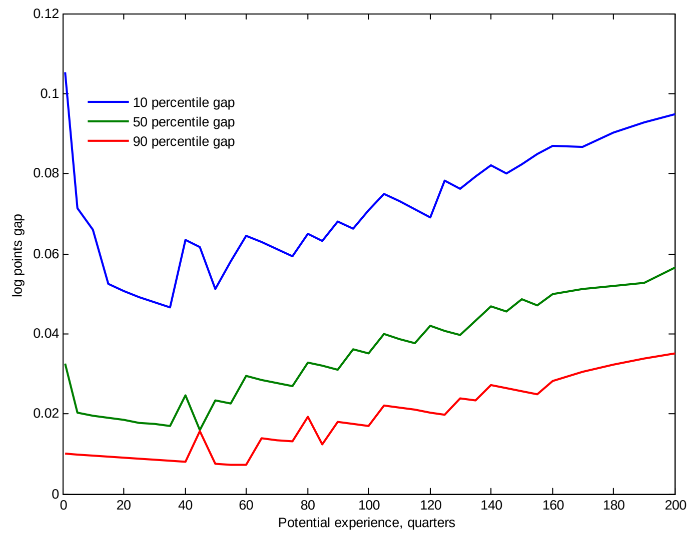

Figure 11: The Differential Impact of η on Profiles of Log Wage Percentiles

To conclude, the evolution of wages over age is different for different parts of the wage distribution - at the lower end the profiles are more concave, due to a greater role of on-the-job search. The negative impact of human capital depreciation assumption on life-cycle profiles of wage percentiles is differential - (i) it is always higher for poorer workers (ii) for relatively rich, it rises mildly with age (iii) for the poor, it is high both for the very young and the very old. In other words, in the presence of skill depreciation, the relatively poor will be much poorer, and especially so when very young and very old. Relatively rich workers will be somewhat poorer, especially so when they are old.

5.6 Flow expected lifetime values

The flow expected lifetime values in the two states are defined in Section 2 by the Bellman equations (7) and (6). The components of the flow values as in (7) and (6) can be loosely labelled as follows:

$$\text{Flow } E(V^{employed}) = \text{current wage} + \text{value of learning} + \text{search option} + \text{separation value}$$
$$\text{Flow } E(V^{unemployed}) = \text{replacement uncome} + \text{value of depreciation} + \text{search option}$$

I use (7) and (6), and the results of Proposition 1 to compute the above totals and the components of expected flow values:

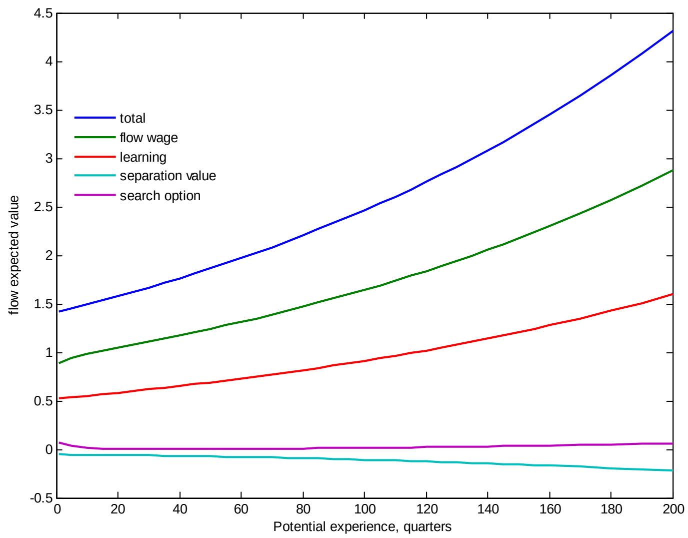

Figure 12: Flow Expected Value Profile in Employment

First, it must be noted that the flow value of employed predicted by the model grows monotonically over the lifetime. A memory-less Poisson process of permanent exit implies that the expected horizon of career is the same in the model for all ages. Human capital grows on average with potential experience. Taken together, these two facts imply that older workers face better value prospects than their younger colleagues. Second, the main component of value growth for the employed workers is flow wage, followed by learning-by-doing. Both these components grow steadily over the lifetime, and their shares in the total value are quite stable - 0.65-0.67 for flow wages, and the share of learning-by-doing is by definition constant, 0.37. The search option comprises a small share of expected value, declining from 0.07 at the beginning of career, to 0.04 at its end. Separation value is always negative, because a worker always becomes worse-off upon separation, and it turns more and more negative over the life-cycle, but relative to the total expected value, its share is stable at 0.04-0.05.

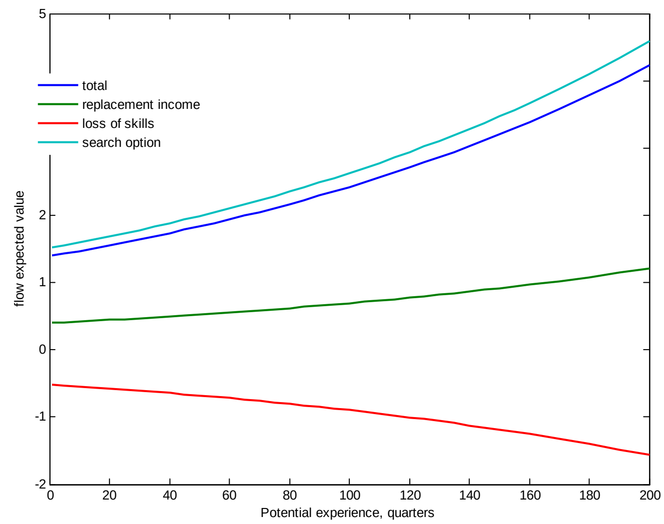

Figure 13: Flow Expected Value Profile in Unemployment

For the unemployed, the relative weights of the components are different than for the employed. Namely, the search option is a major component of value, and it is even somewhat higher than the value itself, compensating for the value losses due to skills depreciation. The drag on value due to skills depreciation increases with experience in absolute terms, however, in relative terms it is constant, by definition, and stands at 0.37. The share of the current replacement income in value is 0.29.

To conclude, the following patterns emerge: in both employment states the relative weights of the components of value are stable over the life-cycle. For the employed, two-thirds of value comes from the current wage, and a bit more than one-third comes from learning-by-doing. The shares of search option and the drag on value due to possible separation are negligible, and roughly compensate one another. For the unemployed, search option is the main source of value, and together with the replacement income it compensates for the negative impact of skills depreciation, which takes a bit more than one-third of expected flow value. The immediate average losses in expected lifetime value upon exogenous separation are relatively small. However, this can be regarded as the lower bound of the total realized value losses, because it does not take into account the deterioration in productivity that will happen over the unemployment spell. If one regards the parameters of the model as policy instruments, then value decomposition above suggests that policies facilitating search (higher λ_0) are most effective in improving the welfare of the unemployed, while for the employed it is more intensive learning on-the-job (higher ρ).

5.7 Youth Unemployment

Everyone is born unemployed, and spends strictly positive time in unemployment, till the first offer arrives, whereupon a worker moves to employment. The time spent in unemployment is exponentially distributed with a mean and a standard deviation of about half a quarter. 16 percent of young workers wait more than one quarter for their first job, and 2.6 percent wait more than half a year.

Relative to a lucky worker, who found his first job quickly and is already working, there are three negative effects acting upon a worker who is still unemployed: (i) he keeps losing human capital (ii) he has no access to on-the-job search, (iii) his career starts later, meaning that a shorter section remains of the average career length. As a result, even after they eventually find a job, the wages of workers who spent a long time in initial unemployment will be lower, than the wages of their employed peers of the same age. As will be shown below, the model implies that this negative impact is substantial, and very persistent. I use the joint distributions of wages, productivities and ages in order to compute the distributions of wages for the average career path, and two counterfactual cases - when a worker spends 5 quarters or 10 quarters in initial unemployment. The way I do it is straightforward - once the distribution of wages within age groups has been obtained (see Section 4 above), inferring the life-cycle wage distributions for delayed entry is rather straightforward - the wages of delayers are the same as the lagged wages of the cohort who started working from the beginning, corrected for the lower initial productivity as a result of human capital depreciation.

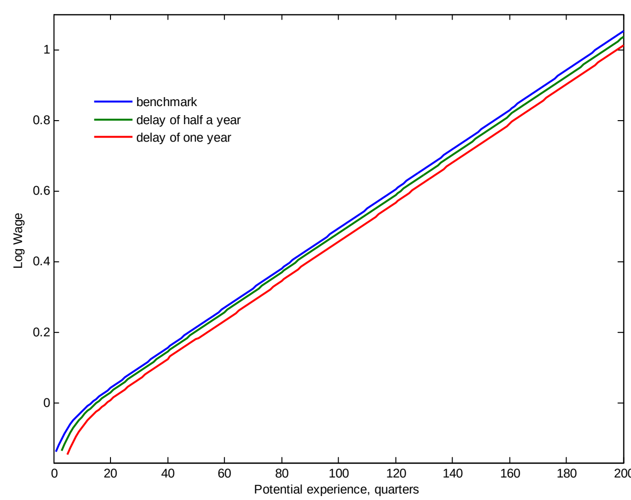

Figure 14: The Impact of Youth Unemployment on Log Wage Profiles

Figure 14 compares the log wage profiles of an average worker, and two workers who spent 2 and 4 quarters waiting for their first job. On average, a worker expects to wait about half a quarter for the first offer, so in this exercise I am regarding the cases in which actual career starts substantially later than expected on average. One can see that the entire profile shifts down and to the right, and this downgrade is more severe, the longer the initial unemployment period. The effect of initial unemployment on wages is permanent, there is no catchup of average wages. For someone who waited half a year for his first job, the average wage will be about 0.02-0.03 log points lower at all ages, relative to employed workers who were already working after spending 1 quarter in the market. For a longer waiting period of one year spent in initial unemployment, the relative losses in log wages amount to 0.03 - 0.07 log points. The gap is somewhat lower towards the end of the career, but is never closed completely.

Though no catchup on average exists, there is catchup for some workers. Taking those who spent half a year in initial unemployment and looking at them 5 years since they found their first job, one finds that 64 percent of them already have a wage that is above the average for that potential experience level (for the benchmark profile the probability to be above the average wage at potential experience five-and-a-half years is 76 percent). The longer initial unemployment duration, the slower the catchup: for a worker who spent an entire year waiting for the first job, taken five years since he found it, the chance to earn a wage above the average is 42 percent, whereas for the benchmark case the chance is 77 percent for a corresponding level of potential experience.

Not only does the average wage go down, but also the prospects of the workers of moving up high in the wage distribution are impacted.

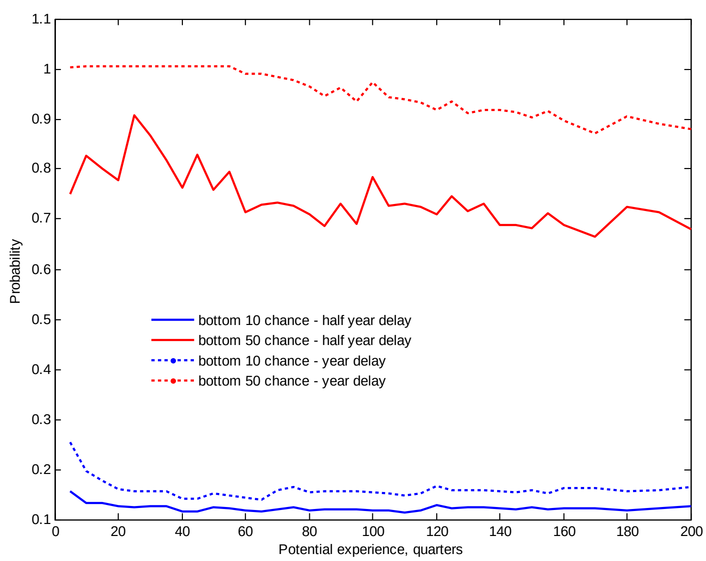

Figure 15: Chances To Be Relatively Poor

Permanent impact of initial unemployment on careers is manifested in the location of workers in within-age wage distribution. Those who live through relatively long initial unemployment spells are downgraded to low percentiles of within-cohort wage distributions. Figure 15 shows that spending half a year in initial unemployment increases the chances to be in the poorest 10 percent of the population at all ages to about 12 percent, and this probability goes up to 16 percent if initial unemployment is one year. The chance to be in the poorest half of the population is over 70 percent for those whose initial unemployment was half a year, and is over 90 percent if initial unemployment lasted one year. Towards the end of the careers the prospects are somewhat better than at the beginning, but the differences are not stark.

The main source of the permanent impact of youth unemployment on wage profiles is productivity loss that occurs while waiting for the first job. As a result of this loss, the profiles of those who started working relatively late are not the benchmark profiles taken with a lag, but also go down because the initial productivity from which they start is lower than for the benchmark profile. The actual delay in career attainment is more than the duration of initial unemployment, and the difference between the two becomes more pronounced, the longer this duration is. For example, when the initial unemployment was half a year, the actual delay is roughly half a year, but when initial duration goes up to one year, the actual delay is already one year and three months.

Finally, the damage of long initial unemployment spells is manifested in a lower wage growth accumulated by the end of a career of average length. Over 40 years of career, the average cumulative wage growth is 0.97 log points. Given initial delay of half a year, cumulative growth goes down to 0.95 log points, and it further declines to 0.94 log points when initial unemployment is one year.

5.8 Plans for future work

I plan to extend the analysis along the following lines:

First, using the joint distribution of potential, actual experience, and piece rate one can project the wage trajectories starting from any given initial condition. Therefore, one can look at the chances to reach certain wage levels by certain age depending on where you are now. In this sense, the model can serve as an instrument for investigating stationary mobility within a given stable wage distribution, analyzed empirically in Bowlus and Robin (2012).

Second, I plan to extend the model to include worker heterogeneity with respect to the rates of human capital accumulation and loss.

Third, I would like to calibrate the model for different subgroups of workers, depending on race, and education level, and analyze the differences.

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